WAVE-DRIVEN ROTATION IN CENTRIFUGAL MIRRORS

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Centrifugal mirrors use supersonic rotation to provide axial confinement and enhanced stability. Usually the rotation is produced using electrodes, but these electrodes have limited the rotation to the Alfven critical ionization velocity, which is too slow to be useful for fusion. Instead, the rotation could be produced using radio frequency waves. A fixed azimuthal ripple is a simple and efficient wave that could produce rotation by harnessing alpha particle energy. This is an extension of the alpha channeling effect. The alpha particle power and efficiency in a simulated devices is sufficient to produce rotation without external energy input. By eliminating the need for electrodes, this opens new opportunities for centrifugal traps.

I. INTRODUCTION

Axisymmetric open magnetic traps make attractive fusion devices due to their simplicity, high beta, and steady state operation. Major issues faced by axisymmetric open traps are magnetohydrodynamic (MHD) stability, microstability, and parallel confinement [1]. One way to address these issues is with supersonic rotation [2]. The rotation shear or centrifugal force can stabilize MHD modes, the large centrifugal potential allows a Maxwellian ion distribution (preventing loss-cone instabilities) and the centrifugal potential and ambipolar potential enhance parallel confinement [3, 4, 5, 6].

While the rotation resolves many issues related to axisymmetric open traps, new obstacles are introduced when one attempts to reach necessary rotation velocities in the plasma. Perhaps the most frustrating has been the Alfven critical ionization velocity (CIV) limitation [7]. The CIV limit depends on the ionization potential of neutrals in contact with the plasma. Even if the plasma is fully ionized, the CIV can limit the rotation velocity at the end insulator or electrode [8]. Without overcoming the CIV limit, centrifugal mirrors could not be considered as viable fusion devices [3].

Despite these difficulties, several experiments have made good progress in centrifugal mirrors. The stabilization of MHD modes by a radial density gradient was demonstrated, confirming theoretical predictions [9]. The PSP-2 experiment in Novosibirsk later showed that the CIV limit could be overcome with extensive vacuum pumping and electrode conditioning [10]. Rotation speeds 40 times larger than the CIV limit were achieved. Recent experiments in Maryland suggest that rotation shear could also stabilize MHD modes in mirror plasmas [6]. This experiment has so far failed to exceed the CIV limitation [11].

It was suggested that the rotation in a centrifugal mirror could be maintained using radio frequency waves rather than electrodes [12, 13, 14]. This would give more flexibility in the design of the end region, which may allow the CIV limit to be overcome. In a fusion plasma, the rotation could be maintained using the alpha particle energy. This is an extension of the alpha channeling concept developed in tokamaks and, more recently, stationary mirrors [15, 16, 17].

Removing alpha particles quickly and redirecting their energy can have a wide range of benefits to centrifugally confined plasmas. As in tokamaks and mirrors, by reducing the alpha particle pressure the fuel density can be increased, which can lead to a doubling of the fusion reactivity. In rotating mirrors, it is especially important to prevent electron heating because the ambipolar potential is proportional to the electron temperature. The ambipolar potential is deconfining for ions, so the lower the electron temperature, the better the ions are confined. Finally, the recirculating power is reduced because the rotation is maintained by the alpha particles. These benefits are in addition to the main effect of no longer needing electrodes, eliminating the CIV limitation.

A simple way to produce the alpha channeling is to use a fixed azimuthal ripple field [14]. This type of wave does not require significant power input, as it has very little energy in the lab frame. In interacting with alpha particles, the wave breaks the adiabatic invariant and azimuthal symmetry so that the particle can scatter radially, converting its kinetic energy potential and vice versa. In order to eliminate the electrodes entirely, then, the alpha particle power retained must exceed the power dissipated by the plasma. Most of the energy loss in a rotating mirror goes into heating the ions and providing the rotation energy. We will show that the alpha particles interacting with stationary waves can sustain the plasma, so that no external energy input is required.

This paper will focus on the energy balance in a rotating mirror fusion reactor with alpha channeling. In Sec. II we will review the power balance of rotating mirrors. Then in Sec. III, we discuss simulations that show stationary waves can convert enough power from alpha particles to maintain rotation. Sec. IV will describe implications and future work.

II. POWER BALANCE

Power dissipated in the supersonically rotating mirror is primarily due to heating and rotating the ions [2]. Consider a plasma rotating so that a deuterium ion moving with the rotation velocity has an energy of 200 keV. When a deuterium atom that is cold in the rest frame is ionized, the ion moves toward the negative electrode by one gyroradius, and the electron toward the positive. The ion appears to acquire 400 keV of energy: in the rotating frame it has 200 keV of kinetic energy, and that frame has a directed energy of 200 keV. Becuase the total energy should be the same, the ion gyrocenter is at a radius with a potential energy of -400 kV compared to the original position. An electron moving with the rotation velocity has an energy of only 54 eV, so we can infer its gyrocenter has potential energy $+108$ V.

In the centrifugal trap, particles cannot leave the plasma without returning most of this energy to the potential. If the mirror ratio $R = 5$, the rotation energy of deuterium at the mirror throat will be 40 keV (assuming isorotation). There is also a centrifugal potential along the field line that the particle must overcome, with magnitude 160 keV. Of the 400 keV initially consumed by to the deuterium ion, the deuteron leaving the trap returns 320 keV to the potential.

We can express the above discussion analytically by assuming a steady state energy balance in the plasma. We will use W_{E0} as the midplane rotation energy of the deuteron, Q_{ie} as the average energy transferred from ions to electrons, and introduce the ambipolar potential Φ_0. Then, making use of the small electron mass and assuming the confining potential is much larger than the temperature [4],

$$W_{E0}\left(1-1/R\right) - e\Phi_0 + T_i + Q_{ie} = W_{E0}, \qquad (1)$$

$$e\Phi_0 + T_e - Q_{ie} = 0. \qquad (2)$$

The overall power balance then requires $W_{E0}/R = T_e + T_i$. The potential energy lost per ion-electron pair is $2W_{E0}/R$, from the rotation and thermal energy.

Some of that energy could be recovered using direct conversion or a shaped end wall (whether made of ring electrodes or an insulator). The shaped end wall makes use of the finite gyroradius of the ions, removing the ions at their potential energy maximum (the maximum distance from the center axis). The energy that can be recovered from the ion by this method is approximately $\sqrt{\pi}W_{E0}/R$ [4]. The ring electrodes that are usually used to create rotation reduce the efficiency of this process because they must be far enough apart to prevent breakdown. A continuous insulating surface would be possible if the rotation were maintained with waves, allowing higher efficiency.

The above discussion has neglected the production of neutrons and alpha particles. However, if the plasma is to burn up fuel, the energy balance may change significantly. Because neutrons are not charged, they leave the trap without overcoming the centrifugal potential. This changes the energy balance of Eq. (1), because the input energy on the right side now includes the deuterium and tritium ions, while the W_{E0} term on the left only includes the alpha particle mass. The potential energy lost after a fusion reaction is therefore $2W_{E0n} + 2W_{E0\alpha}/R$, compared to $2(W_{E0D} + W_{E0T})/R$ if the fuel had been lost at the mirror throat without reacting.

The timescales for parallel ion loss and fusion will govern whether or not alpha particles can sustain the plasma entirely. The ion confinement time is $\nu_{i\parallel} \approx \nu_i e^{1-R}$, using the Pastukhov factor for the confining potential and our previous relationship between W_{E0} and T_i, assuming $T_e \ll T_i$ [18]. The fusion time is simply $\nu_{fus} = n_i \langle\sigma v\rangle/4$. If the alpha channeling efficiency is η, power balance requires,

$$\nu_{fus} W_{\alpha 0}\eta \geq \nu_{fus}\left(W_{E0} + \frac{4W_{E0}}{R}\right) + \nu_{i\parallel}\frac{5W_{E0}}{R}, \quad (3)$$

where W_{E0} is for deuterium and the rotation energy was scaled by the number of nucleons. From this equation we get the requirement,

$$\frac{\nu_{fus}}{\nu_i} \geq \frac{5e^{1-R}}{R\left(\eta W_{\alpha 0}/W_{E0} - 1\right) - 4}. \qquad (4)$$

Assuming $T_i \approx 50\,\mathrm{keV}$ so that $\langle\sigma v\rangle$ is near its maximum, the alpha channeling efficiency must be,

$$\eta \geq \frac{4 + R + e^{8-R}}{70} \qquad (5)$$

We have again assumed $T_i \approx W_{E0}/R$. For $R = 5$, this implies that $\eta > 0.2$, which is satisfied by the suggested waves [14].

Alpha particles that remain in the plasma would typically slow down on electrons before scattering out of the trap. This will have the effect of increasing the electron temperature, and therefore the ambipolar potential, through Eq. (2). Since the centrifugal potential remains the same, the result will be that more electrons and ions can escape, decreasing the confinement time and increasing the dissipated power.

Alternatively, we can remove the alpha particles using radio frequency waves [12, 16]. The particle motion may be described by a diffusion path in phase space if it interacts with the wave repeatedly at random phase. We have suggested that a fixed azimuthal ripple could be used effectively for this purpose [14]. This wave can be made resonant with alpha particles at the cyclotron frequency by using a high mode number, $|m| \geq 20$. When an alpha particle interacts with this wave, it does not change its rest-frame energy, but the particle can scatter radially. If the particle moves inward, it gains energy from the potential and becomes more deeply trapped. As it moves outward, it loses energy and may exit the plasma through the loss cone. In steady state this leads to a radial current of alpha particles. In the next section we will demonstrate that alpha particles interacting with these waves convert enough energy to the radial potential to maintain the plasma rotation.

III. SIMULATION

In order for zero frequency waves to be resonant with alpha particles, the mode number must be $m \approx -\Omega_i/\Omega$, where Ω_i is the cyclotron frequency and Ω is the rotation frequency. For practical fusion devices, this implies $|m| \geq 20$. In a vacuum, such a mode would decay like $r^{|m|}$. The effect of the rapid decay is mitigated by the fact that the plasma is localized near the outer wall in a layer of width $a \ll r$ [4]. Because only a small amplitude wave is necessary to remove alpha particles within a slowing down time, the radial decay is not prohibitive to alpha channeling [14].

To accurately model the plasma response to the ripple, we considered a uniform cylindrical plasma shell and an antenna surface with externally imposed currents. We then found the solution to the two-fluid cold MHD equations in the rotating frame [14]. The resulting fields were used in a simulation that solved the full equations of motion of alpha particles.

In order to remove particles from across phase space, four wave regions were used, positioned at locations with varying field strength (Fig. 1). The wave mode numbers range from $m = -19$ to $m = -30$, and the mirror ratio at the RF region is from 1 to 1.5. The ripple magnitude at the outer edge of the plasma is 5-10

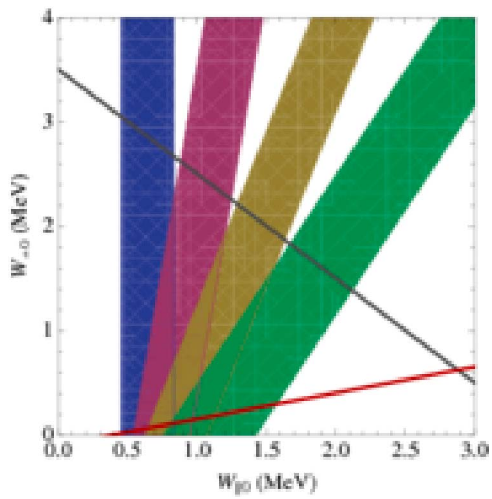

Figure 1: Phase space showing the resonance regions of the waves simulated. The diagonal line at 3.5 MeV indicates the alpha particle birth energy. The upward line by the x-axis indicates the boundary of the alpha particle loss cone.

Gauss. Other wave properties are detailed in Ref. [14].

We simulated the particles in a rotating mirror with a midplane field of 2 Tesla, deuterium rotation energy of 200 keV, mirror ratio of 5, outer radius of 1 m, and midplane density of 1.5×10^{13} cm^{-3}. These parameters are not optimized for power production.

The simulations show that a significant fraction of alpha particles can be removed within the slowing down time (Fig. 2). 32% of the particles were removed by the waves after 1 s, and these particles converted 64% of their kinetic energy to potential energy. Overall, the alpha channeling efficiency is $\eta = 0.21$, satisfying the requirement derived earlier for energy balance.

IV. CONCLUSION

We have shown that alpha particles can be used to drive rotation in centrifugal mirrors. The alpha particles interact with waves that have zero frequency in the lab frame, and can scatter radially. The particle becomes more trapped if it moves inward and gains energy, and can exit if it moves outward losing energy, creating a steady state radial current of alpha particles.

We found the alpha channeling efficiency necessary to sustain the plasma without external energy sources. By solving for the plasma response and simulating the alpha particles, we demonstrated that this efficiency could be achieved using four stationary waves with high azimuthal mode number.

There are a number of energy loss mechanisms that

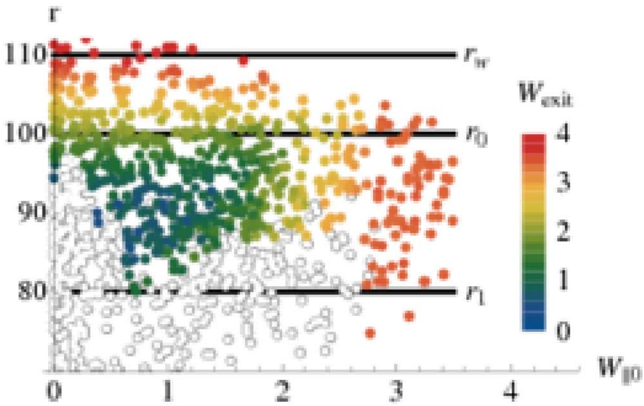

Figure 2: The energy of simulated alpha particles leaving the trap versus gyrocenter and parallel energy at birth. Open circles represent particles remaining in the device after 1 s. The plasma extends from r_1 to r_0 and at r_w is the wall.

were not accounted for in this calculation. For example, at lower rotation speeds charge exchange losses can become significant [4]. In addition, there could be heating from sheared rotation, and radial energy transport could become important. On the other hand, the use of shaped end insulators to recover ion and alpha particle energy was also not included [4]. A deeper calculation of these effects is necessary to determine whether energy balance can be achieved. Because there is still room to improve the alpha channeling efficiency (for example, with a larger ripple magnitude to reach ions near the center), we expect that the results described here will hold once additional losses are included.

The key benefit to producing the rotation using an alpha particle current is that electrodes, which have limited the plasma rotation to the Alfven CIV, are not necessary. This may allow centrifugal traps to achieve their potential as fusion devices identified in early theoretical studies [3].

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