corrected version of some slides available at http://w3.pppl.gov/~hammett/talks/2009/Imperial09-kmri-Hammett.pdf

Angular Momentum Transport in Astrophysical Accretion Flows

Greg Hammett, Princeton Plasma Physics Lab

On behalf of

Eliot Quataert, Berkeley

Prateek Sharma, Berkeley

Jim Stone, Princeton

CMSO, UNH, June 5, 2007

acknowledgments: most slides by Quataert & Sharma

Accretion

- Inflow of matter onto a central object (generally w/ angular momentum)

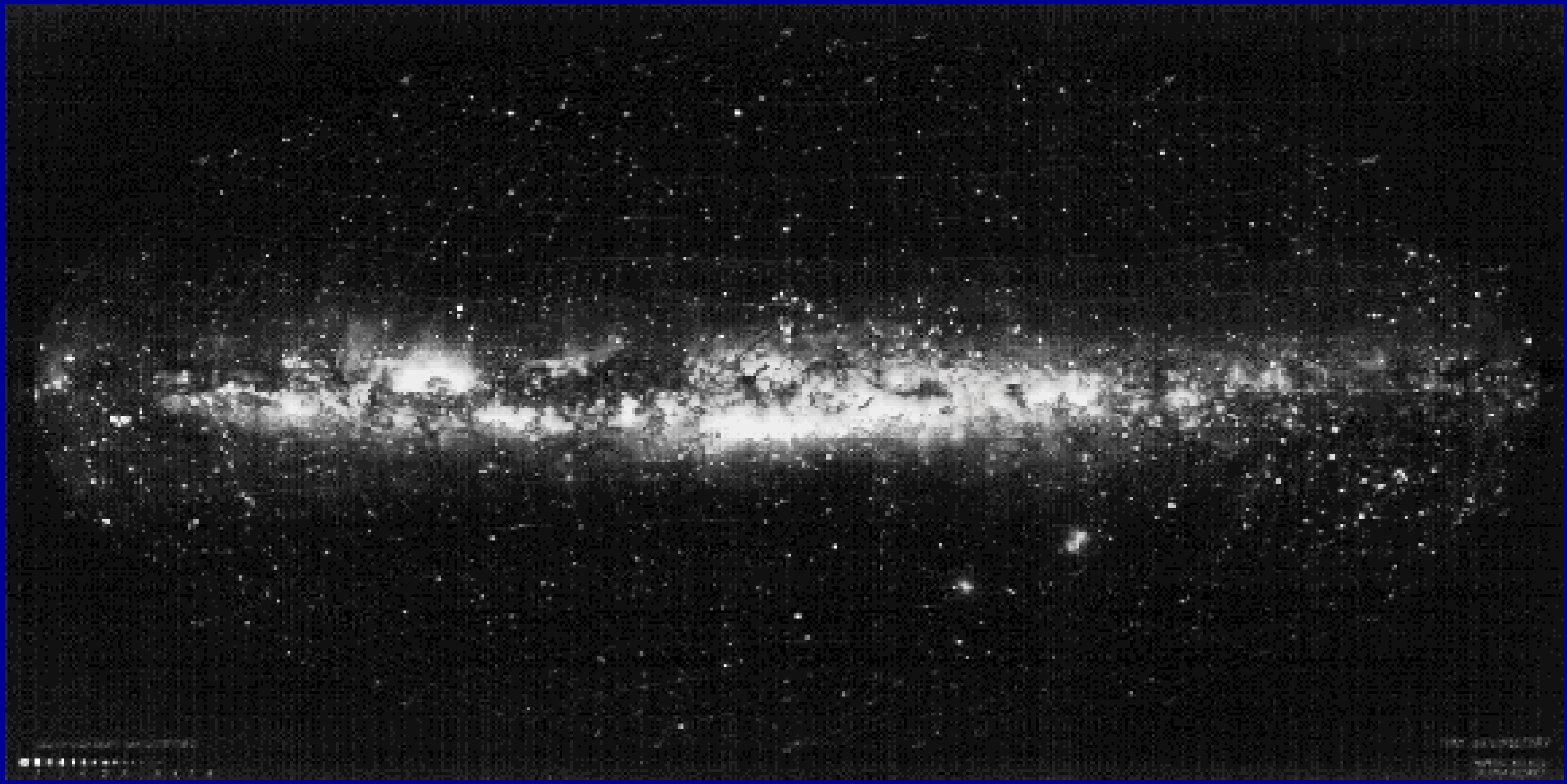
- Central to
  - Star & Planet Formation
  - Galaxy Formation
  - Compact Objects: Black Holes, Neutron Stars, & White Dwarfs

- Energy Released:

$$\dot{E} = \frac{GM\dot{M}}{2R} \equiv \epsilon \dot{M} c^2$$

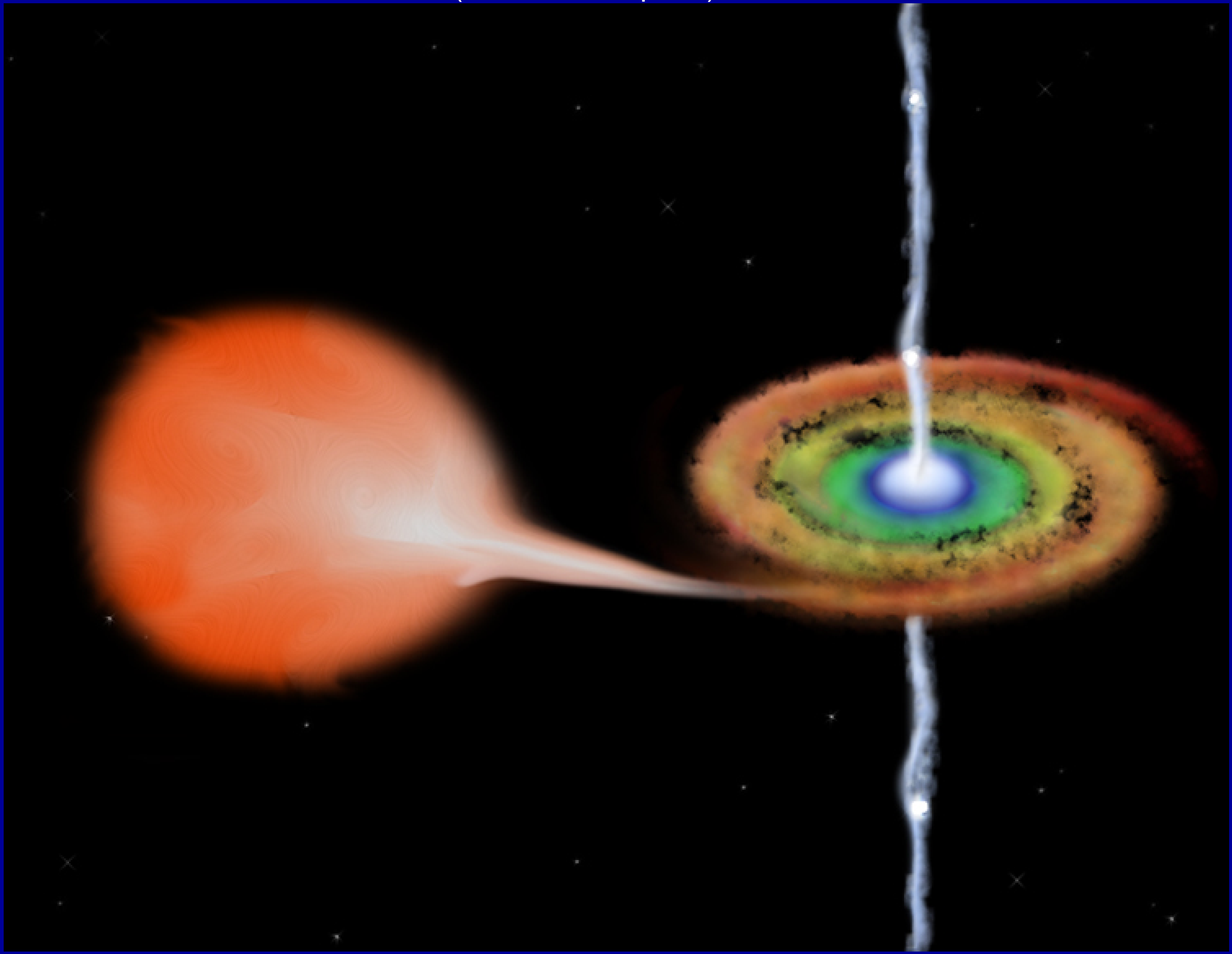
  - sun: $\varepsilon \sim 10^{-6}$
  - BH ($R \sim 2GM/c^2$): $\varepsilon \sim 0.25$ (can be << 1; more later)
  - Fusion in Stars: $\varepsilon \sim 0.007$
  - Accretion onto Black Holes & Neutron Stars is Responsible for the Most Energetic Sources of Radiation in the Universe

The Milky Way: A Thin Disk



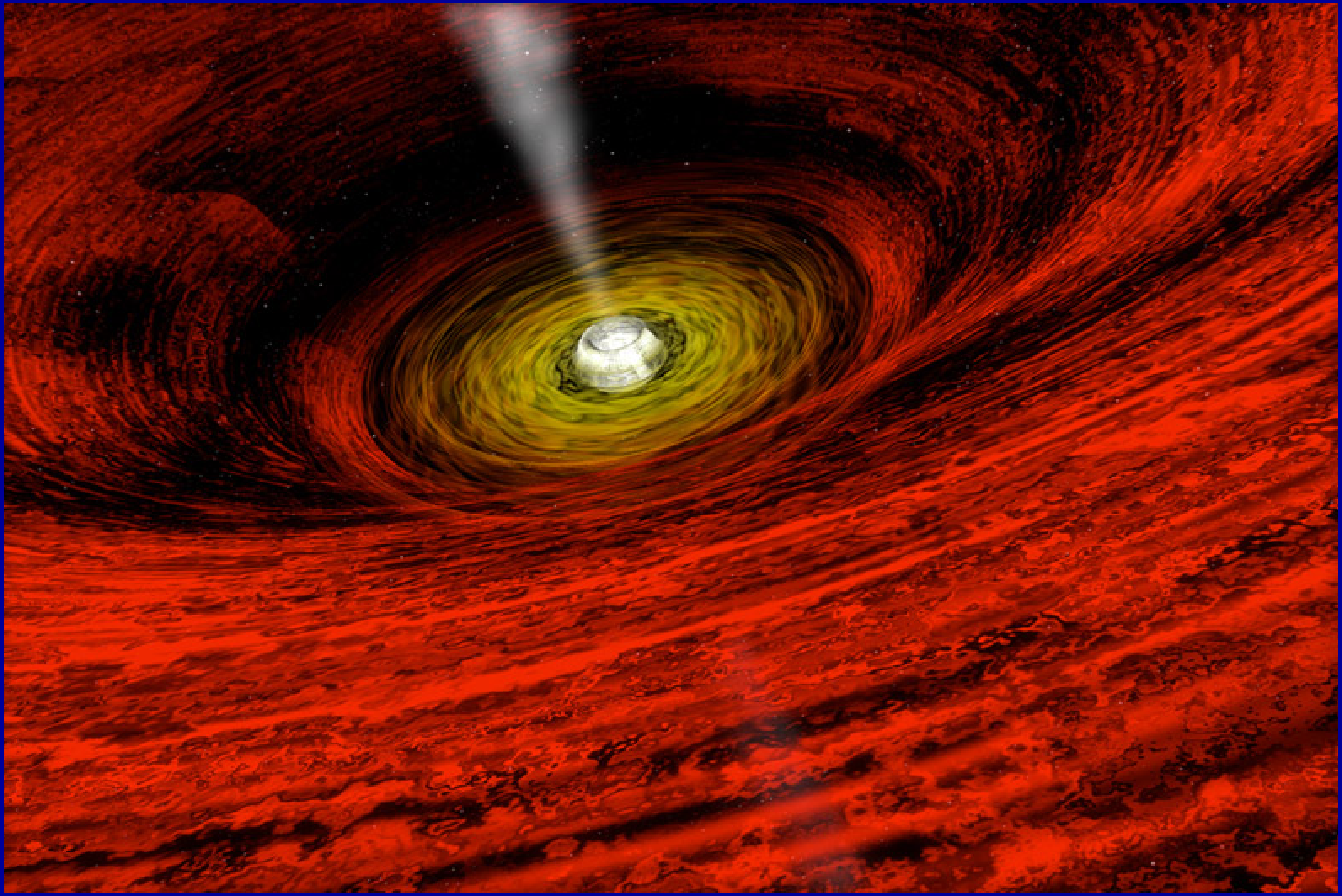
Star orbiting black hole & feeding accretion disk

(artist's conception)



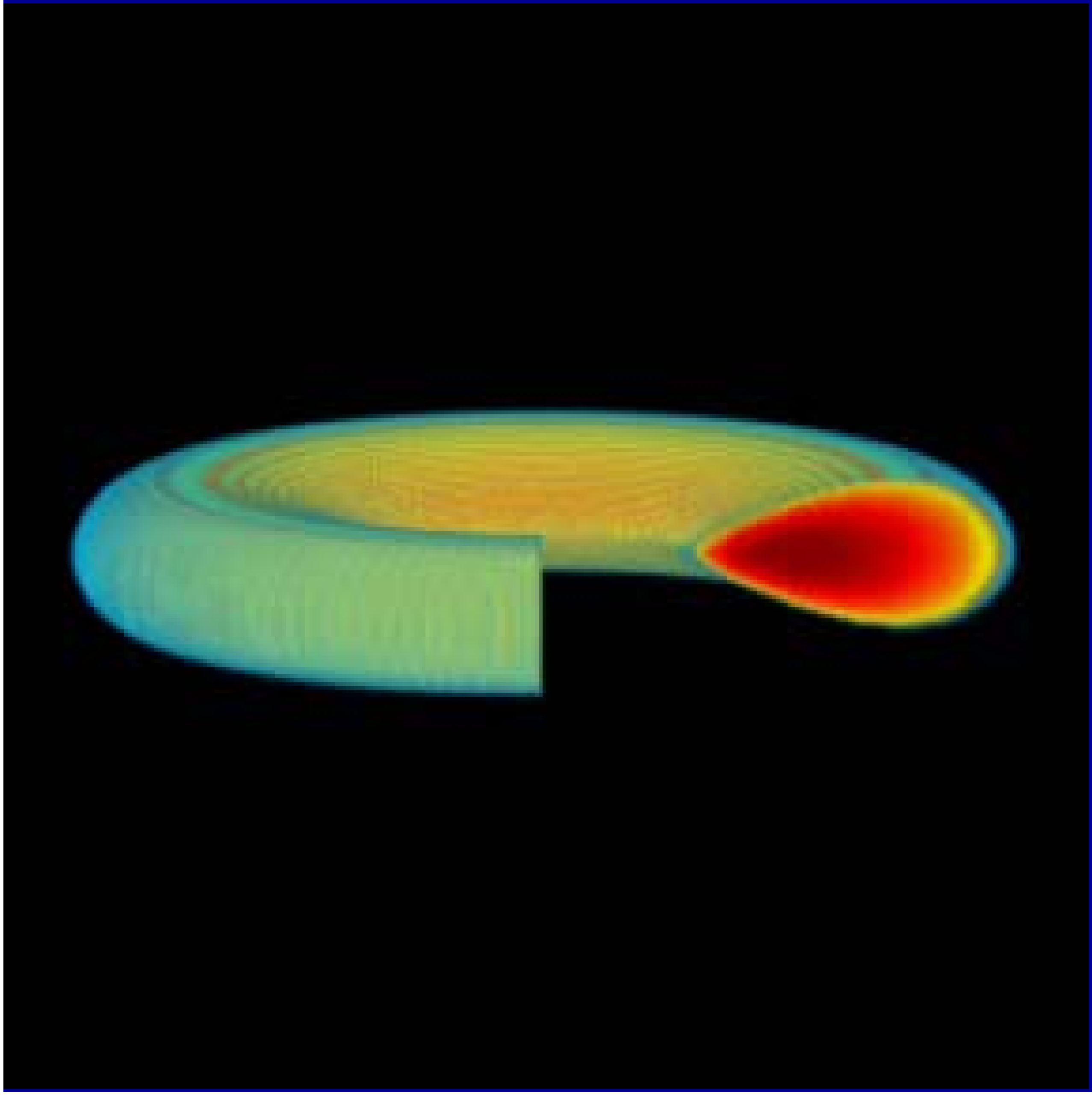
NASA/CXC/SAO A.Hobart   http://chandra.harvard.edu/resources/illustrations.html

Black Hole Neighborhood. (artist's conception)



NASA/CXC/SAO A.Hobart   http://chandra.harvard.edu/resources/illustrations.html

A 3-D Global MHD Simulation



Simulation by Hawley et al.
http://astsun.astro.virginia.edu/~jh8h/

MHD simulations of MRI turbulence very successful. Need to study it in collisionless regime applicable to Sgr A*

Outline

- Accretion Disks: Basic Physical Picture
- MHD of Disks: Angular Momentum Transport
- Collisionless Accretion Flows (BHs & NSs)
  - Astrophysical Motivation
  - Disk Dynamics in Kinetic Theory
    - A mechanism for strong electron heating
      (Sharma et al. astro-ph 07)

Accretion: Physical Picture

- Simple Consequences of Mass, Momentum, & Energy Conservation

- Matter Inspirals on Approximately Circular Orbits

  - $V_r << V_{orb}$ $\quad t_{inflow} >> t_{orb}$
  - t_{inflow} ~ time to lose angular momentum ~ viscous diffusion time
  - $t_{orb} = 2\pi/\Omega$; $\Omega = (GM/r^3)^{1/2}$ (Keplerian orbits; like planets in solar system)

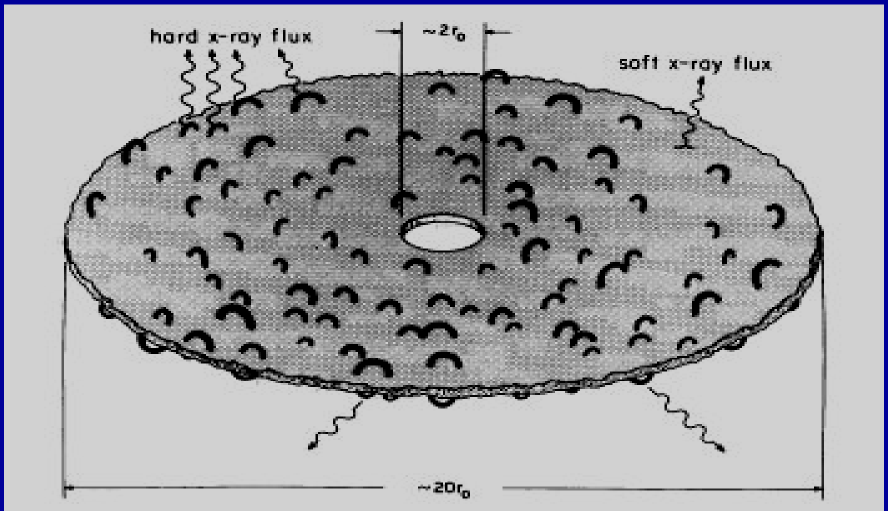
- Disk Structure Depends on Fate of Released Gravitational Energy

  - t_{cool} ~ time to radiate away thermal energy of plasma

  - Thin Disks: $t_{cool} << t_{inflow}$ (plasma collapses to the midplane)
  - Thick Disks: $t_{cool} >> t_{inflow}$ (plasma remains a puffed up torus)

Geometric Configurations

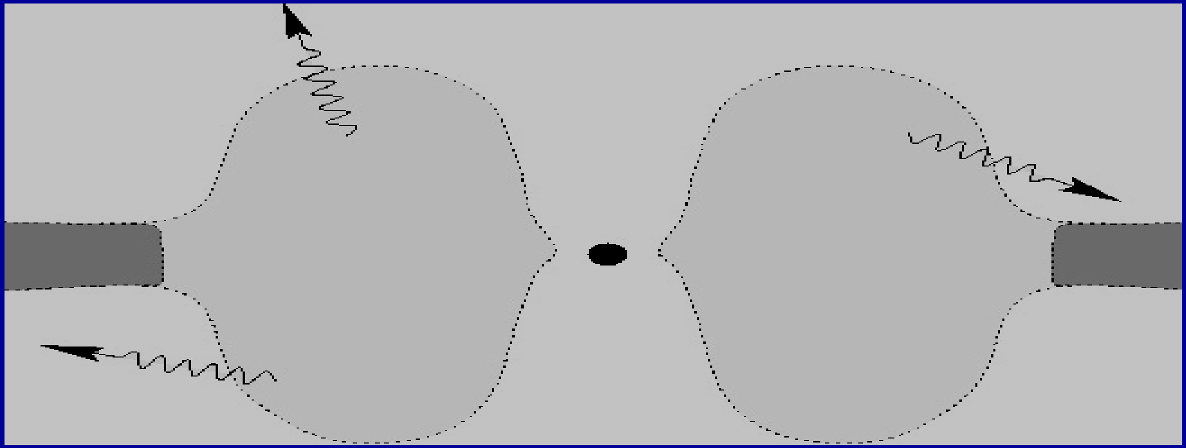
Galeev et al. 1979



e.g., solar system
Milky Way disk

thin disk: energy radiated away

(relevant to star & planet formation, galaxies, and luminous BHs/NSs)

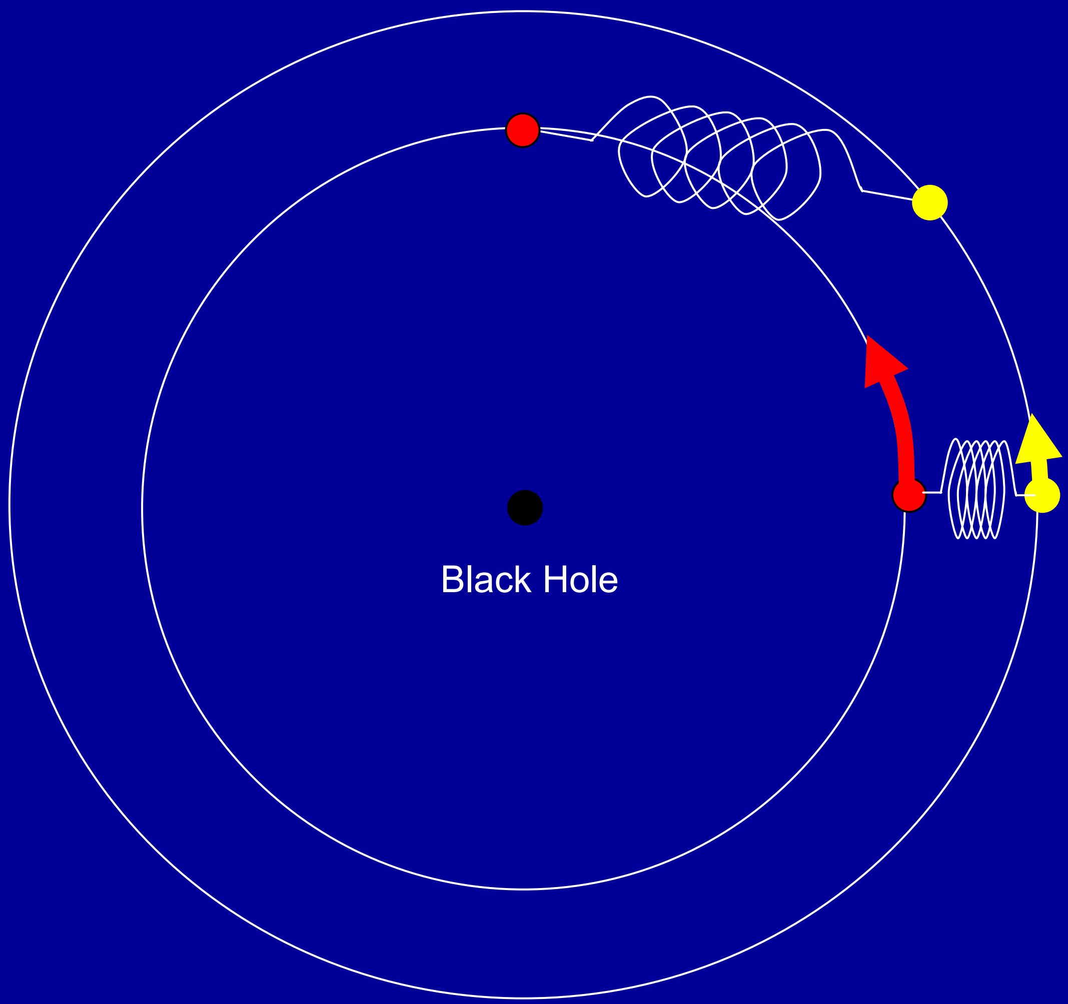


e.g., our Galactic Center (more on this soon)

thick disk (torus; ~ spherical): energy stored as heat

(relevant to lower luminosity BHs/NSs)

Magneto-Rotational Instability explains how accretion disks accrete (Balbus & Hawley, 1991)

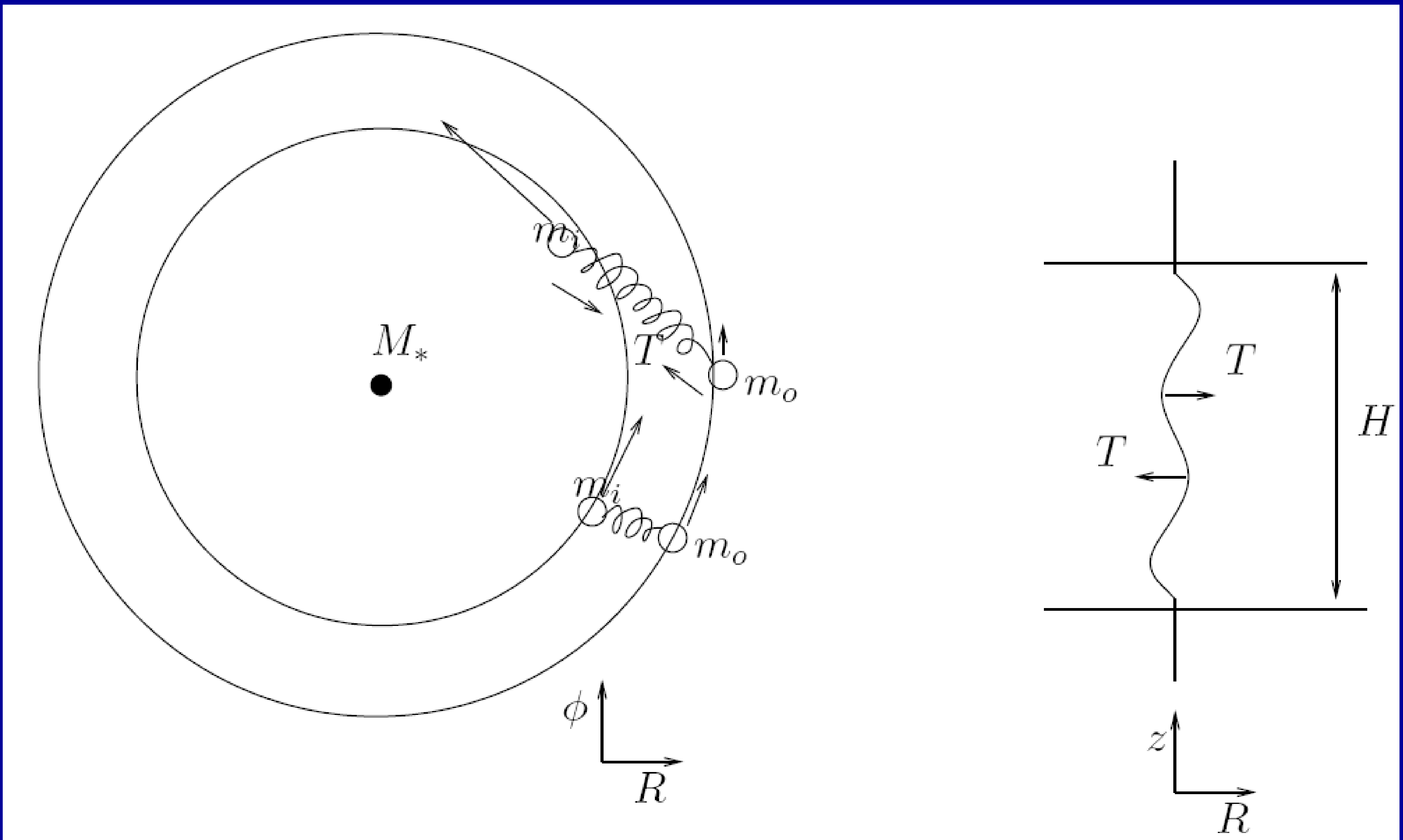


Inner particle orbits faster,
Spring stretches out
Spring force slows inner particle
and accelerates outer particle
Causing inner particle to fall in
and outer particle to go out
Exponentially amplified.

Magnetic fields
Are like springs

spring analogy by Toomre

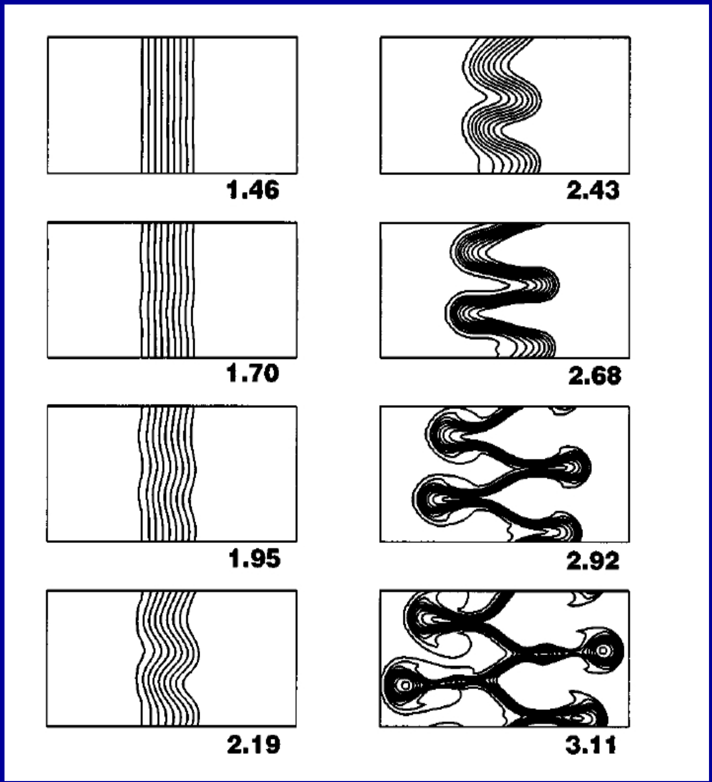
Side view: magnetic field stretching acts like springs & transfer angular momentum



Angular Momentum Transport by MHD Turbulence

(Balbus & Hawley 1991)

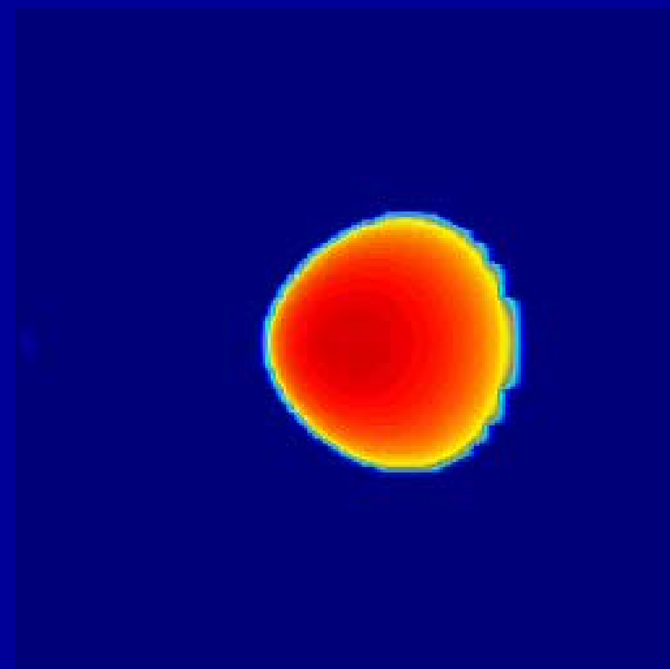
- A differentially rotating plasma with a weak field ($\beta > 1$) & $d\Omega^2/dR < 0$ is linearly unstable in MHD (magnetorotational instability -- "MRI")
- magnetic tension transports ang. momentum, allowing plasma to accrete
- MRI may also be relevant in stars (e.g., solar interior, tachocline)
- Experiments underway to study the MRI in the lab (e.g., PPPL); may have been detected in liquid dynamo expt at UMD (Sisan et al. 2004)



Balbus & Hawley 1998

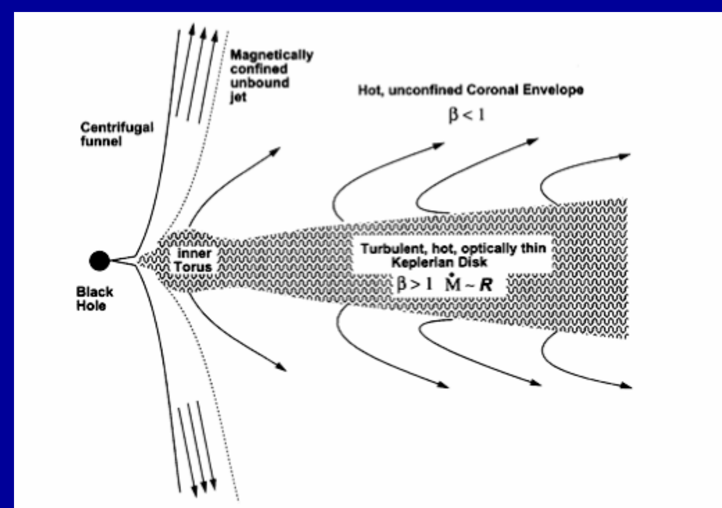
early evolution of MRI

Implications for Global Disk Dynamics



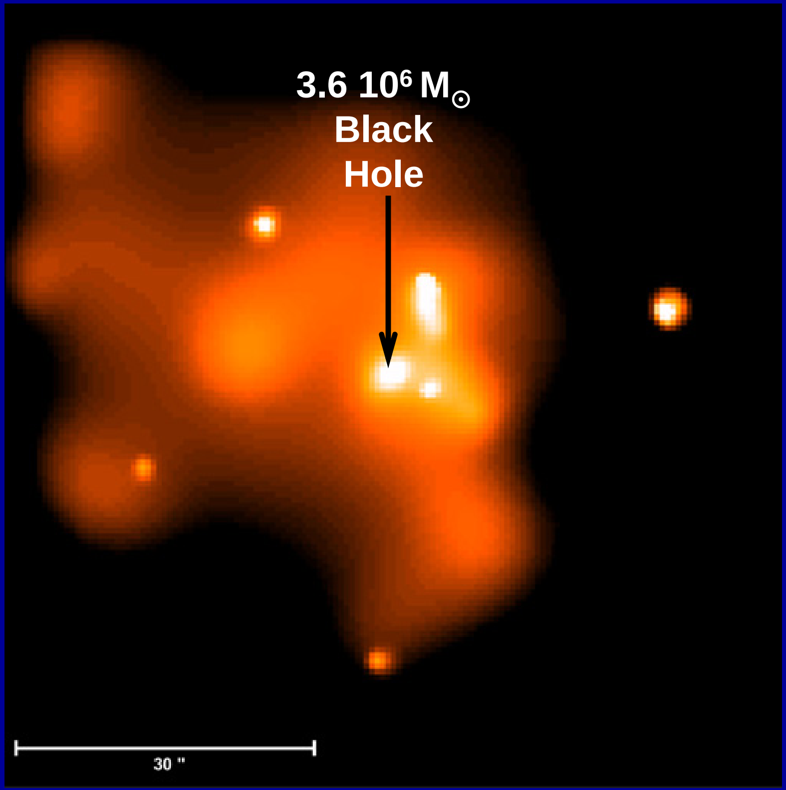
John Hawley

- Instability saturates as MHD turbulence: $\beta \sim 10$ in disk with a $\beta << 1$ corona (global sims for thick disks to date)
- Note: nonlinear saturation does not modify $d\Omega/dr$, source of free energy (instead drives inflow of plasma bec. of Maxwell stress $B_r B_\varphi$)
- Era of 1st principles numerical simulations (radiation transport, full General Relativity, Hall effects, neutrals + ions, kinetic effects, …)



An Astrophysical Context: Our Galactic Center

Galactic Center (*Chandra*)



Ambient Gas: $n \approx 100$ cm^{-3}
$T \approx$ 1-2 keV

- Ambient gas should be grav. captured by the BH
- Estimates give

$$\dot{M}_{\rm captured} \approx 10^{-5}\,{\rm M}_\odot\,{\rm yr}^{-1}$$

rate at which gas is captured at large radii

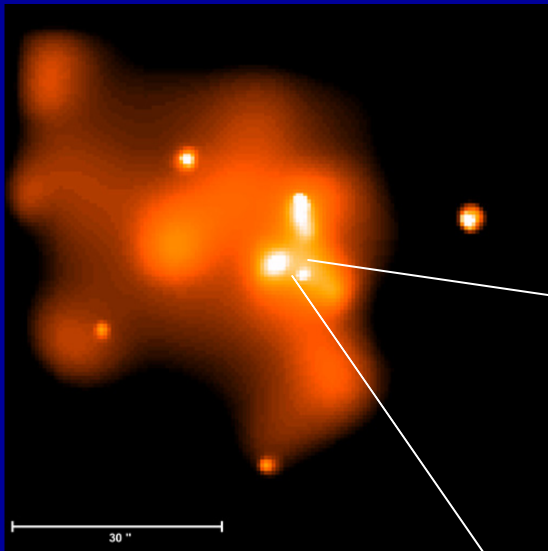
- But then

$$L_{\rm observed} \approx 10^{-5}\,\dot{M}c^2$$

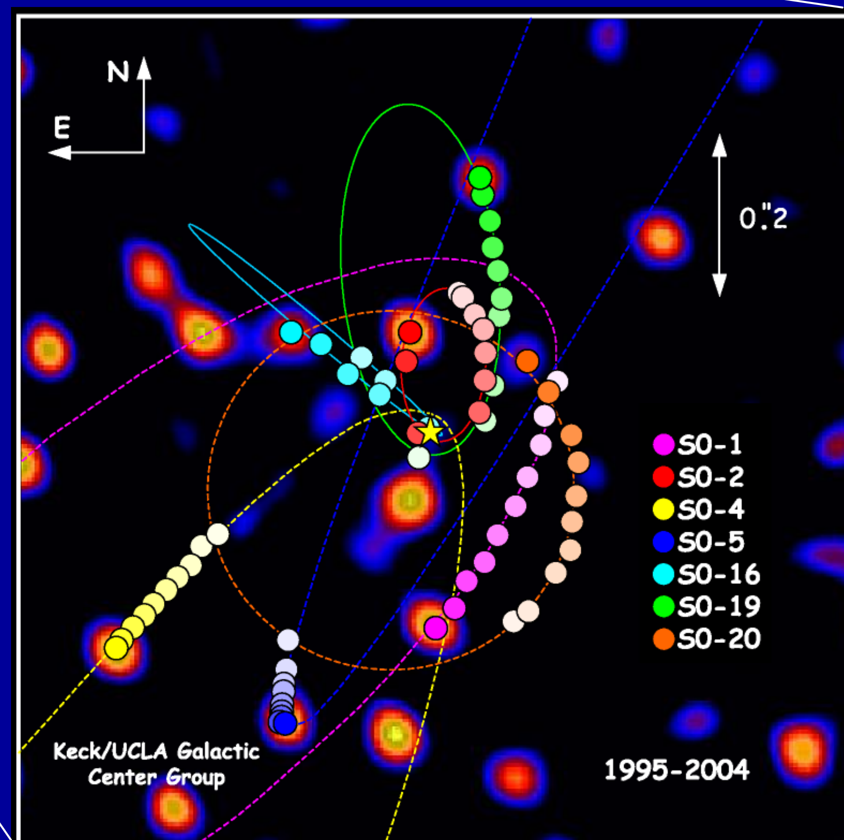
<< ~ 10% efficiency in luminous BHs

http://chandra.harvard.edu/

Galactic Center BH



Chandra



3.6×10^6 M_o black hole

Bondi radius ~ 0.07 pc (2''), n~100/cc, T~1-2 keV

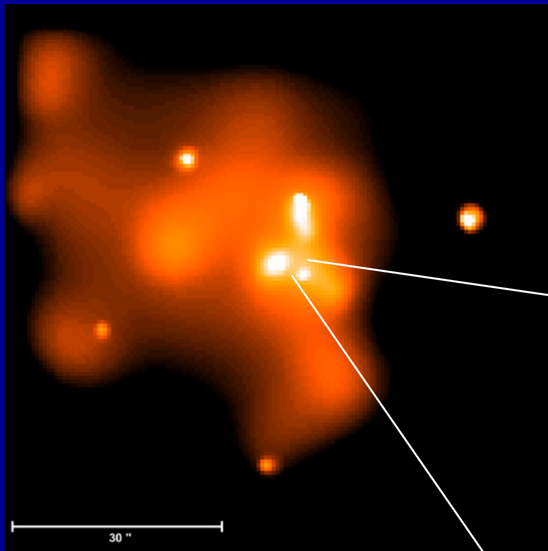
$\dot{M} \sim 10^{-5}$ M_o /yr by stellar outflows

$L_{obs} \sim 10^{-5} \times (0.1\ \dot{M}c^2)$
Why low luminosity? low $\dot{M}$ or low radiative efficiency

Collisionless, magnetized plasma at r ♎ Bondi radius; $\rho_i \ll H$, $\lambda_{mfp} \gg H$

Schödel et al., 2002, A.M. Ghez et al. 2003 http://www.astro.ucla.edu/~ghezgroup/gc

Galactic Center BH



Chandra

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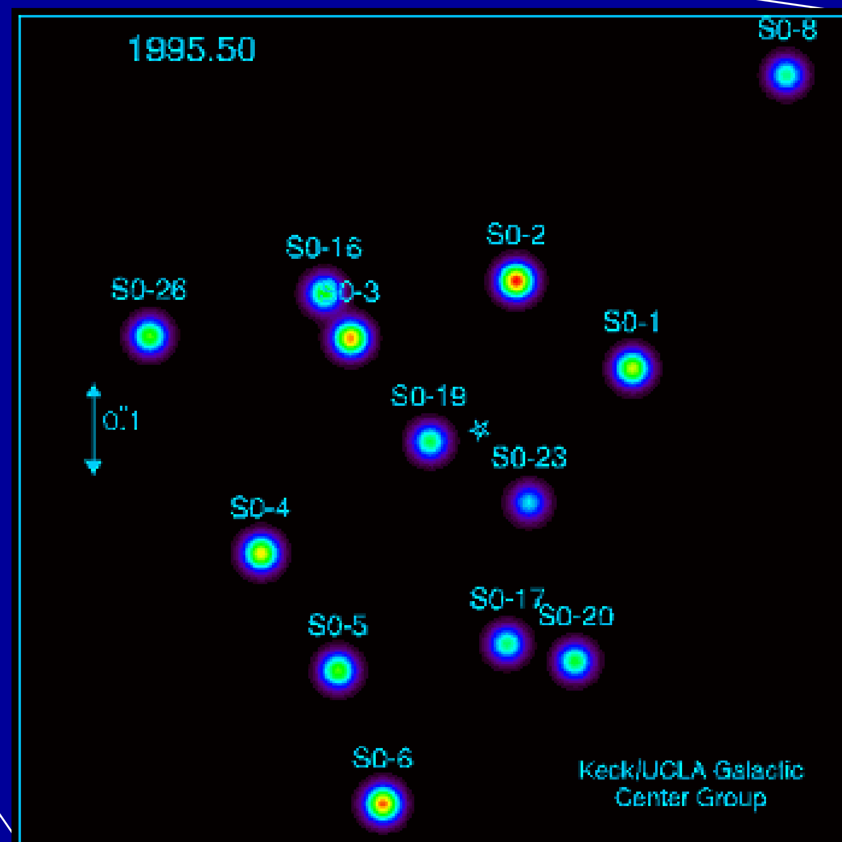
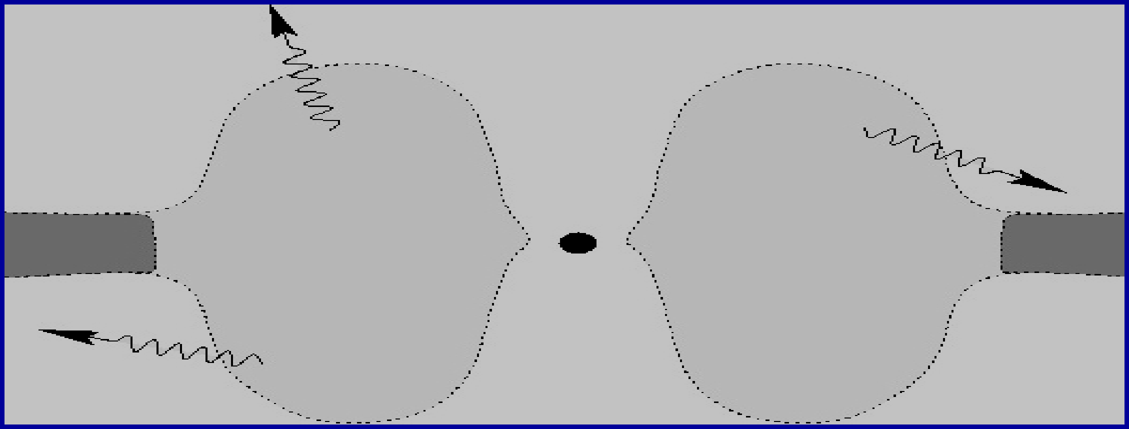


Schödel et al., 2002, A.M. Ghez et al. 2003 http://www.astro.ucla.edu/~ghezgroup/gc

Thick Disks: Radiatively Inefficient

- **At low densities (accretion rates), cooling is inefficient**
- **Grav. energy $\Rightarrow$ thermal energy; *not radiated*** $L \ll \dot{M} c^2$
- **kT ~ GMm$_p$/R: T$_p$ ~ 10$^{11\text{-}12}$ K > T$_e$ ~ 10$^{10\text{-}11}$ K near BH**
- ***Collisionless plasma*: e-p collision time >> inflow time**



- Initial Models (ADAFs) had
  (e.g., Narayan & Yi 1994)

$$\dot{M}_{\rm BH} \sim \dot{M}_{\rm captured}$$

Efficiency ~ 10^{-5} in GC

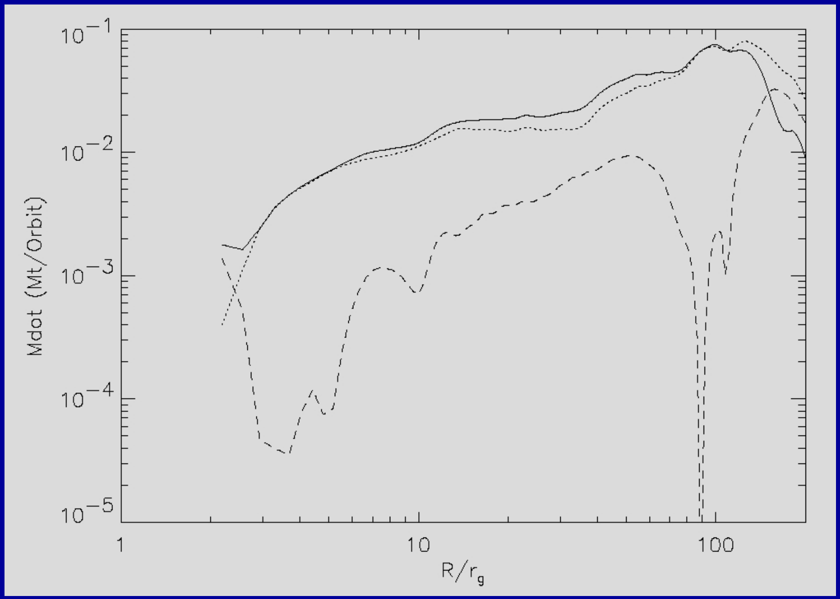
Low efficiency because electron heating is assumed to be very inefficient (electrons radiate, not protons)

- Very little mass supplied at large radii accretes into the black hole (outflows/convection suppress accretion)
  (e.g., Igumenschev & Abramowicz 1999; Stone et al. 1999; Blandford & Begelman 1999; Quataert & Gruzinov 2000)

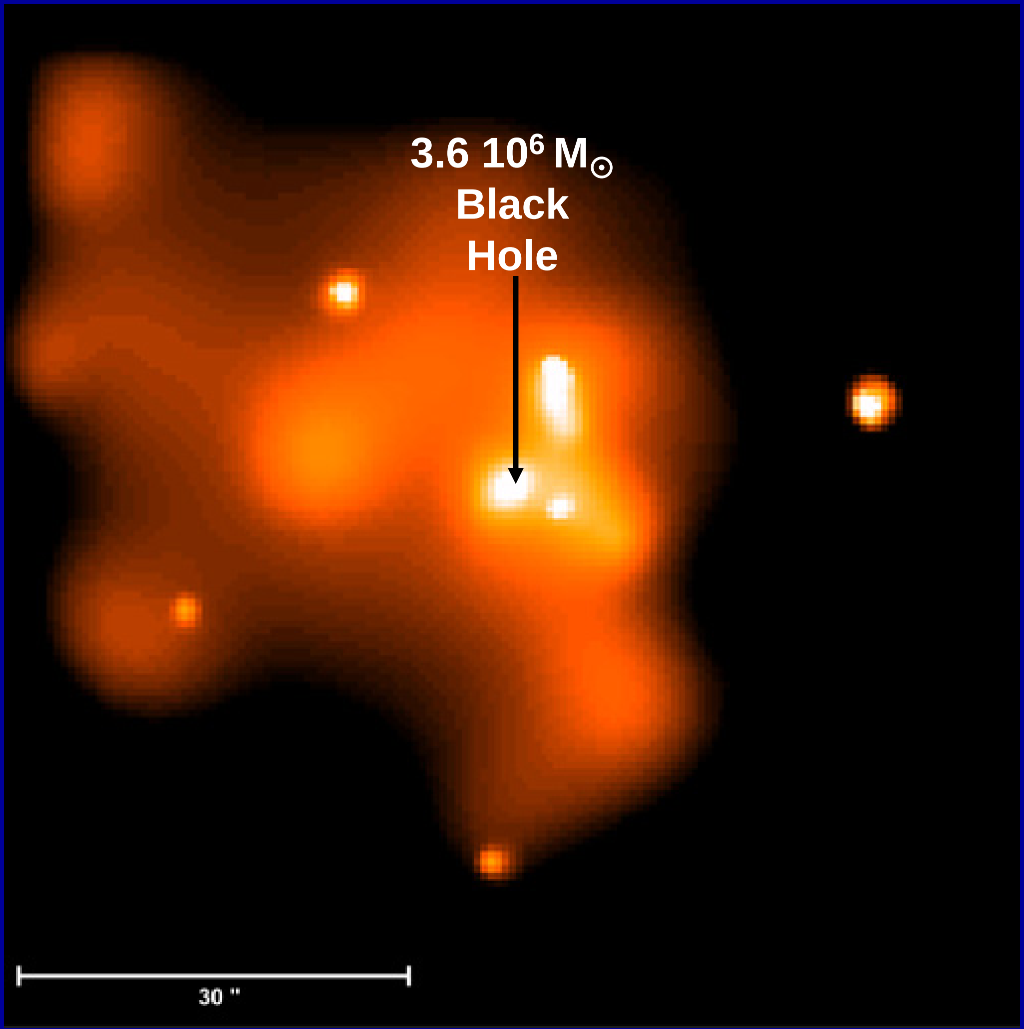
$$\dot{M}_{BH} \ll \dot{M}_{\rm captured}$$

Low luminosity because very little gas makes it to the BH

Hawlwy, Balbus, & Stone 2001



The (In)Applicability of MHD?



Hot Plasma Gravitationally Captured
By BH $\Rightarrow$ Accretion Disk

Observed Plasma

(R ~ 10^{17} cm ~ 10^5 $R_{horizon}$)

T ~ few keV   n ~ 100 cm^{-3}

mfp ~ 10^{16} cm ~ 0.1 R

e-p thermalization time ~ **1000 yrs**

>>

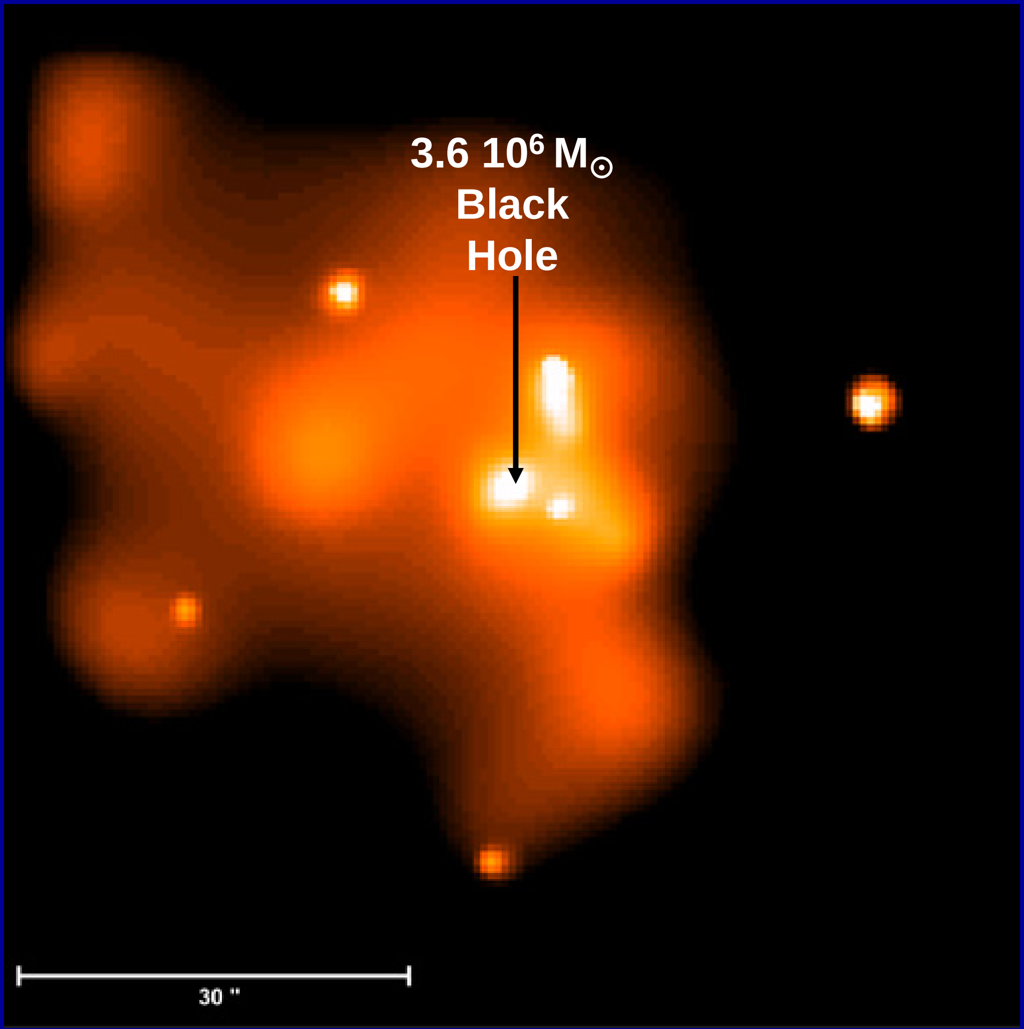
inflow time ~ R/c_s ~ **100 yrs**

electron conduction time ~ **10 yrs**

<<

inflow time ~ R/c_s ~ **100 yrs**

The (In)Applicability of MHD?



Hot Plasma Gravitationally Captured
By BH ⇒ Accretion Disk

Estimated Conditions
Near the BH

$T_p \sim 10^{12}$ K
$T_e \sim 10^{11}$ K
$n \sim 10^6$ cm^{-3}
B ~ 30 G

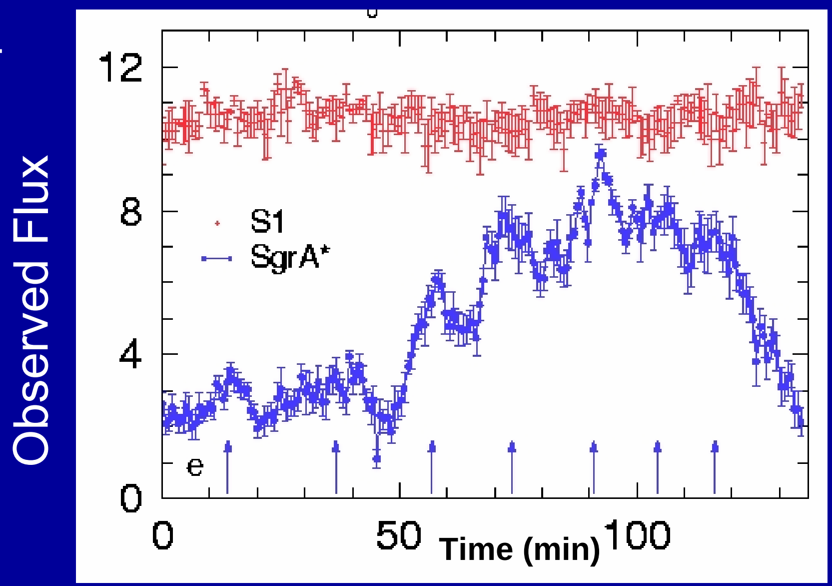
proton mfp ~ 10^{22} cm
>>> $R_{horizon}$ ~ 10^{12} cm

⇒

need to understand
accretion of a magnetized
collisionless plasma

Major Science Questions

- Macrophysics: Global Disk Dynamics in Kinetic Theory
  - e.g., how adequate is MHD, influence of heat conduction, …
- Microphysics: Physics of Plasma Heating
  - MHD turbulence, reconnection, weak shocks, …
  - **electrons produce the radiation we observe**
- Analogy: Solar Wind
  - macroscopically collisionless
  - thermally driven outflow w/ T_p & T_e determined by kinetic microphysics



Nonlinear Evolution Simulated Using Kinetic-MHD

- Large-scale Dynamics of collisionless plasmas: expand Vlasov equation retaining “slow timescale” (compared to cyclotron period) & “large lengthscale” (compared to gyroradius) assumptions of MHD (e.g., Kulsrud 1983)

- Particles efficiently transport heat and momentum along field-lines

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0,$$

$$\rho \frac{\partial \mathbf{V}}{\partial t} + \rho \left(\mathbf{V} \cdot \nabla \right) \mathbf{V} = \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi} - \nabla \cdot \mathbf{P} + \mathbf{F_g},$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{V} \times \mathbf{B} \right),$$

$$\mathbf{P} = p_\perp \mathbf{I} + \left(p_{||} - p_\perp \right) \hat{\mathbf{b}}\hat{\mathbf{b}},$$

Evolution of the Pressure Tensor

$$\rho B \frac{d}{dt}\left(\frac{p_\perp}{\rho B}\right) = -\nabla\cdot(\hat{\mathbf{b}} q_\perp) - q_\perp \nabla\cdot\hat{\mathbf{b}}$$

adiabatic invariance of $\mu \sim mv^2_\perp/B \sim T_\perp/B$

$$\frac{\rho^3}{B^2}\frac{d}{dt}\left(\frac{p_{||}B^2}{\rho^3}\right) = -\nabla\cdot(\hat{\mathbf{b}} q_{||}) + 2q_\perp \nabla\cdot\hat{\mathbf{b}},$$

$q = 0$ CGL or Double Adiabatic Theory

$$q_{\perp,||} \approx \frac{n v_{th}}{|k_{||}|} \nabla_{||} T_{\perp,||}$$

Closure Models for Heat Flux (temp gradients wiped out on ~ a crossing time)

Pressure Anisotropy

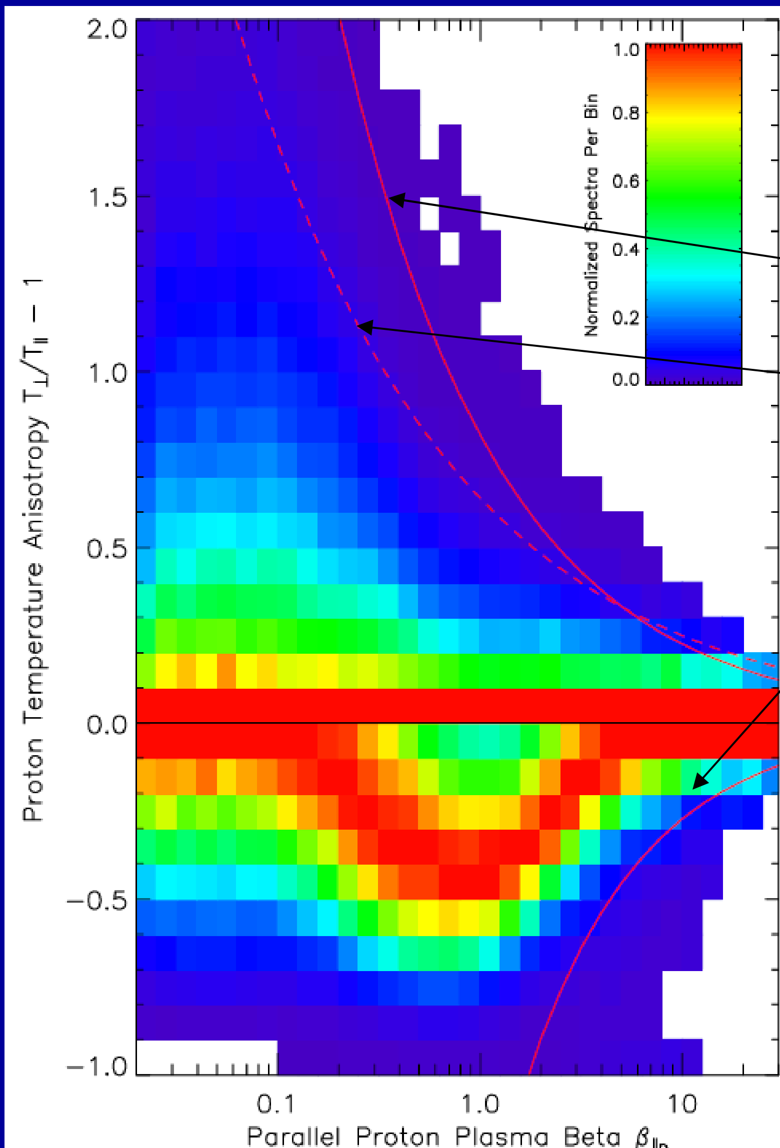
$$\mu \propto T_\perp / B = \text{constant} \;\Rightarrow\; T_\perp > T_\| \text{ as B } \uparrow$$

- $T_\perp \neq T_\|$ unstable to small-scale (~ gyroradius) modes that act to isotropize the pressure tensor (velocity space anisotropy)
  - e.g., mirror, firehose, ion cyclotron, electron whistler instabilities
- waves w/ frequencies ~ Ω_{cyc} violate μ invariance & pitch-angle scatter
  - **provide effective collisions & set mean free path of particles in the disk**
  - impt in other macroscopically collisionless astro plasmas (solar wind, clusters, …)
- Use "subgrid" scattering model in disk simulations

$$\frac{\partial p_\perp}{\partial t} = \ldots - \nu(p_\perp, p_\|, \beta)\left[p_\perp - p_\|\right]$$

$$\frac{\partial p_\|}{\partial t} = \ldots - \nu(p_\perp, p_\|, \beta)\left[p_\| - p_\perp\right]$$

Limits on Pressure Anisotropy



$$\left|\frac{p_\perp}{p_\parallel} - 1\right| \le \frac{S}{\beta^\alpha}$$

mirror: S=7, α=1 (to break adiabatic invariance)

ion-cyclotron: S=0.35, α=0.45 for $\gamma/\Omega_i = 10^{-4}$

mirror dominates IC for $\beta \sim$ 10-100

firehose:S>2, α=1

Pressure anisotropy reduced by pitch-angle scattering if anisotropy exceeds threshold.

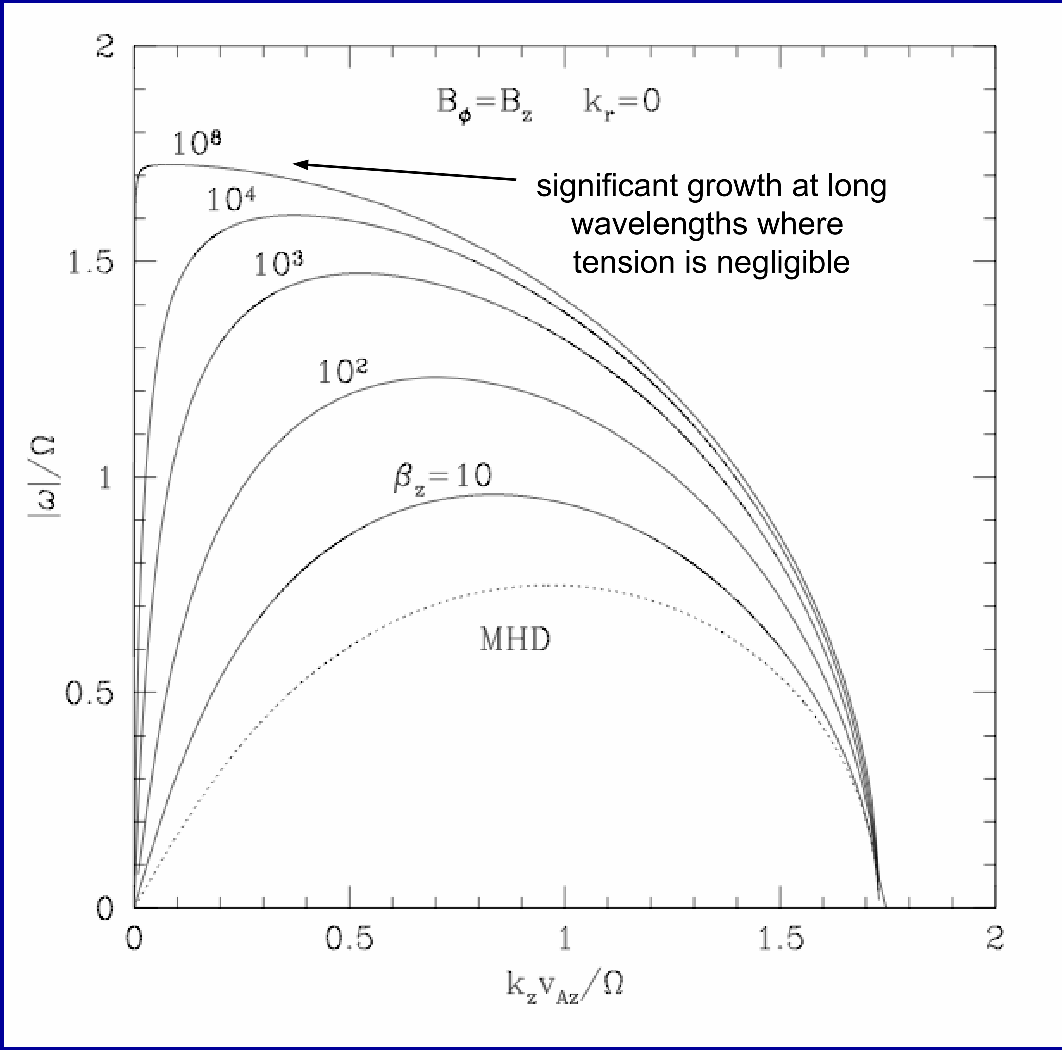
For electrons with $p_\perp > p_\parallel$ electron whistler instability will isotropize: S=0.13, α= 0.55 ($\gamma/\Omega = 5 \times 10^{-8}$) *[using WHAMP code]*

[Kasper et al. 2003, Gary & coworkers]

Examples from Space Physics

- Solar wind at 1 AU statistically at firehose instability threshold *[Kasper et al., Wind]*
- Magnetic Holes in SW & magnetopause, a signature of mirror modes *[Winterhalter et al.,Ulysses]*
- Mirror mode signatures at Heliopause, *[Liu et al.,Voyager1]*
- Above can be interpreted from μ conservation in expanding/compressing plasmas
- Small-scale instabilities driven by pressure anisotropy mediate shock transition in collisionless plasmas
- SW an excellent laboratory for collisionless plasma physics
- Since much of astrophysical plasma (except in stars) is collisionless, a lot of applications in astrophysics; e.g., X-ray clusters, accretion disks, collisionless shocks.

The MRI in a Collisionless Plasma

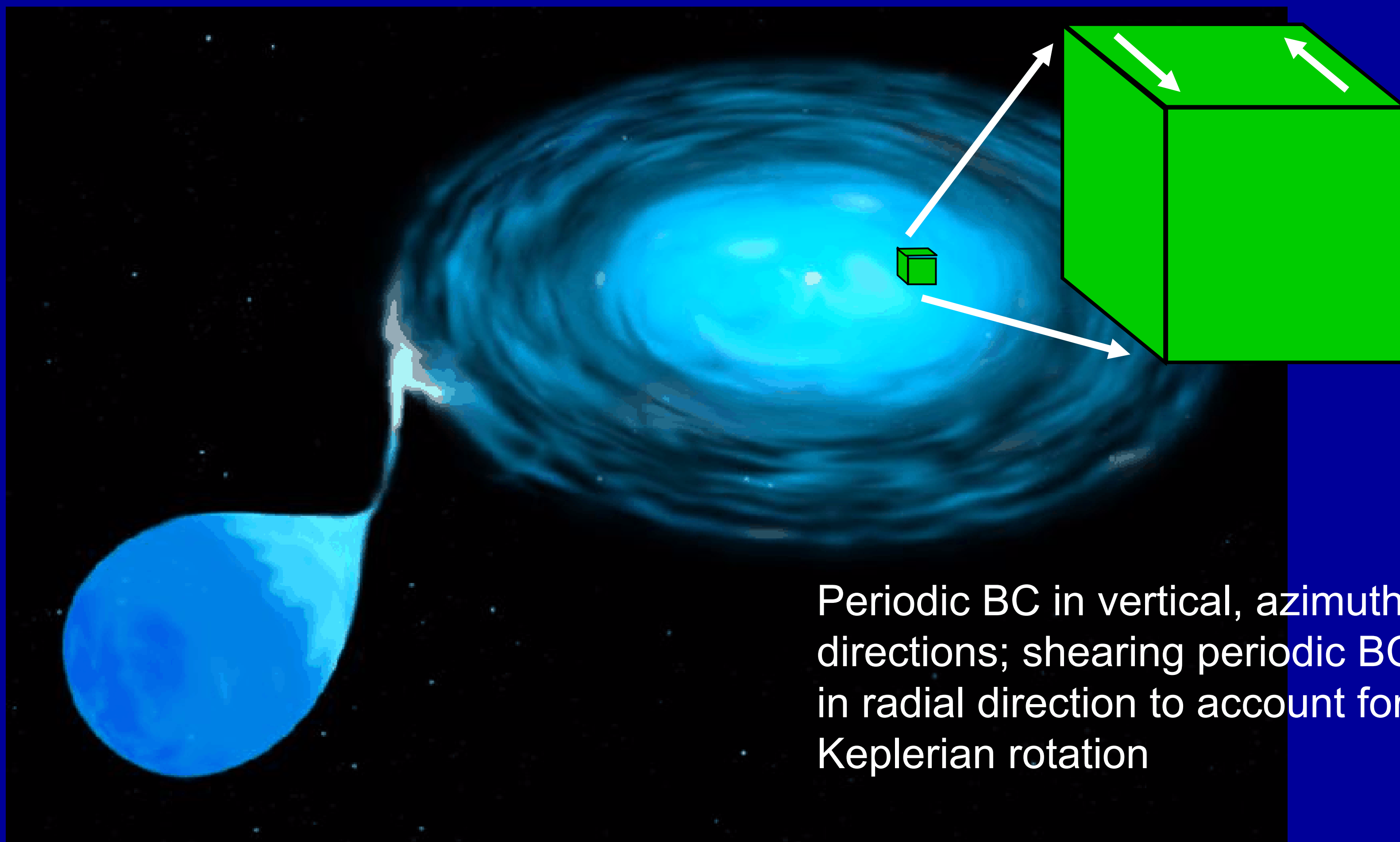


Quataert, Dorland, Hammett 2002; also Sharma et al. 2003; Balbus 2004

angular momentum transport
via anisotropic pressure (viscosity!)
in addition to magnetic stresses

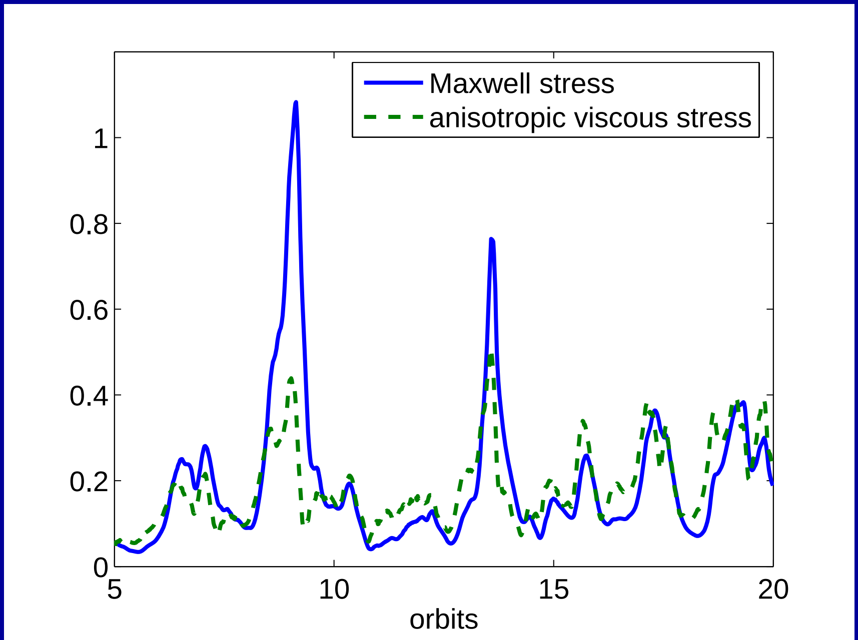
$$F_\varphi \propto \left(\frac{B_z B_\varphi}{B^2}\right)\left(\delta p_\parallel - \delta p_\perp\right)$$

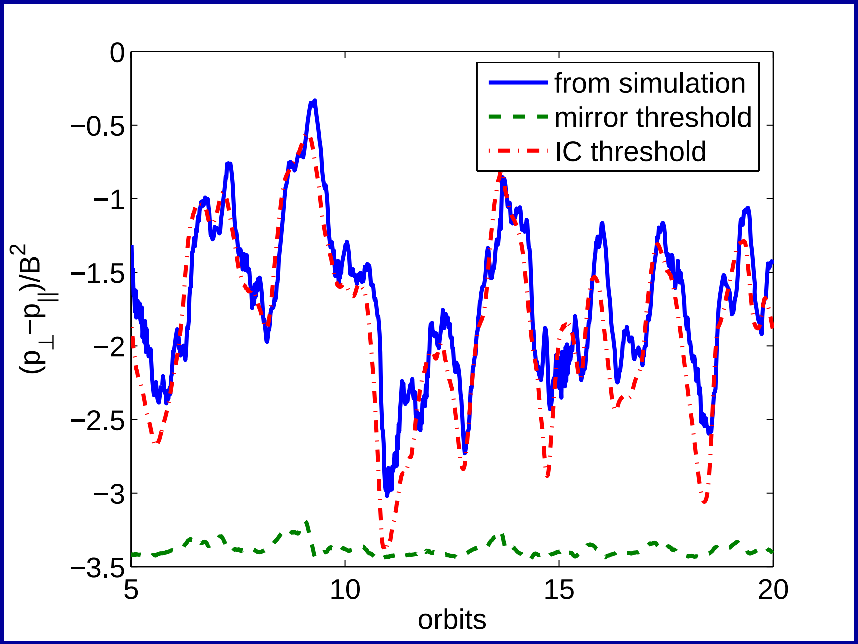
Shearing Box Simulations



Periodic BC in vertical, azimuthal directions; shearing periodic BC in radial direction to account for Keplerian rotation

Pressure Anisotropy

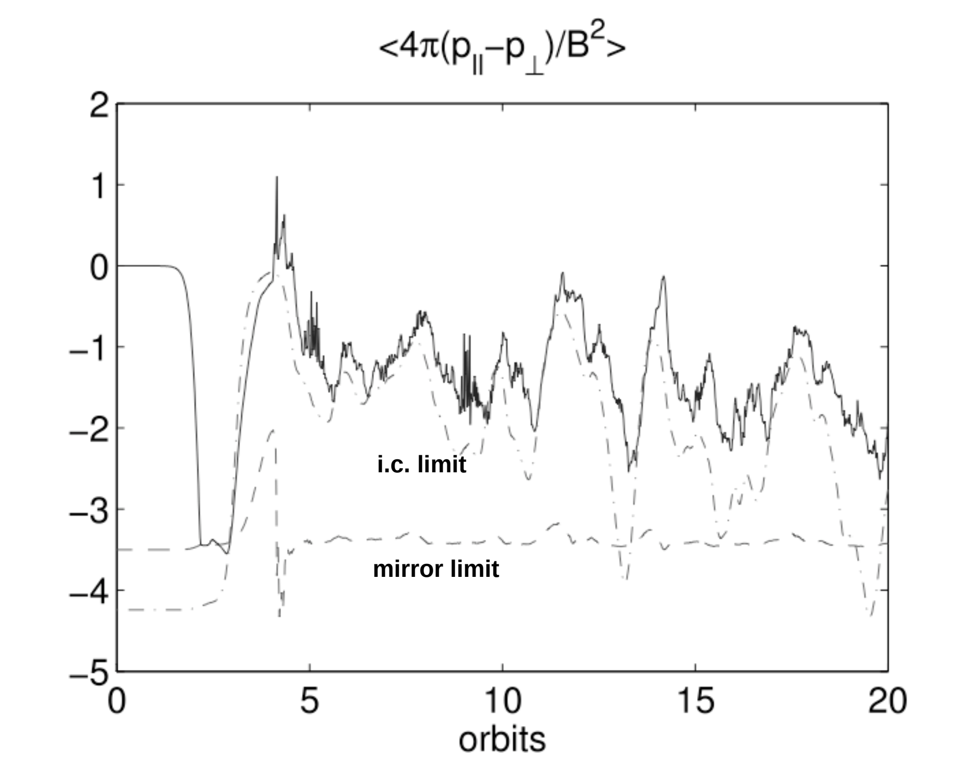




Anisotropic stress ~ Maxwell stress (can dominate at β>>1)
Anisotropic pressure => ‘viscous’ heating at large scales (heating associated with anisotropic stress)
Ion pressure anisotropy limited by IC instability threshold (with $\gamma/\Omega \sim 10^{-4}$)
Will electrons also be anisotropic? Yes, collision freq. is really tiny
Electron pressure anisotropy reduced by electron whistler instability

Local Simulations of the MRI in a Collisionless Plasma

volume-averaged pressure anisotropy



Sharma et al. 2006

Rate of Angular Momentum Transport Enhanced Relative to MHD (by factor ~ unity)

Net Anisotropic Stress (i.e, viscosity) ~ Maxwell Stress

anisotropic stress is a significant source of plasma heating

Heating by Anisotropic Stress

$$\frac{3}{2}\frac{dp}{dt} = -\left(\nabla \cdot \mathbf{P}\right) : \nabla \vec{\mathrm{v}}$$

Pressure tensor heating

$$= -\frac{p_{\|} - p_{\perp}}{B^2} \vec{B}\vec{B} : \nabla \vec{\mathrm{v}}$$

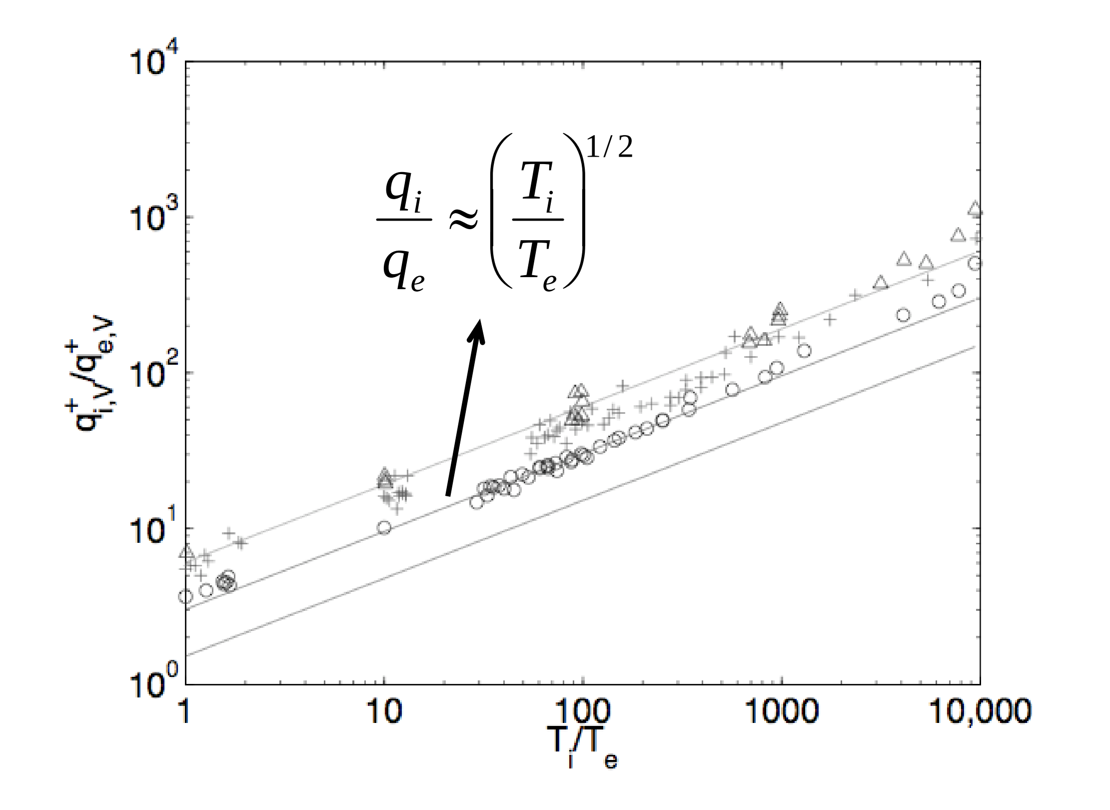
$$\propto \frac{\sqrt{p}}{B} \vec{B}\vec{B} : \nabla \vec{\mathrm{v}}$$

Anisotropy limit set by
Velocity-space instabilities

$$\frac{dT_e}{dt} \propto \sqrt{T_e}$$

Even if electrons start cold, they will be rapidly heated to a temperature independent of initial conditions, becoming comparable to ion temperature

Heating by Anisotropic Stress



Sharma et al. 2007

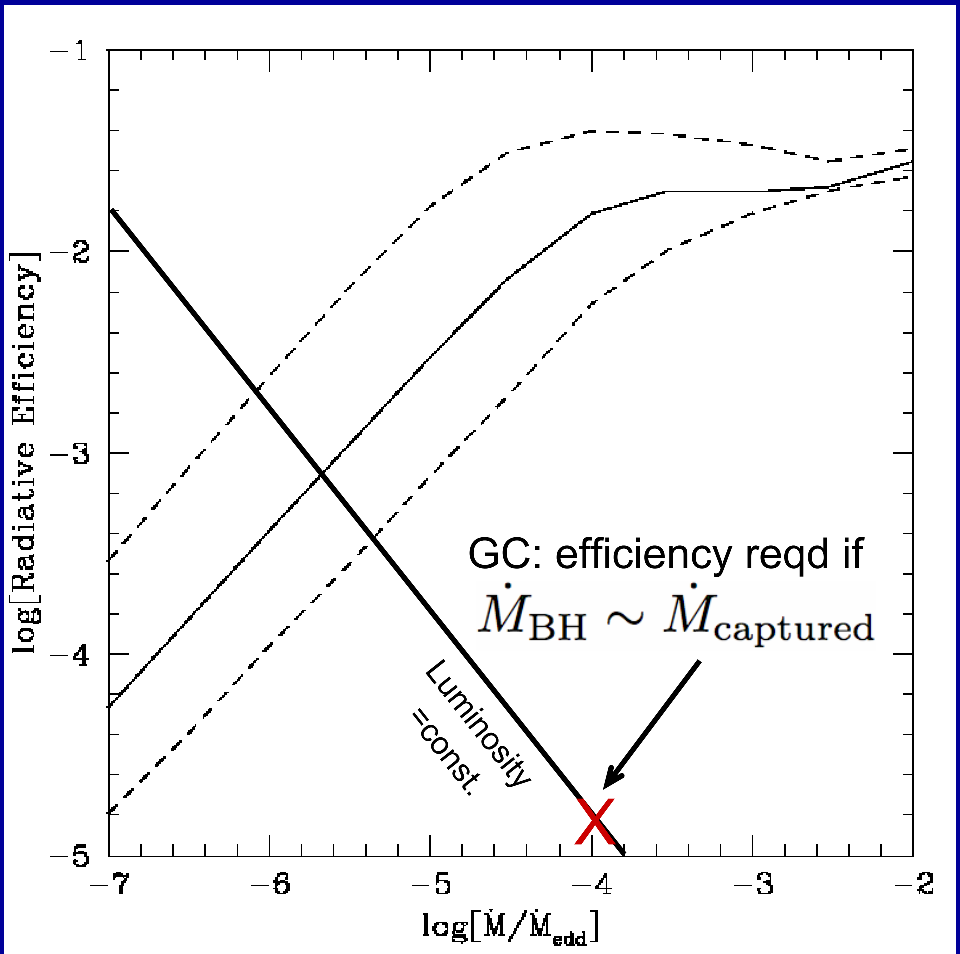
$$q_s \propto P_{r\phi} \frac{d\Omega}{d\ln r} \propto \Delta p_s$$

ion cycl. & e- whistler instability thresholds

$$\frac{\Delta p_s}{p_s} \sim \beta_s^{-1/2}$$

$$\rightarrow \Delta p_s \propto T_s^{1/2}$$

Astrophysical Implications



Sharma et al. 2007

'viscous' heating mediated by high freq. instabilities crucial source of electron heating in hot accretion flows

$\Rightarrow$ **low accretion rate** required to explain the low luminosity of most accreting BHs

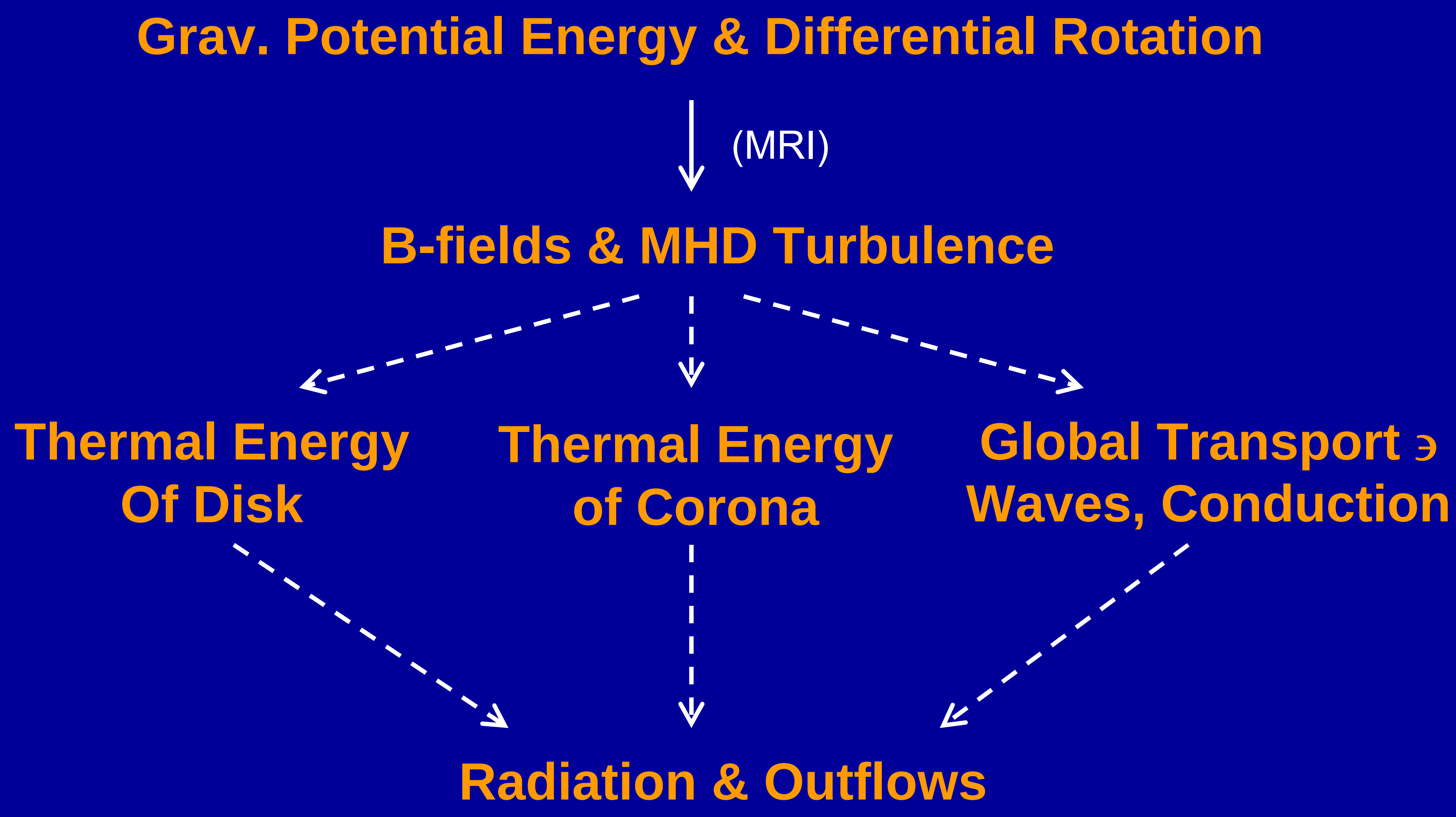
consistent w/ inferences from global MHD sims

Summary

- Disk Dynamics Determined by Angular Momentum & Energy Transport
- Angular momentum transport via MHD turbulence initiated by the MRI
- Thick disks; radiatively inefficient; gravitational energy stored as heat
  - T ~ GeV; macroscopically collisionless; relevant to low-luminosity BHs/NSs
- Kinetic Theory of Accretion Flows (Thick Disks)
  - Anisotropic stress ~ Maxwell Stress
    - Pitch angle scattering by velocity-space instabilities crucial to limit anisotropy
  - Significant electron heating via anisotropic stress (‘viscosity’)
    - $\Rightarrow$ large radiative efficiency; low accretion rates (<< captured at large radii) required to explain the low luminosities of many accreting BHs

LAST SLIDE

Energy Flow in Disks



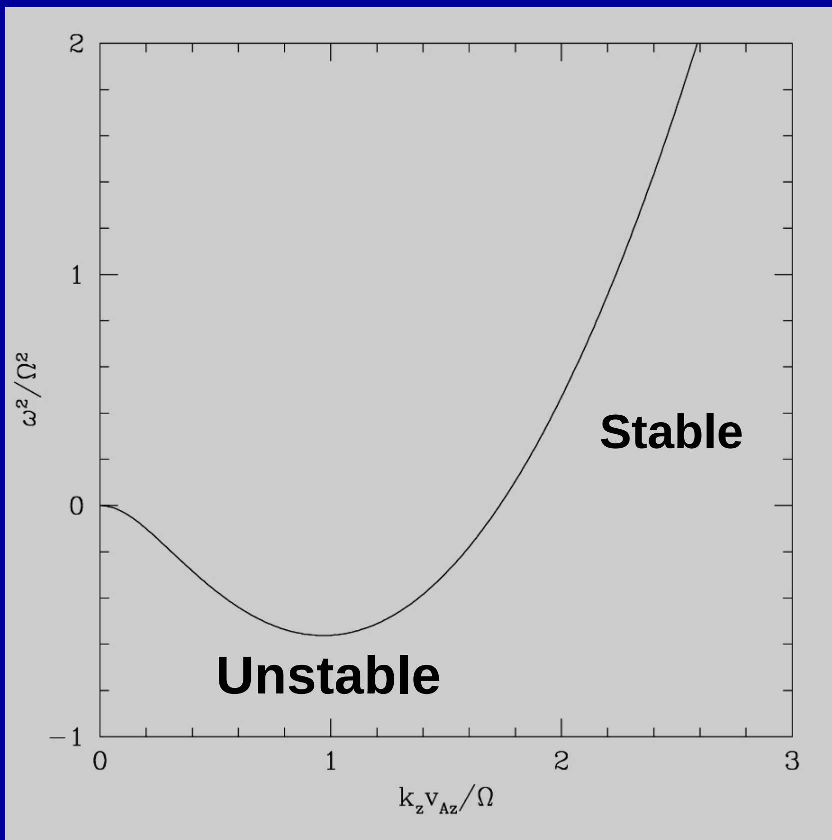
Details of Energy Flow Determine Dynamics/Structure of Disk

Angular Momentum Transport in Disks

- Accretion requires angular momentum transport
  - accretion proceeds on $t_{inflow} \sim t_{vis} \sim r^2/\nu$ (viscous diffusion time)
- In thin disks, inter-particle collisions are insufficient
  - mfp << H $\Rightarrow$ t_{vis} >> observed accretion times ($\nu \sim$ mfp*v_{th})
- Enhanced Transport due to “Turbulence” invoked for ~ 30 years
- Disks appear hydrodynamically stable
  - Stable by Rayleigh criterion: $\Omega \propto r^{-3/2}$ & $l \propto r^{1/2}$
  - 3D nonlinear simulations suggest disks are nonlinearly stable; see also Ji’s talk on experiments

Linear Instability

Frequency^2

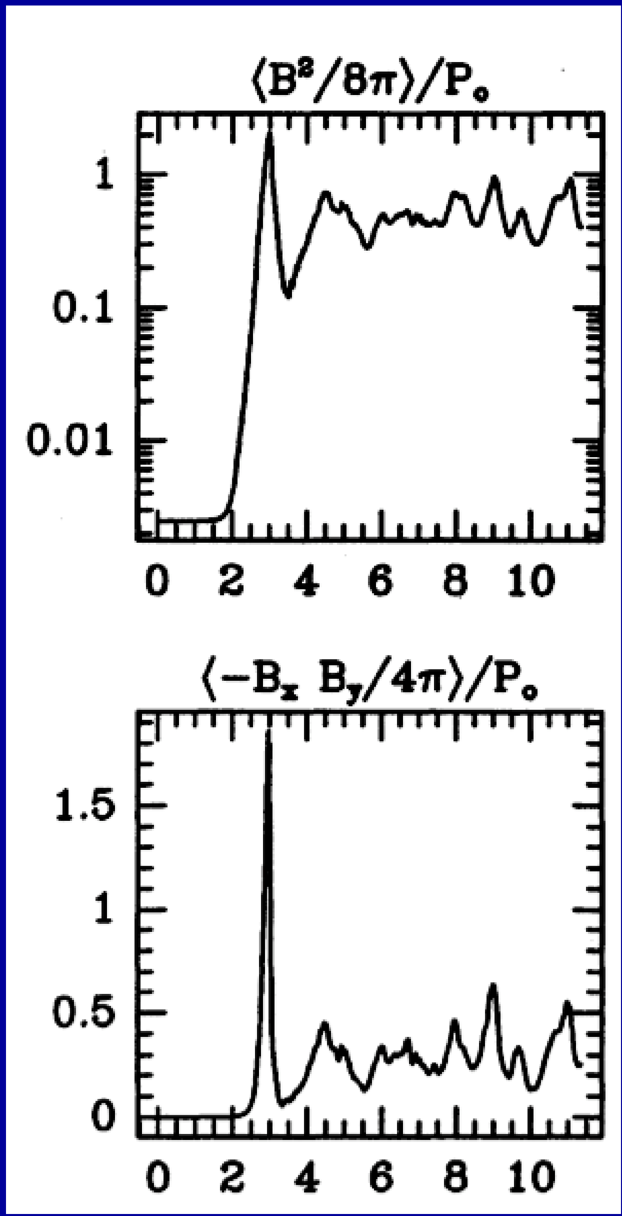


$k_z v_A/\Omega$ (Alfven Freq/Disk Freq)

Local Simulations

Magnetic Energy

Magnetic Stress



Orbits