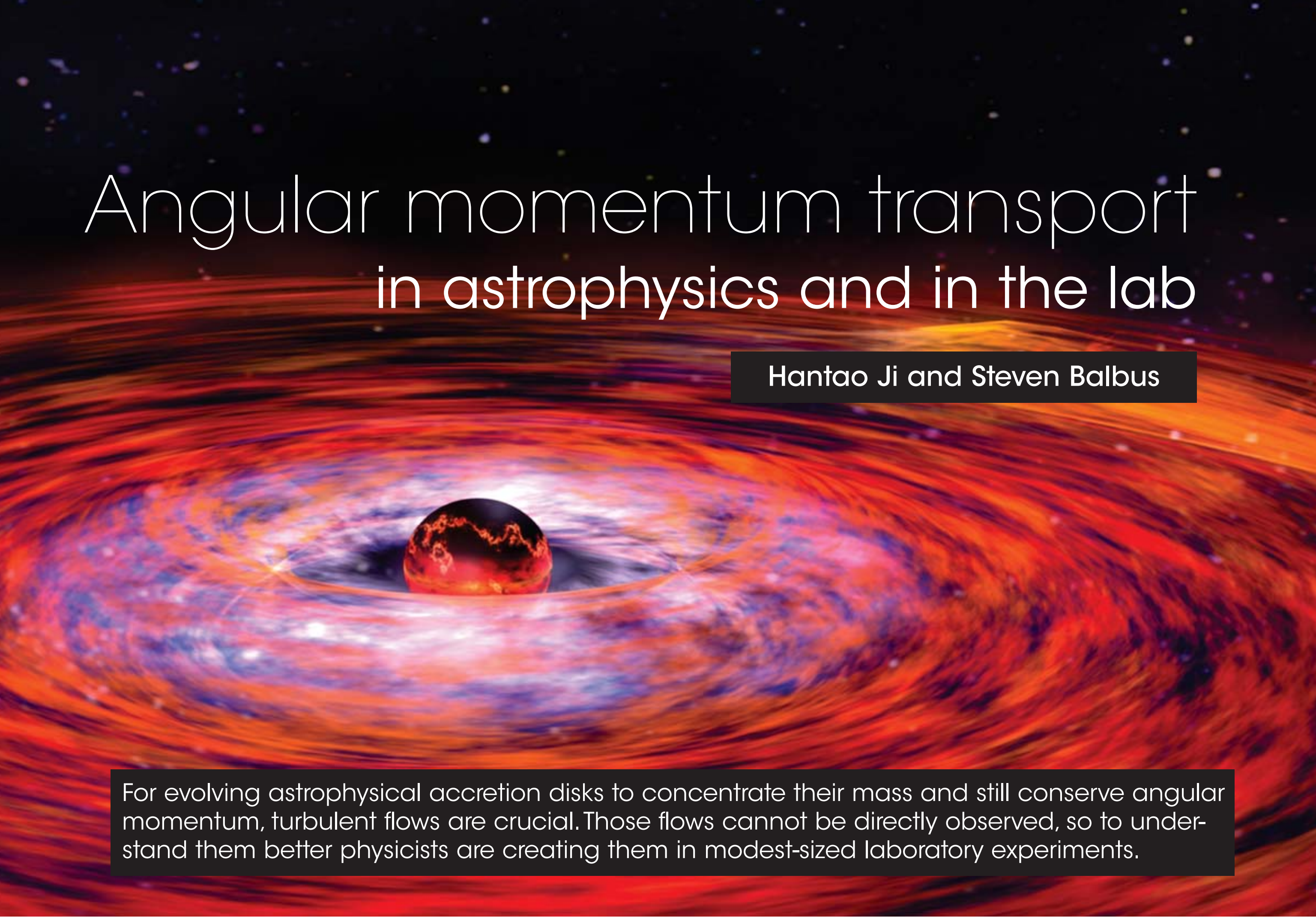
Angular momentum transport in astrophysics and in the lab

Hantao Ji and Steven Balbus

For evolving astrophysical accretion disks to concentrate their mass and still conserve angular momentum, turbulent flows are crucial. Those flows cannot be directly observed, so to understand them better physicists are creating them in modest-sized laboratory experiments.



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The universe is full of galaxies, stars, and planets: gravitationally bound, compact structures that started out as well-dispersed bodies of gas and dust. The initially dilute matter contains at least some angular momentum, and were it to gravitationally contract without loss of that angular momentum, the effective $1/r^2$ repulsive potential that arises from the centrifugal term in the equation of motion would eventually become very large. Thus even a small amount of initial angular momentum would strongly restrict any contraction. How then is angular momentum removed (transported radially outward) so that contraction can proceed over many orders of magnitude? That, in a nutshell, is the classical angular momentum problem of astrophysics—a puzzle that endured for many decades.

Typically, the contraction proceeds until a reservoir of angular momentum accumulates. Since it is much easier to lose energy by radiation than to lose angular momentum, the reservoir takes the form of a thin, rotationally supported disk. That "accretion disk" is so named because a parcel of fluid in the disk will somehow shed its angular momentum with time and slowly spiral inward, causing the central region of the disk to gain mass.

Nowadays the astrophysical consensus is that angular momentum transport in accretion disks is mediated by magnetic fields. Theoretically, even a weak magnetic field can destabilize the orbital gas motion and render the flow turbulent via the so-called magnetorotational instability (MRI). Under those conditions, angular momentum is efficiently transported outward through the disk as matter spirals in, energy dissipates, and the disk radiates. Numerical simulations of accretion disks routinely show that process.

The MRI, to be discussed in detail below, is a robust prediction of magnetohydrodynamic (MHD) models that resolves a longstanding problem. But there can be no substitute for direct observation, particularly when turbulence is involved. Therefore, physicists are working intently to create a laboratory environment in which they can study the onset and development of the MRI in a real system and investigate the nonlinear hydrodynamical stability of disk-like flows. Such controlled experiments promise to provide valuable physical insights on the behavior of rotating fluids—both magnetized and unmagnetized.

Accretion disks are important to the process of star formation. Protostellar accretion disks are often sufficiently large that they can be directly imaged by

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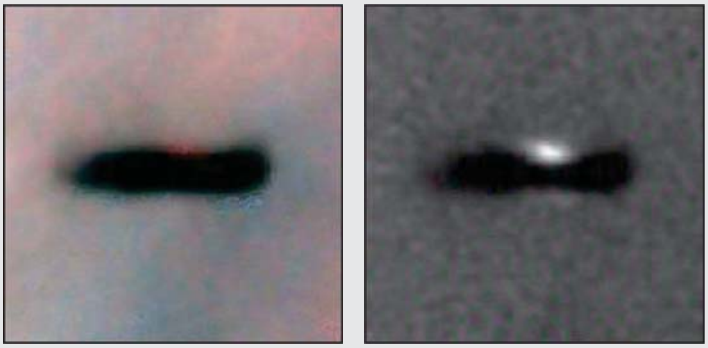


Figure 1. A protostellar disk in the Orion Nebula, viewed edge-on by the *Hubble Space Telescope*. The left image was generated with a camera that is sensitive to radiative emission lines from the gaseous disk; the right image was taken with a filter that blocks those lines. With the disk gas less prominent, reflected light from the central star is clearly visible. (Courtesy of Mark McCaughrean, C. Robert O'Dell, and NASA.)

high-angular-resolution instruments such as the *Hubble Space Telescope*. Figure 1, for example, shows both the disk and, via scattered light, the embedded forming star. Disks also form by mechanisms other than gravitational collapse. Indeed, the modern era of accretion-disk theory began in the 1960s with detailed investigation of mass transfer from one star to another in close binary star systems, environments in which the disks play an important evolutionary role.[1]

Keplerian disks

Disks around compact objects—white dwarfs, neutron stars, and black holes—are of particular interest. They are often extremely luminous; indeed, manifested as quasars at the centers of massive galaxies, they are some of the brightest objects in the universe. The dual requirements of copious dissipational heating and outward angular momentum transport suggest that the gas in the disks is highly turbulent.

In the 1970s Nikolai Shakura and Rashid Sunyaev developed a phenomenology in which they simply assumed that some sort of anomalous turbulent stress was present from some undetermined cause that extracted angular momentum from the fluid elements and transported it outward.[2] To many, their scenario was quite plausible. The disks under consideration are supported almost entirely by rotational forces. As a consequence, the rotating gas follows a law similar to Kepler's law for planetary motion: Its speed is proportional to the reciprocal square root of the distance from the center. The gas in those so-called Keplerian disks experiences strong shear, and shear is often associated with the onset of turbulence in terrestrial flows.

Under some conditions, the forces of turbulent stress have properties in common with enhanced viscosity and may be treated as such. Assuming that the characteristic scale of turbulence in a radially extended disk is the thickness H, one can separate the scale of the turbulence from the much larger scale of the local disk radius R and regard the turbulence as local. By then averaging over intermediate scales that are large compared with H but small compared with R, one might be able to formulate a sensible viscous model for turbulent accretion disks.

The magnetorotational instability

The onset of turbulence is a notoriously contentious subject—a mixture of mathematical muscle, laboratory experimentation, and physical reasoning. In fact, that contentiousness at least partially motivates this article. For the case in which gas rotates about a central point mass in near Keplerian orbits, the Coriolis force is strongly stabilizing and offsets the destabilization associated with shear flow. Local linear-stability calculations with a simple equation of state for the gas indicate propagating waves rather than instability. The notion that differential rotation (that is, with nonconstant angular velocity) in Keplerian disks breaks down from laminar to turbulent flow is not supported by any sort of straightforward or strongly compelling mathematical argument or computational demonstration.

An important advance came in 1991 when one of us (Balbus) and John Hawley made the simple point that since magnetic fields are pervasive, any modestly ionized accretion disk was likely to be at least weakly magnetized.[3] By including a weak magnetic field in the local stability analysis (here, "weak" means that the magnetic field has little effect on the equilibrium rotation), Balbus and Hawley showed that disks are subject to a powerful disruptive instability whenever the angular velocity decreases with increasing radius. Thus differentially rotating MHD fluids are strongly unstable, whereas no such behavior attends a field-free hydrodynamic (HD) fluid. The instability is generally known as the magnetorotational instability, or MRI.

But that was not the first look at the stability of rotating magnetized fluids. Indeed, in his classic 1961 text *Hydrodynamic and Hydromagnetic Stability* (Clarendon Press), Subrahmanyan Chandrasekhar discussed his own work and an earlier calculation by noted Russian physicist Evgeny Velikhov, who considered a cylindrical, liquid-metal system much like those being used today to shed light on the physics of accretion disks. Unfortunately, Chandrasekhar carried out a rather imposing global analysis on a system governed by a very local instability, and he mistakenly dropped an important term in an extension of the work that was meant to include dissipational physics.[4] Thus the significance of the MRI went unappreciated for decades.

Nowadays, even though the importance of the MRI is well recognized, astrophysicists remain interested in HD processes. Protostellar disks are accreting systems, but they are large, and thus heat slowly from shear-driven dissipation; cool, and thus have little thermal ionization; and dusty, and thus trap any residual charge carriers on grains. How the MRI might or might not work in protostellar disks is an active area of research on star formation, and answering the question of whether hydrodynamical shear by itself leads to turbulence is pivotal to an understanding of those important objects. The question has motivated intense efforts to study astrophysically relevant flows in the laboratory. We will turn to some of those experiments shortly, but first we review some key elements of HD and MHD stability analyses.

Go with the flow

The online version of this article presents a simple model describing a magnetic field–free disk whose fluid elements execute circular motion in equilibrium. In seeking to understand whether the HD flow is stable or unstable, the most direct route is to study the behavior of small excursions from circular motion in a frame rotating with the equilibrium angular velocity of a particular parcel of fluid. In such a rotating frame, Coriolis and centrifugal terms must be included in the analysis. The former is a stabilizing influence, the latter destabilizing. For excursions within the orbital plane, a straightforward normal mode analysis[5] yields purely oscillatory solutions for the displacement, with an associated "epicyclic" angular frequency κ given by

$$\kappa^2 = \frac{2\Omega}{R}\frac{d(R^2\Omega)}{dR}. \qquad (1)$$

Here, Ω is the equilibrium angular velocity at cylindrical radius R. If the angular momentum per unit mass, $R^2\Omega$, increases with radius, κ is real and a perturbed fluid element oscillates stably about its equilibrium circular orbit. By contrast, if $R^2\Omega$ decreases with radius, the fluid element's displacement grows exponentially and the disk is unstable, a result first obtained by Lord Rayleigh in 1916. In the laboratory, Rayleigh unstable flows can become turbulent. But for astrophysical disks, $R^2\Omega$ increases with radius; they are not unstable by Rayleigh's criterion. The increasing angular momentum profile is, in effect, the core of the angular momentum problem.

The failure to satisfy the Rayleigh criterion for instability is not in itself a guarantee of stability. For one thing, the Rayleigh criterion strictly applies only to axisymmetric disk perturbations. In fact, for an arbitrary angular velocity profile, no one has come up with a general criterion that will guarantee stability under small displacements. Moreover, the theoretical questions surrounding the stability of finite amplitude perturbations are very difficult. For those reasons, numerical investigation and laboratory experiments are crucial to our understanding of disk physics. The central question is whether there exist nonlinear, nonaxisymmetric perturbations that destabilize a Keplerian rotation profile.

Magnetic fields as springs

The stability analysis of an accretion disk changes markedly in the presence of a magnetic field—even a weak one. In a magnetized fluid, the Lorentz force term has two distinct components: a piece that acts like a pressure gradient and one that acts like a tension. In disk systems, the tension term is more important and, as its name implies, gives rise to a Hooke's law acceleration linearly proportional to the displacement of the fluid but oppositely directed. Ostensibly it provides a stabilizing influence, but as the box on this page shows, that is not always the case.

The form of the magnetic tension results from a fundamental property of any fluid that is an excellent conductor of electricity, as even modestly ionized gases and plasmas are. In a perfectly conducting fluid, magnetic field lines behave as though they were painted onto or frozen into the fluid. That result follows straightforwardly from Faraday's law of induction and the vanishing of the electric field in the local rest frame of the moving fluid.[5] Consider a stationary, uniformly magnetized and homogeneous medium of density ρ, and suppose that the magnetic field $\mathbf{B}$ points along the z axis. If the fluid is perturbed and the displacement field $\boldsymbol{\xi}$ has the plane-wave form $\exp(ikz - i\omega t)$ with $\boldsymbol{\xi}\cdot\mathbf{B} = 0$, then waves will propagate along the magnetic field lines and satisfy the dispersion relation

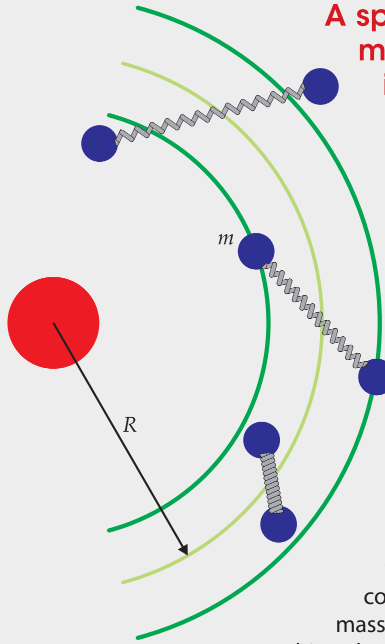
$$\omega^2 = \frac{k^2B^2}{\rho\mu_0} \equiv k^2 v_A^2, \qquad (2)$$

where μ_0 is the usual free-space permeability.

The waves described by $\boldsymbol{\xi}$ are analogous to waves along a string. They propagate at the so-called Alfvén velocity $v_A = B/\sqrt{\rho\mu_0}$, named after the Swedish physicist who first analyzed them. Note the simple but important physical point that even if v_A is small, the wave frequency can be large if k is sufficiently big—that is, if the wavelength is small enough. The quantity kv_A is known as the Alfvén frequency, ω_A.

The crucial step for obtaining the MRI is to combine the two above scenarios and consider a Keplerian disk threaded by a uniform magnetic field parallel to the axis of rotation. One might guess that the resulting dispersion relation would simply produce a frequency response in which κ and ω_A added

A spring model of the magnetorotational instability



The figure to the left shows the evolution (from bottom to top) of two fluid elements (blue) originally orbiting a central element (red) at a radius R, but somehow forced to split off from that orbit. If the fluid elements are threaded by a magnetic field perpendicular to the page, they will be attracted to each other by a restoring force proportional to their mutual displacement. It is as though the two elements were masses connected by a spring. Since the masses are on slightly different orbits, the attractive force will generate torques on them. The inner mass, rotating a bit more rapidly, will be pulled back by the spring, lose angular momentum, and drop to a smaller orbital radius. The outer mass will be sped up, acquire angular momentum, and move outward. The spring will stretch farther—more precisely, the magnetic tension will rise—the torque will increase, and the masses will continue to separate at a yet faster rate. The evolution is a runaway process as long as the spring is not too strong.

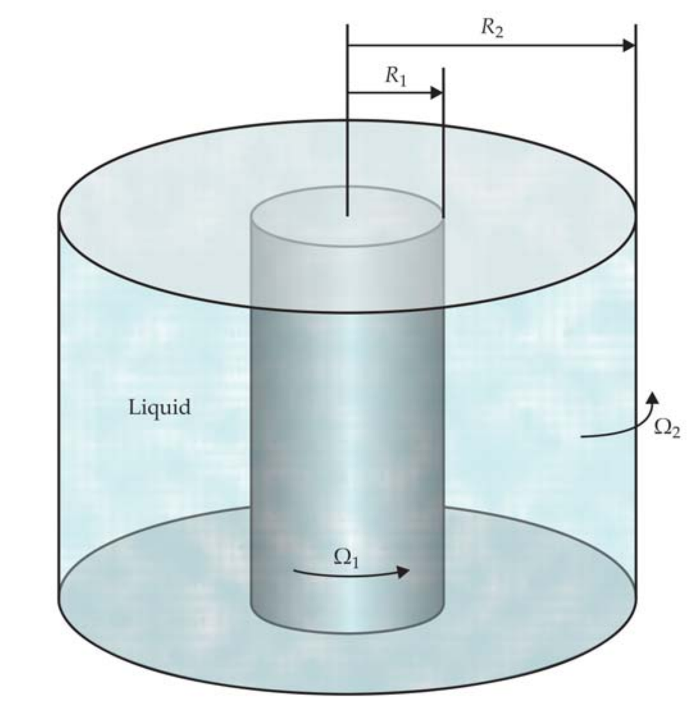


Figure 2. Taylor–Couette flow. Pictured here is a Taylor–Couette cell, in which differentially rotating flow is driven between two concentric cylinders—an inner cylinder that rotates at angular velocity Ω_1 and an outer one rotating at Ω_2. Using either water or liquid metal as a medium and choosing $\Omega_1 > \Omega_2$ and $R_1^2\Omega_1 < R_2^2\Omega_2$, one can set up the conditions relevant for studying the rotational dynamics of accretion disks. The end caps on the top and bottom of the device can profoundly affect the global stability of such flows, but as described in the main text, it is possible to control or minimize that undesirable effect.

in quadrature. In fact, the dispersion relation is much more complicated. For large k, the result is[5]

$$\omega^4 - (\kappa^2 + 2k^2 v_A^2)\,\omega^2 + k^2 v_A^2 \left(k^2 v_A^2 + \frac{d\Omega^2}{d\ln R}\right) = 0. \quad (3)$$

The surprising consequence is that if

$$\frac{d\Omega^2}{dR} < 0, \quad (4)$$

then for wavenumbers satisfying $k^2v_A^2 + d\Omega^2/d(\ln R) < 0$, the disk is unstable. (In contrast, for the unmagnetized fluid, the stability condition is determined by the gradient of angular momentum.) When the inequality holds, the four roots of equation 3 describe two ordinary waves with $\omega^2 > 0$ and two solutions with $\omega^2 < 0$—a damped and an exponentially growing mode. In general, perturbations will be a superposition of those four modes; after just a few rotations of the disk, they will be dominated by the exponentially growing mode. Note that the criterion for instability is most easily met when the magnetic field is weak, not strong.

Because any accretion disk, not just a Keplerian disk, will almost certainly be rotating more rapidly in its interior regions than in its exterior regions, any magnetized disk—even a weakly magnetized one—is dynamically unstable. The reader may enjoy the exercise of beginning with equation 3 and showing that for a Keplerian profile (that is, $\Omega^2 \propto 1/R^3$), $\kappa = \Omega$; the maximum growth rate is $3\Omega/4$, independent of the magnetic field strength; and the maximum growth rate occurs for $k^2 = k_{max}^2 = 15\Omega^2/(16v_A^2)$. In general, the maximum growth rate is $(1/2)|d\Omega/d(\ln R)|$.

Notice that the magnetic field appears only in the combination kv_A. Thus, even if the field is small, the destabilizing magnetic tension force could still be substantial if k is large enough. In reality, when the magnetic field becomes vanishingly weak, k_{max} is so large that the assumption of perfect electrical conductivity breaks down and the would-be unstable mode is, in fact, resistively damped. Larger wavelengths continue to correspond to unstable modes, but the growth times get ever longer as the magnetic field tends to zero.

The MRI is insensitive to the initial field geometry; indeed, it results even when the starting field is purely azimuthal. The instability is easy to demonstrate numerically, and it unquestionably leads to MHD turbulence and enhanced outward angular momentum transport, as observations demand. But calculating the MRI in its fully turbulent manifestation is very challenging, and creating the phenomenon in the laboratory is at least an equal challenge. Still, despite the inherent difficulties, laboratory studies of the MRI would be of great interest. The relevant astronomical observations are at best indirect and not at all straightforward to interpret.

Life at super-high Reynolds number

Interesting astrophysical questions related to accretion disks can be addressed in the lab, even absent a magnetic field. We noted earlier that protostellar disks pose a stern challenge to MRI-based theories of accretion disk turbulence and that an understanding of the effects of HD shear is crucial to an understanding of those astrophysically important objects. Indeed, protostellar disks include low-ionization regions thought to be "MRI-dead."[6] As it happens, the dead zone coincides with the region of the planets in our own solar system, and so the question of HD stability is a practical one.

Early ideas on accretion disk turbulence centered on the notion that at sufficiently high Reynolds number, nonlinear instabilities in any shear flow, including Keplerian rotation, would drive turbulence. (The dimensionless *Re* is a measure of the importance of inertial forces relative to viscous forces; high *Re* means viscosity is less important.)

There is a profound gap between *Re* in a protostellar disk, which might be as high as 10^{14}, and the highest accessible laboratory *Re* of 10^7 or so. But even if experiment could establish with certainty only that stability prevails through the accessible range of *Re*, that would be reasonably compelling evidence for the astrophysical stability of hydrodynamical Keplerian shear. Although surprises are always possible, it is difficult to see what features of simple homogeneous shear would require viscous effects to be less than a part in several million before a fluid flow abruptly breaks down into turbulence. The experimental quest has led numerous groups to

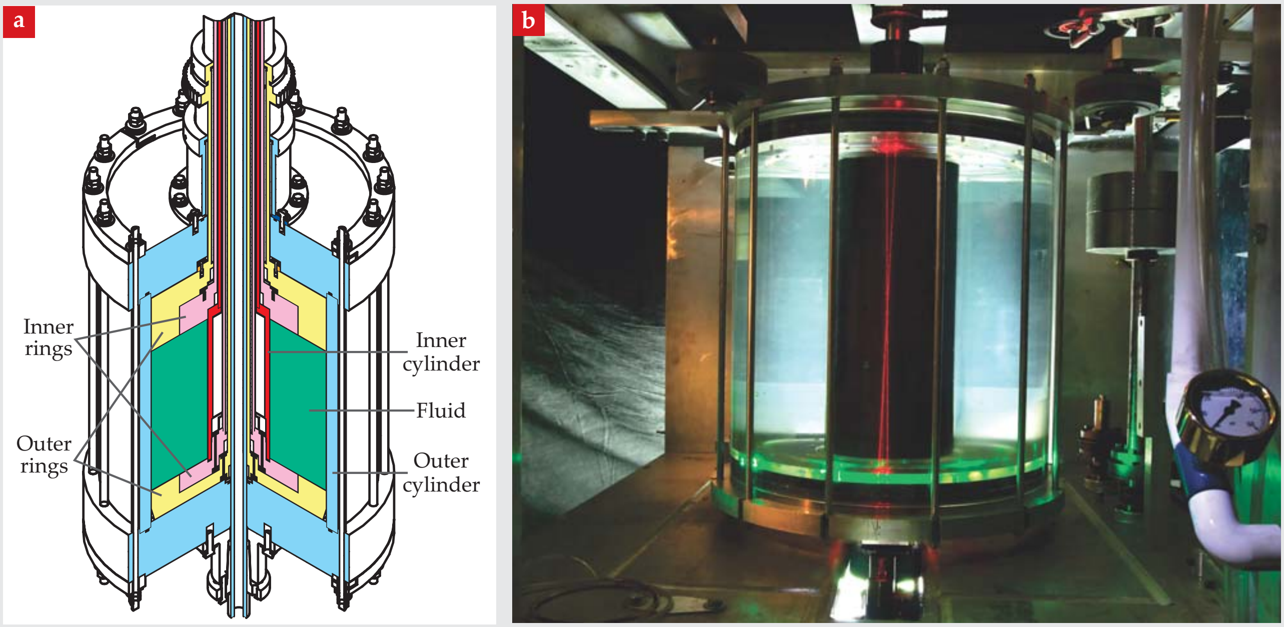


Figure 3. The Princeton magnetorotational instability (MRI) experiments, which use laser Doppler velocimetry to measure local velocities in a fluid. **(a)** This schematic shows the device that, in a 2006 experiment, saw no sign of turbulence in astrophysically relevant flows; illustrated here are the inner and outer cylinders, confined fluid, and a pair of end caps each divided into inner and outer rings. The opposing inner- and outer-ring pairs may be driven at different angular speeds to minimize the complicating Ekman effect discussed in the text. The device is now being used with a liquid gallium mixture to detect the MRI. (Adapted from H. Ji et al., ref. 8.) **(b)** The new Princeton cell shown here uses single, independently rotatable rings at each end. Nozzles on the inner cylinder allow experimenters investigating the onset of hydrodynamic turbulence to introduce controlled perturbations. Both the cells depicted in this figure are about 40 cm in diameter. (Photograph courtesy of Eric Edlund and Elle Starkman.)

the laboratory study of what are called Taylor–Couette flows (see figure 2) in both HD and magnetized fluids.

Hydrodynamic experiments

Controlled laboratory experiments on rotating flows began in 1890, when Maurice Couette measured the viscosity of water in a flow between two concentric cylinders. More than 30 years later, G. I. Taylor analyzed the stability of such flows, and he did precise experiments to confirm his calculations. But only two early investigations, by F. Wendt in 1933 and Taylor in 1936, dealt with centrifugally stable flows with $d(R^2\Omega)/dR > 0$. Unfortunately, those investigations were also in the regime $d\Omega/dR > 0$ and are thus unsuitable for accretion disks. The proposed turbulent disk models[7] based on those works, however, raise an interesting question: Can laboratory turbulence be generated in other relevant astrophysical profiles—for example, quasi-Keplerian flows, for which $d\Omega/dR < 0$ and $d(R^2\Omega)/dR > 0$?

The existence of so-called subcritical transitions, which lead to turbulence in such linearly stable systems as pipe flows, is well established at *Re* of several thousand. Astrophysical flows are characterized by enormous *Re*, but the question remains: Does the dominant rotational background of disk flow stabilize the subcritical transition or is a transition to turbulence still present? If it is present, is the generated turbulence enough to explain the observed accretion rates?

Laboratory experiments designed to address those astrophysical questions have thus far generated conflicting results. Local stress measurements[8] using laser Doppler velocimetry at the Princeton MRI device find no sign of turbulence in quasi-Keplerian flows of water at *Re* up to 2×10^6. The measured transport efficiency is significantly below the requirements dictated by observed disk accretion rates. The conjectured nonlinear instability does not occur at all, occurs at undetectable levels, or requires *Re* in excess of 2×10^6. Even more telling, in a follow-up new experiment, the Princeton group has found that the quasi-Keplerian flows are robustly stable against externally controlled perturbations.

The results of a similarly motivated experiment conducted at the University of Maryland sharply contrast with the Princeton findings. Global torque measurements on the inner cylinder of the Maryland experiment detected large transport levels in quasi-Keplerian flows at similar *Re* to those in the Princeton experiment.[9] Extrapolated to astrophysical parameters, the Maryland results would lead one to conclude that HD turbulence alone is sufficient to explain accretion in astrophysical disks.

The incompatibility of the two observations has naturally led to close examination of the setup and methodology of the two experiments. Not only do they use different diagnostics, they deal with significantly different geometries and axial boundary conditions. In particular, the cylinder end caps, unavoidable in a laboratory setting, are obviously absent in astrophysical disks. Yet they can have profound effects, not just in the vicinity of the caps but throughout the bulk of the flow; they drive a secondary flow known as Ekman circulation and can thus influence global stability. To control that effect,

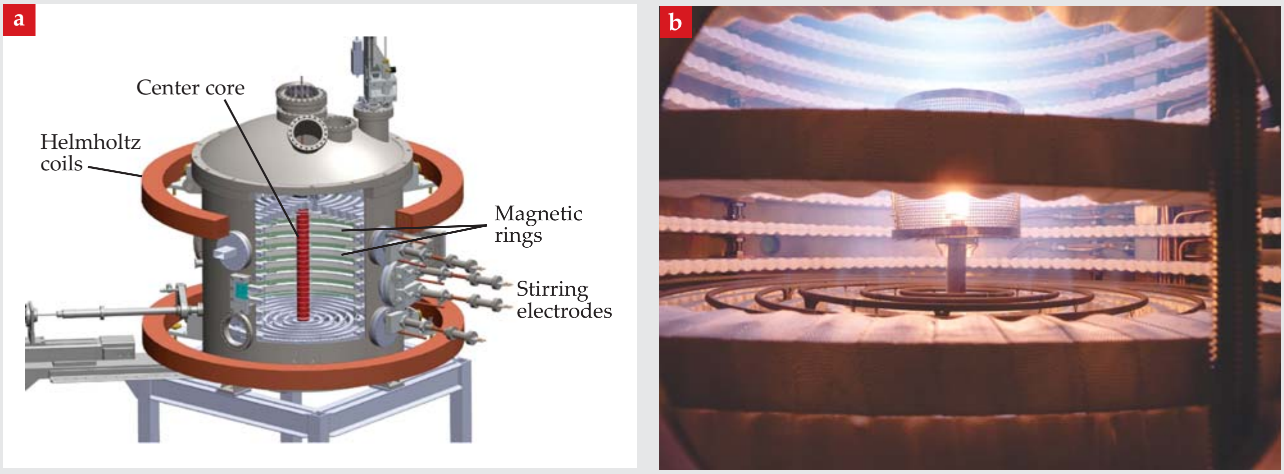


Figure 4. The University of Wisconsin plasma Couette experiment. (a) The magnetic rings shown in this schematic are responsible for confining the plasma. The edge of the plasma is stirred by applying an electric field across the magnetic field; viscous forces then ensure that the entire plasma flows. (Adapted from ref. 18.) **(b)** This photograph shows the glowing plasma, which is made by hot electrons emitted from the central core. The containment vessel is about 1 m in diameter. (Courtesy of Cami Collins and Cary Forest.)

in the relatively compact Princeton cell shown in figure 3a, each end cap is split into two rings, and each ring is driven separately at its own angular speed, which is generally different from those of the inner and outer cylinders. By choosing appropriate speeds for the rings, one can accurately restore Taylor–Couette profiles without Ekman circulation. To avoid the troublesome Ekman effects, the Maryland team took torque measurements only from the middle third of a relatively extended device.

A group from the University of Twente in the Netherlands plans to join the Princeton and Maryland teams in a three-way collaboration hoping to reach a consensus. With help from the Maryland group, torque diagnostics will be added to the Princeton group's new device, shown in figure 3b, for comparisons with laser Doppler velocimetry results. In addition, the Twente researchers will run an independent diagnostic with particle image velocimetry in a long Taylor–Couette apparatus[10] similar to that used at Maryland. Their experiment will measure directly the local stress in quasi-Keplerian flows. All involved in the collaboration are optimistic that a concordant laboratory finding will emerge that bears upon a longstanding astrophysical puzzle.

Magnetohydrodynamic experiments

Studying the MRI in the laboratory is similar in some respects to studying HD turbulence, but the details are very different. Modern MHD Taylor–Couette flow experiments were not proposed until a decade after the rediscovery of the MRI. The Princeton experiment uses a liquid gallium mixture and includes an axial magnetic field of optimal strength to destabilize an otherwise stable, quasi-Keplerian flow.[11] To date, however, the MRI has been difficult to identify unambiguously, even though the required threshold conditions have been exceeded; the predicted level of instability is comparable to the measurement errors.

Two noteworthy events have marked the path on what has been a long, at times frustrating, journey toward demonstrating the MRI in the laboratory. The first is an experiment using spherical Couette flow with a stationary outer sphere.[12] In that setup, the flow had an HD instability even before an axial magnetic field is imposed. When the magnetic field was added, the flow exhibited what seemed to be further instabilities, which were interpreted as manifestations of the MRI. But a recent numerical study suggests that the observations were actually of something called a Shercliff layer instability.[13] That type of instability, confirmed in the Princeton liquid-metal experiment, does not arise in astrophysical disks, but it could conceivably be relevant to planetary cores.

The second event was the theoretical finding, by Rainer Hollerbach and Günther Rüdiger, of another MHD instability of sheared rotation.[14] Sometimes called the helical MRI (HMRI), it requires an azimuthal component of the magnetic field in addition to the standard axial component, so the field lines are helical. The HMRI manifests as a form of traveling wave, and it has been observed in the Potsdam–Rossendorf Magnetic Instability Experiment conducted at the Helmholtz-Zentrum Dresden-Rossendorf research center.[15] The nature of the HMRI has been identified as a weakly destabilized inertial oscillation, but, alas, Keplerian flows are stable.[16]

With improved numerical predictions and more accurate measurements, a positive laboratory identification of the MRI may be near. Then the challenge will be to generate true turbulence by a spectrum of many MRI modes with sufficiently large amplitudes. To help bring that about, the Princeton liquid-metal MRI experiment team is planning an upgrade to broaden the range of experimentally accessible parameters while maintaining well-controlled boundary conditions. Meanwhile, an ambitious facility known as DRESDYN is being designed in Dresden, Germany, to allow an even wider range of accessible parameters.[17]

Several efforts are under way to develop new experiments that go beyond incompressible hydrodynamics and magnetohydrodynamics to explore other physics relevant to astrophysical phenomena. One is the plasma Couette experiment at the University of Wisconsin,[18] illustrated in figure 4. The Wisconsin group uses a novel technique to confine its plasma, and it has already generated rapid rotation. A concept based on a swirling gas flow and an injection-pump scheme is being prototyped at Princeton. Despite such prospects for the laboratory study of astrophysical puzzles, large gaps separate the dimensionless parameters accessible in laboratory experiments and numerical simulations from those of astrophysical systems. Smaller but still problematic gaps separate laboratory and simulation parameters. There will always be a need for powerful analytic theory to help close those gaps and synthesize what physicists have learned from their studies of very different systems.

In astrophysics, the ultimate laboratory must be the universe itself. Future observatories such as the ground-based Atacama Large Millimeter/Submillimeter Array and the spaceborne *James Webb Space Telescope* will surely provide new surprises and change the course of accretion-disk physics research. Those new insights will, in their turn, motivate new laboratory experiments and drive the field of numerical disk simulation.

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