Introduction to stellarator transport

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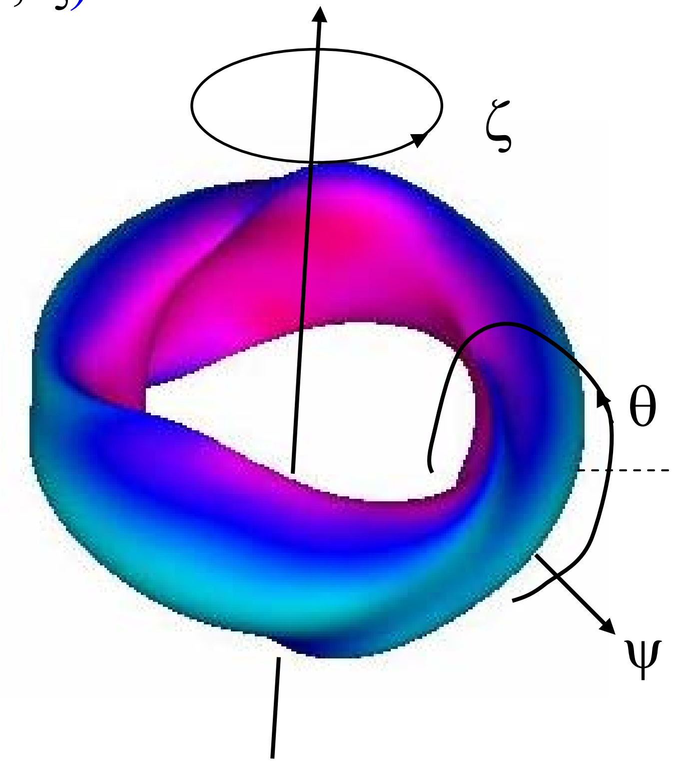
- Geometry, coordinates:

-Parametrize with flux coordinates (ψ or r , θ, ζ) natural to the magnetic field:

$$\vec{\mathbf{B}} = \nabla\psi\times\nabla\theta + \nabla\zeta\times\nabla\psi_p$$
$$= \nabla\psi\times\nabla\alpha_p \qquad (1)$$

with $\alpha_p \equiv \theta - \iota\zeta$, $\psi \equiv \psi_t \equiv B_0 r^2 \equiv$ toroidal flux,
$\iota \equiv d\psi_p/d\psi_t \equiv$ rotational transform $\equiv 1/q$.



-Mod B: $B(\mathbf{x}) = |\vec{\mathbf{B}}| = \sum_{m,n} B_{mn} \cos(n\zeta - m\theta)$

$$\approx B_0(r)[1 - \varepsilon_t(r)c(\Theta) - \delta_h(\mathbf{x})\cos\eta]$$

$$\rightarrow B_0(r)[1 - \varepsilon_t(r)\cos\theta - \varepsilon_h(r)\cos\eta] \quad (2)$$

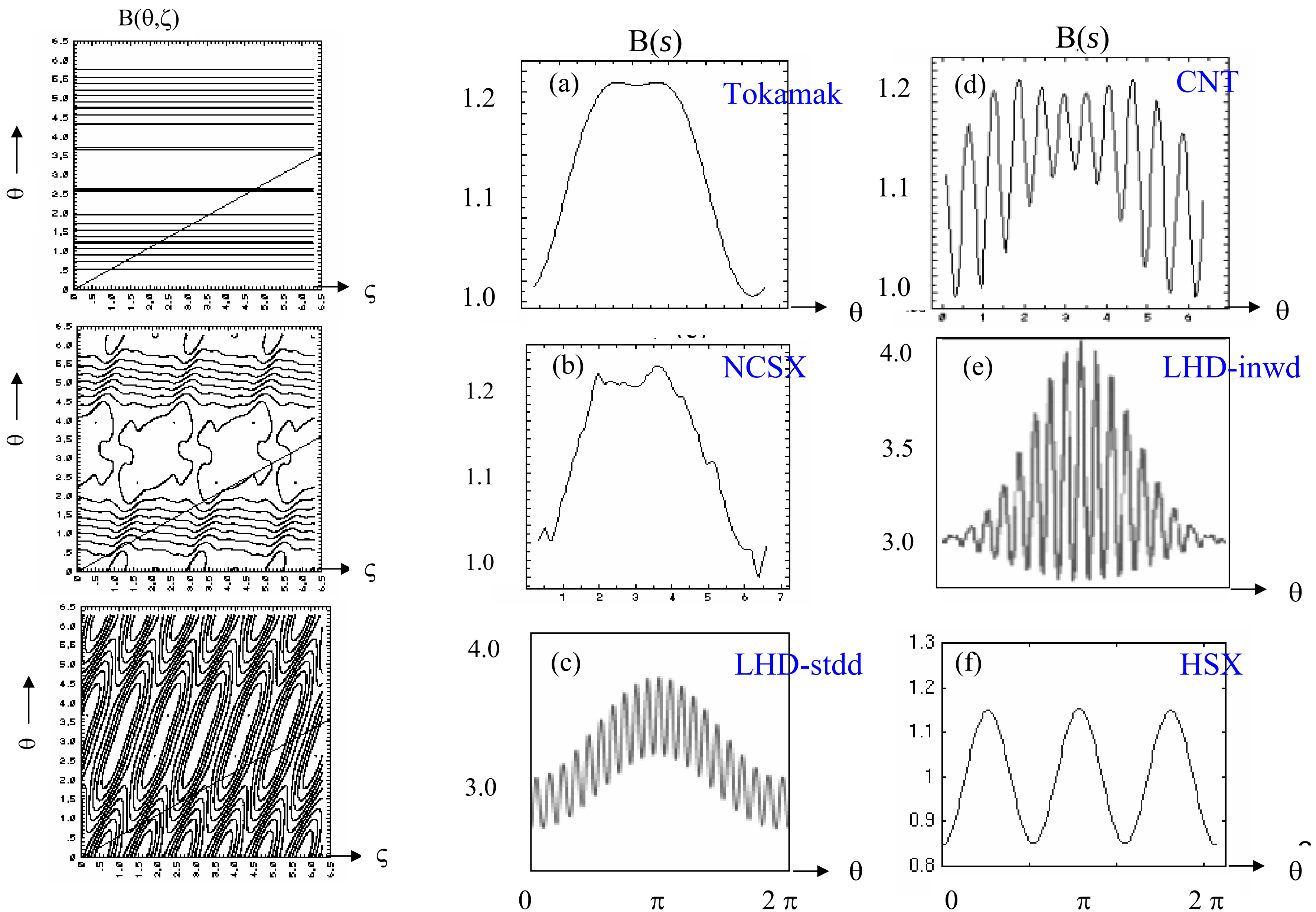
with $\delta_h(\mathbf{x}) \equiv \varepsilon_h(r) k(\mathbf{x})$, $\eta \equiv n\zeta - m\Theta$.

-Determines particle orbits in flux coords.

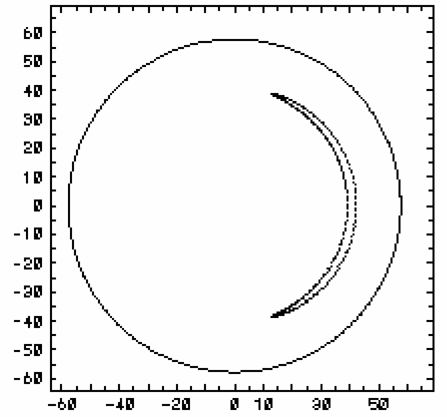
-Parameter $p \equiv \varepsilon_h/\varepsilon_t$, =measure of distance of stellarator from symmetric limits $\varepsilon_h = 0$, $\varepsilon_t = 0$.

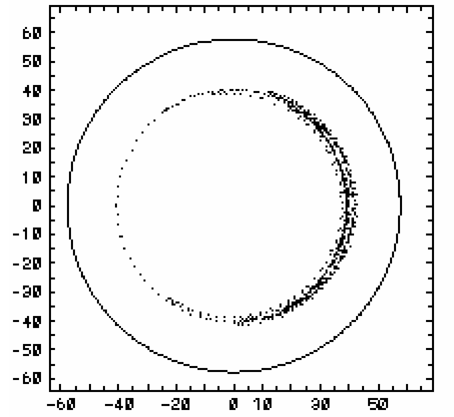
-Magnetic field structure:

(ordered by $p \equiv \varepsilon_h / \varepsilon_t$)

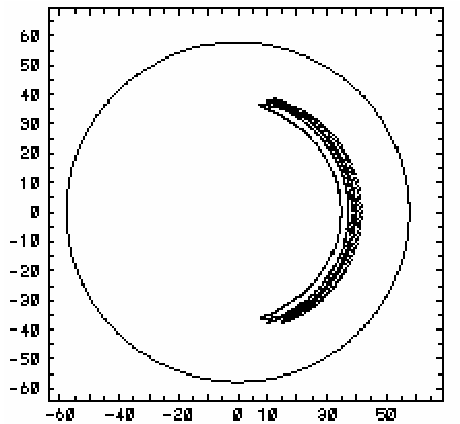


-Particle orbits:



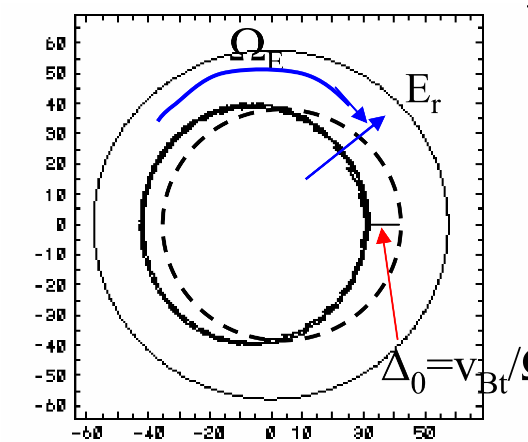


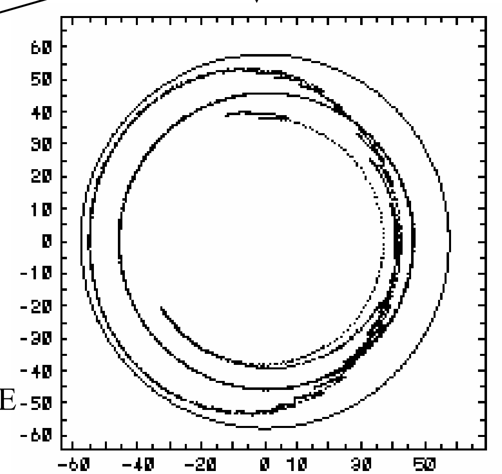
bananas (toroidally trapped), $\varepsilon_h=0$

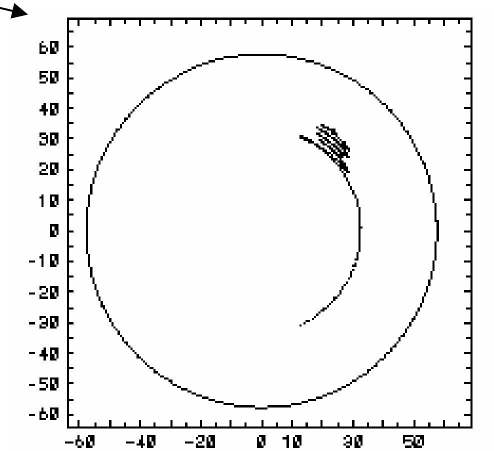


drifting banana, $\varepsilon_h \neq 0$

superbananas (ripple-trapped), $\varepsilon_h \neq 0$

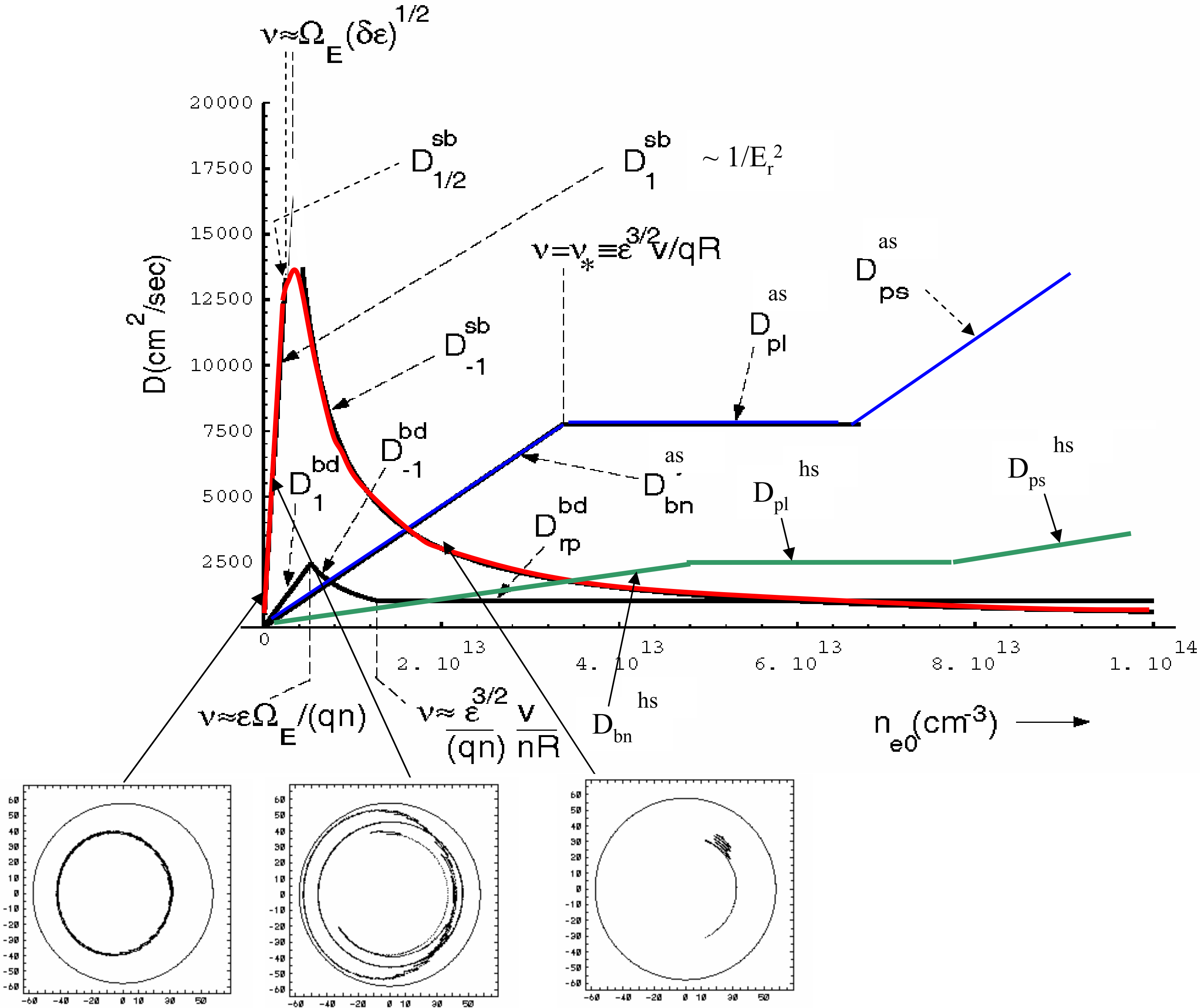






ν

-Neoclassical Transport – Overview:



-Radial diffusion:

$D \approx F\,\tilde{\nu}\,\Delta^2$, (3)

with

$F \equiv$ fraction of particles contributing,

$\tilde{\nu} \equiv$ freq of taking step in random walk

$\Delta \equiv$ radial step size.

-Example-1: Banana regime, tokamak:

$F = F_t \equiv (2\varepsilon_t)^{1/2}$ = frac of toroidally-trapped particles,

$\Delta = \rho_{bt} \equiv$ banana width $\approx v_{Bt}/(v_\parallel/qR) \approx q\rho/\varepsilon_t^{1/2}$,

$\tilde{\nu} = \nu_t \equiv$ toroidal detrapping frequency $= \nu/(2\varepsilon_t)$,

with $v_{Bt} \approx \rho v/2R$ = toroidally-induced radial drift velocity.

$\Rightarrow D_{bn}^{as} \approx (2\varepsilon_t)^{1/2} \nu_t\, \rho_{bt}^{\;2} \approx \nu\, q^2 \rho^2/\varepsilon_t^{3/2}$

-Example-2: Banana regime, straight stellarator:

[A.Pytte, A.H. Boozer, Phys. Fluids **24**, 88 (1981).]

$F = F_h \equiv (2\varepsilon_h)^{1/2}$ =frac of helically-trapped particles,

$\Delta = \rho_{bh} \equiv$ banana width $\approx v_{Bh}/(v_{\|}/L_h) \approx (q_h R/r)\rho\, \varepsilon_h^{1/2}$,

$\tilde{\nu} = \nu_h \equiv$ ripple detrapping frequency $= \nu/(2\,\varepsilon_h)$,

with $L_h \equiv R/n$, $v_{Bh} \approx (\rho v/2)(m\,\varepsilon_h/r)$ = helically-induced radial drift velocity, $q_h \equiv m/n$.

$\Rightarrow D_{bn}^{hs} \approx (2\,\varepsilon_h)1/2\nu_h\, \rho_{bh}^{2} \approx \nu(q_h R/r)^2 \rho^2\, \varepsilon_h^{1/2}$

- Superbanana branch :

- "1/ν-regime" ($\nu_h/\Omega_E > 1$):

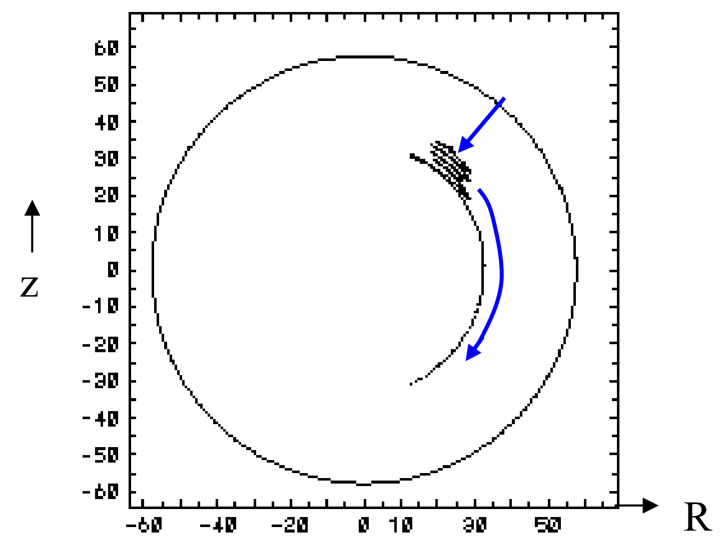
[Galeev, Sagdeev, Furth, Zh.Prikl.Mekh. i Tekhn.Fiz., **3** (1968), Gibson, Mason, Plasma Phys. **11**, 121 (1969), Stringer, Nucl. Fusion **12**, 689 (1972), Connor, R.J. Hastie, Phys. Fluids} **17**, 114 (1974).)]

$\Delta \approx v_{Bt}/\nu_h$, with $v_{Bt} \approx \rho v/2R$ = toroidally-induced radial drift velocity,

$F \approx (2\varepsilon_h)^{1/2}$ = frac of ripple-trapped particles,

$\tilde{\nu} \approx \nu_h \equiv \nu/(2\,\varepsilon_h)$ = detrapping frequency.

$$\Rightarrow D_{-1} \approx (2\,\varepsilon_h)^{1/2}\,\nu_h\,(v_{Bt}/\nu_h)^2 \approx (2\,\varepsilon_h)^{3/2}\,v_{Bt}^{\;2}/\nu$$



D_{-1} has strong energy dependence, $\sim K^{7/2}$,
and is indep of $\Omega_E \sim E_r = -\partial_r\Phi$.
$D_{-1i}/D_{-1e} \sim (M_i/M_e)^{1/2} >> 1$.

-Well-depth parameter :

$y=$ 0, deeply ripple-trapped particle

1, marginally-trapped particle

>1, non-ripple-trapped particle.

-For model B-field (2), have

$$y = [K/\mu B_0 - 1 + \varepsilon \cos\theta + \delta_h]/(2\delta_h), \qquad (4)$$

with $\eta \equiv n\zeta - m\theta$=ripple phase, $K \equiv (E - e\Phi)$= kin.energy

-Diffusion in y due to pitch-angle scattering:

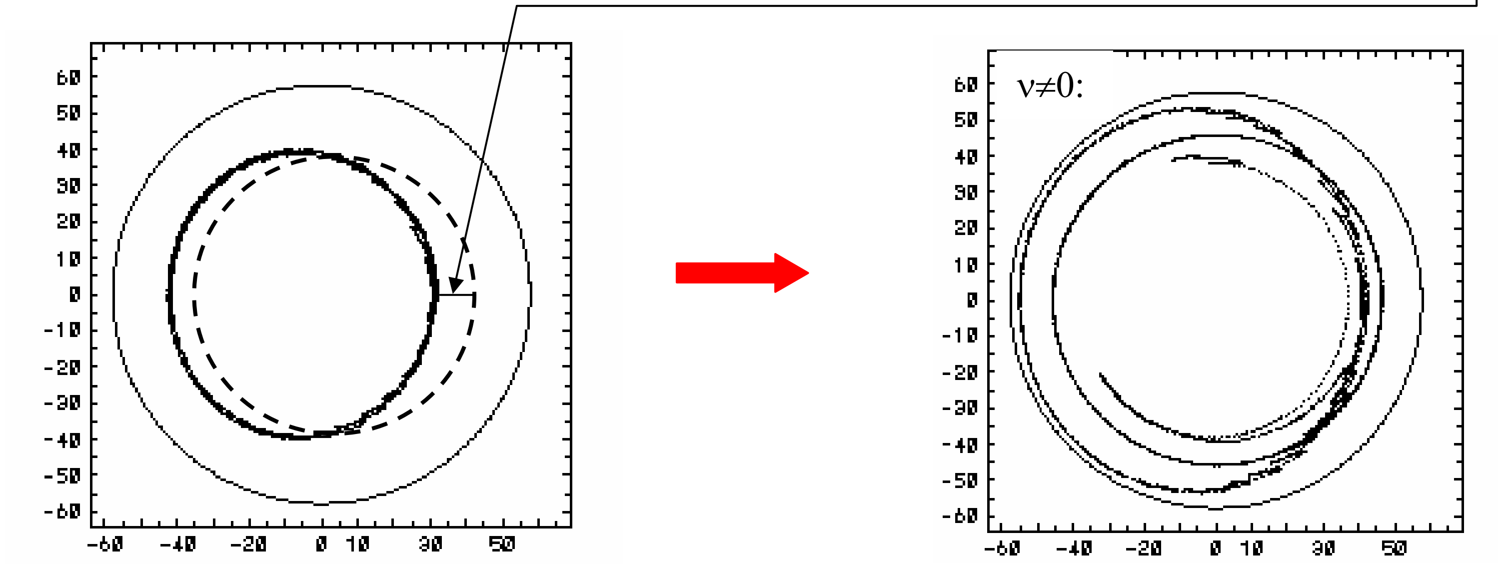
$<(\delta y)^2> \approx \nu_h t$, with $\nu_h \equiv \nu/(2\delta_h)$.

$\Rightarrow$time τ_h to detrap from ripple-well for $\delta y \approx 1$:

$\tau_h \approx 1/\nu_h$.

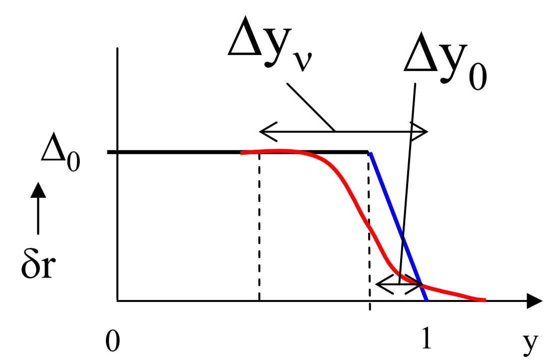
- “ν^1, $\nu^{1/2}$ superbanana regimes” ($\nu_h/\Omega_E < 1$) :

-Collisions perturb orbits from ν=0 superbananas, having sb width Δ_0=v_{Bt}/Ω_E .



-sb’s within a distance Δy_0=1/p of y=1 detrap collisionlessly, making sb excursion $\delta r(y)$ continuous. ($p \equiv \delta_h/\varepsilon$):

-For $\nu_h/\Omega_E > p^{-2}$, a collisional boundary layer is formed, of width $\Delta y_\nu = (\nu_h/\Omega_E)^{1/2}$, swamping Δy_0.



- "ν^1 sb-regime" ($\nu_h/\Omega_E < p^{-2}$):

[Galeev, Sagdeev, Sov. Phys. Usp. **12**, 810 (1970)]

$$\Delta = \Delta_0\ ,\ F \approx F_0 \equiv (2\delta_h)^{1/2}\ \Delta y_0\ ,\ \ \tilde{\nu} \approx \nu_h/(\Delta y_0)^2\ ,$$

$$\Rightarrow D_1 \approx \nu p (2\delta_h)^{-1/2} v_{Bt}^{\ 2}/\Omega_E^{\ 2}$$

- "$\nu^{1/2}$ sb-regime" ($p^{-2} < \nu_h/\Omega_E < 1$):

[Galeev, Sagdeev, Sov.Phys.Usp. **14**, 810 (1969),Galeev, Sagdeev, Furth, Rosenbluth, Phys. Rev. Letters **22**, 511 (1969).]

$$\Delta = \Delta_0\ ,\ F \approx F_\nu \equiv (2\delta_h)^{1/2}\ \Delta y_\nu = (\nu/\Omega_E)^{\ 1/2}\ ,\ \ \tilde{\nu}\ \approx \nu_h/(\Delta y_\nu)^2 = \Omega_E$$

$$\Rightarrow D_{1/2} \approx \nu^{1/2}\ v_{Bt}^{\ 2}/\Omega_E^{\ 3/2}$$

-Banana-drift branch:

- “stochastic regime”

[Goldston, White and Boozer, Phys.Rev. Lett. **47**, 647 (1981).]

- “ν^{1},ν^{-1} bd-regimes”

[Linsker, Boozer, Phys. Fluids **25**, 143 (1982).]

- “banana-plateau regime”

[Boozer, Phys. Fluids **23**, 2283 (1983).]

-Ambipolar roots & radial electric field E_r:

-Ambipolarity Condition:

$$0=\Sigma_{s=i,e}\, e_s\, \Gamma_s(E_r) \qquad (15)$$

-Symmetric contributions to Γ_s intrinsically ambipolar.
-Nonsymmetric contributions in (15) determine $E_r(r)$.

Radial Fluxes – sb branch:

$$\begin{bmatrix}\Gamma_s\\ Q_s\end{bmatrix} = -\ \frac{2n_s}{\sqrt{\pi}} \int dx\, x^{1/2}\, e^{-x} \begin{bmatrix}1\\ T_s x\end{bmatrix} D_q(x,E_r)\ [\ \frac{n'_s}{n_s} - \frac{e_s E_r}{T_s} + (x-\frac{3}{2})\frac{T'_s}{T_s}\ ], \quad (16)$$

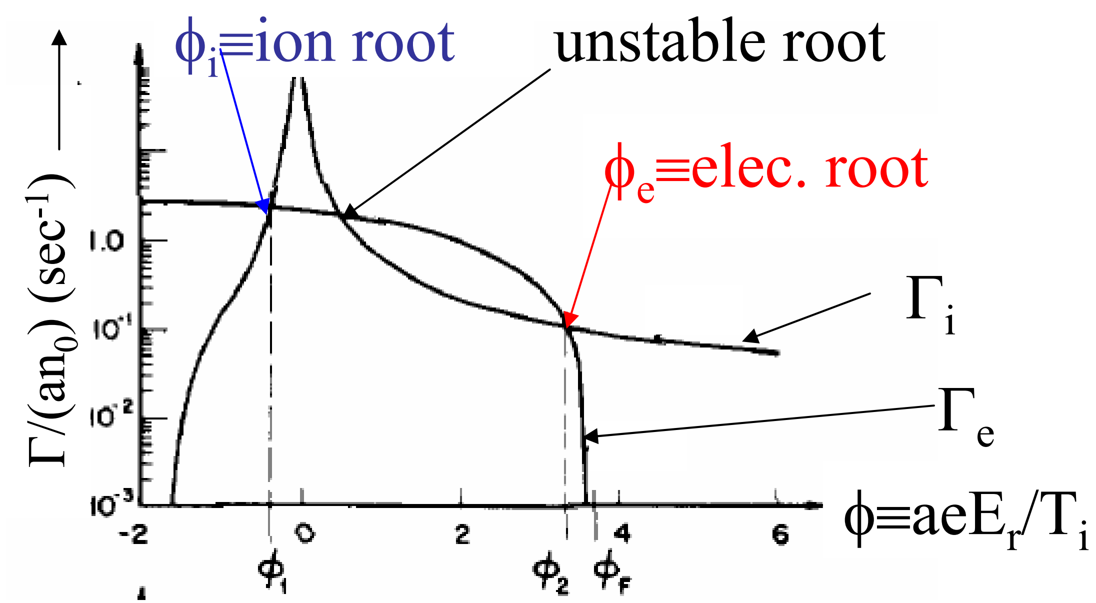
with $x \equiv K/T$, K=kinetic energy, q=-1,1/2,1 = power of ν in D.
Do energy-integration $\int_0^\infty dx$ over ν-regimes, with

$D_{-1}(x)=\sigma_{-1}(2\varepsilon_h)^{1/2}V_{Bt}^{2}/\nu \sim x^{7/2}$, (18)
$D_{1/2}(x)=\sigma_{1/2}\nu^{1/2}\, V_{Bt}^{2}/\Omega_E^{3/2} \sim x^{5/4}$,
$D_1(x)=\sigma_1 \nu p(2\varepsilon_h)^{-1/2}\, V_{Bt}^{2}/\Omega_E^{2} \sim x^{1/2}$.

-Roots of the ambipolarity condition:

(1)Galeev, Sagdeev, Furth, Rosenbluth, [Phys. Rev. Letters **22**, 511 (1969)] found $D_{-1,1/2}$, and found a single root, with $E_r<0$, electrons in the $1/\nu$ regime, holding in the ions, which are in the $\nu^{1/2}$ regime (the "ion root").

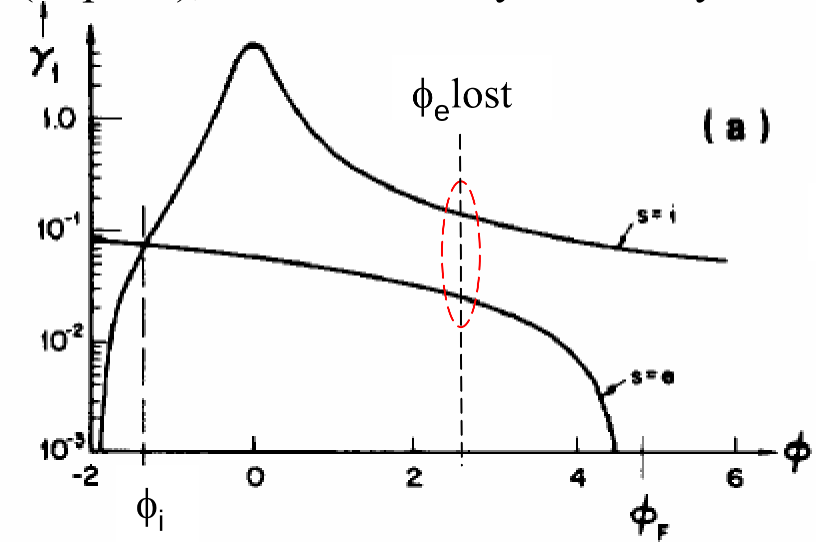
(2)Multiple roots: Mynick, Hitchon [Nucl. Fusion **23**, 1053 (1983)] using model with $D_{-1,1/2,1}$, and found multiple roots of (15), 2 stable and 1 unstable.



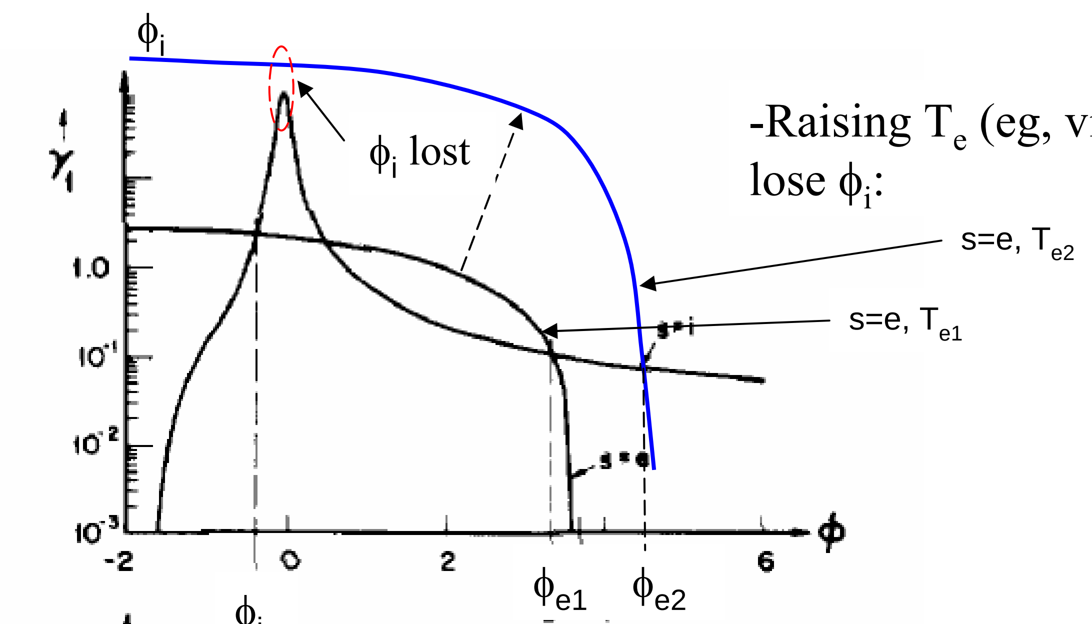
Elec. root experimentally observed:
[Idei et al., Phys. Rev. Lett **71** 2220 (1993),
Maassberg, Beidler, et al.,Phys. Plasmas 7, 295 (2000),
Castejon, et al, Nucl. Fusion **42**, 271 (2002).
Ida, et. al, Phys. Rev. Letters **86**, 5297 (2001)]

-Root loss:

-As parameters change (eg, as r changes), Eq.(15) can lose its real roots (in pairs), so that one may have only an ion, or an electron root.



-Eg, raising n_0 increases ν, broadens Γi peak:



-Raising T_e (eg, via ECRH), keeping T_i, can lose ϕ_i:

-Root jumping:

-Ambipolarity constraint (15) is algebraic, solved at each r. Doesn't answer which root is selected, or what happens if different roots occur at nearby r.

$\Rightarrow$Need a p.d.e. to evolve E_r in (r,t). Done in
[Shaing, Phys.Fluids 27, 1567 (1984),
Hastings, Houlberg, Shaing, Nucl. Fusion, **25**, 445 (1985)]:

$$\partial_t\left[\varepsilon_0 \frac{c^2}{v_A^2}\frac{m}{nq}\ E_r\right] = -\frac{1}{V'}\left[\partial_r V'\, D_E \partial_r\, E_r\right] + \Sigma_s e_s \Gamma_s\, (E_r).$$

$D_E \equiv$ "electric diffusion coef", obtained by solving the kinetic eqn which gives Γ_s to higher order in $\delta r/a$.

-**Internal transport barriers via root-jumping:**

-When root jumps occur, provide rapidly-changing $E_r \Rightarrow$possibility of ITB via flow-shear.

-Observed on W7-AS [Stroth, et, al., PRL, **86**, 5910 (2001)] , LHD [Ida, et al., PRL **91**, (2003).], CHS [Minami, etal, Nucl. Fusion **44**, 342 (2004).]