Nonlocal Closures for Plasma Fluid Simulations

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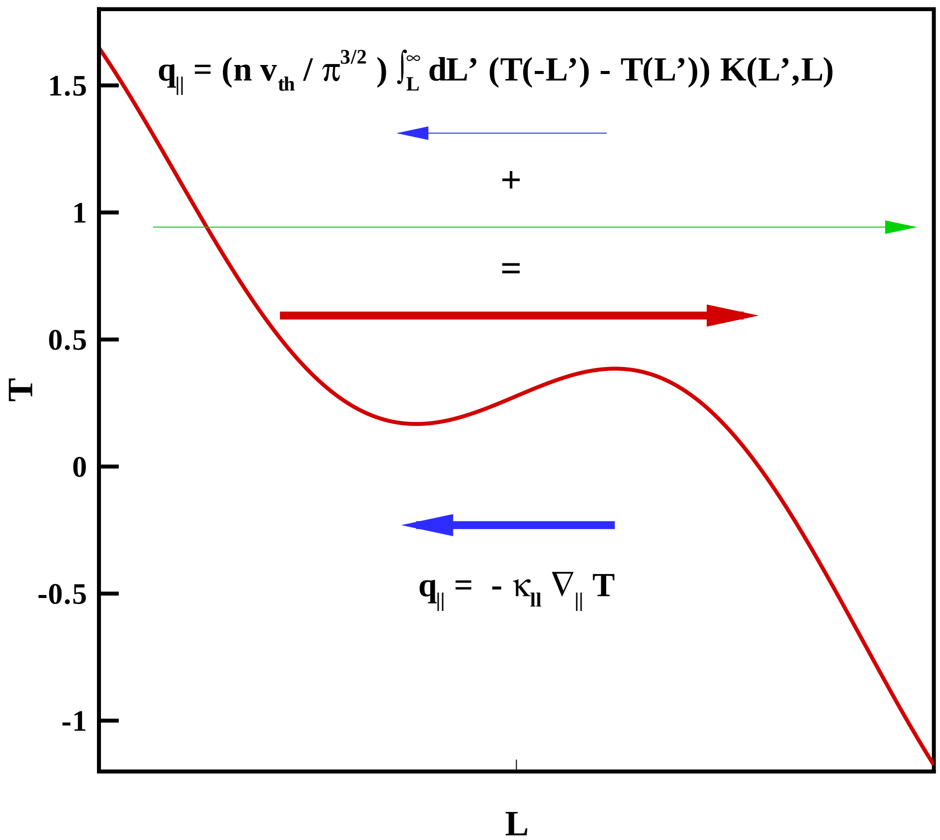
Nonlocal closure relations required for fluid simulations of high-performance plasmas.

- *Local closures* $\rightarrow$ depend on local gradients of T and $\vec{u}$.
- *Nonlocal closures* $\rightarrow$ depend on perturbed T and $\vec{u}$ all along magnetic field lines.



Outline

- Derive nonlocal closures from gryo/bounce-averaged kinetic equation.
- Emphasize continuous mapping from collisional to nearly collisionless regimes for nonlocal closures which embody:
    1. Landau,
    2. collisional, and
    3. particle trapping physics in
    4. general toroidal geometry.
- Use massively parallel semi-implicit implementation of nonlocal closures to make long time scale fluid simulations possible.
- Apply nonlocal closures to studies of heat flow dynamics in high-performance, toroidal fusion experiments.

Close fluid equations with kinetically derived $\vec{q}$ and $\mathbf{\Pi}$.

- Species evolution equations and closure moments for five moment model:

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot n\vec{u} = 0 \quad \rightarrow \text{density}$$

$$mn\left(\frac{\partial}{\partial t} + \vec{u} \cdot \vec{\nabla}\right)\vec{u} = en(\vec{E} + \frac{1}{c}\vec{u} \times \vec{B}) - \vec{\nabla}p - \underline{\vec{\nabla} \cdot \mathbf{\Pi}} + \vec{R} \quad \rightarrow \text{flow}$$

$$\frac{3}{2}n\left(\frac{\partial}{\partial t} + \vec{u} \cdot \vec{\nabla}\right)T = -p\vec{\nabla} \cdot \vec{u} - \underline{\mathbf{\Pi}{:}\vec{\nabla}\vec{u}} - \underline{\vec{\nabla} \cdot \vec{q}} + Q \quad \rightarrow \text{temperature}$$

$$\underbrace{\vec{q} \equiv \int d^3v' \frac{1}{2}mv'^2\vec{v}' f,}_{\text{heat flow}} \qquad \underbrace{\mathbf{\Pi} \equiv \int d^3v' m[\vec{v}'\vec{v}' - \frac{v'^2}{3}\mathbf{I}]f\,.}_{\text{viscous stress tensor}}$$

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- Parallel dynamics dominant in magnetized fusion plasmas:

$$\underbrace{\vec{q}_{\parallel} \equiv -T \int d^3v' (\frac{3}{2} - \frac{mv'^2}{2T}) v'_{\parallel} f\,.}_{\text{parallel conductive heat flux}}$$

Take Chapman-Enskog-like approach to derive closures.

- Chapman and Enskog proposed following form for f [1]:

$$f = f_M + F = \underbrace{n(\frac{m}{2\pi T})^{\frac{3}{2}} \exp(-\frac{mv'^2}{2T})}_{\text{dynamic Maxwellian}} + \underbrace{F(\vec{x},\vec{v},t)}_{\text{kinetic distortion}}.$$

- Use fluid moment equations to rewrite df_M/dt in full kinetic equation

$$\frac{dF}{dt} - C(F + f_M) = -\frac{df_M}{dt}.$$

- Gyro-average ($\rho_L \nabla_\perp << 1$) and focus on parallel dynamics:

$$\begin{aligned}
\frac{\partial F}{\partial t} + \vec{v}_\parallel \cdot \vec{\nabla} F - C(f) = & \\
-\frac{m}{T}(v_\parallel^2 - \frac{v_\perp^2}{2})(\hat{\mathbf{b}}\hat{\mathbf{b}} - \frac{\mathbf{I}}{3}):\vec{\nabla}\vec{u} f_M & \quad \} \quad \text{flow drive, drift terms} \\
+\vec{v}_\parallel \cdot \left(\vec{\nabla} \cdot \mathbf{\Pi}_\parallel - \vec{R}\right) \frac{f_M}{p} & \quad \} \quad \text{stress drive} \\
+L_1^{1/2}(\frac{d \ln T}{dt} + \vec{\nabla} \cdot \vec{u}) f_M - L_1^{3/2} \vec{v}_\parallel \cdot \vec{\nabla} T \frac{f_M}{T}. & \quad \} \quad \text{temperature drives}
\end{aligned}$$

Parallel component of closures dominant.

- Temperature change due to slow, resistive evolution obeys

$$\vec{\nabla} \cdot (\vec{q}_{||} + \vec{q}_{\perp}) = \vec{\nabla} \cdot \vec{q}_{||} \quad - \vec{\nabla} \cdot \underbrace{\kappa_{\perp}}_{\kappa_{\perp}/n \approx 1(m^2/s)} \vec{\nabla}_{\perp} T \quad \approx 0.$$

- Diffusive, Braginskii closure sets [2]

$$\vec{q}_{||} = -\kappa_{||} \vec{\nabla}_{||} T \qquad \text{with} \qquad \kappa_{||} >> \kappa_{\perp}.$$

- For long-mean-free-path plasmas [3,4,5,6,7]

$$q_{||} = \frac{n^{eq} v_{th}}{\pi^{3/2}} \int_L^{\infty} dL' \left[T(-L') - T(L') \right] K(L', L).$$

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2 S. I. Braginskii, *Transport Processes in a Plasma* (edited by M. A. Leontovich, Consultants Bureau, New York, 1965), Vol. 1.

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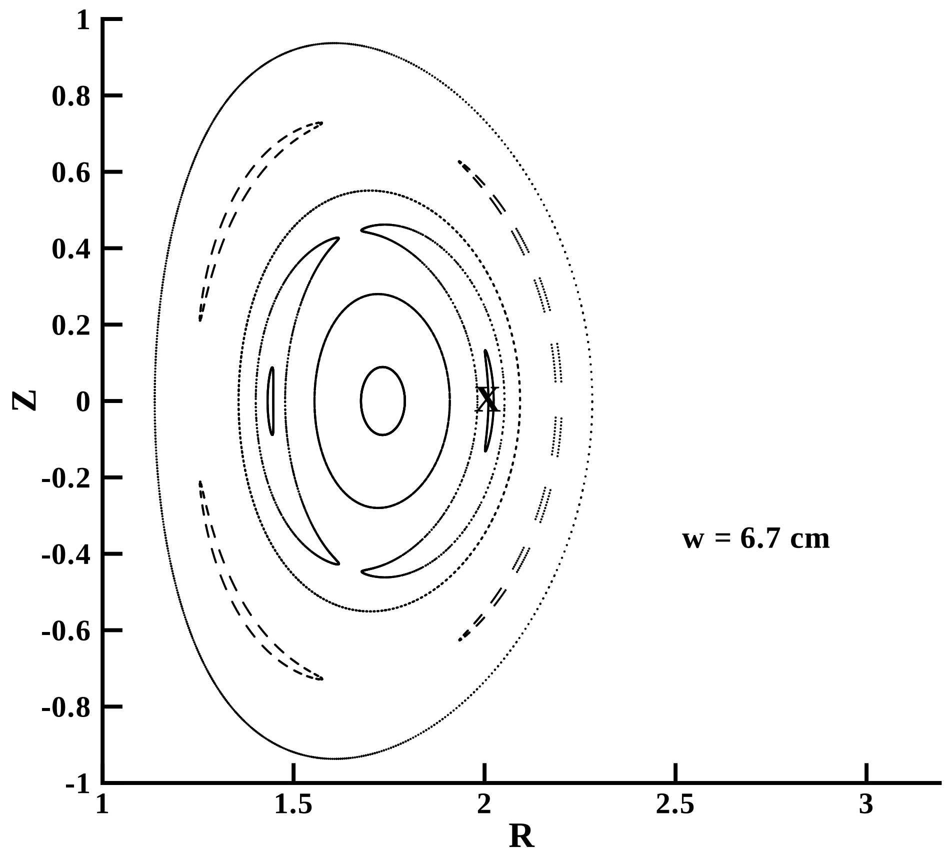
6 Z. Chang and J. D. Callen, Phys. Fluids **4**, 1167 (1992).

7 P. B. Snyder, G. W. Hammet, and W. Dorland, Phys. Plasmas **4**, 3974 (1997).

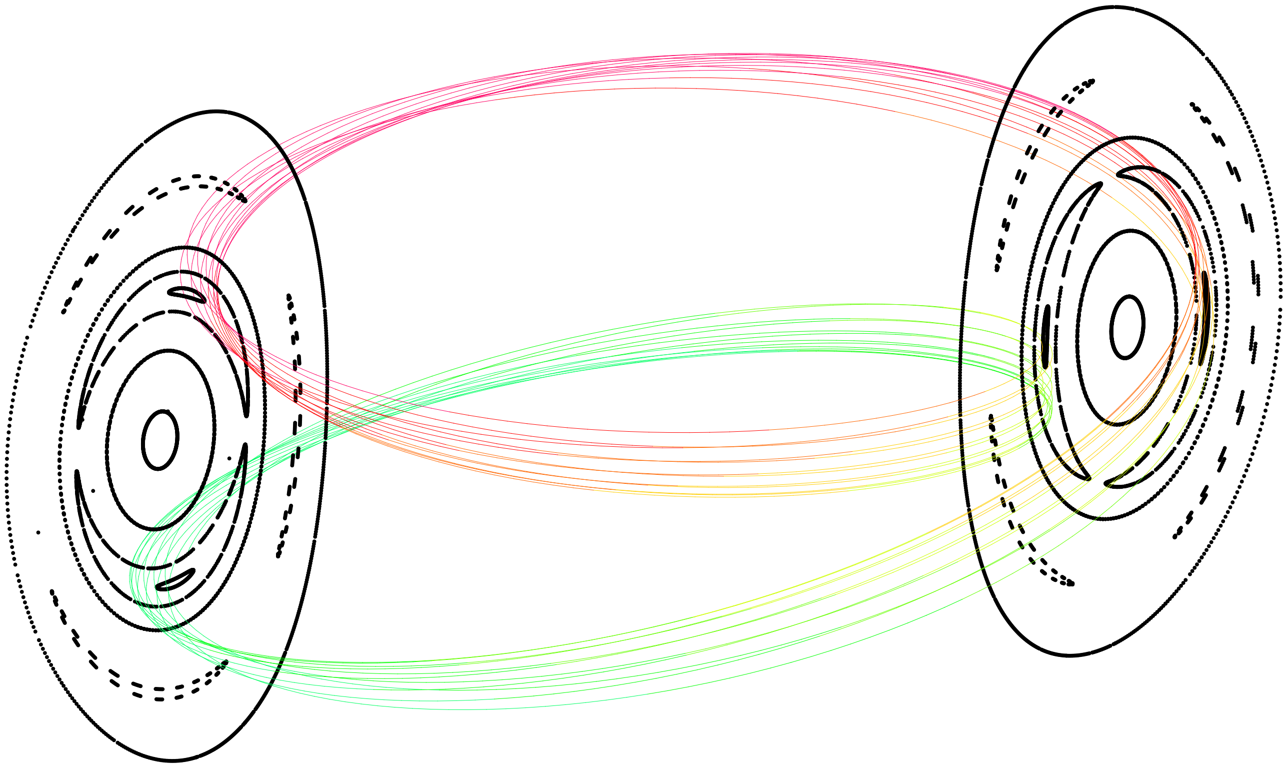
Changing magnetic topology results in large $q_{\|}$.

- $q_{\|}$ flattens T inside islands reducing heat confinement and possibly drives neoclassical tearing modes (NTMs).



Changing magnetic topology results in large $q_{\|}$.

- Particles see T perturbations of scale length, L_T, which is comparable to the collision length, L_ν.

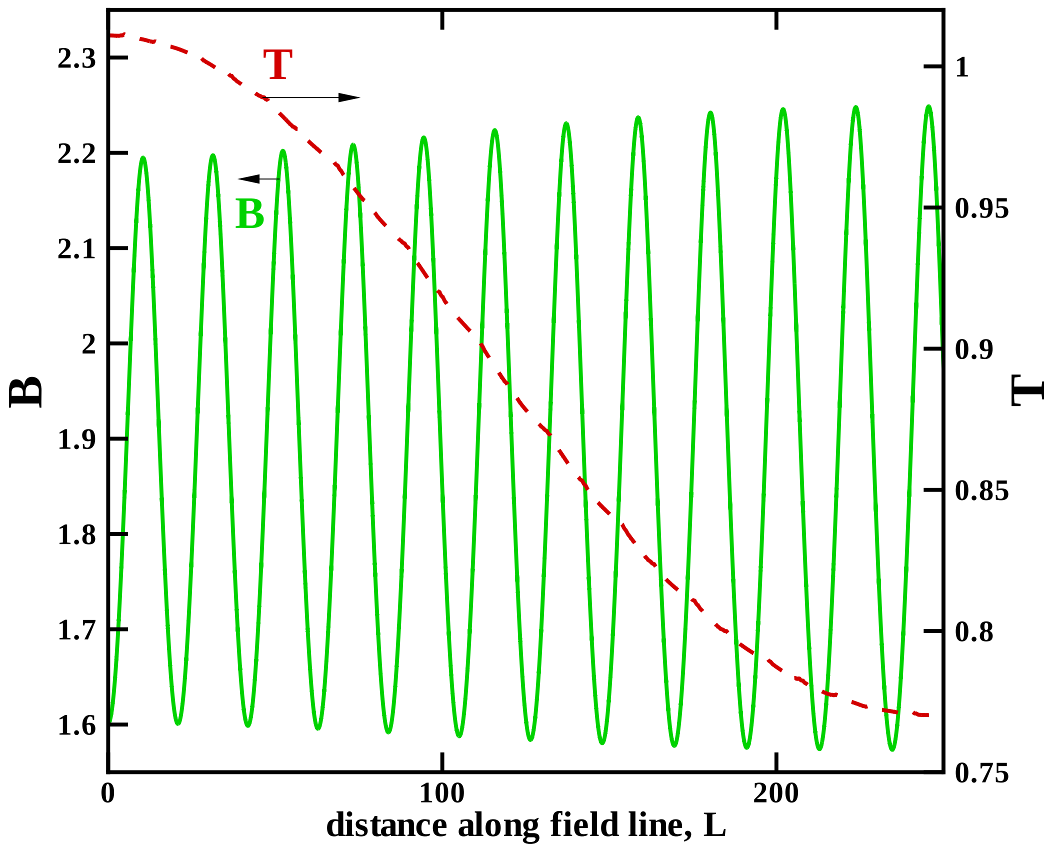


Nonlocal closures involve multiple parallel scale lengths.

- $L_T \equiv (\nabla_{\|} \ln T)^{-1}$

  $L_\nu \equiv v_{th}/\nu_0 \quad \Rightarrow \quad \underbrace{L_T \sim L_\nu \sim 100m >> l}_{\text{moderately collisional}}$ ($T = 1$ keV, $n = 10^{20} m^{-3}$)

  $l \equiv (\nabla_{\|} \ln B)^{-1}$



Average over bounce motion to handle short scale length.

- Expand $\vec{v}_{||} \cdot \vec{\nabla} = \vec{v}_{||} \cdot (\vec{\nabla}_l + \vec{\nabla}_L)$ and apply bounce-average operator, $\langle\rangle \equiv \oint dl/v_{||}$, to annihilate $(\vec{v}_{||} \cdot \vec{\nabla}_l)F^1$:

$$\left\langle (\frac{\partial}{\partial t} + \vec{v}_{||} \cdot \vec{\nabla}_L)F^0 \right\rangle - \langle C(F^0 + f_M^0)\rangle =$$
$$-\left\langle \frac{m}{T}(v_{||}^2 - \frac{v_\perp^2}{2})(\hat{\mathbf{b}}\hat{\mathbf{b}} - \frac{\mathbf{I}}{3}){:}(\vec{\nabla}_l \vec{u}^1 + \vec{\nabla}_L \vec{u}^0) f_M^0 \right\rangle$$
$$+\left\langle \vec{v}_{||} \cdot \left(\vec{\nabla}_l \cdot \mathbf{\Pi}_{||}^1 - \vec{R}^1\right) \frac{f_M^0}{p^0} \right\rangle$$
$$-\left\langle L_1^{3/2} \vec{v}_{||} \cdot \vec{\nabla}_L T^0 \frac{f_M^0}{T^0} \right\rangle .$$

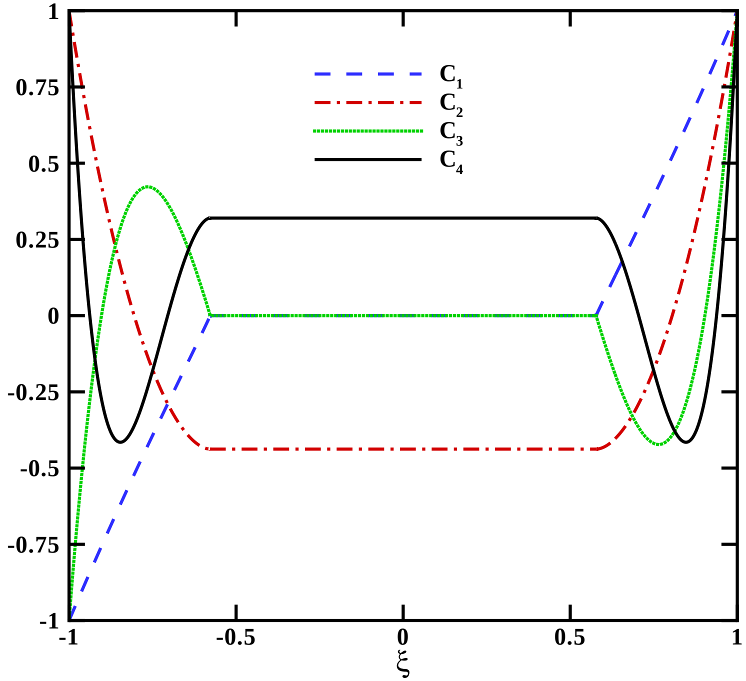
- Balance free-streaming and collisions with temperature gradient drive for nonlocal $q_{||}$ closure,

$$\left\langle \vec{v}_{||} \cdot \vec{\nabla}_L F^0 - C(F^0 + f_M^0) \right\rangle = -\left\langle L_1^{3/2} \vec{v}_{||} \cdot \vec{\nabla}_L T^0 \frac{f_M^0}{T^0} \right\rangle .$$

Expand in eigenfunction basis to handle pitch angle variable, ξ.

- Solve separated eigenvalue equation involving bounce-averaged pitch-angle scattering operator [4,8,9]

$$\frac{\partial}{\partial \xi}\frac{(1-\xi^2)}{\xi}\left(\oint dl \frac{v_\parallel}{v}\right)\frac{\partial C_n}{\partial \xi} + \lambda_n J C_n = 0, \quad \text{where} \quad \xi = \pm\sqrt{1 - \mathrm{v}_\perp^2 \mathrm{B}/\mathrm{v}^2}.$$



4 E. D. Held, J. D. Callen, and C. C. Hegna, Phys. Plasmas **10**, 3933 (2003).

8 J. G. Cordey, Nucl. Fusion **16**, 499 (1976).

9 E. D. Held, CPTC report UW-CPTC 99-5 (1999).

Solve for F and calculate nonlocal $q_{\|}$ closure.

- Write $\vec{F}$ as vector of coefficients, $F^0 = \sum_{n=1}^{N} F_n C_n(\xi)$ satisfying

$$\mathbf{L}\vec{F} + \mathbf{A}v\frac{\partial \vec{F}}{\partial L} = \vec{g}.$$

- Spatially nonlocal expression for $q_{\|}$ [3,4]:

$$q_{\|}(L) = \frac{n^{eq} v_{th}}{\pi^{3/2}} \int_L^\infty dL' \left[T(-L') - T(L')\right] K(L', L),$$

$$K(L', L) = \frac{1}{v_{th}^3} \int \left\langle d^3 v\right\rangle \left(L_1^{(\frac{3}{2})}\right)^2 e^{-v^2/v_{th}^2} C_1^2(\xi) \sum_{i=1}^{N/2} a_i e^{-|\bar{k}_i|(L-L')}.$$

- Collisional limit yields $q_{\|} = \underbrace{-\kappa_{\|}} \nabla_{\|} T$.

modified by particle trapping and enhanced collisional effects

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3 E. D. Held, J. D. Callen, C. C. Hegna, and C. R. Sovinec, Phys. Plasmas **8**, 1171 (2001).

4 E. D. Held, J. D. Callen, and C. C. Hegna, Phys. Plasmas **10**, 3933 (2003)

Continuous map for $q_{\parallel}$ as collisionality varies.

- Heat flow response due to sinusoidal T perturbations reduced by particle trapping in collisional and nearly collisionless regimes.