Numerical implementation of parallel electron closures in NIMROD [1]

E.D. Held and J. W. James

CEMM-APS 2001

Long Beach, CA

[1]Research supported generously by the US DOE in the form of grant DE-FG03-01ER54618 and DE-FG03-01ER54660.

Outline

- **Analytic form of parallel electron closures.**

- **Implementation of electron closure model in NIMROD.**

- **Electron heat transport in presence of magnetic island.**

Heat flow involves nonlocal integration along field lines.

- The parallel heat flow definition is

$$q_{\parallel} \equiv T \int d^3v \left(\frac{v^2}{v_{th}^2} - \frac{5}{2} \right) v_{\parallel} f.$$

- Heat flow moment appears as drive:

$$\begin{pmatrix} \nu & v\epsilon_{0,1}\frac{\partial}{\partial L} & 0 & 0 & \dots & & \\ v\epsilon_{1,0}\frac{\partial}{\partial L} & \lambda_1\nu & v\epsilon_{1,2}\frac{\partial}{\partial L} & 0 & & & \\ 0 & v\epsilon_{2,1}\frac{\partial}{\partial L} & \lambda_2\nu & v\epsilon_{2,3}\frac{\partial}{\partial L} & 0 & \dots & \\ \vdots & & \vdots & & & & \\ \dots & & & & 0 & v\epsilon_{N,N-1}\frac{\partial}{\partial L} & \lambda_N\nu \end{pmatrix} \begin{pmatrix} F_0 \\ F_1 \\ F_2 \\ \vdots \\ F_N \end{pmatrix} = \begin{pmatrix} -\frac{4}{3}L_1^{(\frac{1}{2})}\frac{\partial q_{\parallel}}{\partial L}\frac{f_M}{p} \\ \frac{2}{3}vL_1^{(\frac{3}{2})}\frac{\partial T}{\partial L}\frac{f_M}{T} \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

- Explicit treatment using $q_{\parallel}$ and T at previous timestep.

$$q_{\parallel}^{n+1}(L') = \int^{L'} dL \left[T^n(L' - L) - T^n(L' + L) \right] \frac{\partial K_{q_{\parallel}}}{\partial(\ln L)} + G(q_{\parallel}^n(L')),$$

$$K(s, L_{mfp}) = n_0 v_{th} s^3 \left(s^2 - \frac{5}{2} \right)^2 \sum_{i=0}^{N} a_i \exp\left(-s^2 - \bar{k}_i L \right).$$

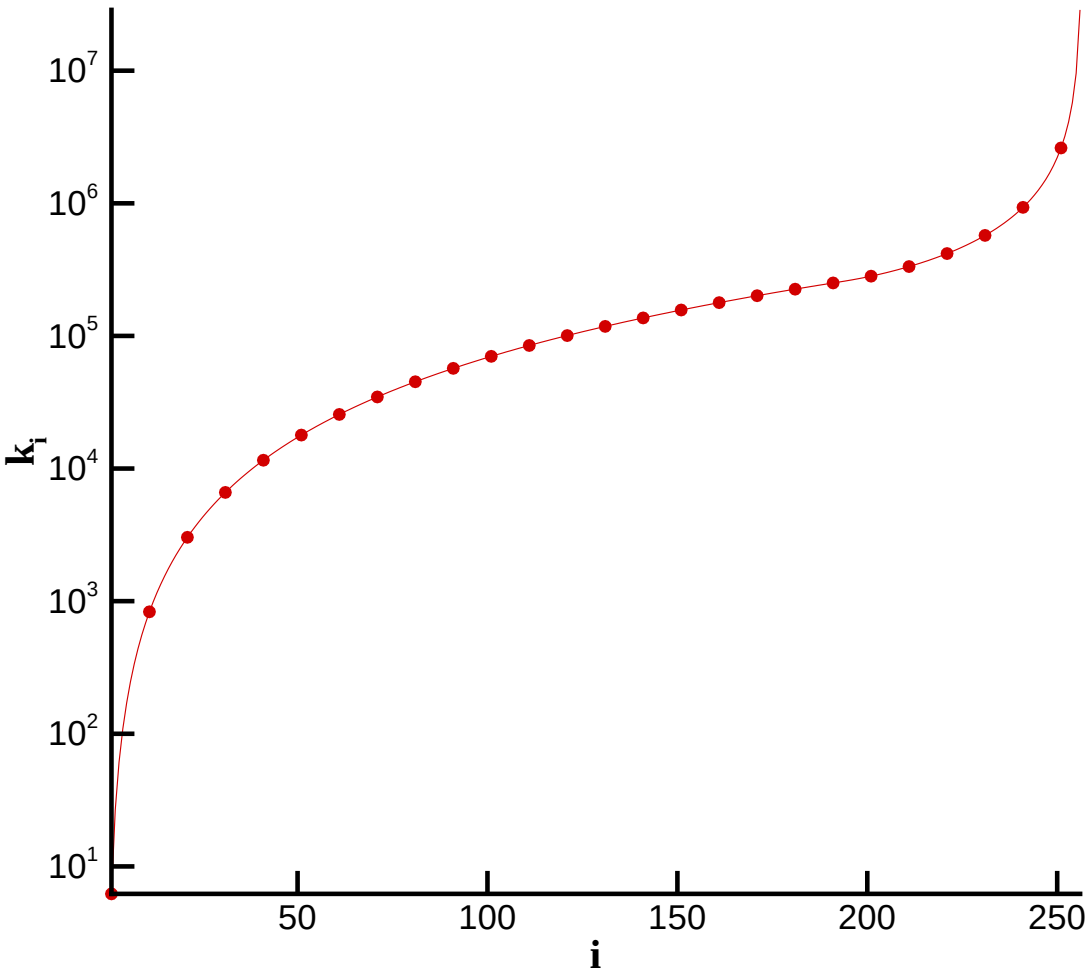


Figure 1: k_i's resulting from expansion in basis of 512 Legendre polynomials.

Collisional information contained in coefficients, a's.

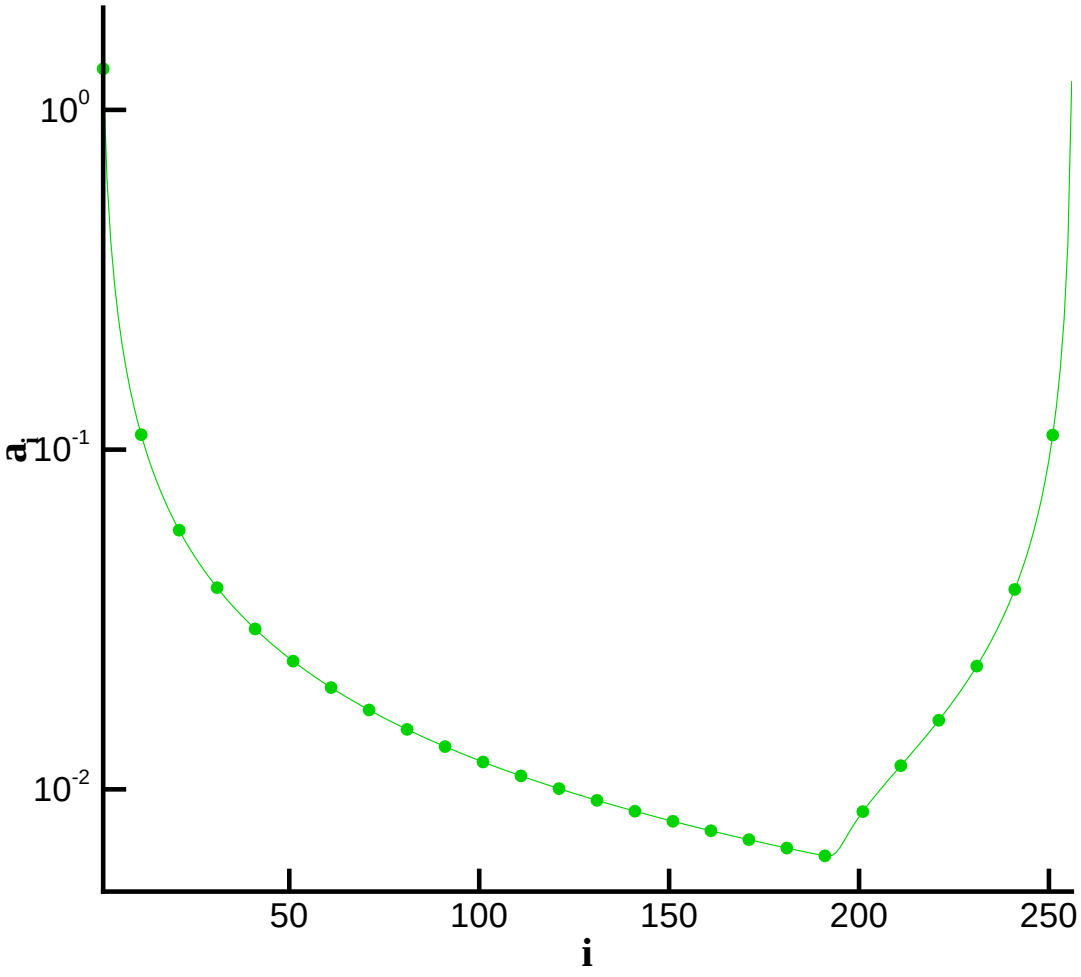


Figure 2: a_i's resulting from expansion in basis of 512 Legendre polynomials.

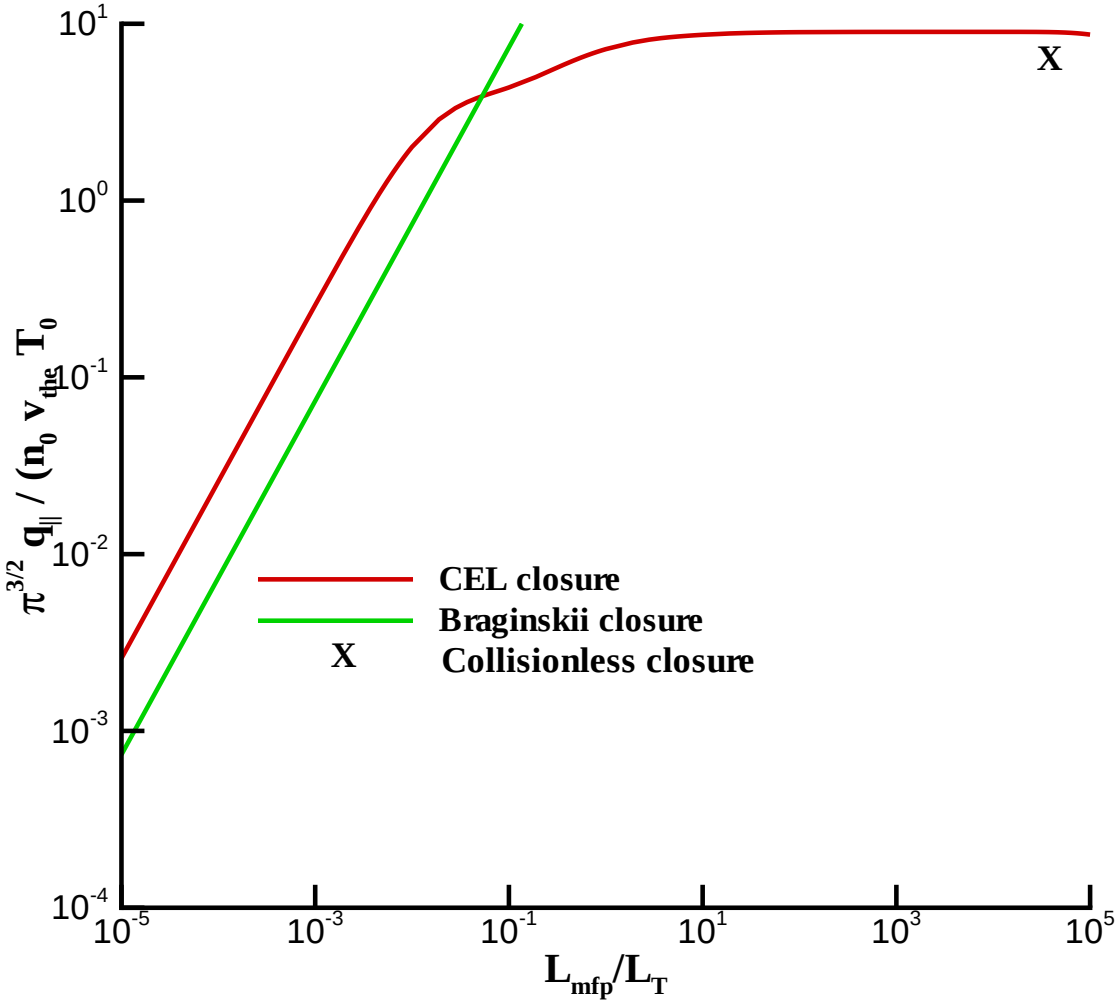


Figure 3: Heat flux for homogeneous magnetic field and sinusoidal temperature perturbations.

Nearly collisionless closure truncates more rapidly than collisionless closure.

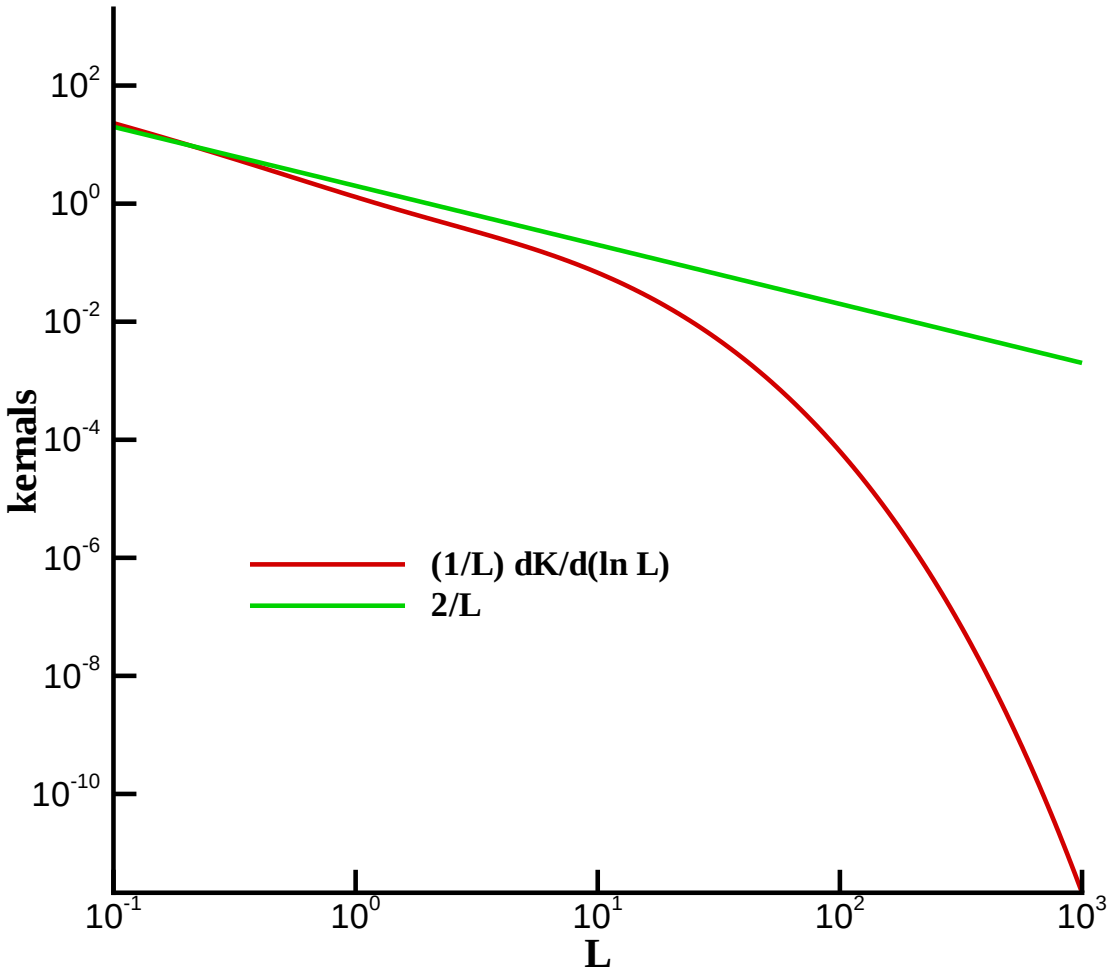


Figure 4: Kernal of nearly collisionless closure falls off more rapidly than kenral of colliisonless closure. Here it was assumed that $L_{mfp} = 1$.

Islands in slab geometry good testbed for heat flux closures.

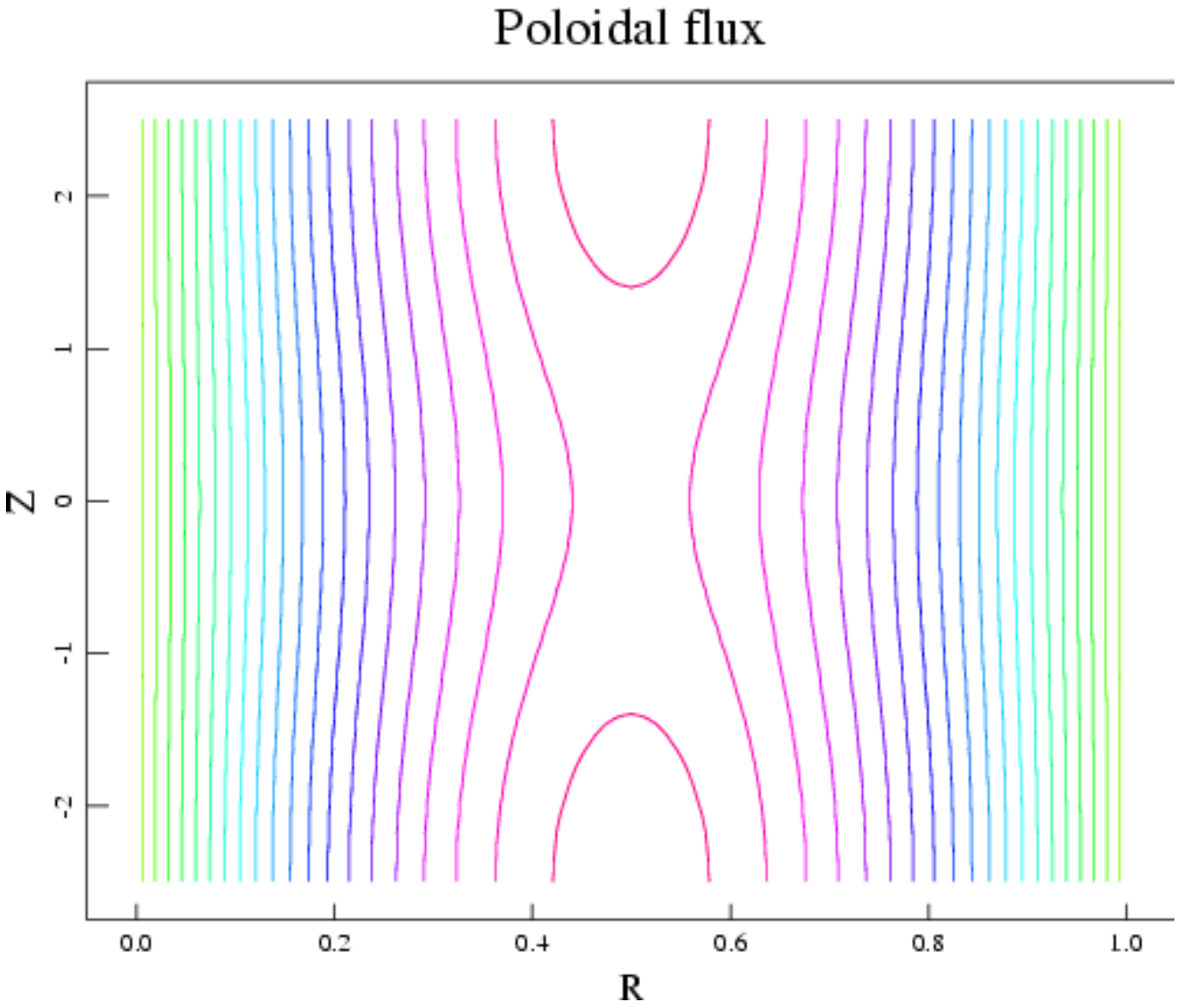


Figure 5: Single-helicity island in slab geometry periodic in Z and into plane.

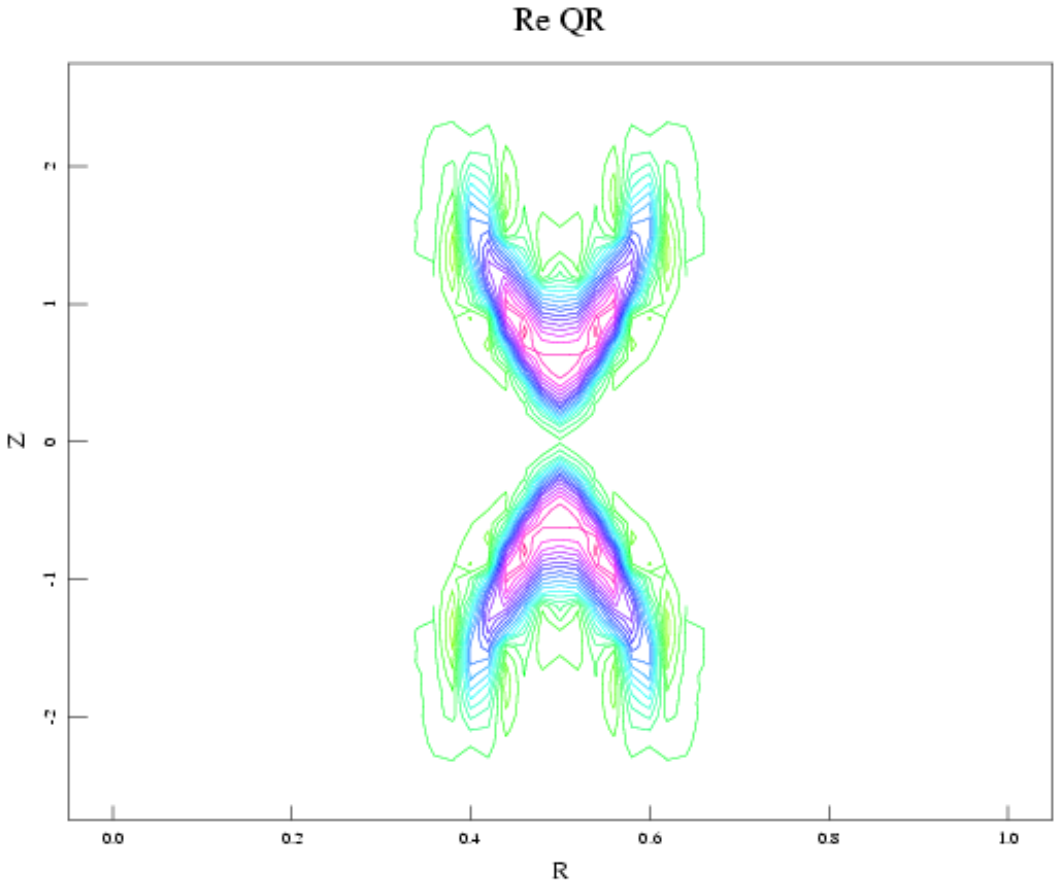


Figure 6: Radial component of flow of heat inside island due to rapid free-streaming/diffusive transport along field lines.

Diffusive and CEL closures take different forms.

- **Diffusive heat flow uses**

  $$\vec{\nabla} \cdot \vec{q} = \vec{\nabla} \cdot \kappa \vec{\nabla} T = \kappa_\perp \nabla^2 T + \vec{\nabla} \cdot (\kappa_\parallel - \kappa_\perp) \hat{\mathrm{b}} \hat{\mathrm{b}} \cdot \vec{\nabla} T,$$

  where $\kappa_\parallel$ and $\kappa_\perp$ are the parallel and perpendicular scalar conductivities.

- **CEL closure can be written in several different forms:**

  $$\vec{\nabla} \cdot \vec{q} = \kappa_\perp \nabla^2 T - \vec{\nabla} \cdot \kappa_\perp \hat{\mathrm{b}} \hat{\mathrm{b}} \cdot \vec{\nabla} T + \vec{\nabla} \cdot \vec{q}_\parallel,$$

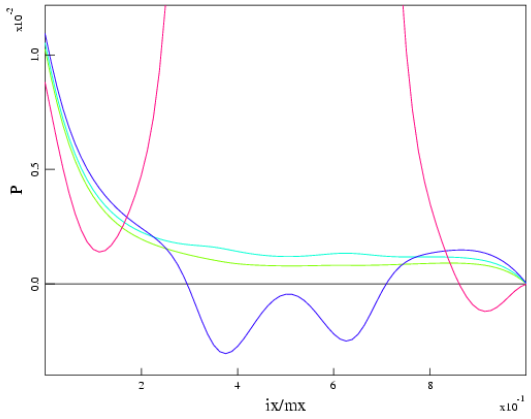
  where $\vec{q}_\parallel = q_\parallel \hat{\mathrm{b}}$ hence,

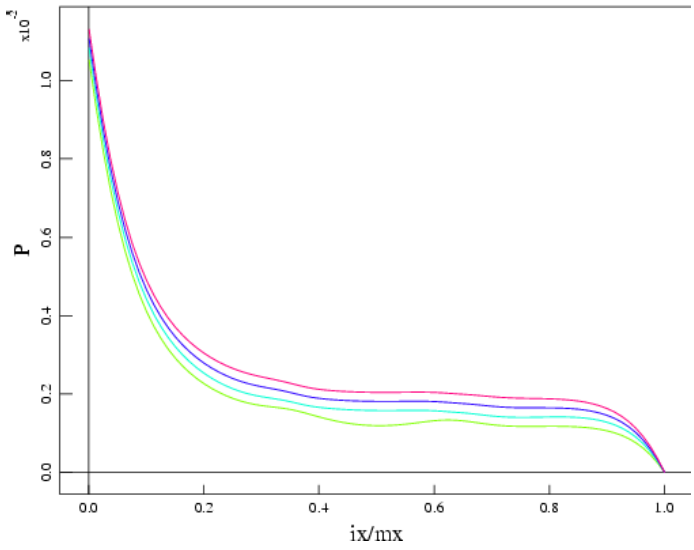
  $$\vec{\nabla} \cdot \vec{q}_\parallel = \hat{\mathrm{b}} \cdot \vec{\nabla} q_\parallel + q_\parallel \hat{\mathrm{b}} \cdot \vec{\nabla} \hat{\mathrm{b}}.$$

- **Pressure equation stabilized numerically with self-adjoint, semi-implicit operator on left side.**

  $$\left[\mathrm{I} - dtf \left(\kappa_\perp \nabla^2 + \vec{\nabla} \cdot (\kappa_\parallel - \kappa_\perp) \hat{\mathrm{b}} \hat{\mathrm{b}} \cdot \vec{\nabla}\right)\right] \Delta T = dt \left[\kappa_\perp \nabla^2 T - \vec{\nabla} \cdot \kappa_\perp \hat{\mathrm{b}} \hat{\mathrm{b}} \cdot \vec{\nabla} T + \vec{\nabla} \cdot \vec{q}_\parallel\right]$$

Semi-implicit operator critical for numerical stability.





Parallel heat flow robustly flattens pressure across O-point.

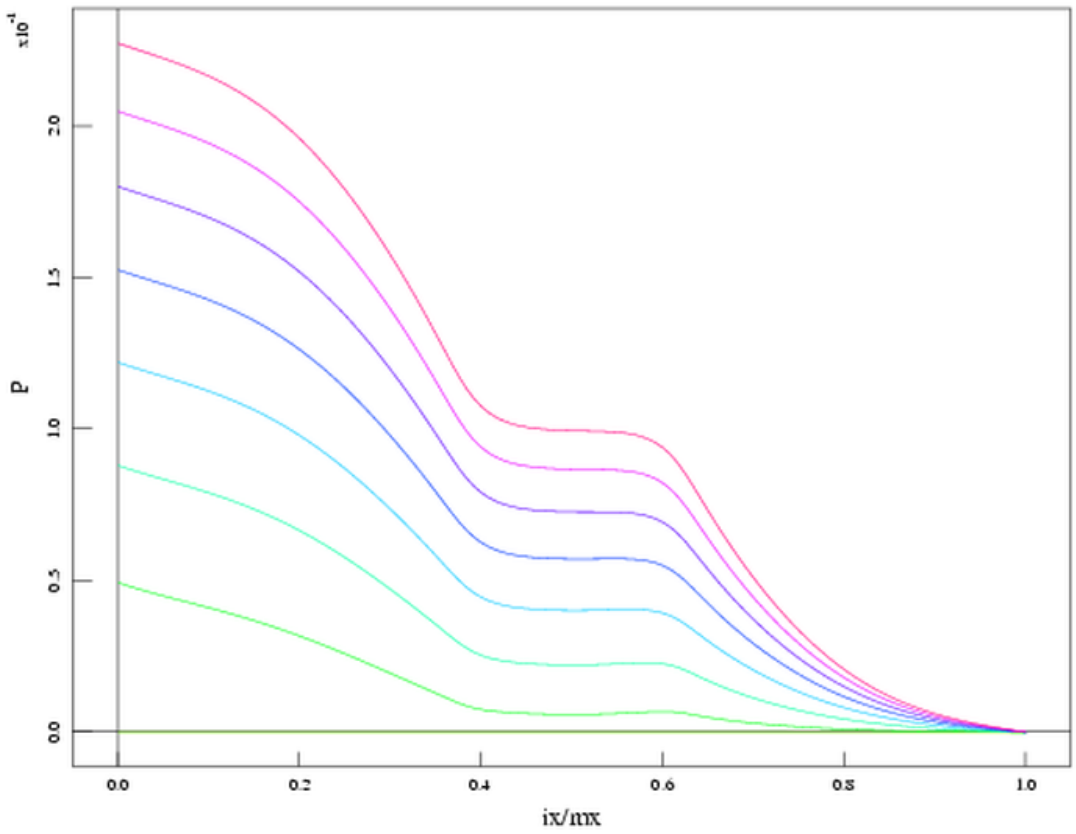


Figure 7: Flattening inside island due to rapid free-streaming/diffusive transport along field lines.

Conclusions

- **Robust temperature flattening from CEL heat flow closure.**

  10^2 larger parallel heat flow with $v_{the}/\pi^{3/2} \sim 10^6$ and $n_e = 5 \times 10^{13}$ in CEL model compared to diffusive model with $\kappa_{\|} = 10^7$.

- **Processors integrate $\approx 10^2 - 10^3$ m for single heat flux calculation to converge.**

  $T_e = 100$eV, $v_{the} = 5 \times 10^6$ and $L_{mfp} = 2 - 3$m.

- **CEL heat flow time $>$ pressure equation solve time**

  100 processors on 40 X 40 grid and cubic finite elements spend 80% of CPU time on CEL closures

- **More work needed for production runs.**