Progress on Hybrid Kinetic-MHD Simulation in NIMROD

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Outline

- the equations of motion
- diagnosing the particles
- effects of beta reduction and other parameters
- effects of the hot particles and the benchmark

Hot Particle Pressure Minor Impact on MHD Equations

- in the limit $n_h \ll n_0$, $\beta_h \sim \beta_0$, and quasi neutrality , the hybrid-kinetic momentum equation is

$$\rho\left(\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U}\nabla\cdot\mathbf{U}\right) = \mathbf{J}\times\mathbf{B} - \nabla\cdot\underline{\mathbf{p}}_b - \nabla\cdot\delta\underline{\mathbf{p}}_h$$

  where the subscripts b, h denote the bulk plasma and hot particles.

- assume CGL form $\delta\underline{\mathbf{p}}_h = \begin{pmatrix} p_\perp & 0 & 0 \\ 0 & p_\perp & 0 \\ 0 & 0 & p_\parallel \end{pmatrix}$

- the pressure moment at a position $\mathbf{x}$ is[1]

$$\begin{aligned} \delta p(\mathbf{x}) &= \int m(v - V_h)^2 \delta f(\mathbf{x}, \mathbf{v}) d^3 v \\ &= \sum_{i=1}^{N} m(v_i - V_h)^2 g_0 w_i \delta^3(x - x_i) \end{aligned}$$

  where sum is over the particles, m mass of the particle, $g_0 w_i$ is the perturbed phase density[2], and V_h is the hot flow velocity

[1]S. E. Parker and W. W. Lee, 'A fully nonlinear characteristic method for gyro-kinetic simulation', *Physics of Fluids B*, **5**, 1993

[2]G. Hu and J. A. Krommes, 'Generalized weighting scheme for δf particle simulation method', *Physics of Plasmas*, **1**, 1994

The Particle Equations of Motion

- use the drift kinetic equations of motion

$$\dot{\mathbf{x}} = v_\parallel \hat{\mathbf{b}} + \frac{m}{eB^4}\left(u^2 + \frac{v_\perp^2}{2}\right)\left(\mathbf{B}\times\nabla\frac{B^2}{2}\right) + \frac{\mathbf{E}\times\mathbf{B}}{B^2}$$
$$m\dot{v}_\parallel = -\hat{\mathbf{b}}\cdot(\mu\nabla B - e\mathbf{E})$$

- for the slowing down distribution function

$$f_{eq} = \frac{P_0 \exp(\frac{P_\zeta}{\psi_0})}{\varepsilon^{3/2} + \varepsilon_c^{3/2}}$$

where $P_\zeta = g\rho_\parallel - \psi$ is the canonical toroidal momentum and ε is the energy, ψ_0 is the total flux, and ε_c is the critical slowing down energy

$$\begin{aligned}\dot{\delta f} &= f_{eq}\left\{\frac{mg}{e\psi_0 B^3}\left[\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)\delta\mathbf{B}\cdot\nabla B - \mu_0 v_\parallel \mathbf{J}\cdot\mathbf{E}\right]\right.\\ &\quad \left. + \frac{\delta\mathbf{v}\cdot\nabla\psi_p}{\psi_0} + \frac{3}{2}\frac{e\varepsilon^{1/2}}{\varepsilon^{3/2} + \varepsilon_c^{3/2}}\mathbf{v}_D\cdot\mathbf{E}\right\}\end{aligned}$$

where

$$\begin{aligned}\mathbf{v}_D &= \frac{m}{eB^3}\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)(\mathbf{B}\times\nabla B) + \frac{\mu_0 m v_\parallel^2}{eB^2}\mathbf{J}_\perp\\ \delta\mathbf{v} &= \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \mathbf{v}_\parallel\cdot\frac{\delta\mathbf{B}}{B}\end{aligned}$$

Hot Particle Pressure Diagnostics

- equilibrium hot particle pressure used as diagnostic for deposition
- note more peaked hot particle pressure

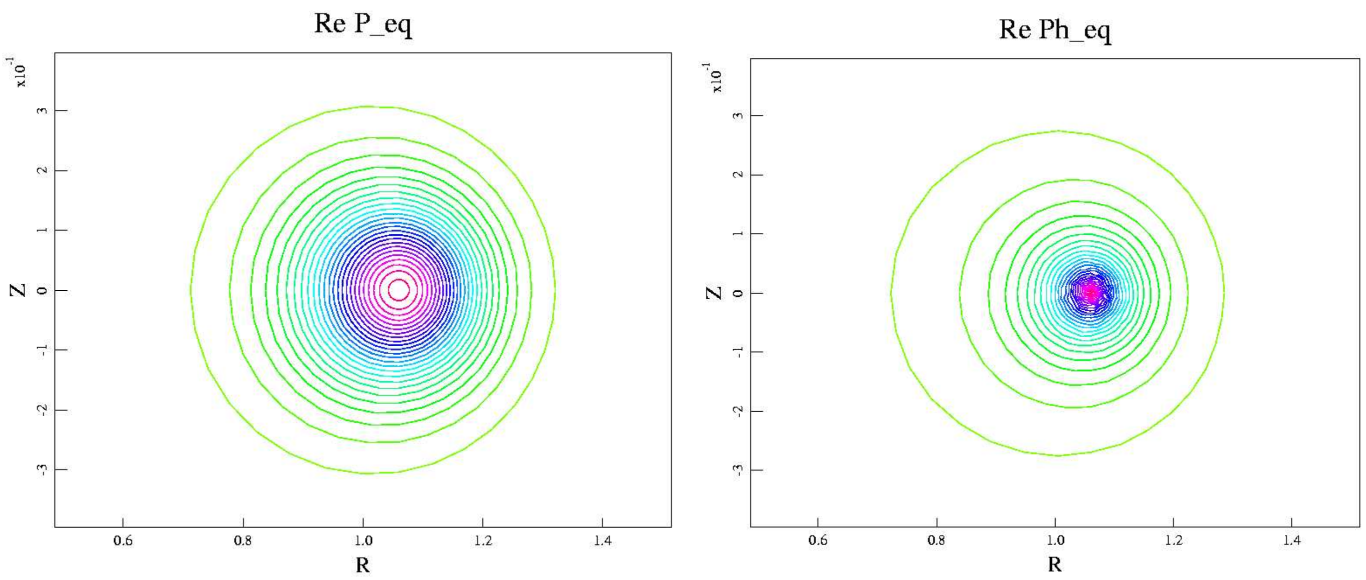

Figure 1: hot particle pressure is relatively smooth compared to equilibrium pressure

- required $2M$ particles on 32×32 grid
  - **if** the grid were uniform $\simeq 2000$/cell
  - but nonuniform grid, factor of $50 - 100$ difference in cell size between smallest and largest cells (cells near axis vs. outer cells)
  - particle requirements are not suprising

- profiles used in benchmark are not as smooth

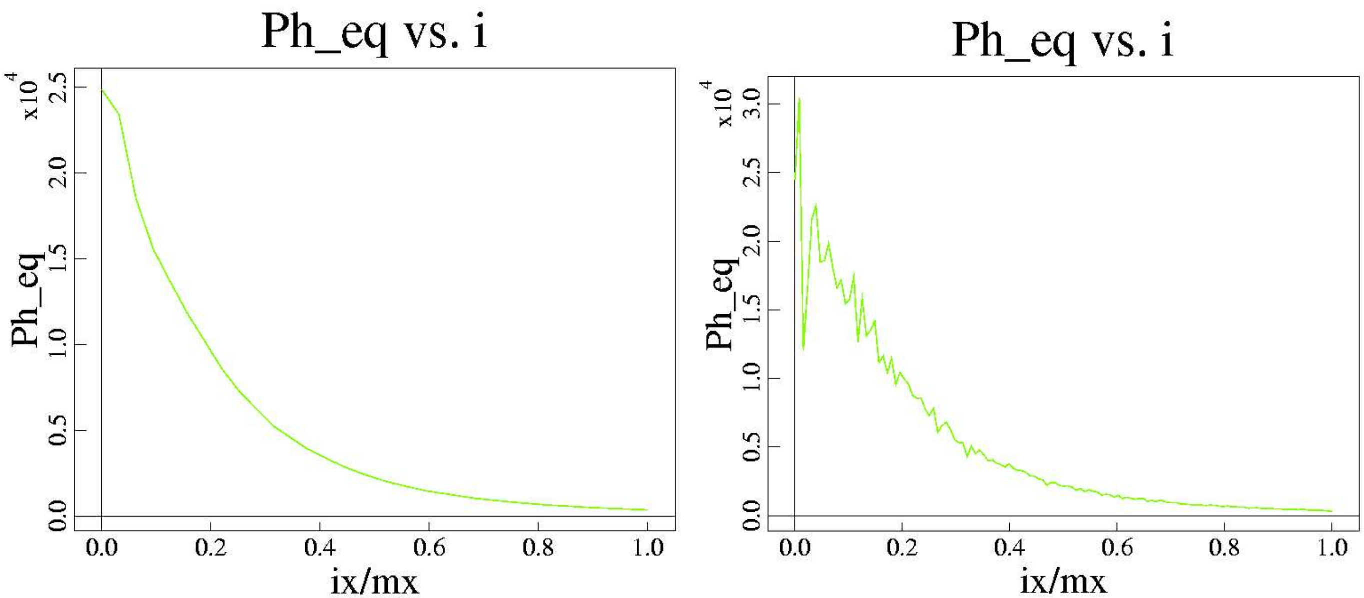

Figure 2: lower marked density of benchmark case shows noise near axis

  - $1M$particles on 128×128 grid (factor ~ 32 lower particle resolution)
  - not so bad because we are not concerned with axis
  - could use some sort of smoothing filter

Hot particle flow is not negligible

- canonical slowing down distribution showed negligible hot particle flow $\sim 10^4 m/s$
- bounce averaged slowing down distribution $f_0^{ba} = \frac{P_0 \exp(-\frac{\langle\psi\rangle}{\psi_0})}{\varepsilon^{3/2} + \varepsilon_c^{3/2}}$ where $\langle\psi\rangle = \langle g\rho_\parallel\rangle - P_\zeta$ and $\langle g\rho_\parallel\rangle = R_0 \frac{mv}{e}\sqrt{1 - \frac{\mu B_0}{\varepsilon}} sign(v_\parallel)$
- shows significant flow $\sim 10^5 m/s \sim \mathcal{O}(v_A)$

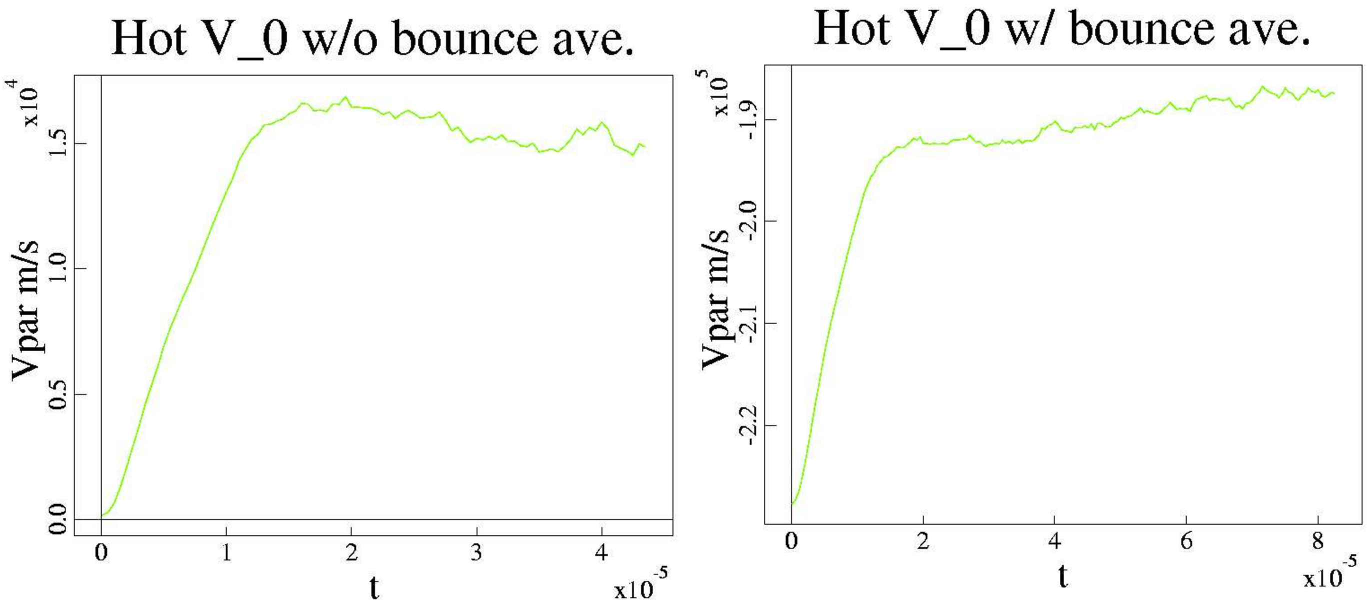

Figure 3: bounce averaged hot particle flow shows significant flow

β_h scan

- $S = 1.26e5$ and $Pr = 25$
- wrong parameters - baseline growth rate too small!

$\beta_h (\% \beta_{tot})$	$\gamma \tau_A$ w/o particles	$\gamma \tau_A$ w/ particles	ω / ω_A
0	.017	.017	0
30	.015	.0167	0
60	.012	.014	0
90	.009	.0308	.0357
100	.004	no data	no data

Table 1: γ vs. β_h w/ & w/o particles

- trend looks promising
- although no oscillation detected for smaller β_h, runs with particles show mode tilt

* bounce averaged distribution not used

* hot particle flow not taken into account

* current also not taken into account

Significant change of physics at $\beta_h = 90\%\beta_{tot}$

$\beta_h(\%\beta_{tot})$	$p_{eq\ max}$	$p_{h\ max}$
30	4.e-4	3.5e-5
60	2.e-4	4.5e-5
90	1.e-3	1.7e-3

Table 2: maximum amplitude of p_{MHD} and p_h

- significant change in relative amplitudes at $\beta_h = 90\%\beta_{tot}$
- $\beta_h = 90\%\beta_{tot}$ shows clear oscillations

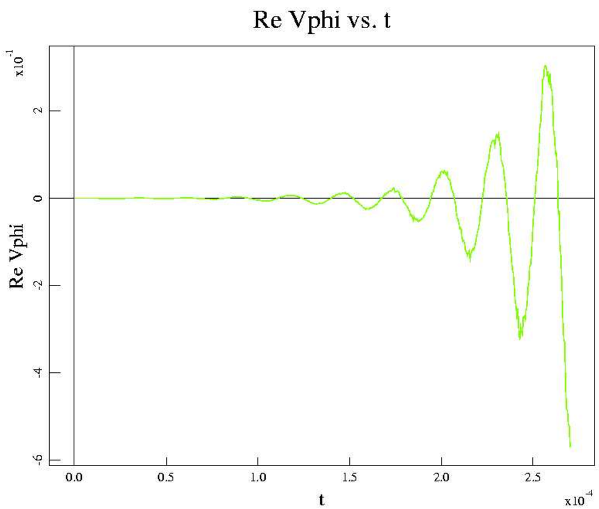

Figure 4: time trace of V_ϕ shows growth and oscillations

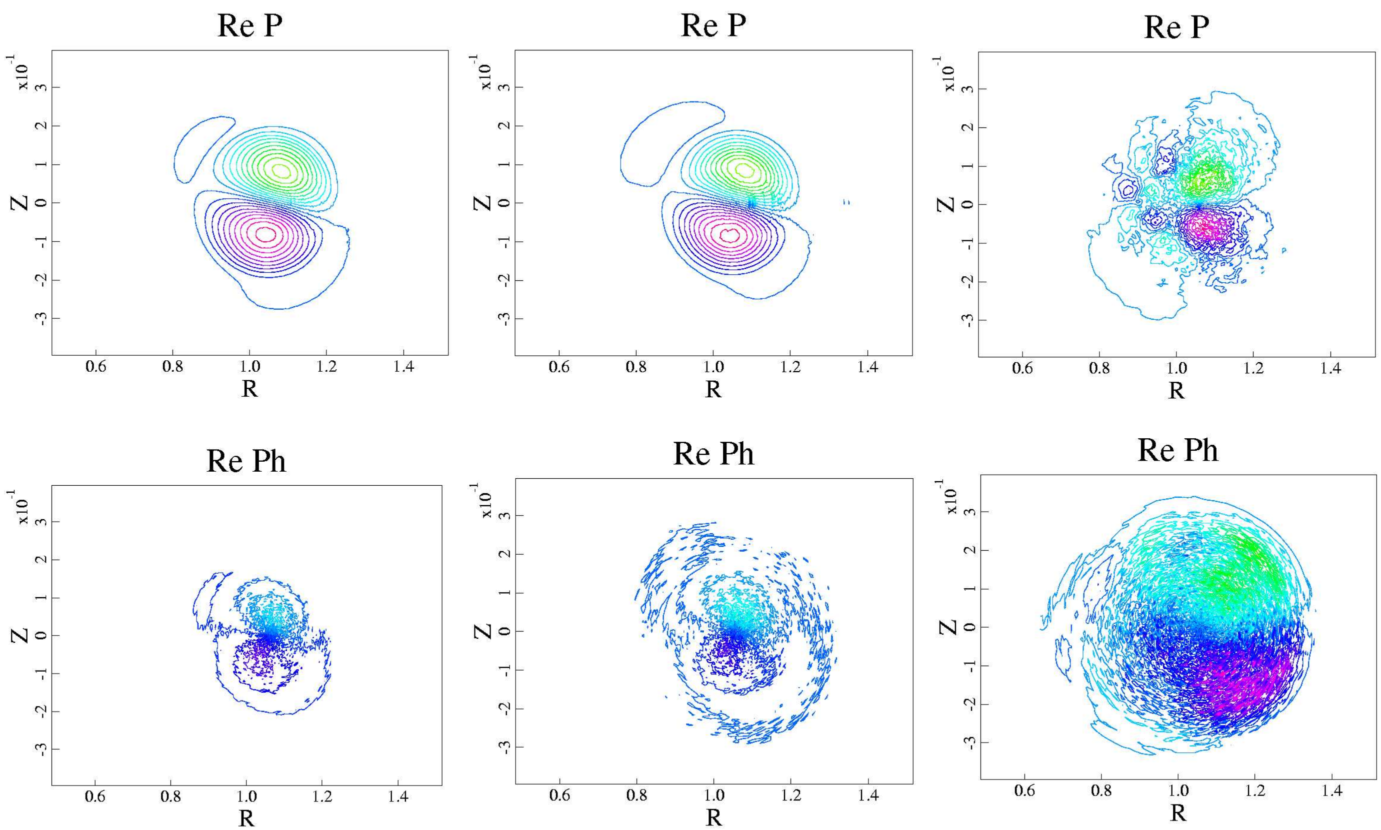

Figure 5: p_{MHD} & p_h $(1,1)$ contours for $\beta_h = [30, 60, 90]\%\beta_{tot}$ show marked difference at $\beta_h = 90\%$

Still To Do

- run with the correct parameters ($S \simeq 1.e5, Pr = 1?$)
- run with bounce average distribution
- correct for hot particle flow
- include current
- filter for particle noise