

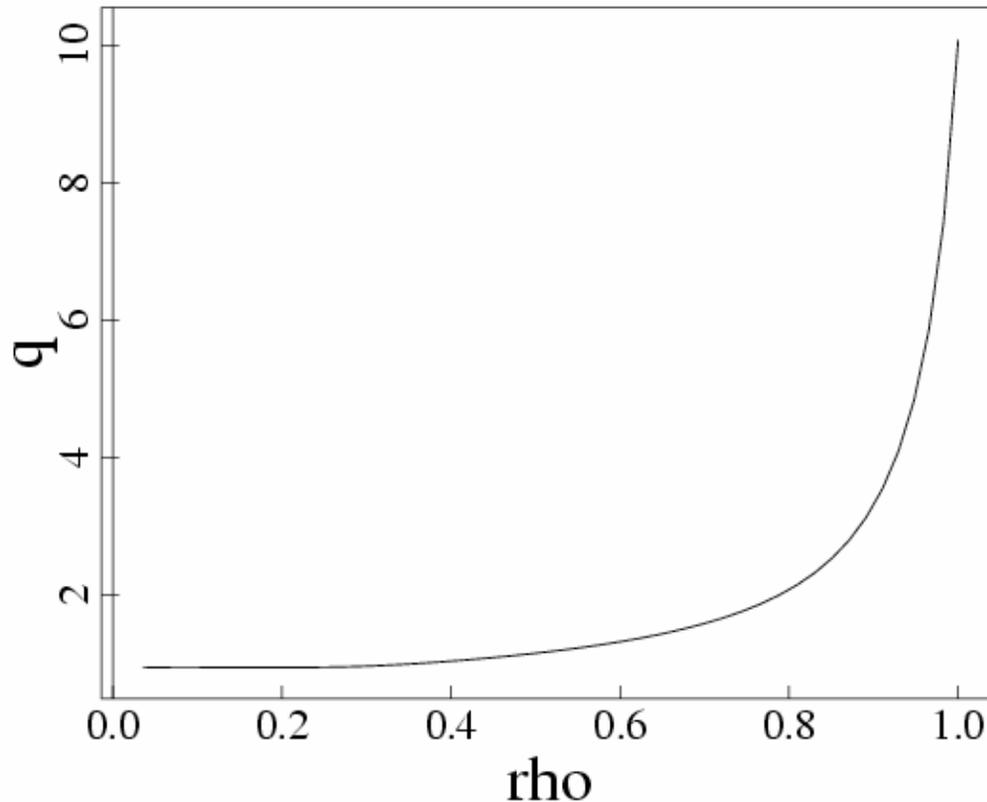
CDXU Benchmark Case

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CDXU is Low Aspect Ratio Tokamak



- Equilibrium produced from TSC calculation (*jsolver* file)
- $R/a = 1.4$
- $q(0) = 0.95$
- $q(a) \sim 10$
- Benchmark with M3D:
 - Linear growth rate and eigenfunction
 - Nonlinear evolution

Benchmark Parameters

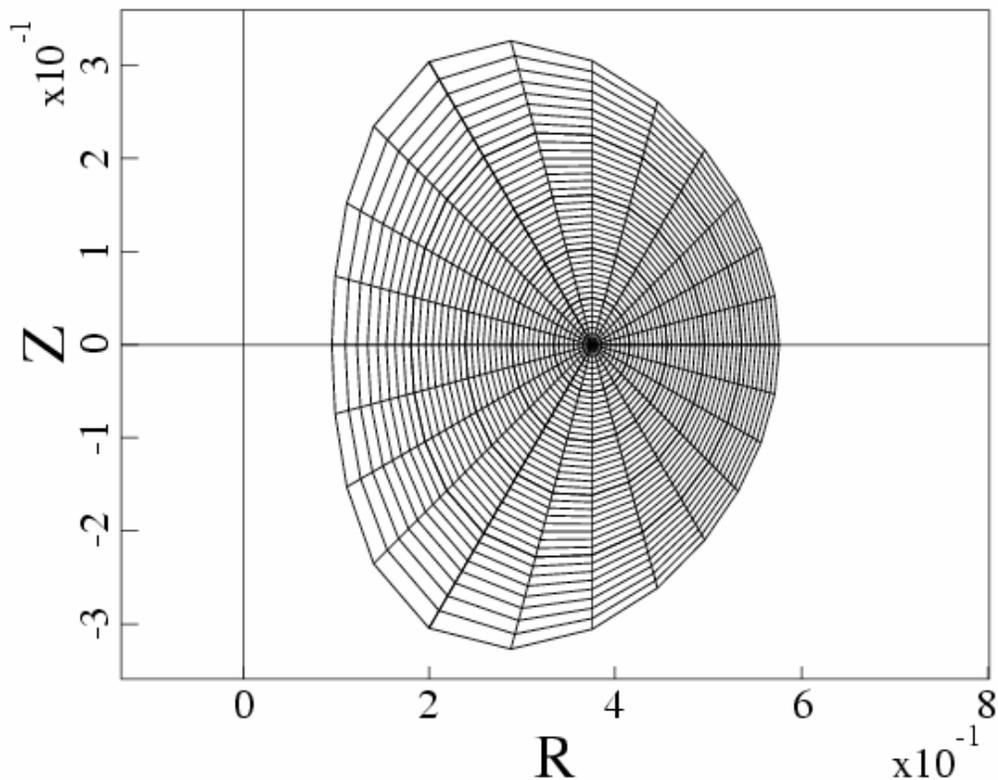
- $S(0) = 1.97 \times 10^4$
- $\beta \sim 10^{-2}$ (low- β)
- $\eta \sim T_{\text{eq}}^{-3/2}$: fixed profile ($\eta(\mathbf{r},t)$ also considered)
- $\text{Pr} = 10$: flat profile ($\text{Pr} = 100$ and shaped profiles also considered)
- Density evolution:
 - Fixed equilibrium profile for linear case (little or no effect on NIMROD results)
 - Full 3-D evolution for non-linear cases
- Thermal conduction:
 - Isotropic: $\kappa_{\perp} = 200 \text{ m}^2/\text{sec}$ (CDXU confinement time)
 - Anisotropic: $\kappa_{\parallel} = 10^8 \text{ m}^2/\text{sec}$
 - Adiabatic and $\kappa_{\perp} = 1 \text{ m}^2/\text{sec}$ cases also considered

Benchmarking Issues

- Units
- Codes don't produce the same output
- Evolution of "Equilibrium"
- Results strongly dependent on dissipation *parameters* and *profiles*
- $n = 1$ is not a "standard" 1/1 sawtooth mode
 - Strong $m > 1$ components appear at inboard edge
- Higher- n modes are more unstable than $n = 1$
 - Low n
 - Localized at inboard edge
 - High n
 - Move toward outboard edge
- Role of resistivity evolution?
- Is sufficient toroidal resolution possible?
- Is there a better benchmark case?

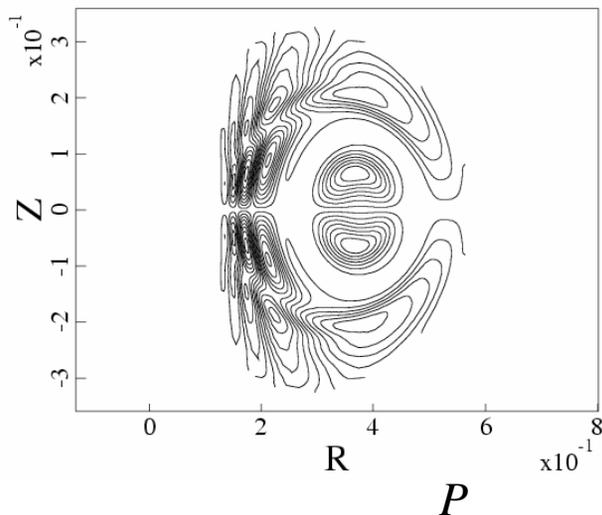
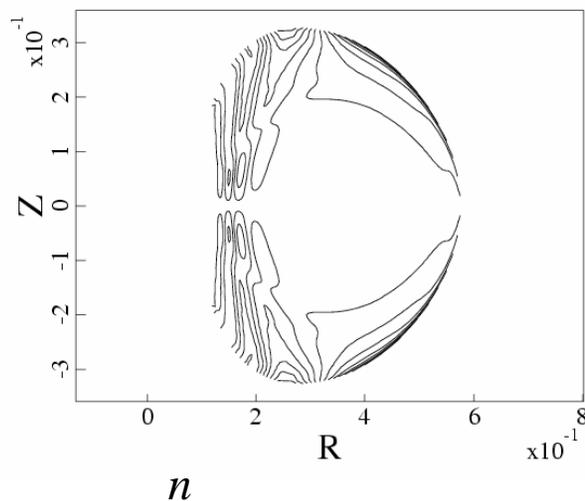
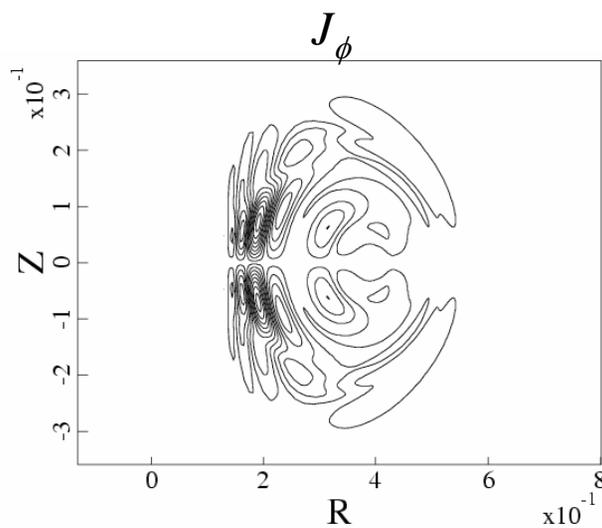
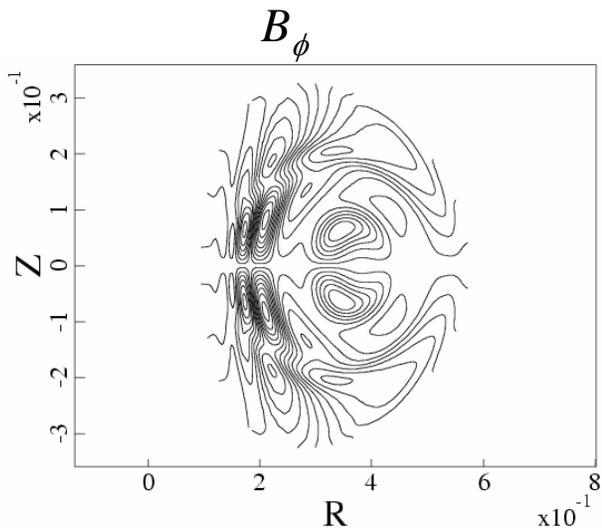
The NIMROD Poloidal Grid

Finite Element Mesh



- 40×24 grid
- 4th order finite elements
- Conducting wall

The “Standard” Case



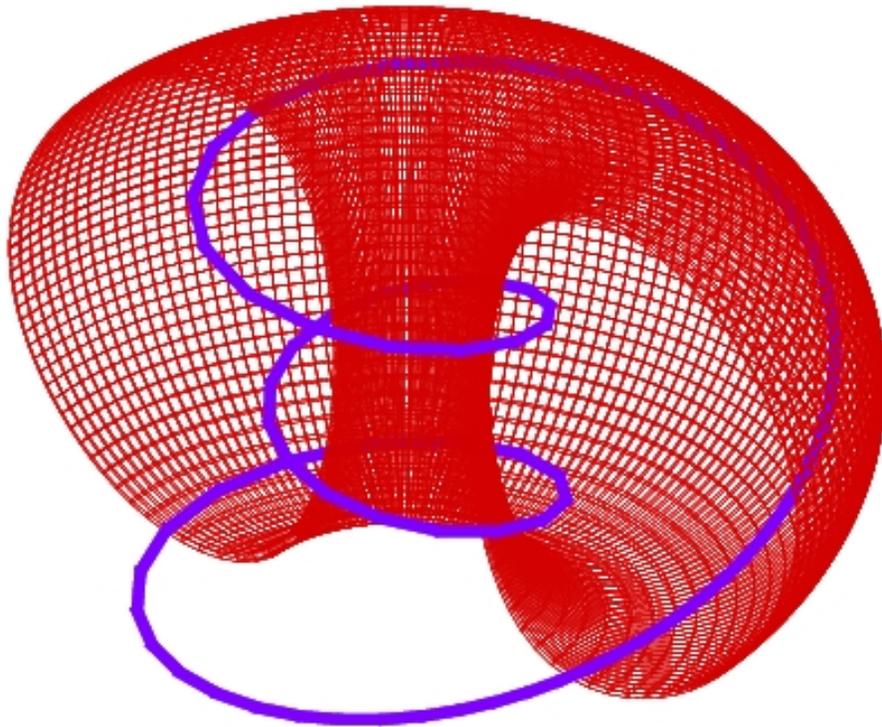
- $n = 1$
- $Pr = 10$
- $\nu = \text{const.}$
- $\eta \sim T_{\text{eq}}^{-3/2}$ (fixed)
- Isotropic heat flux:
 - $\kappa_\perp = 200 \text{ m}^2/\text{sec}$
- Small $m = 1$
- Higher m components at inboard edge
- Density evolution (no effect on linear behavior)

$$\gamma\tau_A = 3.7 \times 10^{-4}$$

The Variations

- “Standard” case: $Pr = 10$ (flat), $\kappa_{\perp} = 200 \text{ m}^2/\text{sec}$ (isotropic)
 - $\gamma\tau_A = 3.7 \times 10^{-4}$
- Variation 1: $Pr = 10$ (flat), $\kappa_{\perp} = 1 \text{ m}^2/\text{sec}$ (isotropic)
 - $\gamma\tau_A = 1.8 \times 10^{-3}$
- Variation 2: $Pr = 10^2$ (flat), $\kappa_{\perp} = 200 \text{ m}^2/\text{sec}$ (isotropic)
 - *Stable!*
- Variation 3: $Pr = 10^2$ (flat), $\kappa_{\perp} = 1 \text{ m}^2/\text{sec}$ (isotropic)
 - $\gamma\tau_A = 4.5 \times 10^{-4}$
- Variation 4: $Pr = 10$ (flat), $\kappa_{\perp} = 200 \text{ m}^2/\text{sec}$, $\kappa_{\parallel} = 10^8 \text{ m}^2/\text{sec}$
 - *Marginal*
- Variation 5: $Pr = 10$ (flat), $\kappa_{\perp} = 1 \text{ m}^2/\text{sec}$, $\kappa_{\parallel} = 10^8 \text{ m}^2/\text{sec}$
 - *Marginal*
- Variation 6: $Pr = 10^2$ (flat), adiabatic pressure
 - $\gamma\tau_A = 1.7 \times 10^{-4}$
- Variation 7: $Pr = 10$ (flat), adiabatic pressure
 - $\gamma\tau_A = 2.4 \times 10^{-3}$

Mode Structure?

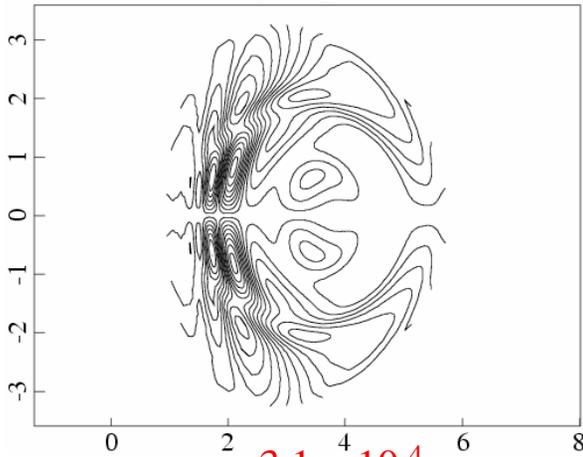


Equilibrium field line with pitch
 $m = 3, n = 1$

- Low aspect ratio
- Low- n field lines make more turns on inboard side
- Mode localized along equilibrium field line will have more structure on inboard side
- Higher- n ?????

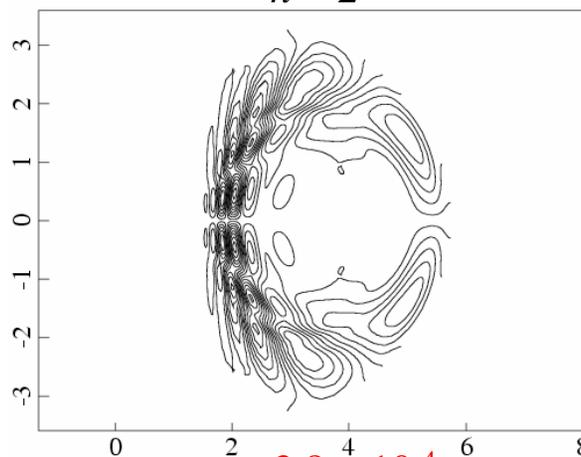
Effect of toroidal mode number n

$n = 1$



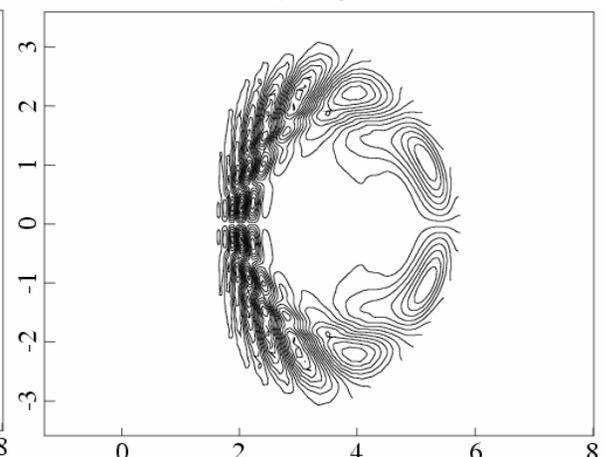
$\gamma\tau_A = 3.1 \times 10^{-4}$

$n = 2$



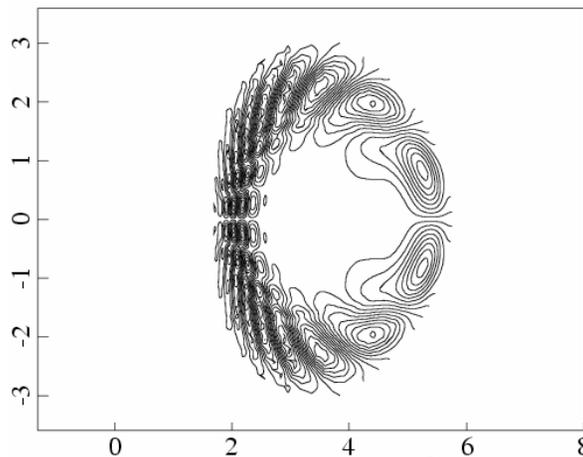
$\gamma\tau_A = 3.8 \times 10^{-4}$

$n = 3$



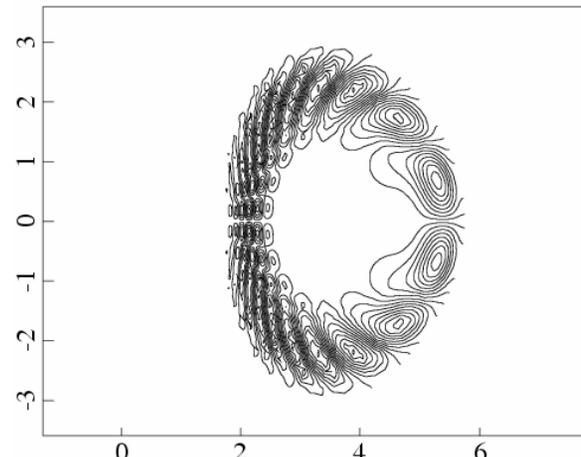
$\gamma\tau_A = 3.4 \times 10^{-3}$

$n = 4$



$\gamma\tau_A = 4.9 \times 10^{-3}$

$n = 5$

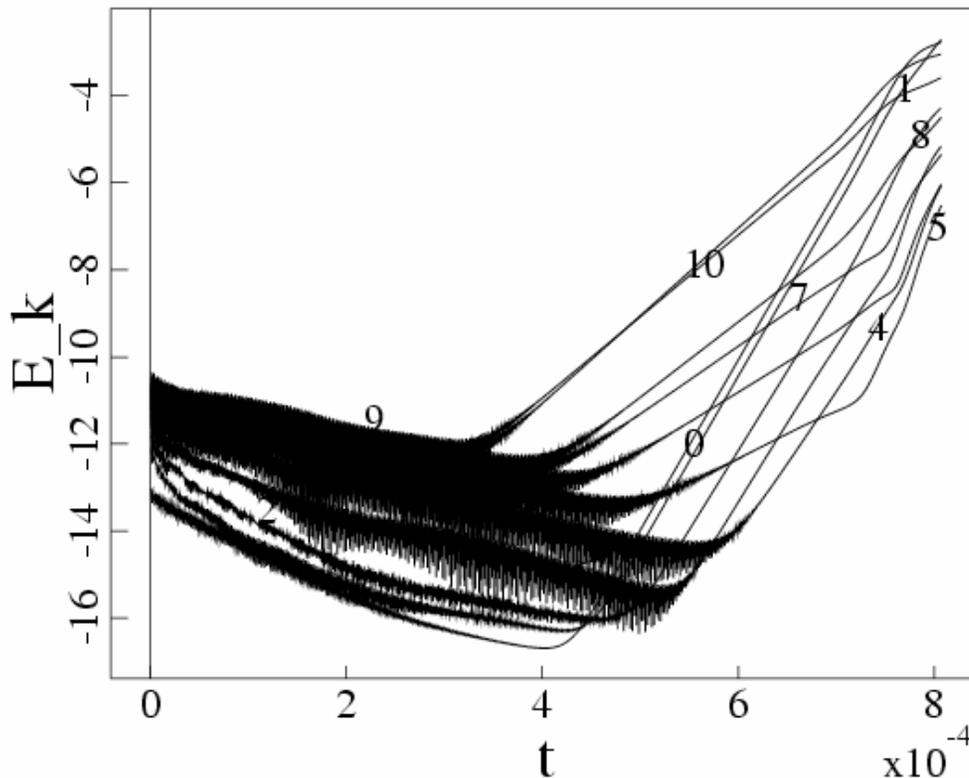


$\gamma\tau_A = 6.1 \times 10^{-3}$

- Edge of discharge
- Move slightly outboard with increasing n
- Growth rate increases with n

Nonlinear “Standard” Case

Kinetic Energy vs. t



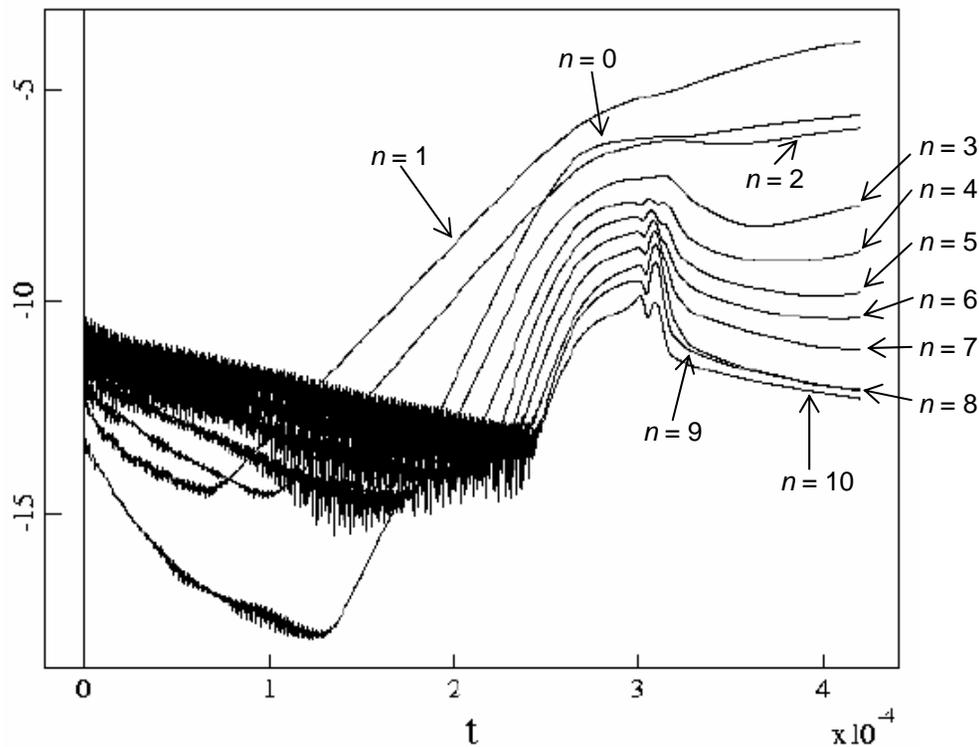
- $n = 10, 9, \dots$ most unstable
- Lower n (e.g., $n = 1$) now *nonlinearly driven*:

$$\mathcal{V}_{n=1} \sim \mathcal{V}_{n=9} + \mathcal{V}_{n=10}$$

- Completely different from linear picture
- Can there ever be enough toroidal resolution?

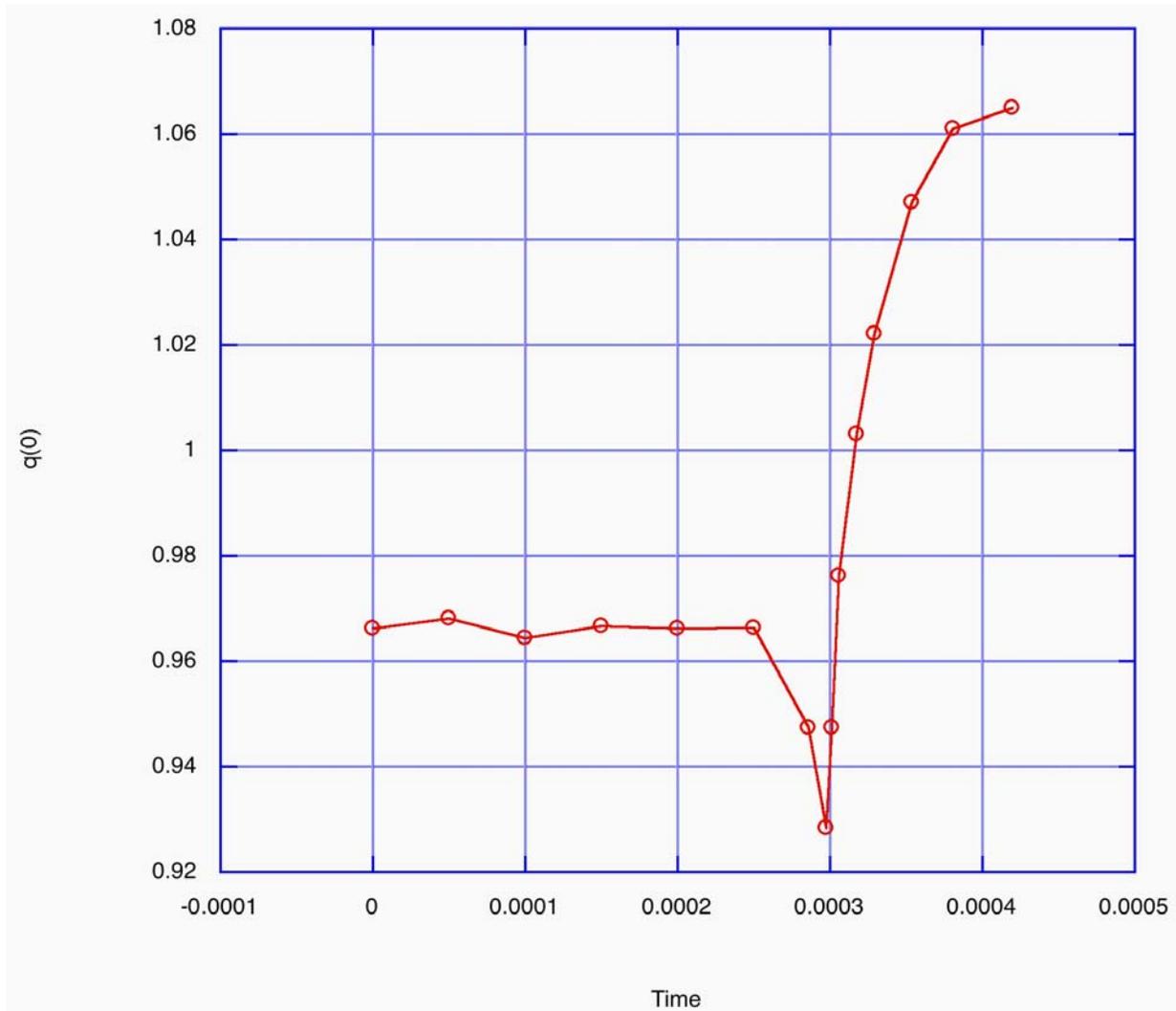
“Alternative” Nonlinear Case:

$$q(0)=0.97$$



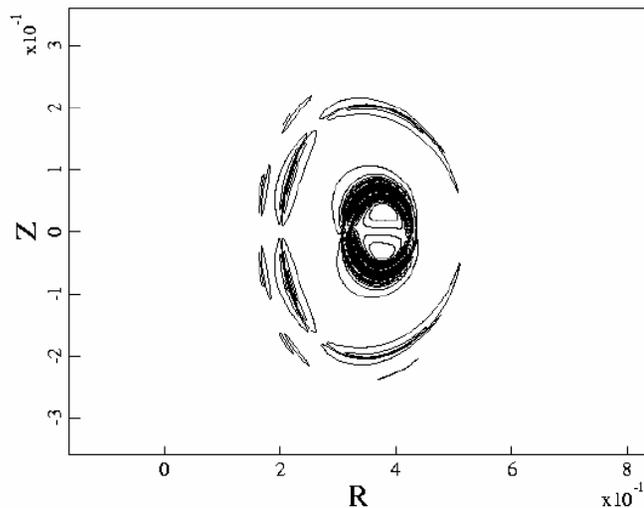
- *Linear* $n = 1$ is dominantly $1/1$ (“sawtooth”)
- *Nonlinear* $n = 1$ changes from $1/1$ to $2/1$
- Finite sized $2/1$ mode growth nonlinearly in Rutherford regime
- No indication of high- n instability

Evolution of $q(0)$



$n = 1$ mode Changes After Reconnection

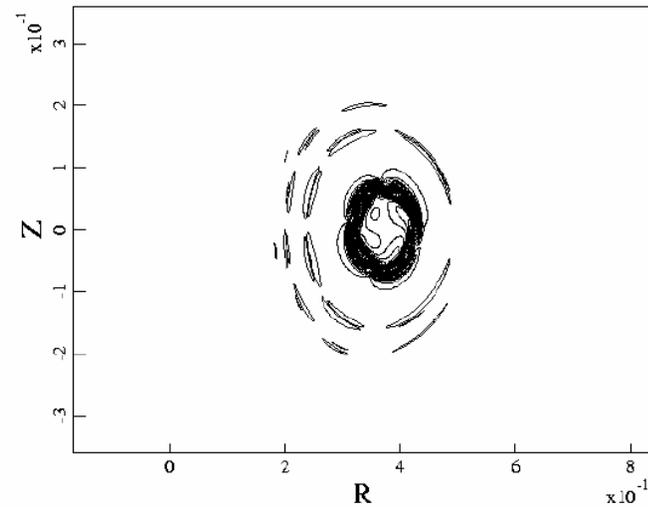
Before sawtooth saturation



$t = 2.5 \times 10^{-4}$ sec.

1/1 with harmonics

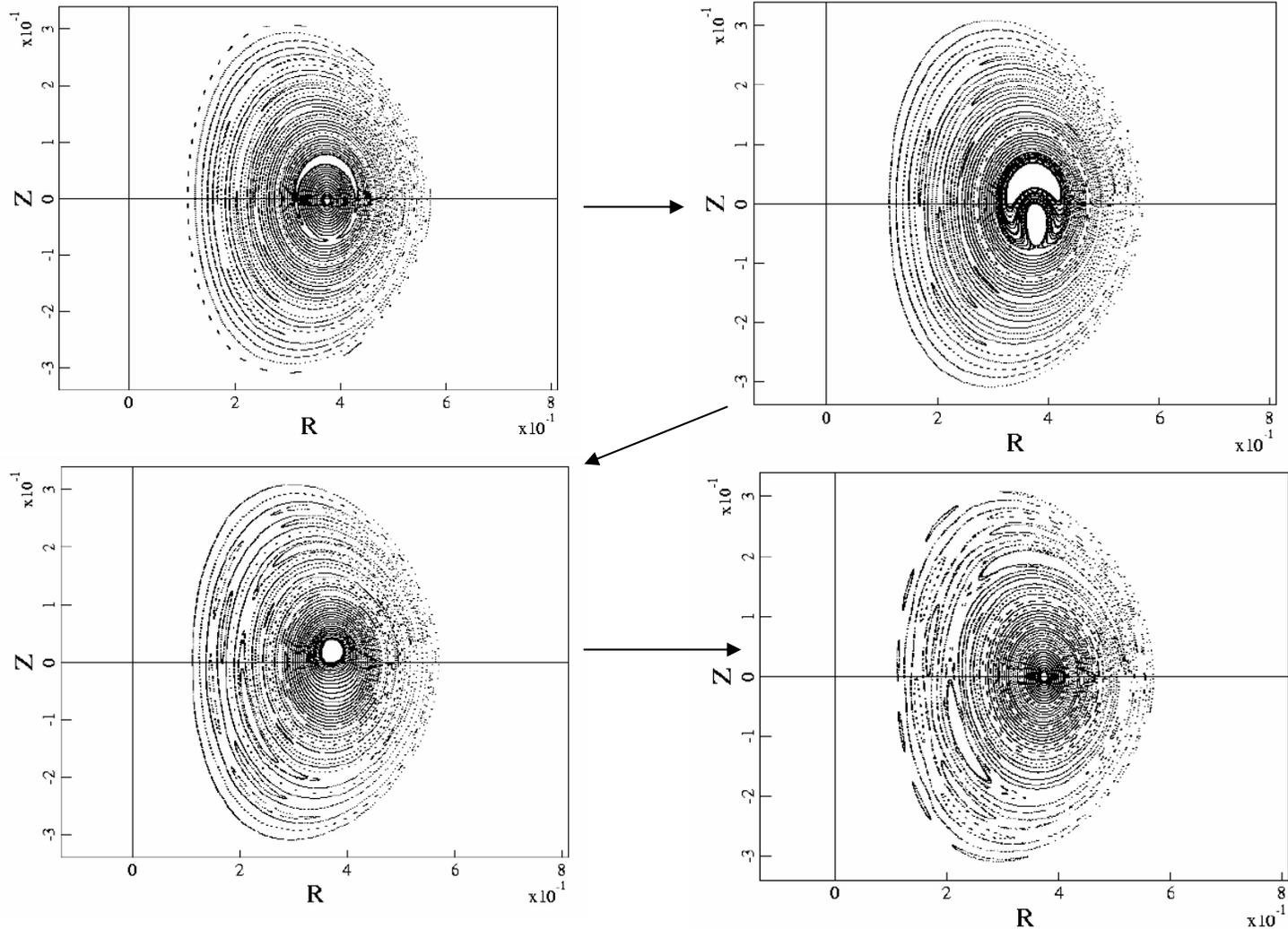
After sawtooth saturation



$t = 4.20 \times 10^{-4}$ sec.

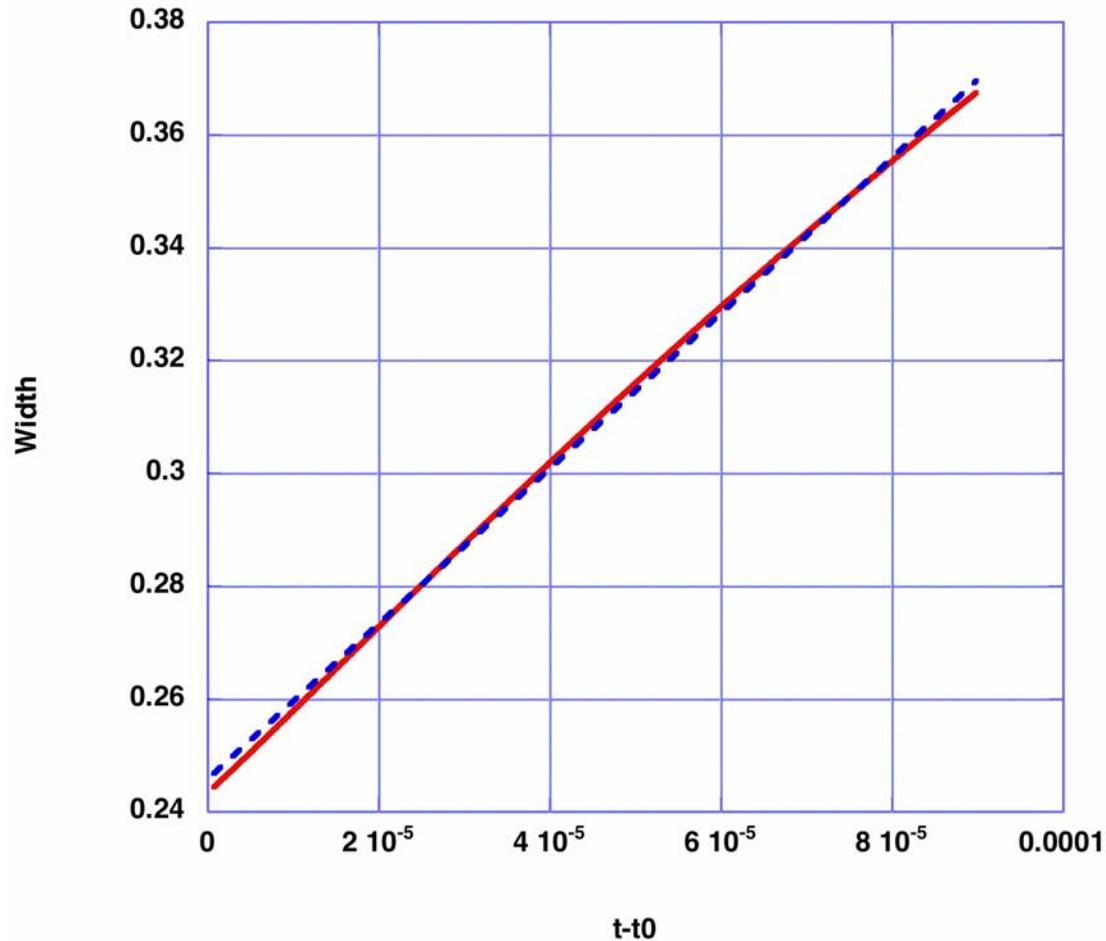
2/1 with harmonics

1/1 Reconnection and 2/1 Growth

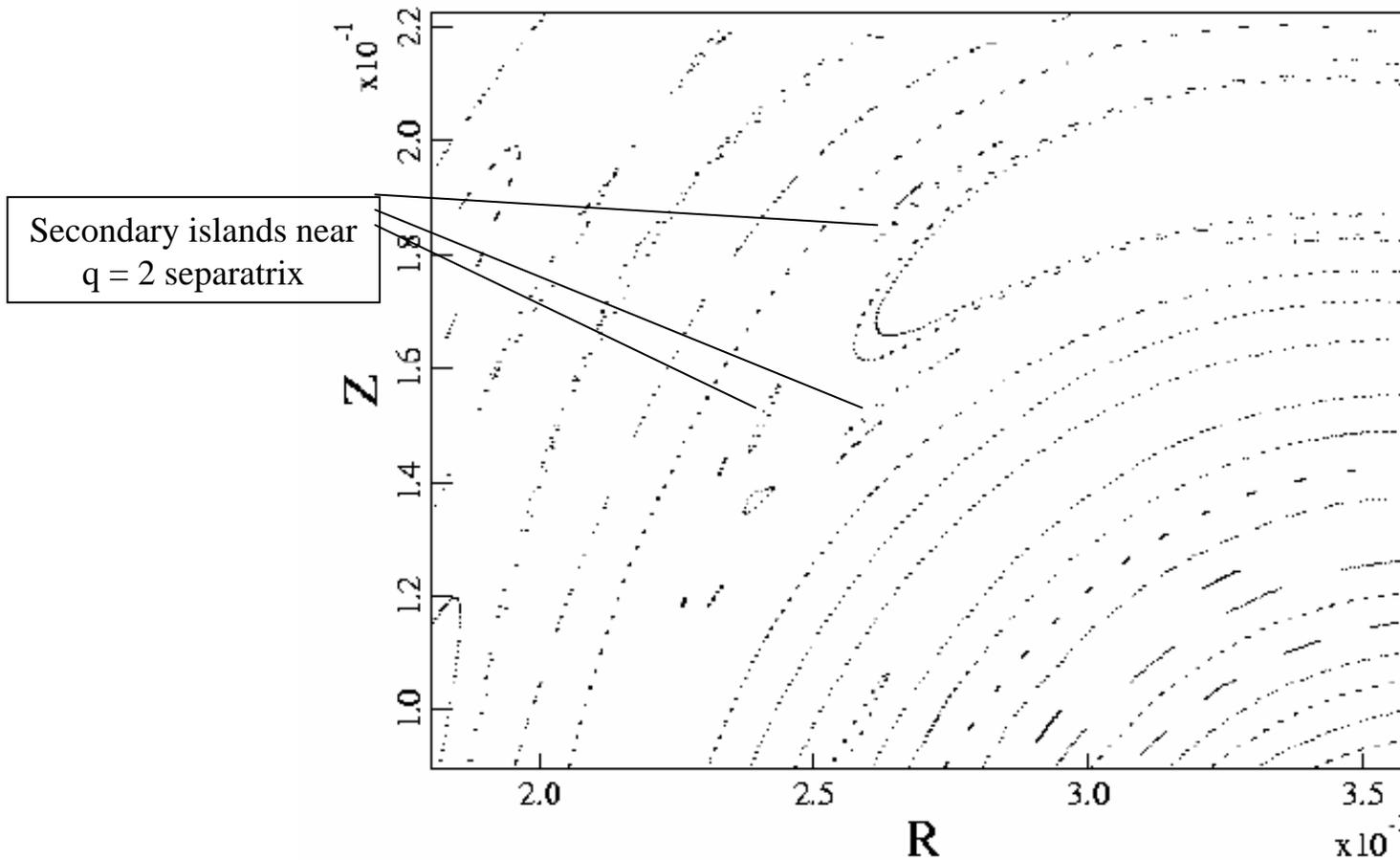


Growth of 2/1 Island after 1/1 Reconnection

2/1 Island Width vs. Time



Field Line Structure near $q = 2$ Surface



Remarks

- At agreed parameters, low- $q(0)$ CDXU profile has non-standard stability properties:
 - “Sawtooth mode” (1/1) is at best marginally unstable
 - Outer parts of q -profile drive instability at higher m
 - Growth rate increases with increasing n , mode structure becomes more complex
- Can we ever achieve sufficient toroidal resolution for this case for nonlinear calculation?
- Poor case to study non-linear sawtooth evolution?
- Is $q(0) = 0.95$ physically relevant to CDXU?
 - $q(0) = 0.97$ case exhibits beautifully dominant sawtooth, no high- n instability, rich nonlinear dynamics
 - Would CDXU ever achieve $q(0) = 0.95$?