Free-Boundary Magnetohydrodynamic Simulations of DIII-D Tokamak Plasmas with NIMROD

NIMROD

Non-Ideal MHD with Rotation
Open Discussion Project

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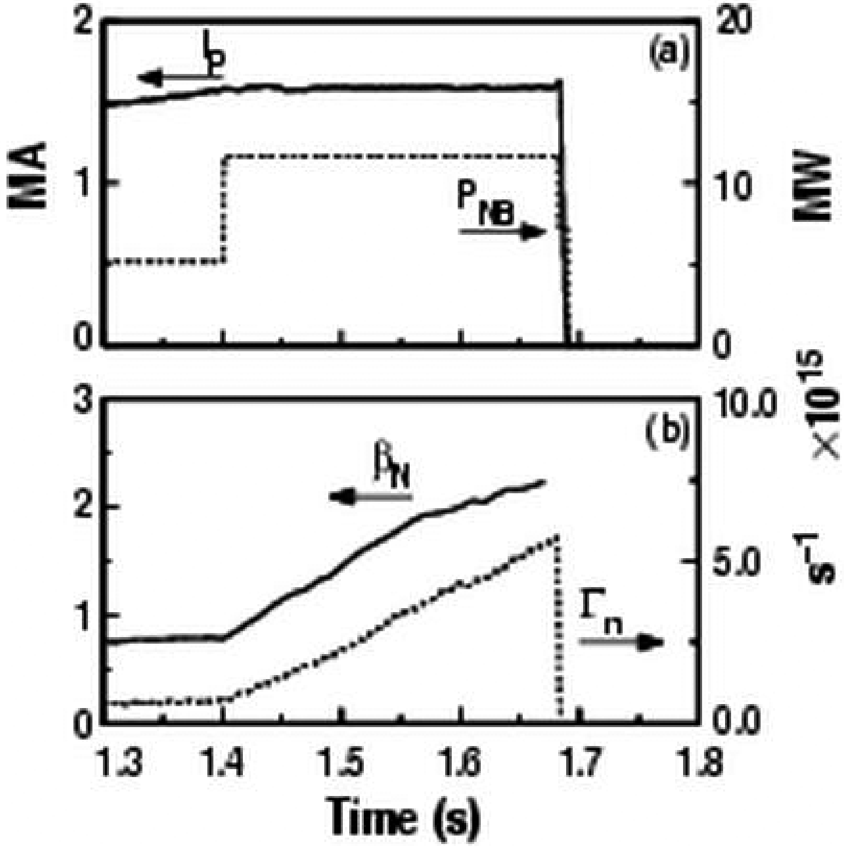
Outline

- Motivation
  - High-beta disruption discharge on DIII-D tokamak
  - Simple analytic theory

- NIMROD Modeling
  - Fixed boundary
  - Free-boundary using equilibria above marginal limit
  - Free-boundary using "best-fit" equilibria

Acknowledgements: C. Sovinec, J. Callen (UW-M),
A. Turnbull, M. Chu (GA)

DIII-D SHOT #87009 Observes a Plasma Disruption During Neutral Beam Heating At High Plasma Beta



Callen et.al, Phys. Plasmas 6, 2963 (1999)

Mode Passing Through Instability Point Has Faster-Than-Exponential Growth

- **In experiment mode grows faster than exponential**
- **Theory of ideal growth in response to slow heating**
  (*Callen, Hegna, Rice, Strait, and Turnbull, Phys. Plasmas 6, 2963 (1999)*):

Heat slowly through critical β:

$$\beta = \beta_c (1 + \gamma_h t)$$

Ideal MHD:

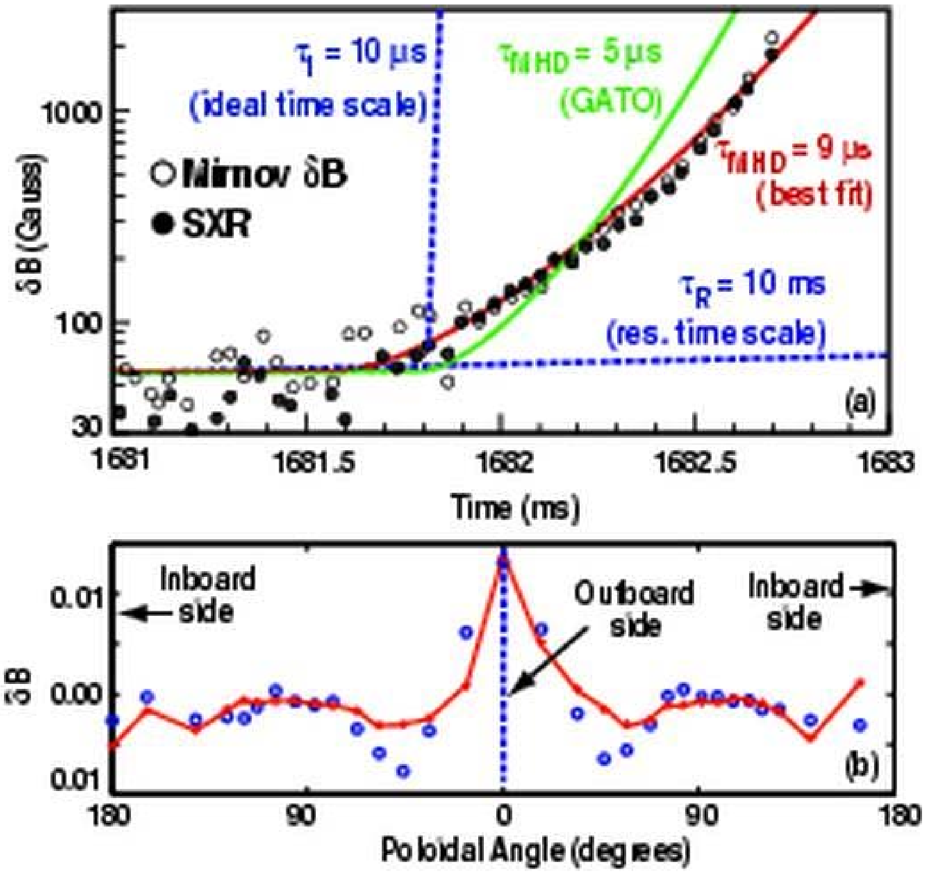
$$\omega^2 = -\hat{\gamma}^2_{MHD} (\beta / \beta_c - 1) \quad \Rightarrow \quad \gamma(t) = \hat{\gamma}_{MHD} \sqrt{\gamma_h t}$$

Perturbation growth:

$$\frac{d\xi}{dt} = \gamma(t)\xi \quad \Rightarrow \quad \xi = \xi_0 \exp[(t / \tau)^{3/2}], \quad \tau = (3/2)^{2/3} \hat{\gamma}_{MHD}^{-2/3} \gamma_h^{-1/3}$$

DIII-D SHOT #87009 Observes a Mode on Hybrid Time Scale As Predicted By Analytic Theory

- **Growth is slower than ideal, but faster than resistive**



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Resistive MHD Equations Used to Numerically Model Disruption

- **MHD Equations Solved:**
  - **Density Equation:**

$$\frac{\partial n}{\partial t} + \nabla \cdot n\mathbf{V} = 0$$

  - **Momentum Equation**

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V}\right) = \mathbf{J}\times\mathbf{B} - \nabla p - \mu\nabla^2\mathbf{V}$$

  - **Resistive MHD Ohm's Law:**

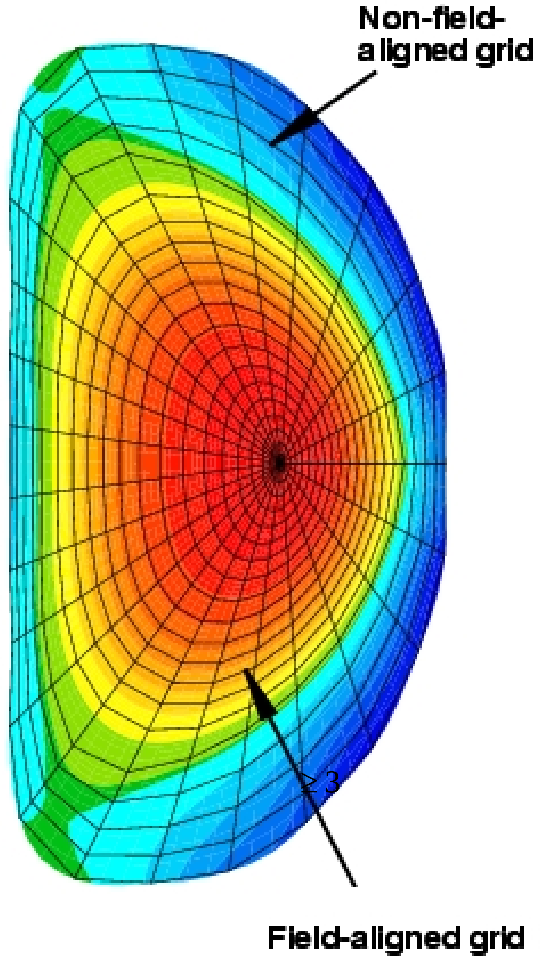
$$\mathbf{E} = \underbrace{-\mathbf{V}\times\mathbf{B}}_{\text{Ideal MHD}} + \underbrace{\eta\mathbf{J}}_{\text{Resistive MHD}}$$

  - **Temperature Equations:**

$$\frac{\partial T_\alpha}{\partial t} + \mathbf{V}_\alpha\cdot\nabla T_\alpha + \gamma T_\alpha \nabla\cdot\mathbf{V}_\alpha = -(\gamma-1)\nabla\cdot\mathbf{q}_\alpha + (\gamma-1)Q_\alpha$$

$$\text{Currently:}\quad \mathbf{q}_\alpha = -\kappa_{\parallel}\mathbf{bb}\cdot\nabla T - (\kappa_{\perp} - \kappa_{\parallel})\nabla T$$

Spatial Discretization Uses Finite-Elements in Poloidal Plane, Pseudospectral in Toroidal Angle



- **Can parallelize by FE blocks and by toroidal mode number**
- **Lagrangian elements of arbitrary polynomial degree (specified at runtime)**
  - **Spectral convergence needed for realistic conditions: Error ~ h^{p+1}**

High S:

Use polynomial degree ≥ 3

High $\kappa_{||}/\kappa_{perp}$:

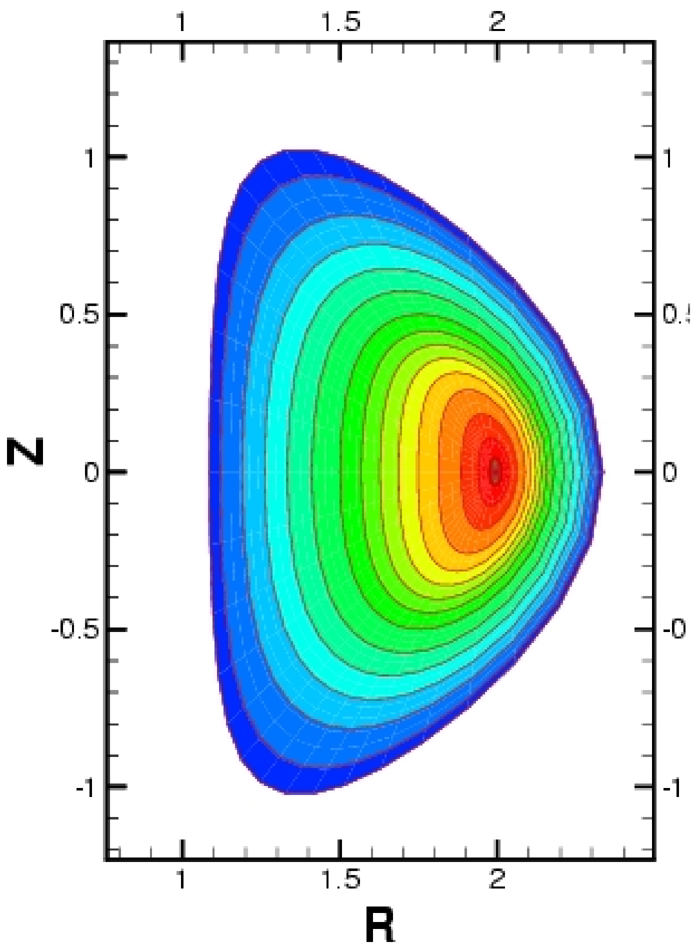
Use polynomial degree ≥ 4

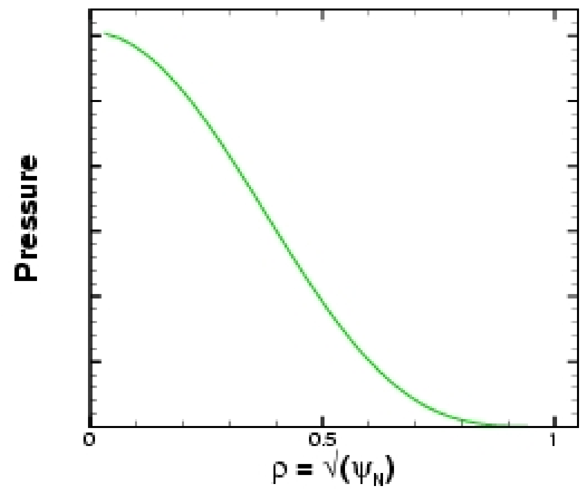
See Sovinec et.al. ***submitted to Journal of Computational Physics***

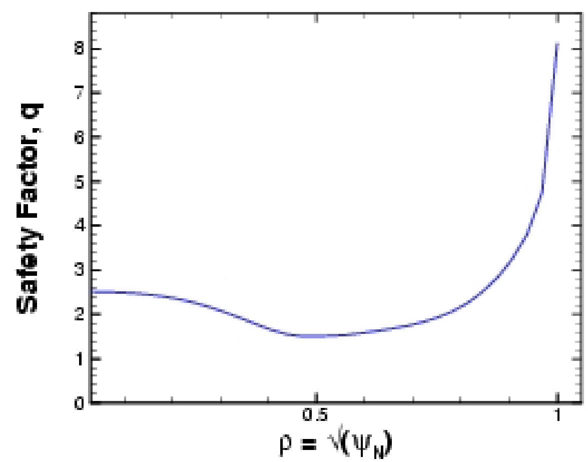
(Draft at http://nimrodteam.org)

Initial Simulations Performed Using Fixed Boundary Equilibria

- Use q and pressure profile from experiment
- Negative central shear

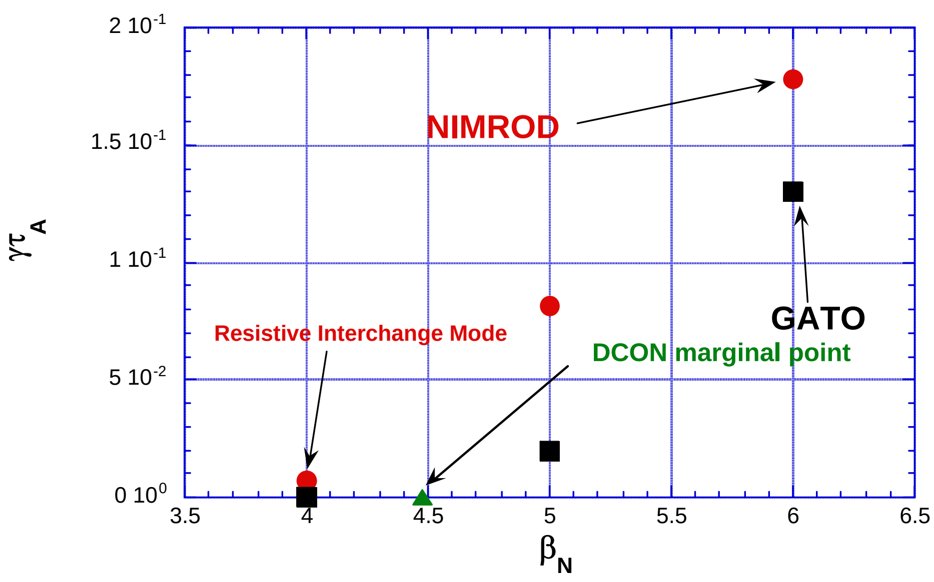






Fixed Boundary Simulations Require Going to Higher Beta

- Conducting wall raises ideal stability limit
  - Need to run near critical β_N for ideal instability NIMROD gives slightly larger ideal growth rate than GATO
- NIMROD finds resistive interchange mode below ideal stability boundary



Nonlinear Simulations Find Faster-Than-Exponential Growth As Predicted By Theory

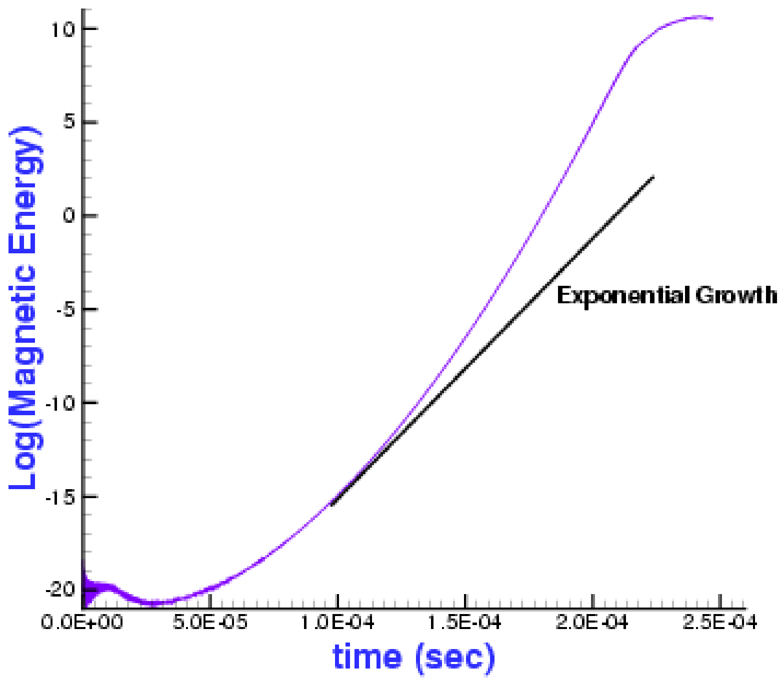
- **Initial condition: equilibrium below ideal marginal β_N**
- **Use resistive MHD**
- **Impose heating source proportional to equilibrium pressure profile**

$$\frac{\partial P}{\partial t} = \quad ..... \quad + \gamma_H P_{eq}$$

$$\Rightarrow \quad \beta_N = \beta_{Nc}\left(1 + \gamma_H t\right)$$

- **Follow nonlinear evolution through heating, destabilization, and saturation**

Log of magnetic energy in n = 1 mode vs. time
$S = 10^6$ Pr = 200 $\gamma_H = 10^3$ sec^{-1}



Scaling With Heating Rate Gives Good Agreement With Theory

- **NIMROD simulations also display super-exponential growth**
- **Simulation results with different heating rates are well fit by $\xi \sim \exp[(t-t_0)/\tau]^{3/2}$**
- **Time constant scales as**

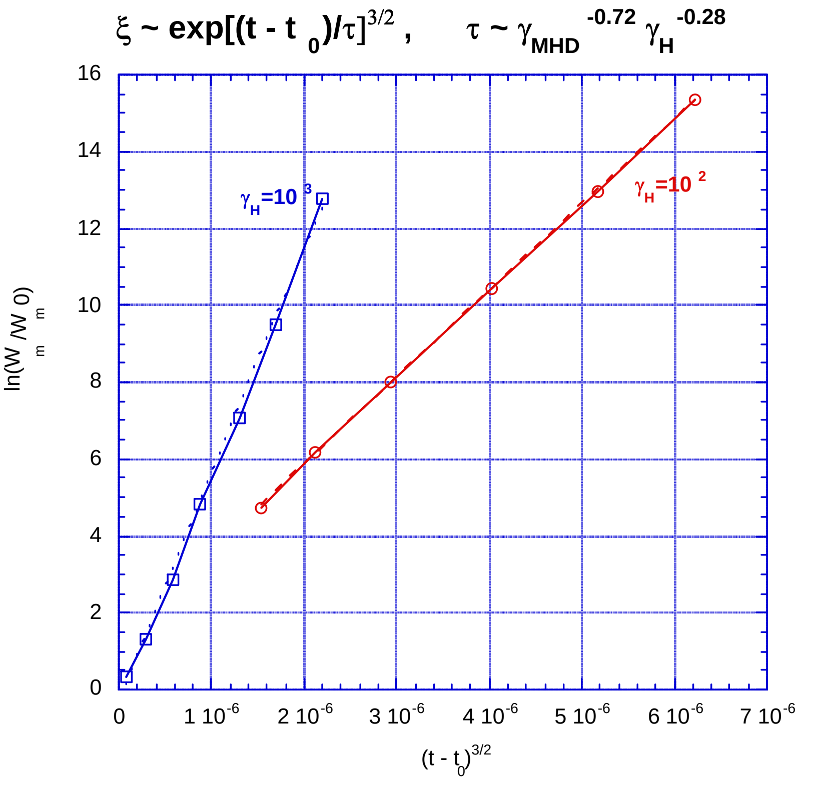
$$\tau \sim \gamma_{MHD}^{-0.72} \gamma_H^{-0.28}$$

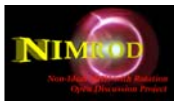
- **Compare with theory:**

$$\tau = (3/2)^{2/3} \hat{\gamma}_{MHD}^{-2/3} \gamma_h^{-1/3}$$

- **Discrepancy possibly due to non-ideal effects**

Log of magnetic energy vs. $(t - t_0)^{3/2}$ for 2 different heating rates



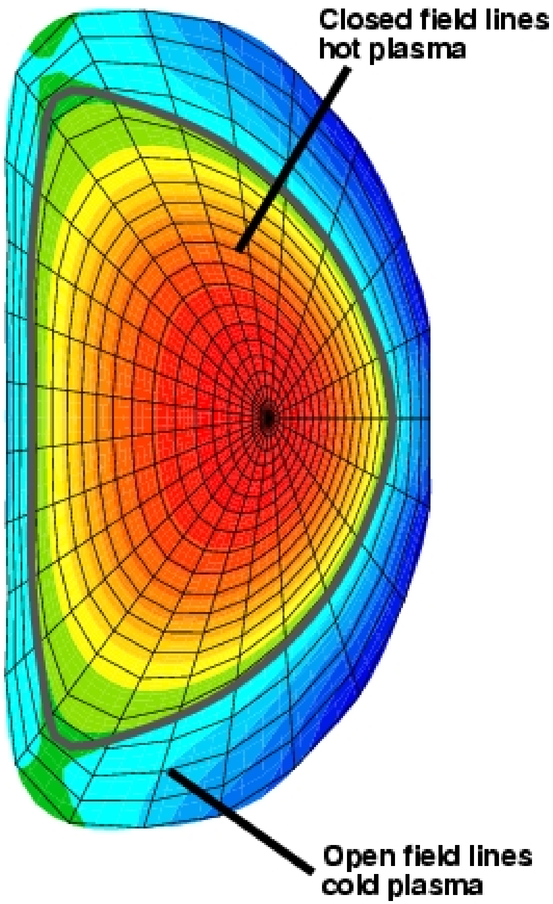


Free-Boundary Simulations Models
"Halo" Plasma as Cold, Low Density Plasma

- **Typical DIII-D Parameters:**
  T_{core}~10 keV **T_{sep}~1-10 eV**
  n_{core}~5×10^{19} m^{-3} **n_{sep}~ 10^{18} m^{-3}**

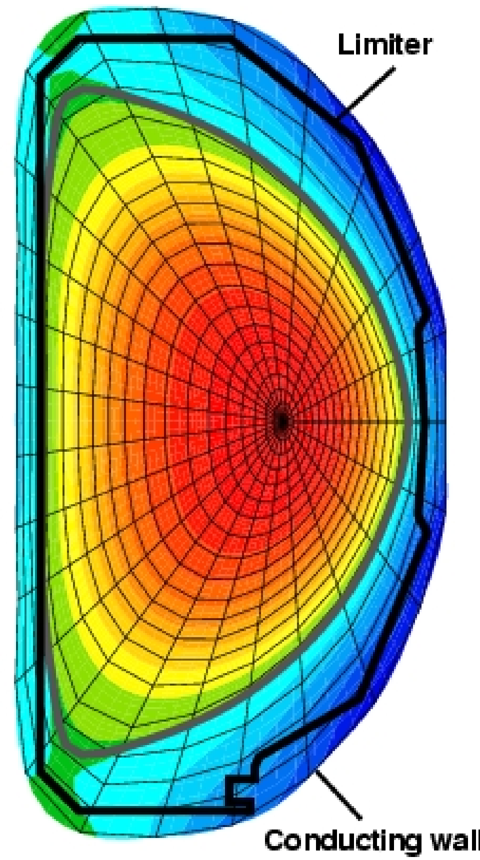
- **Spitzer resistivity: $\eta \sim T^{-3/2}$**
  - **Suppresses currents on open field lines**
  - **Large gradients 3 dimensionally**

- **Requires accurate calculation of anisotropic thermal conduction to distinguish between open and closed field lines**



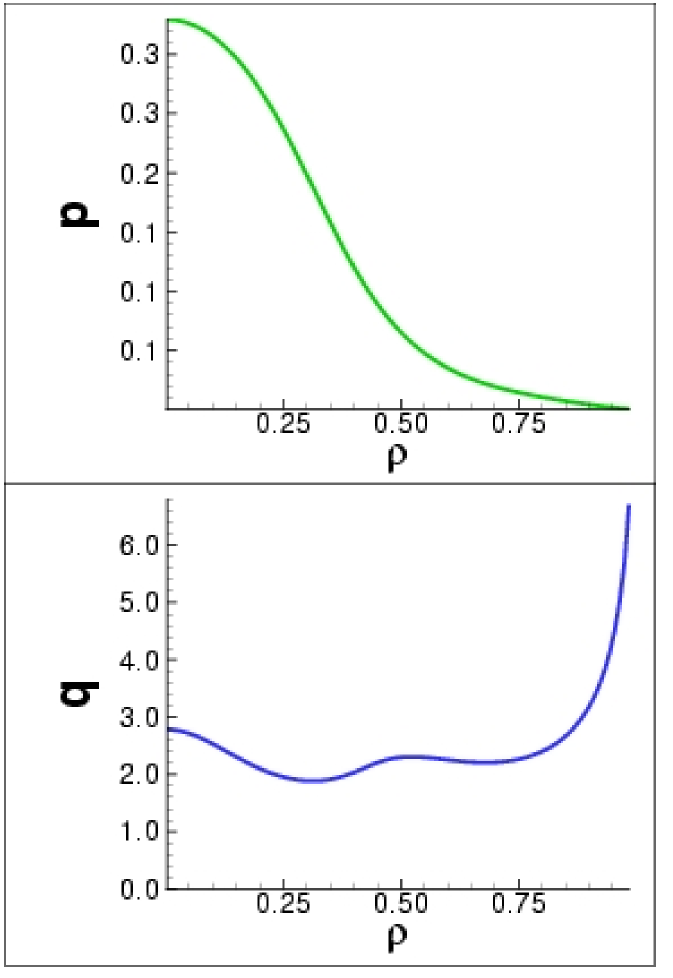
Goal of Simulation is to Model Power Distribution On Limiter during Disruption

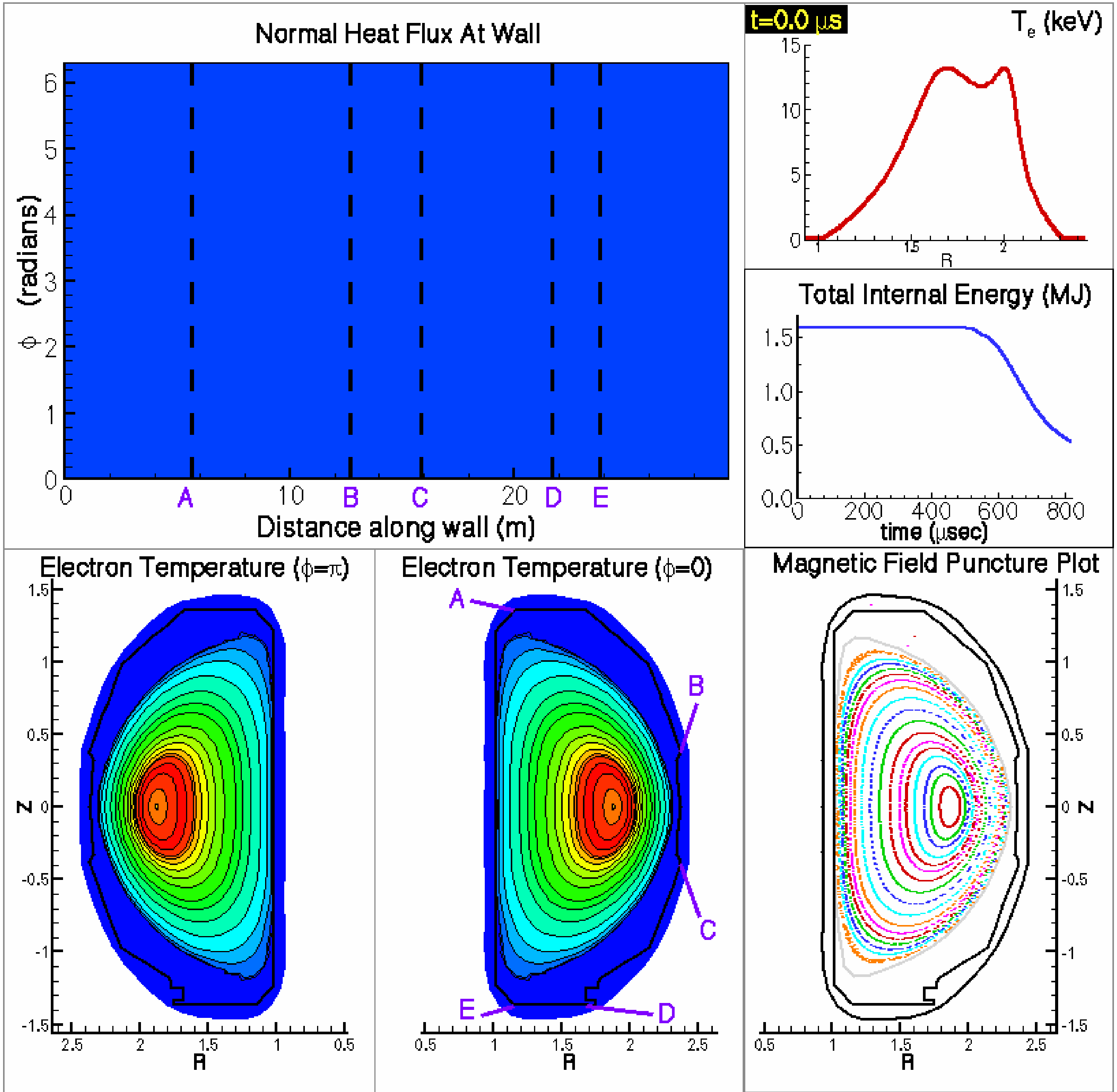
- **Plasma-wall interactions are complex and beyond the scope of this simulation**
- **Boundary conditions are applied at the vacuum vessel, NOT the limiter.**
  - **Vacuum vessel is conductor**
  - **Limiter is an insulator**
- **This is accurate for magnetic field:**
  - **B_n=constant at conducting wall**
  - **B_n can evolve at graphite limiter**
- **No boundary conditions are applied at limiter for velocity or temperatures.**
  - **This allows fluxes of mass and heat through limiter**
  - **Normal heat flux is computed at limiter boundary**



Free-Boundary Simulations Based on EFIT Reconstruction

- **Pressure raised 8.7% above "best fit" EFIT**
- **Above ideal MHD marginal stability limit**
- **Simulation includes:**
  - ***n* = 0, 1, 2**
  - **Anisotropic heat conduction (with no T dependence)** $\kappa_{par}/\kappa_{perp}=10^8$
- **Ideal modes grow with finite resistivity ($S = 10^5$)**

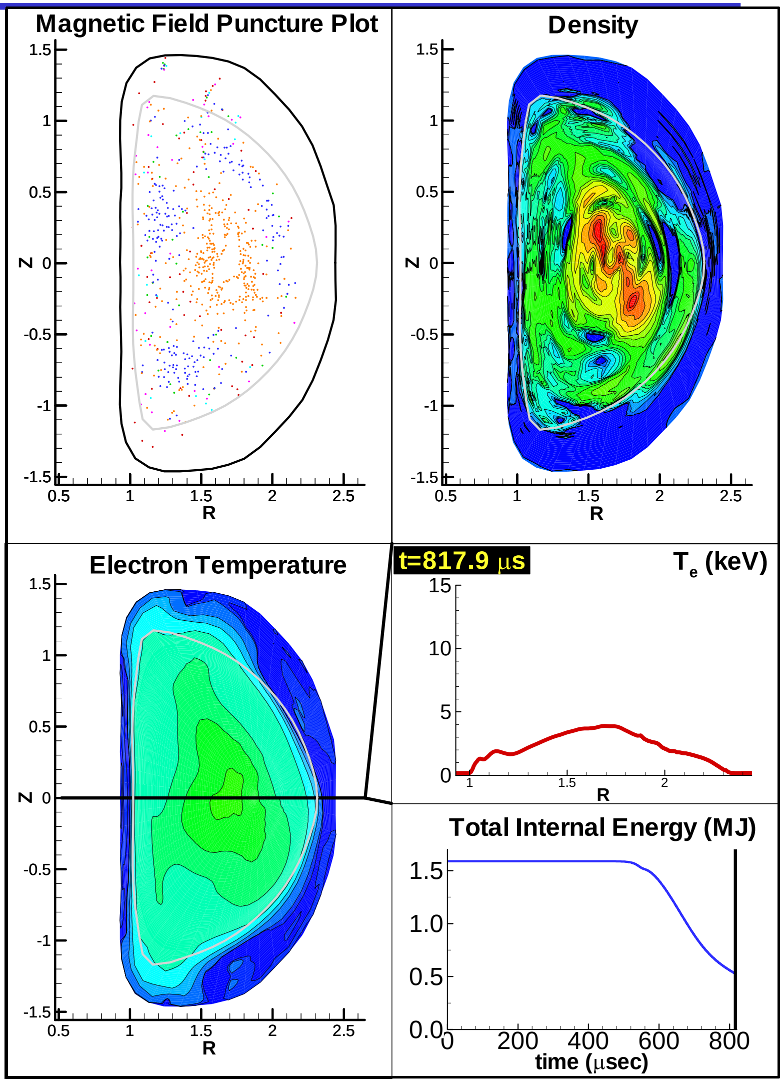




Normal Heat Flux At Wall
φ (radians)
Distance along wall (m)
A
B
C
D
E
t=0.0 μs
Tₑ (keV)
R
Total Internal Energy (MJ)
time (μsec)
Electron Temperature (φ=π)
Electron Temperature (φ=0)
Magnetic Field Puncture Plot
Z
R

Initial Simulations Above Ideal Marginal Stability Point Look Promising

- **Because magnetic field becomes stochastic, heat lost to wall preferentially at divertor by parallel heat conduction**
- **Disruption is very different from conventional model of plasma hitting the wall.**

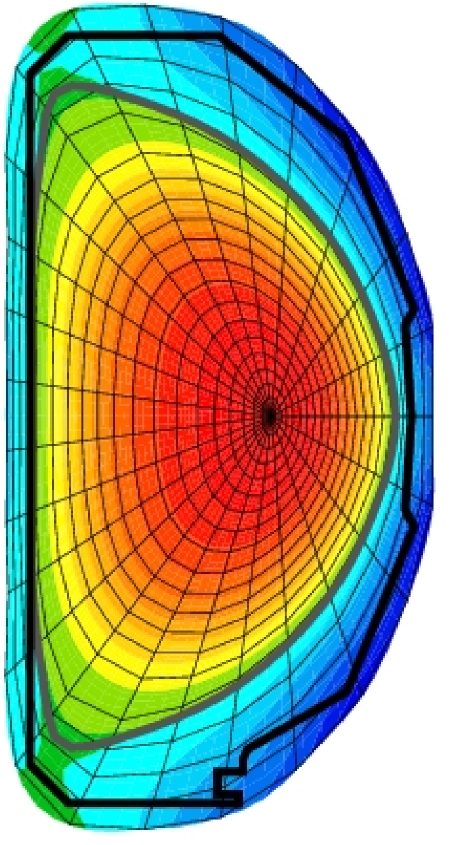


More Challenging Simulations Include Greater Resolution and Heating

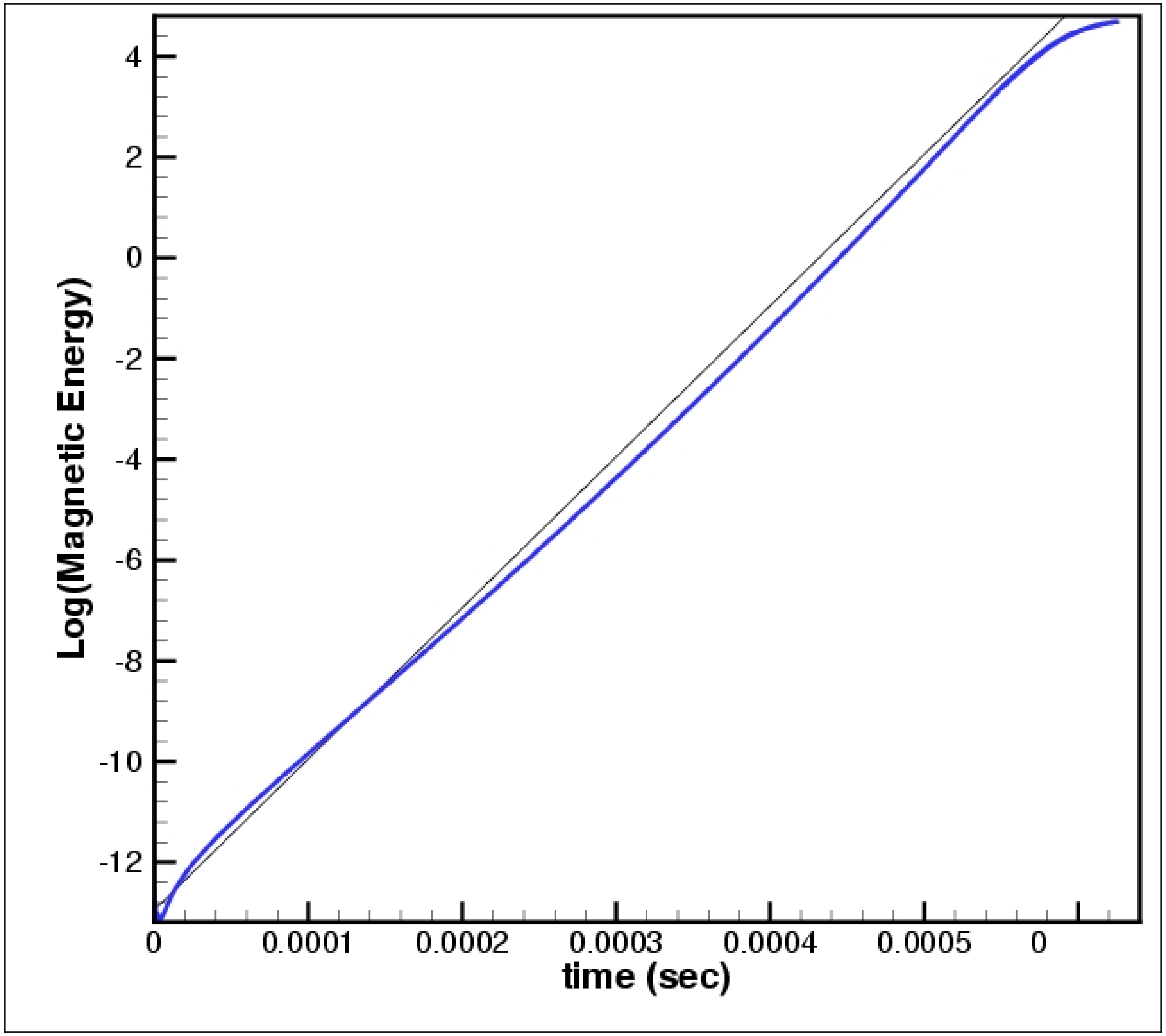
- **Evolving electron and ion temperatures, but heating is applied just to ions**

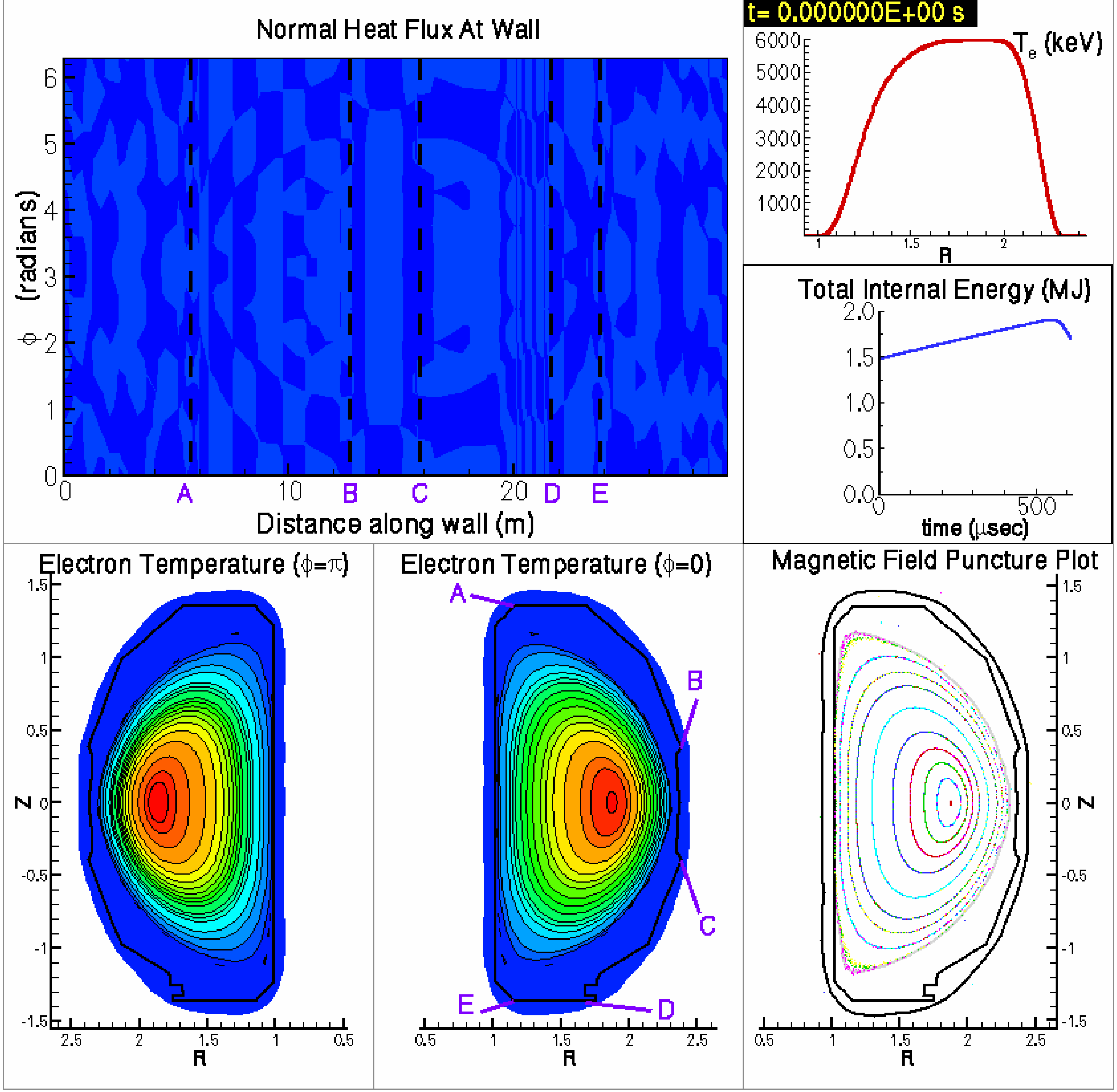
- **Simulation is started with "best fit" EFIT. Submarginal to ideal MHD as given by DCON.**

- **Simulation includes:**
  - **n = 0-6**
  - **Anisotropic heat conduction (with no T dependence)** $\kappa_{par}/\kappa_{perp}=10^8$



Faster Than Exponential Growth ?





Normal Heat Flux At Wall
φ (radians)
Distance along wall (m)
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t= 0.000000E+00 s
T_e (keV)
R
Total Internal Energy (MJ)
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Electron Temperature (φ=π)
Electron Temperature (φ=0)
Magnetic Field Puncture Plot
Z
R

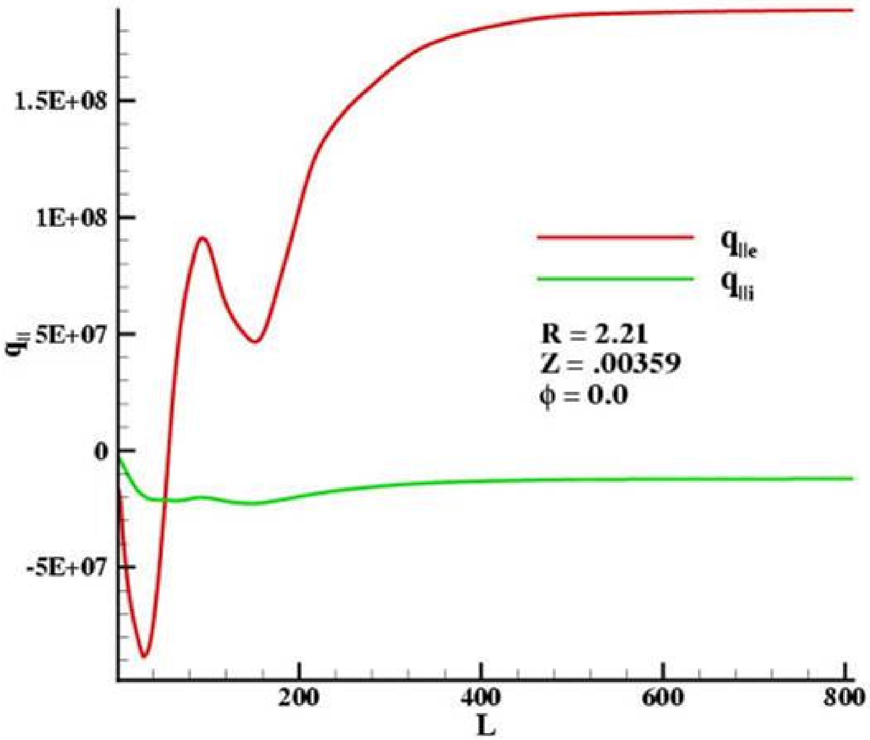
Future Directions

- Direct comparison of code against experimental diagnostics
- Increased accuracy of MHD model
  - Temperature-dependent thermal diffusivities
  - More aggressive parameters
  - Resistive wall B.C. and external circuit modeling
- Extension of fluid models
  - Two-fluid modeling
  - Electron heat flux using integral closures
  - Energetic particles
- Simulations of different devices to understand how magnetic configuration affects the wall power loading

Nonlocal Effects Important For Quantitative Calculation of Heat Flux

- **Collision scale lengths in this case can be many kilometers**
- **Nonlocal $q_{||}$ closure addresses free-streaming, collisional, and particle trapping effects in long-mean-free-path regime**

$$q_{||} = \int_0^{\infty} dL' \, [T(L-L') - T(L+L')] \frac{\partial K}{\partial \ln L'}$$



Conclusions

NIMROD's Advanced Computational Techniques Allows Simulations Never Before Possible

- **Heating through β limit shows super-exponential growth, in agreement with experiment and theory (although fixed boundary works better)**
- **First successful case of initial-value code using "best-fit" equilibria directly.**
- **Simulation of disruption event shows qualitative agreement with experiment.**
- **Loss of internal energy is due to rapid stochastization of the field, and not a violent shift of the plasma into the wall.**
- **Heat flux is localized poloidally and toroidally in a narrow "beam" as X-point gets shifted towards divertor.**