Update on M3D Results from the CDX-U Cross-Code Benchmark

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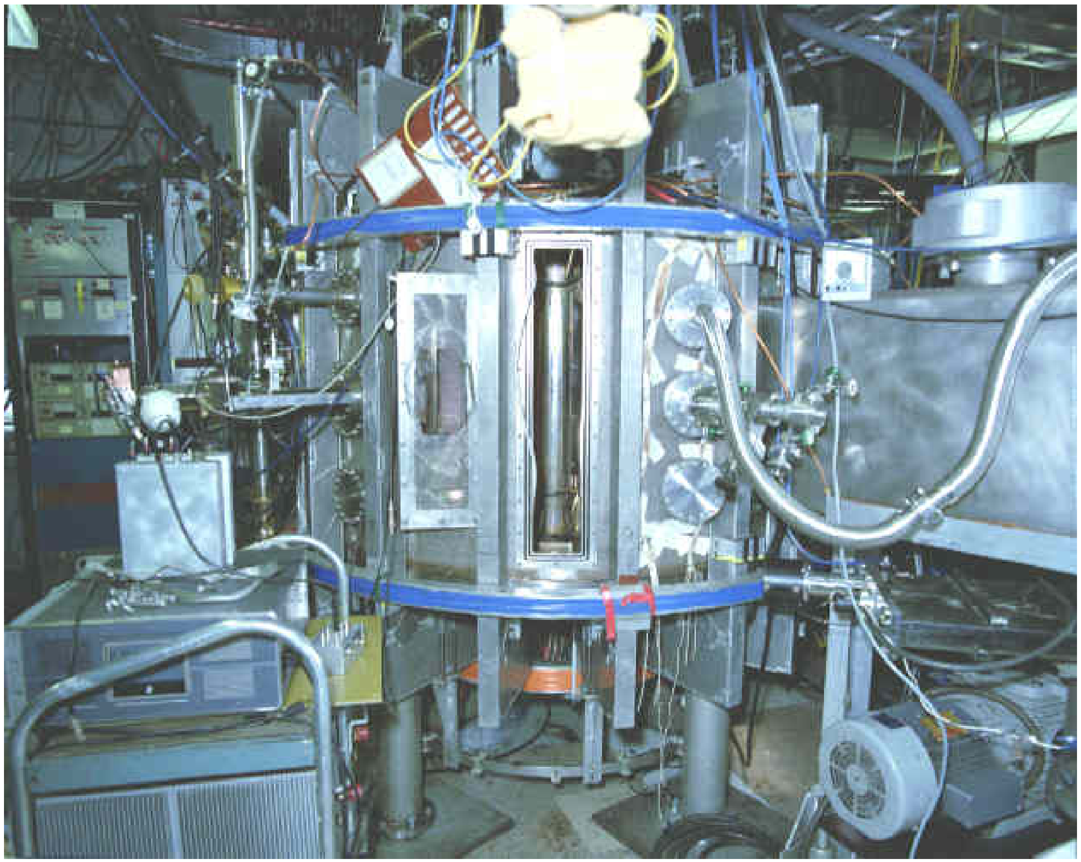
Center for Extended MHD Modeling Meeting
Savannah, Georgia
November 14, 2004

Outline

I. The CDX Device
   A. Machine Parameters
   B. Constructing Equilibria
   C. Benchmark Parameters
II. First case: $q_0 = 0.922$
   A. Linear Eigenmodes
   B. Nonlinear Evolution
      1. Mode Coupling
      2. Stabilizing Effects
III. Second case: $q_0 = 1.04$
   A. n>1 Eigenmodes
   B. Nonlinear Evolution
      1. Characterizing the Modes
      2. Heat Conduction Effects
IV. Conclusions
V. Topics for Further Study

Characteristics of the Current Drive Experiment Upgrade (CDX-U)

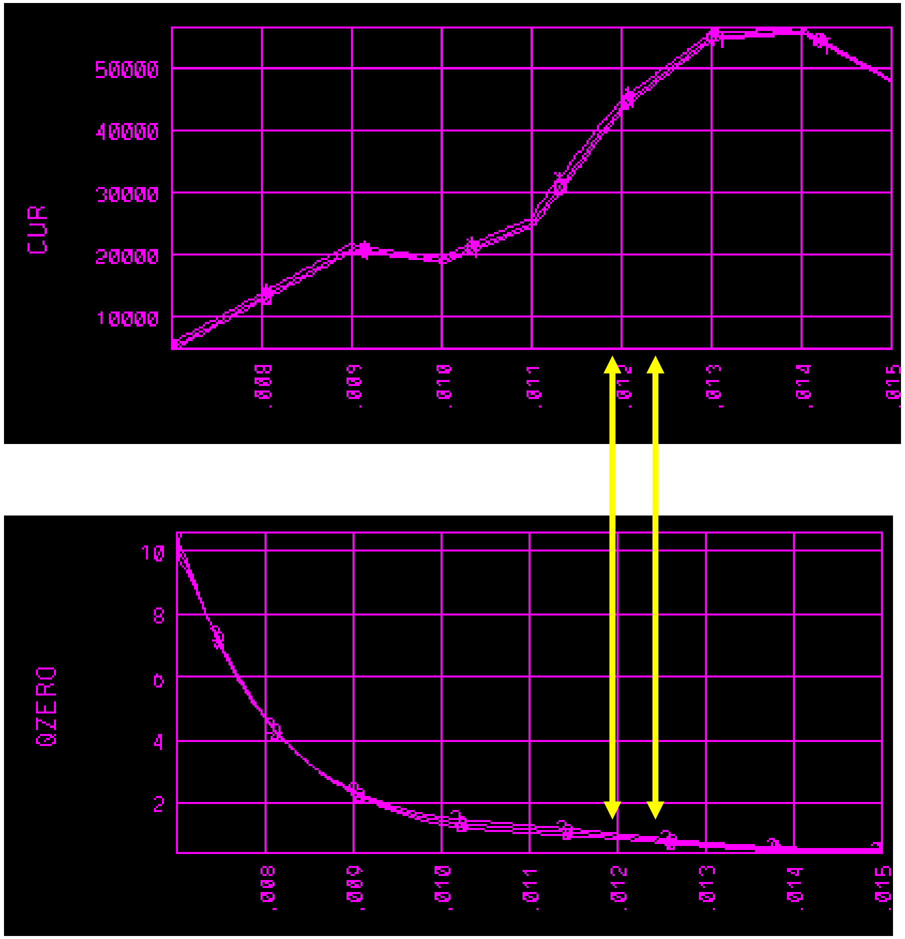


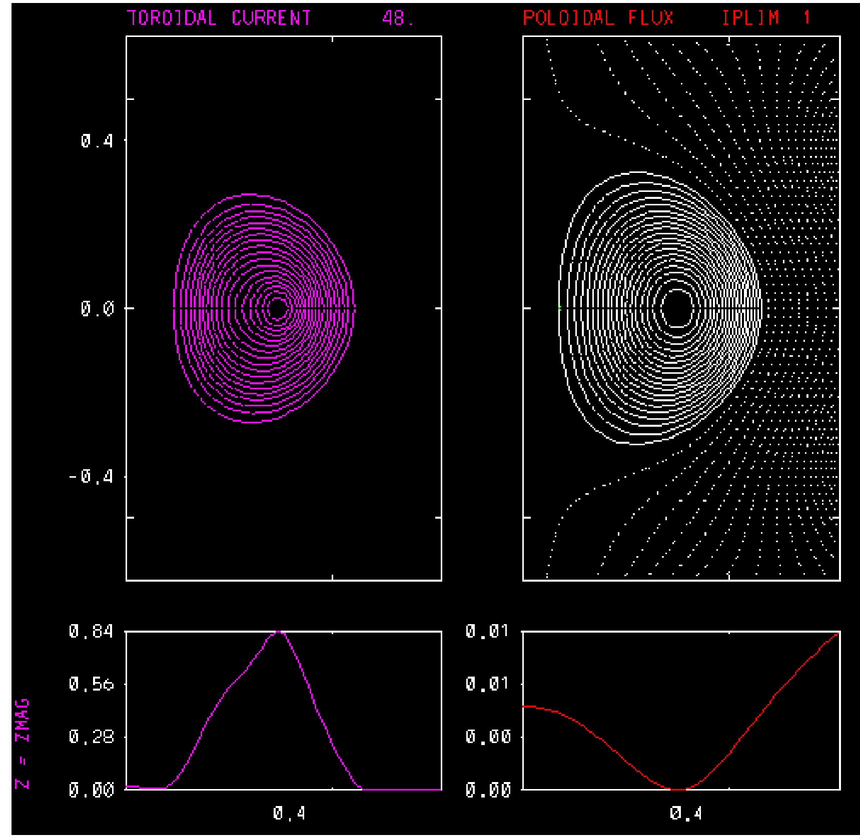
- Low aspect ratio tokamak (R_0/a = 1.4 – 1.5)
- Small (R_0 = 33.5 cm)
- Elongation $\kappa \sim 1.6$
- $B_T \sim 2300$ gauss
- $I_p \sim 70$ kA
- $n_e \sim 4\times10^{13}$ cm^{-3}
- $T_e \sim 100$ eV $\rightarrow$ S $\sim 10^4$
- Discharge time ~ 12 ms

- Soft X-ray signals from typical discharges indicate two predominant types of low-*n* MHD activity:
  - sawteeth
  - “snakes”

Generating Equilibria

Transport timescale code TSC follows axisymmetric evolution of typical CDX-U discharge.





S. Jardin

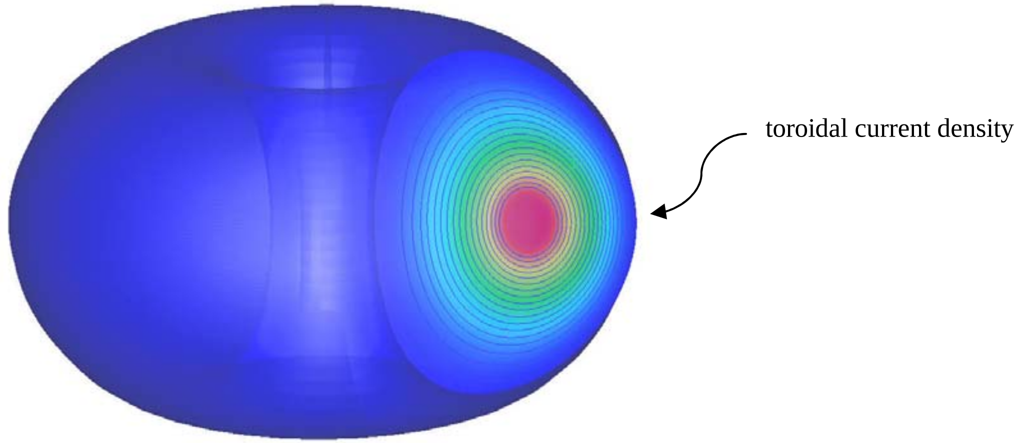
Equilibria at t=12.40 ms (as q_0 drops to 0.92) and t=12.00 ms (q_0=1.04) are used to initialize 3D runs.

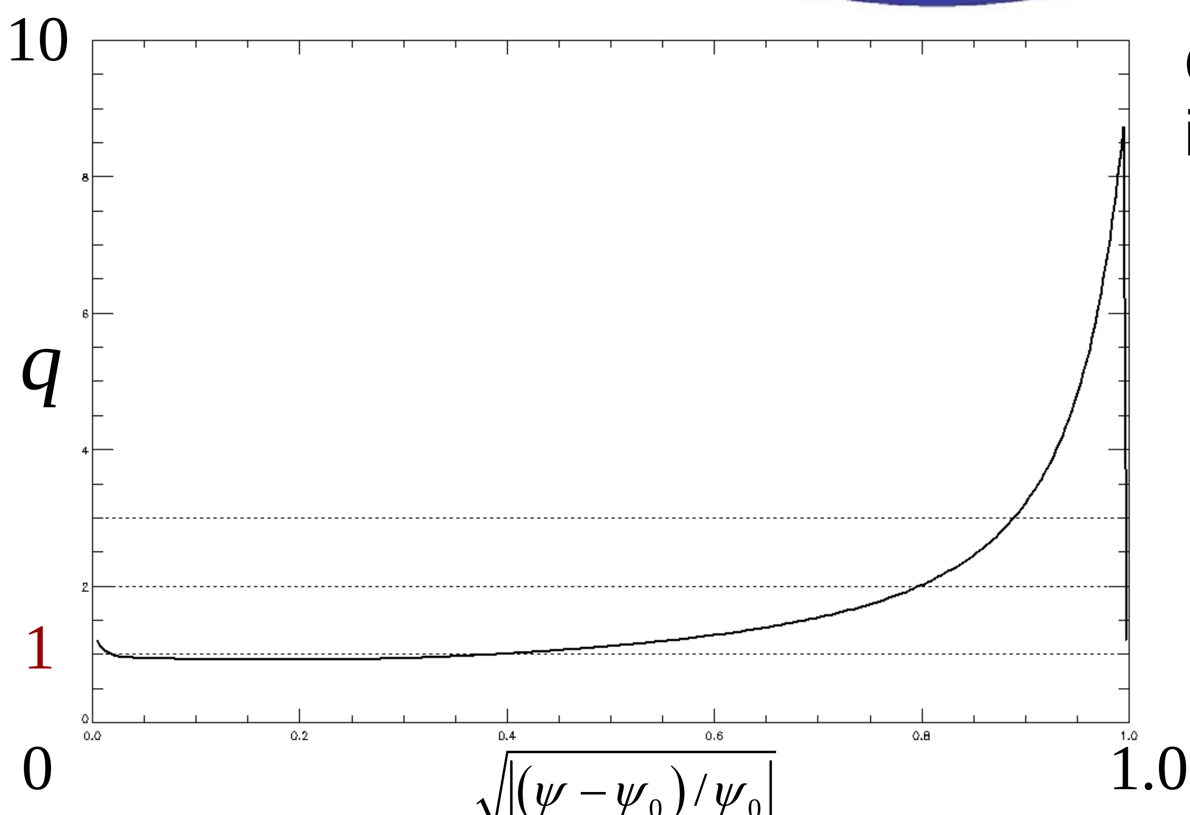
Baseline Parameters

Lundquist Number *S*	~2×10^4 on axis.
Resistivity η	Spitzer profile $\propto T_{eq}^{-3/2}$, cut off at $100\times\ \eta_0$
Prandtl Number *Pr*	10 on axis.
Viscosity μ	Constant in space and time.
Perpendicular thermal conduction $\kappa_\perp$	0
Parallel thermal conduction $\kappa_{\|\|}$	0
Peak Plasma β	~ 3×10^{-2} (low-beta).
Density Evolution	Turned on for nonlinear phase.

Case 1: $q_0 < 1$

- Equilibrium taken from a TSC sequence (Jsolver file).
- $q_{\min} \approx 0.922$
- $q(a) \sim 9$



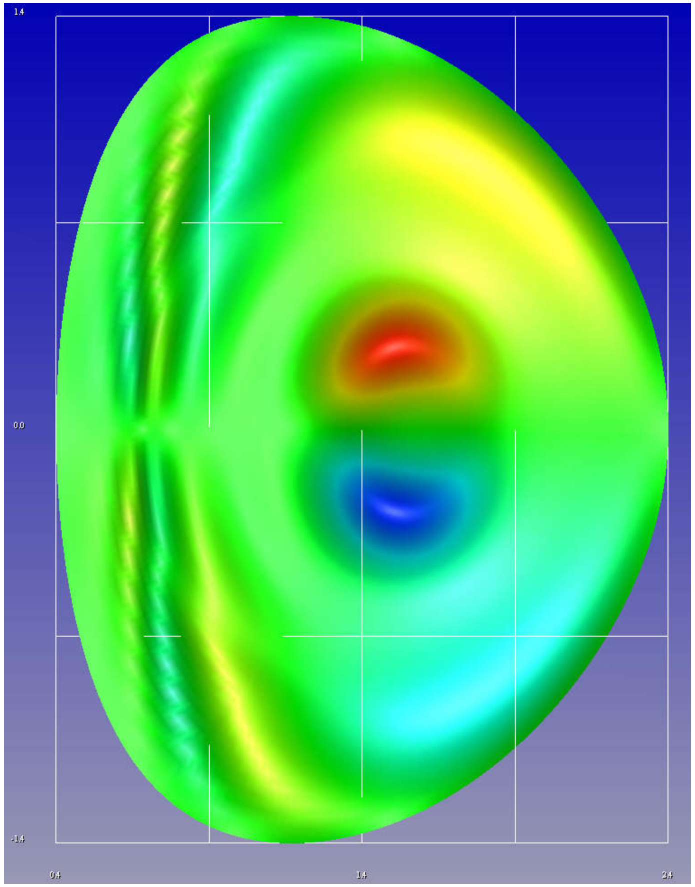


Questions to investigate:

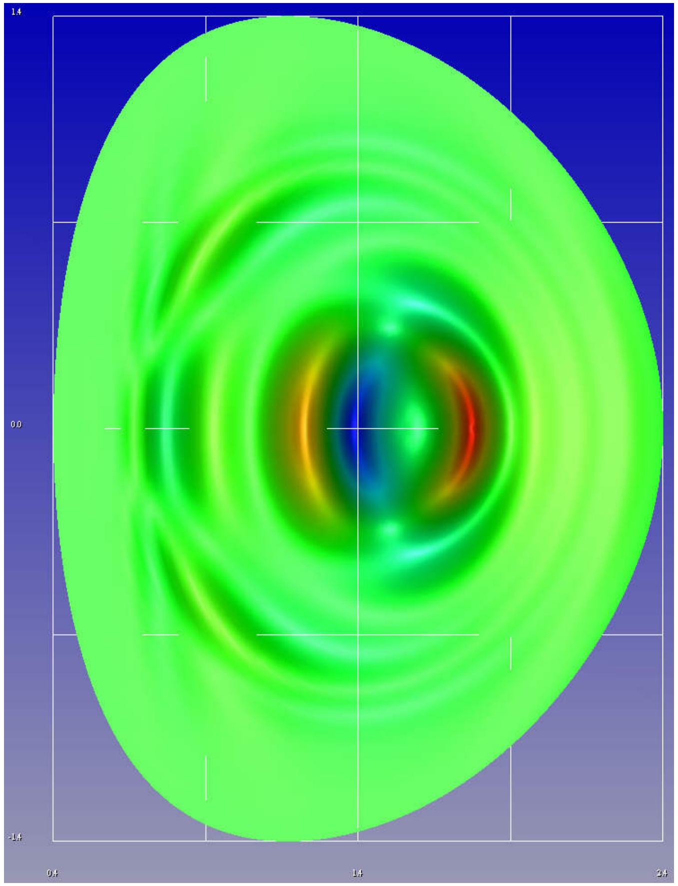
- Linear growth rate and eigenfunctions
- Nonlinear evolution
  - disruption?
  - stagnation?
  - repeated reconnections?

Low Aspect Ratio:
n=1 Eigenmode

Incompressible velocity stream function U

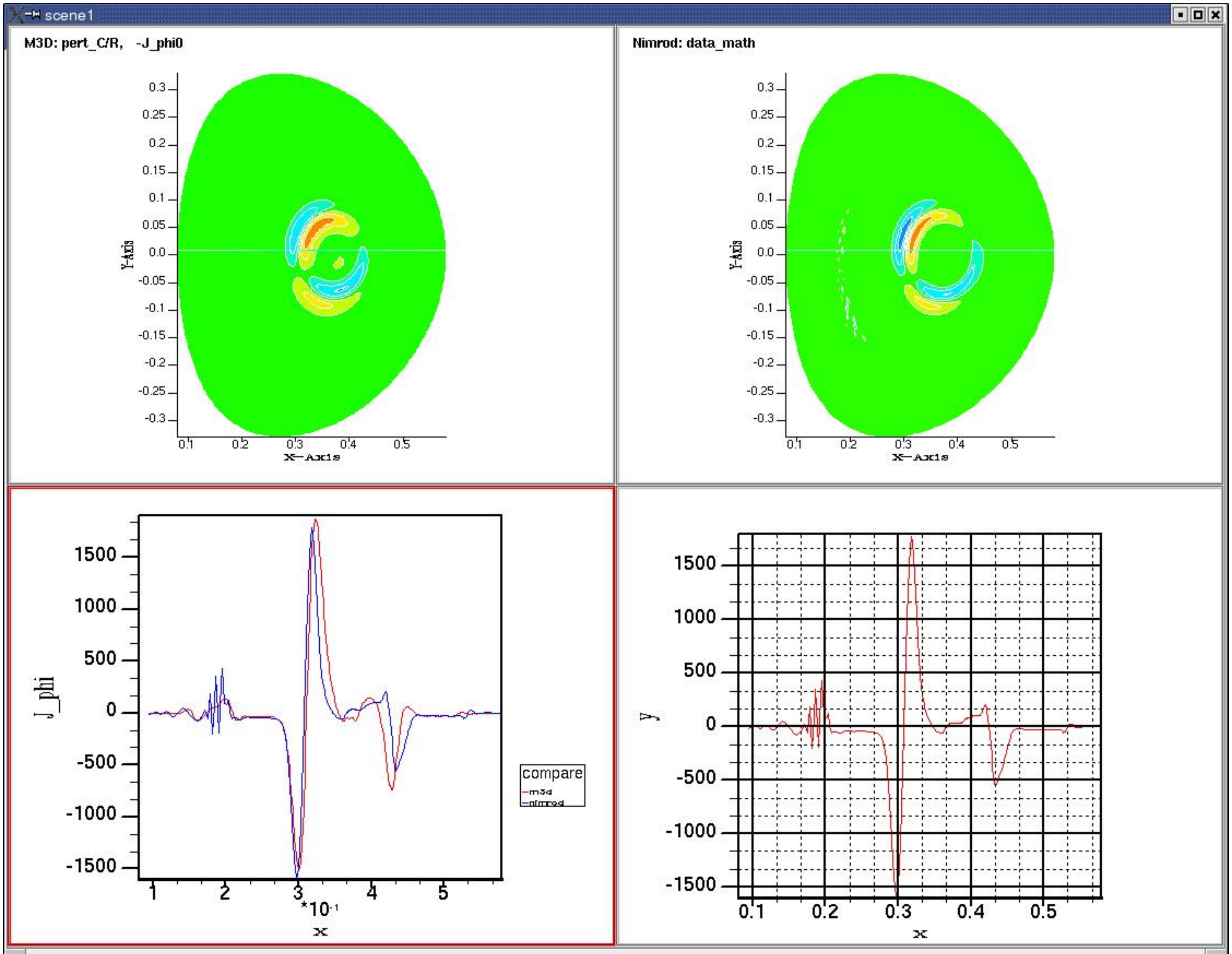


Toroidal current density J_ϕ



$\gamma \tau_A = 8.61 \times 10^{-3} \rightarrow$ growth time = 116 τ_A

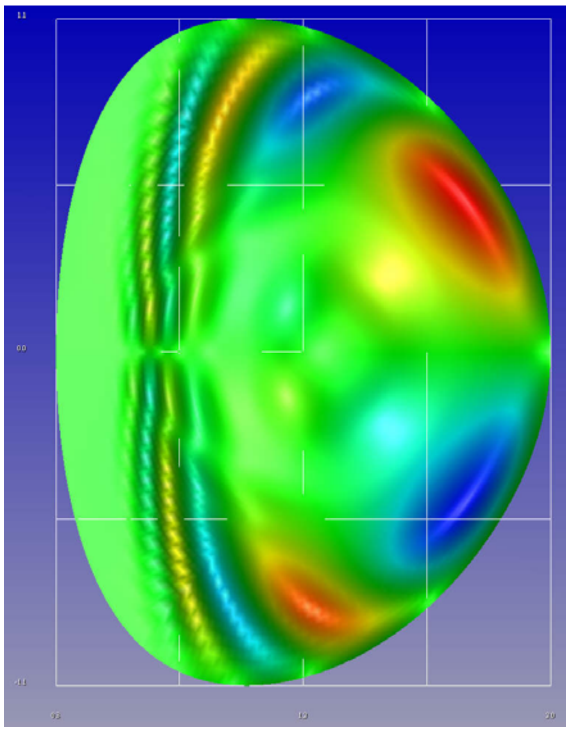
Predicted Eigenmode Agrees with NIMROD Result



Low Aspect Ratio:
Higher *n* Eigenmodes

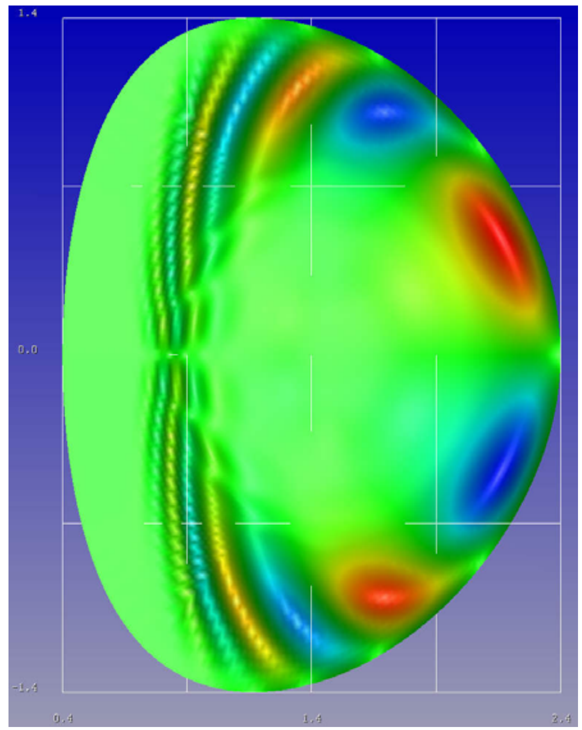
Incompressible velocity
stream function *U*

$n = 2$



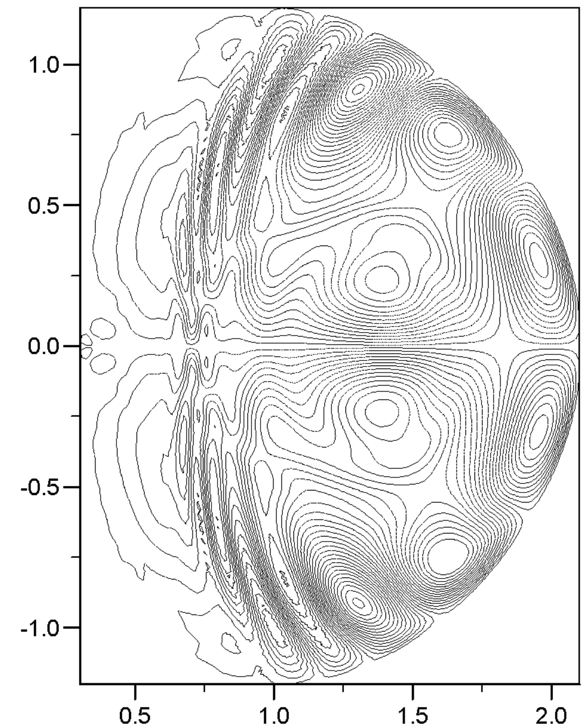
m ≥ 5
$\gamma \tau_A = 1.28 \times 10^{-2}$

$n = 3$



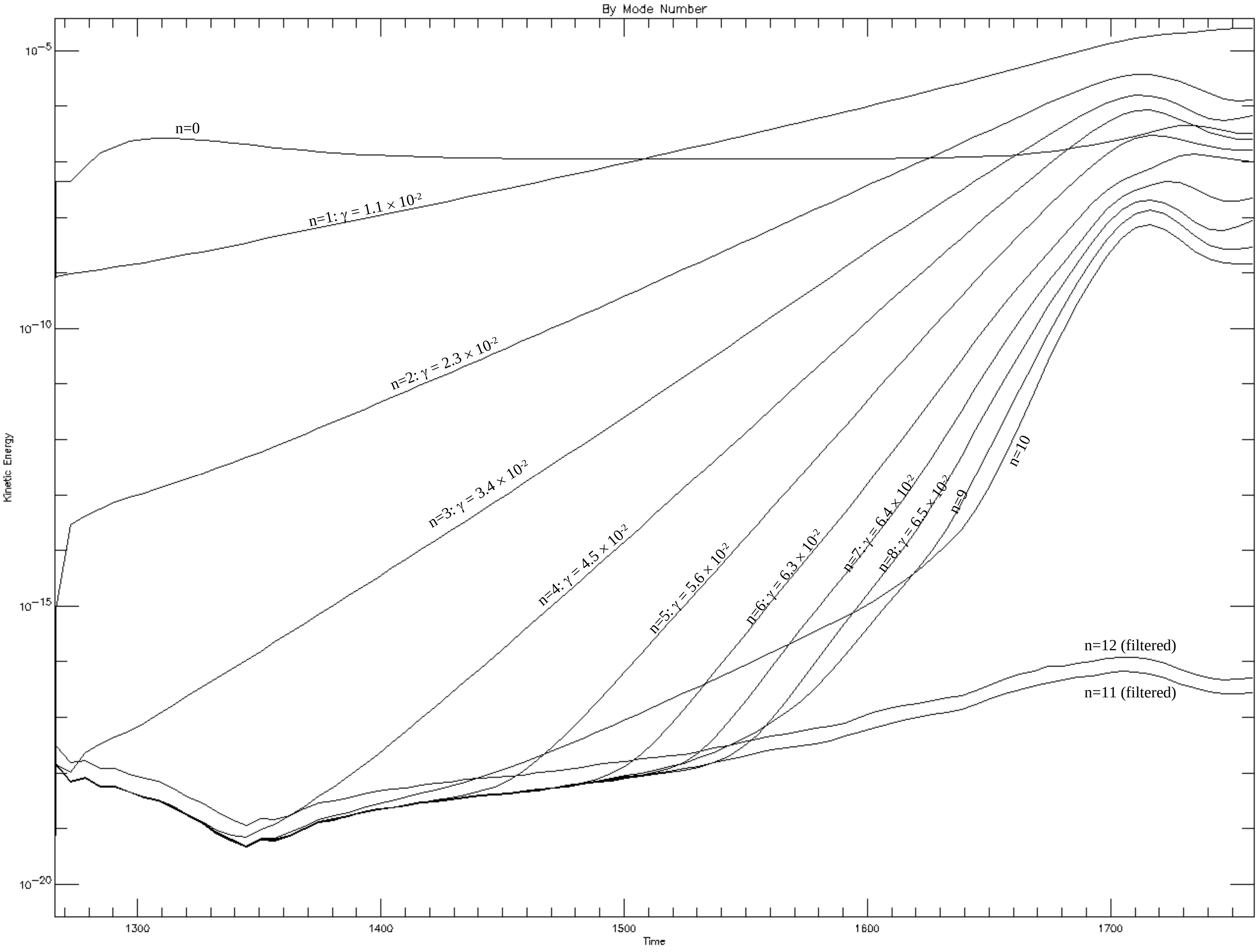
m ≥ 7
$\gamma \tau_A = 1.71 \times 10^{-2}$

$n = 4$ (projected)



m ≥ 8
not converged

Low Aspect Ratio: Nonlinear Kinetic Energy History

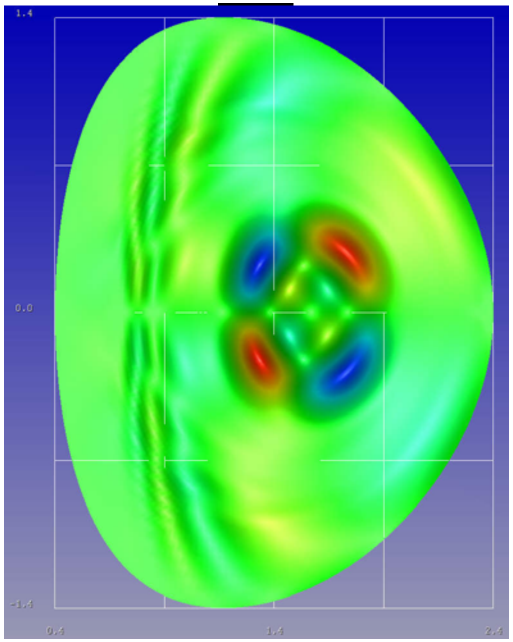


“Linear” high-*n* modes are driven, not eigenmodes

Incompressible velocity stream function *U*

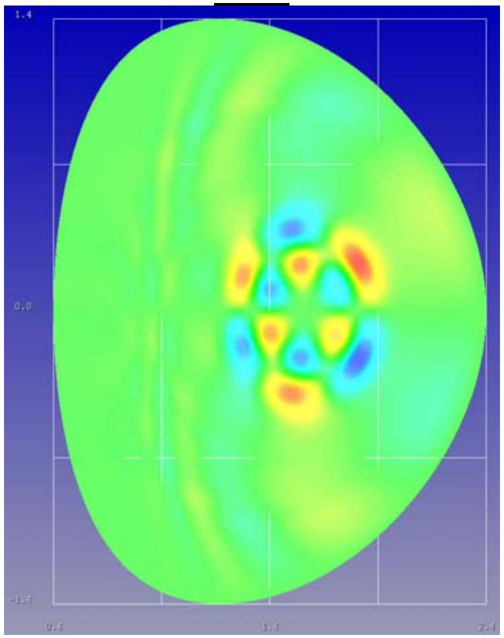
Component of “linear” mode
in nonlinear run

<u>*n*=2</u>



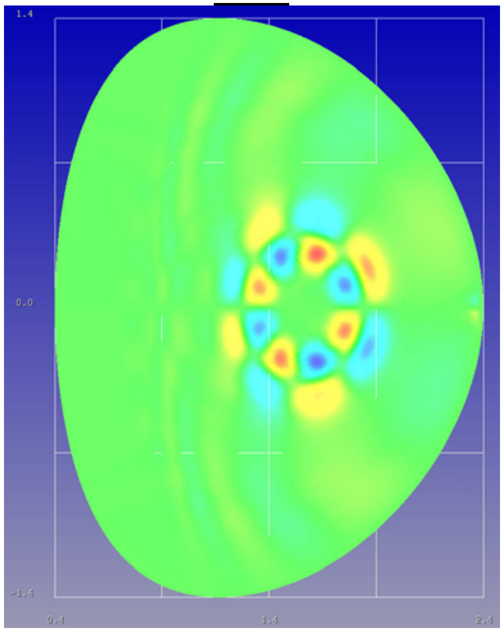
m = 2
$\gamma = 2.3 \times 10^{-2}$

<u>*n*=3</u>



m = 3
$\gamma = 3.4 \times 10^{-2}$

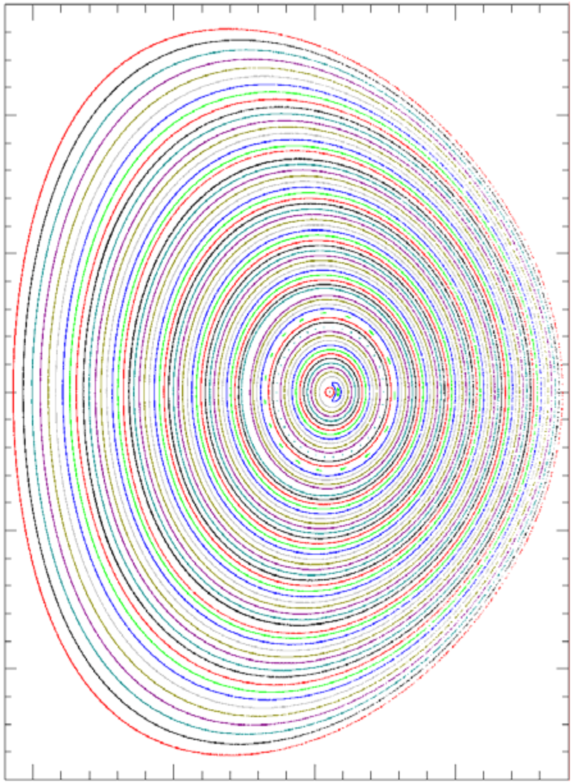
<u>*n*=4</u>



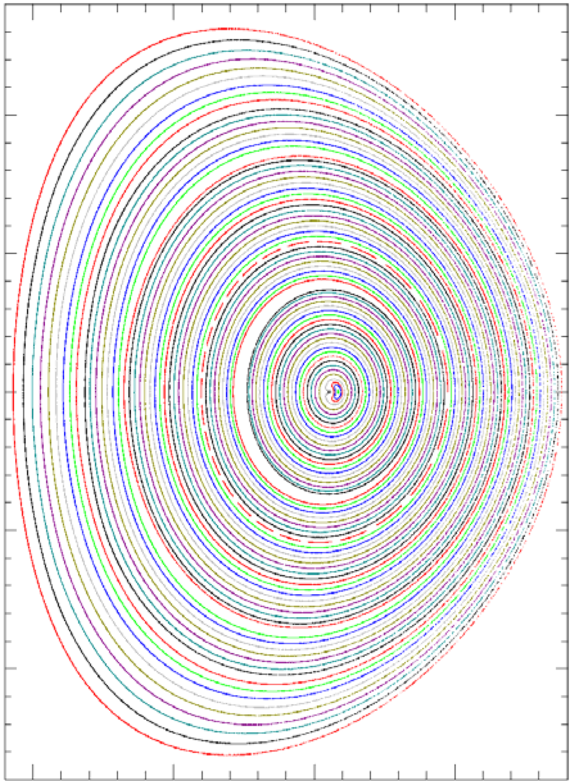
m = 4
$\gamma = 4.5 \times 10^{-2}$

Low Aspect Ratio: Nonlinear Time Series

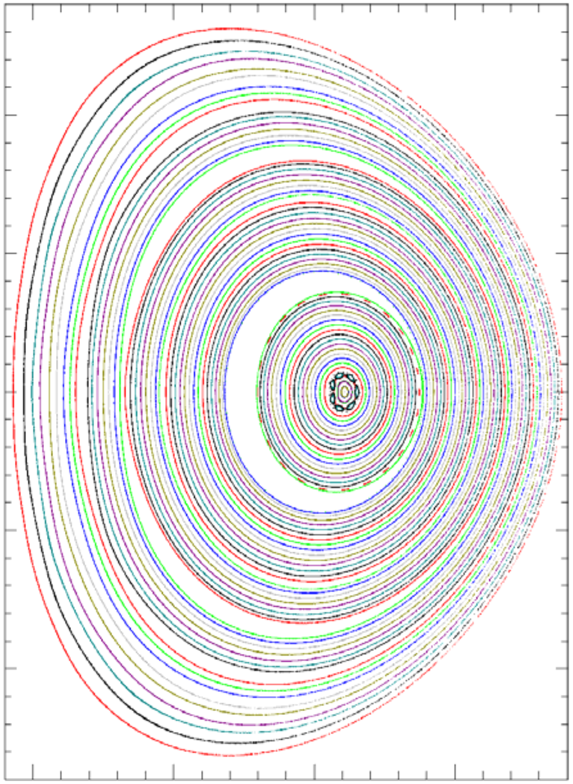
Poincaré Plots



t = 1266.17, chopped × 4.470358



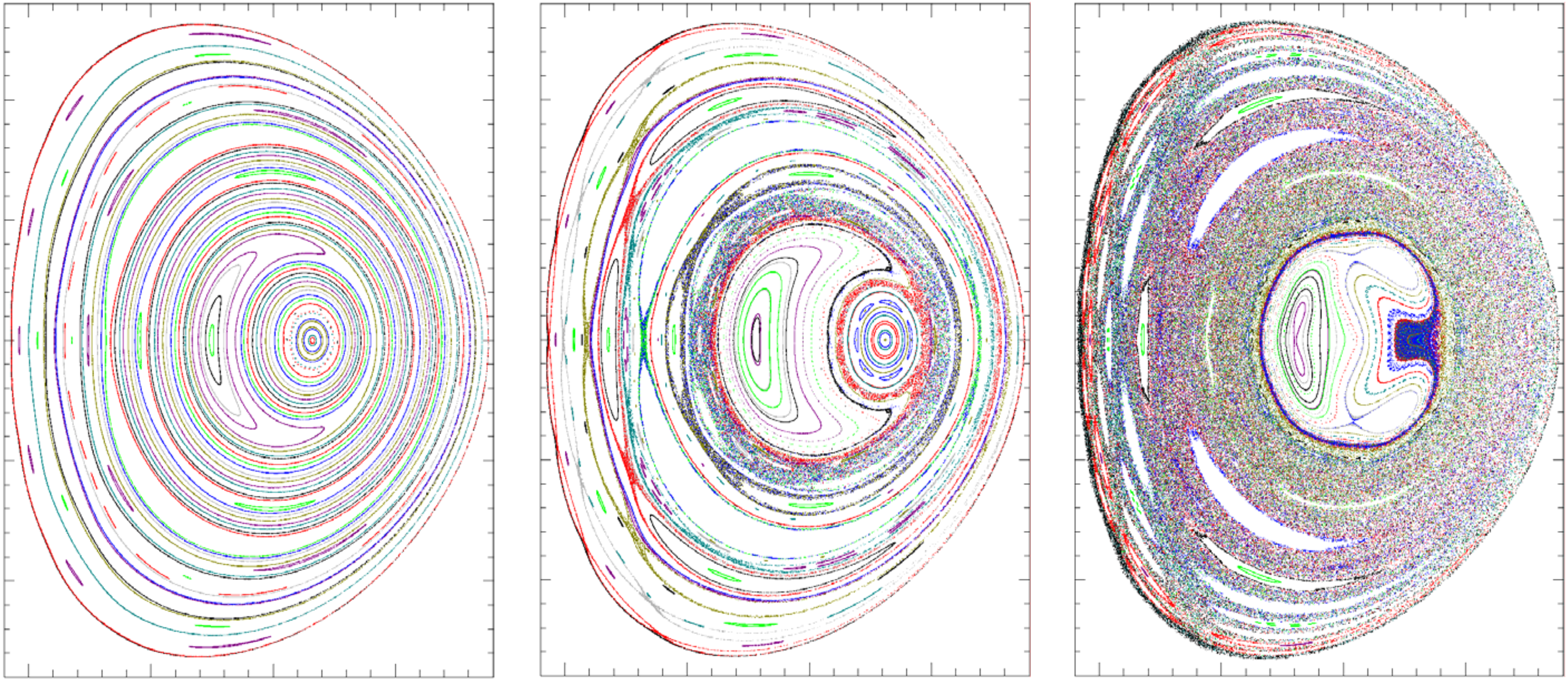
t = 1404.57



t = 1548.68

Low Aspect Ratio: Nonlinear Time Series

Poincaré Plots, Continued



t = 1620.62 t = 1686.41 t = 1758.34

Disruption occurs before completion of sawtooth crash.

Summary of the q_0<1 Case

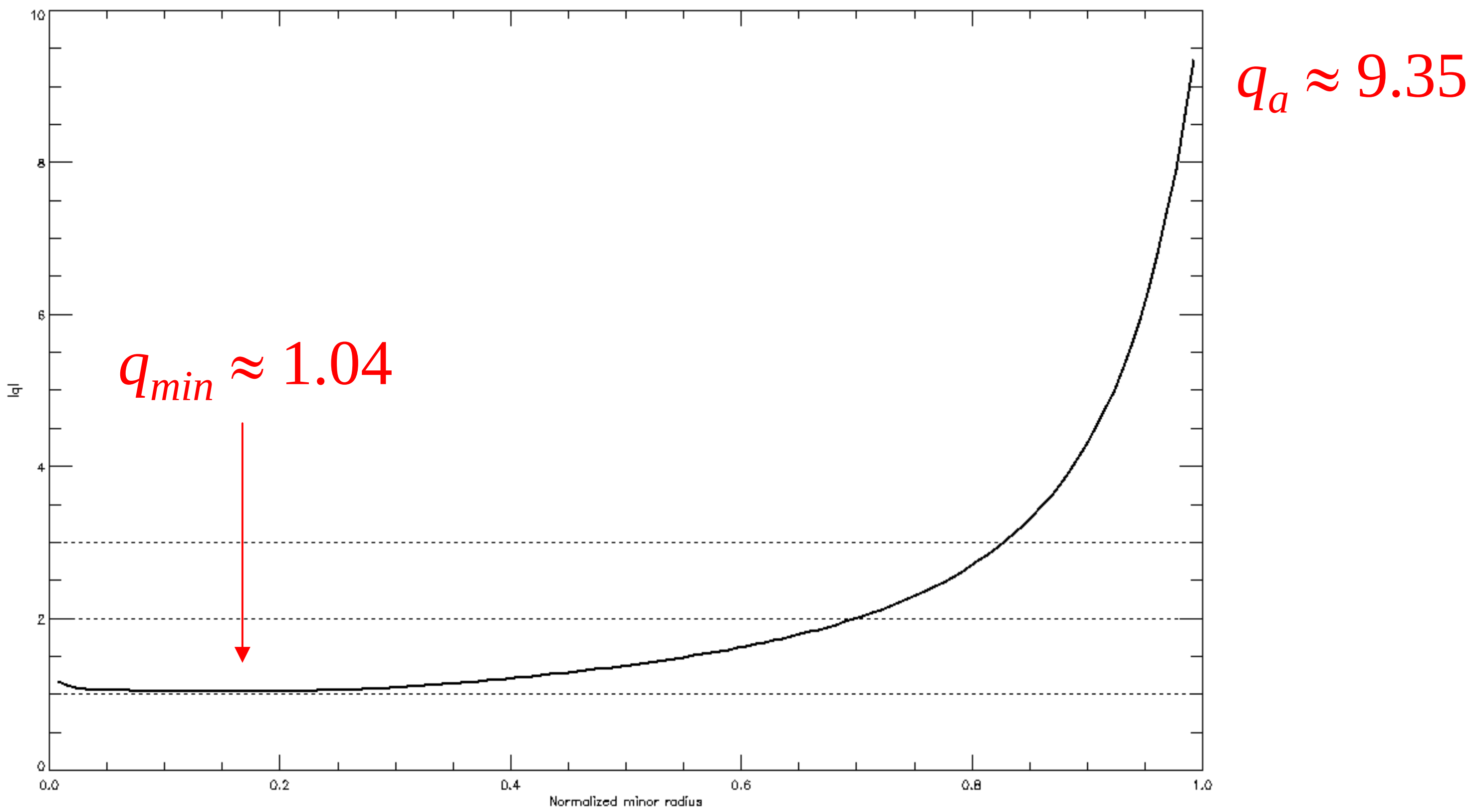
- All toroidal modes of the q_{min}= 0.92 CDX equilibrium are linearly MHD-unstable.
    - *n* =1 is an internal kink mode
    - *n* >1 are ballooning instabilities
    - Higher *n* modes have higher growth rates.

- Nonlinear MHD evolution beginning with just an *n*=1 perturbation disrupts within a sawtooth crash time.
    - High poloidal mode number *m* components of the *n*=1 mode interact to create islands, stochasticity in outer region.
    - *n* =1 mode couples to and drives higher *n* modes at *q*=1 rational surface to create further stochasticity.

- Adding large parallel thermal conductivity (via artificial sound wave) has a stabilizing effect on higher *n* modes, but not on *n*=1.
- Adding the ω^* term to the MHD equations does not appreciably alter the growth rates of either the *n* =1 or the *n* >1 modes.

Case 2: $q_0 > 1$

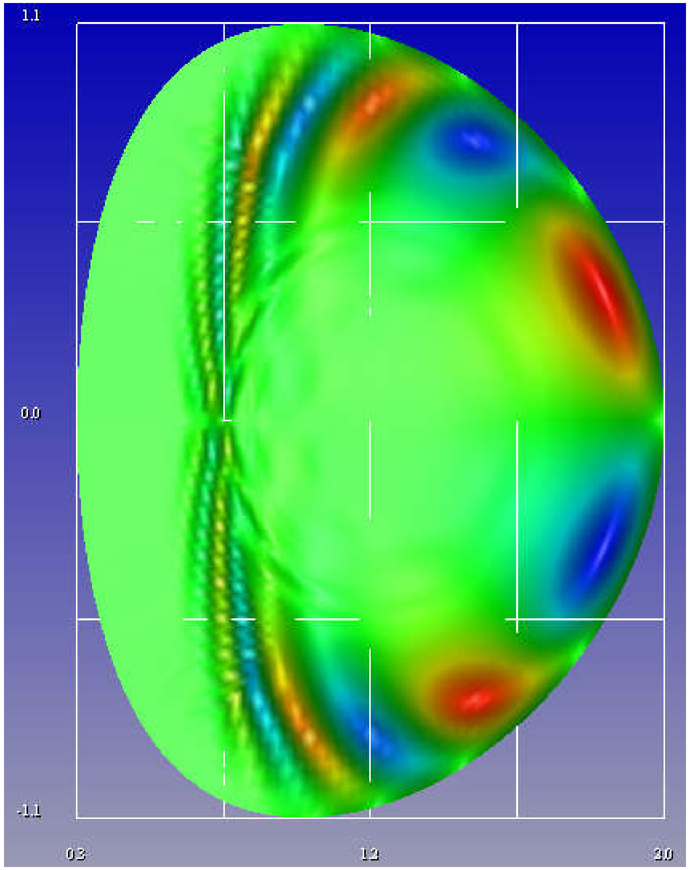
Equilibrium taken from earlier in same TSC sequence as case 1.



Eigenmodes

(*U*, incompressible part of velocity stream function)

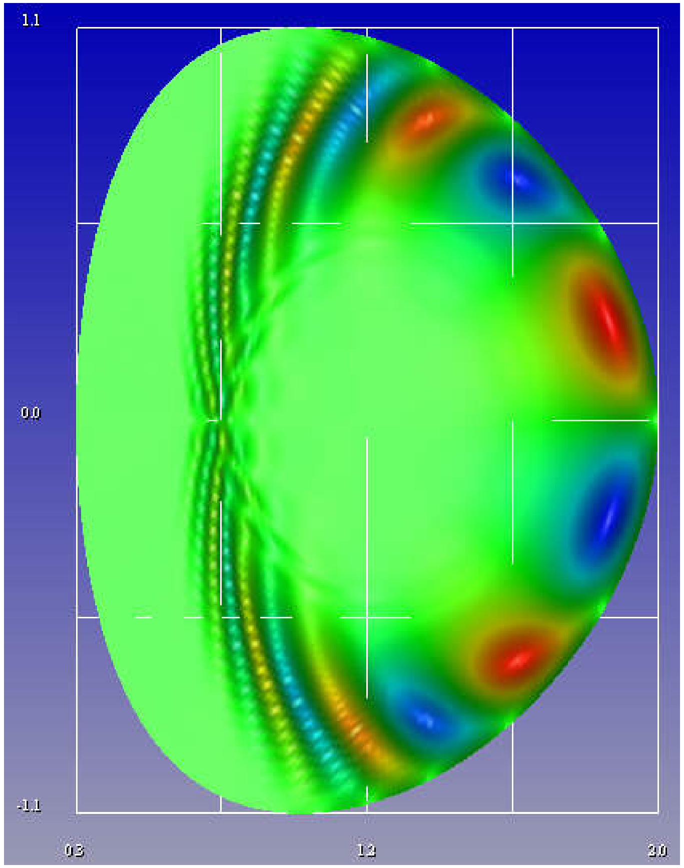
$n = 3$



$m \approx 7$

$\gamma \tau_A \approx 1.42 \times 10^{-2}$

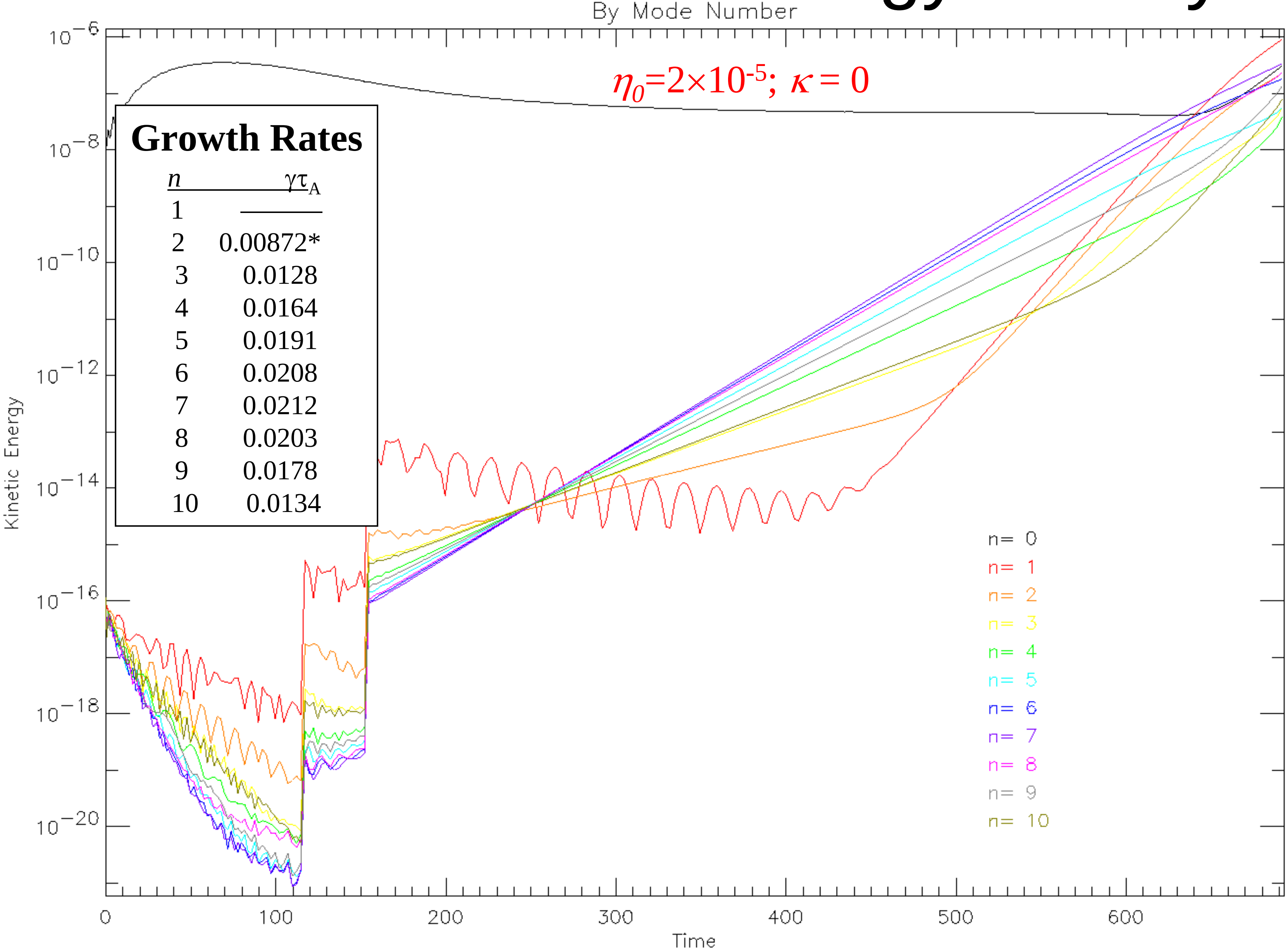
$n = 4$



$m \approx 9$

$\gamma \tau_A \approx 1.87 \times 10^{-2}$

Nonlinear Kinetic Energy History



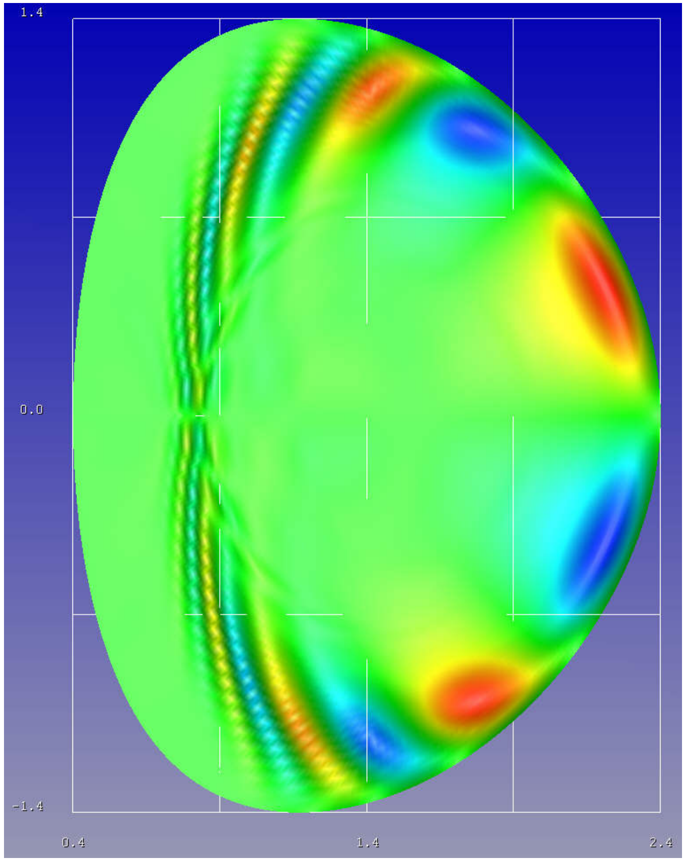
Growth Rates

n	$\gamma\tau_A$
1	———
2	0.00872*
3	0.0128
4	0.0164
5	0.0191
6	0.0208
7	0.0212
8	0.0203
9	0.0178
10	0.0134

Modes Observed Nonlinearly at t=219.6

(*U*, incompressible part of velocity stream function)

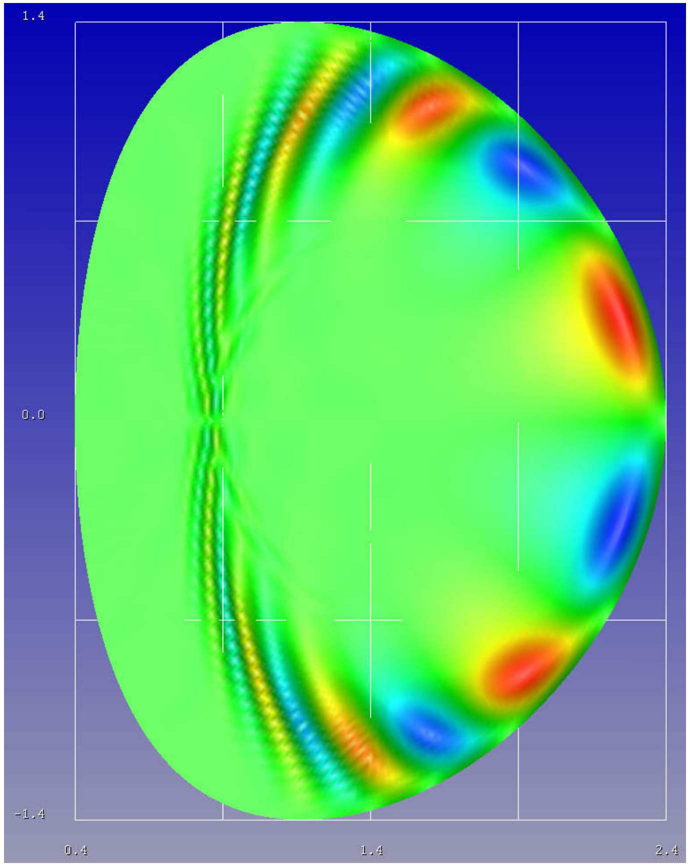
$n = 3$



$m \approx 8$

$\gamma \tau_A \approx 1.28 \times 10^{-2}$

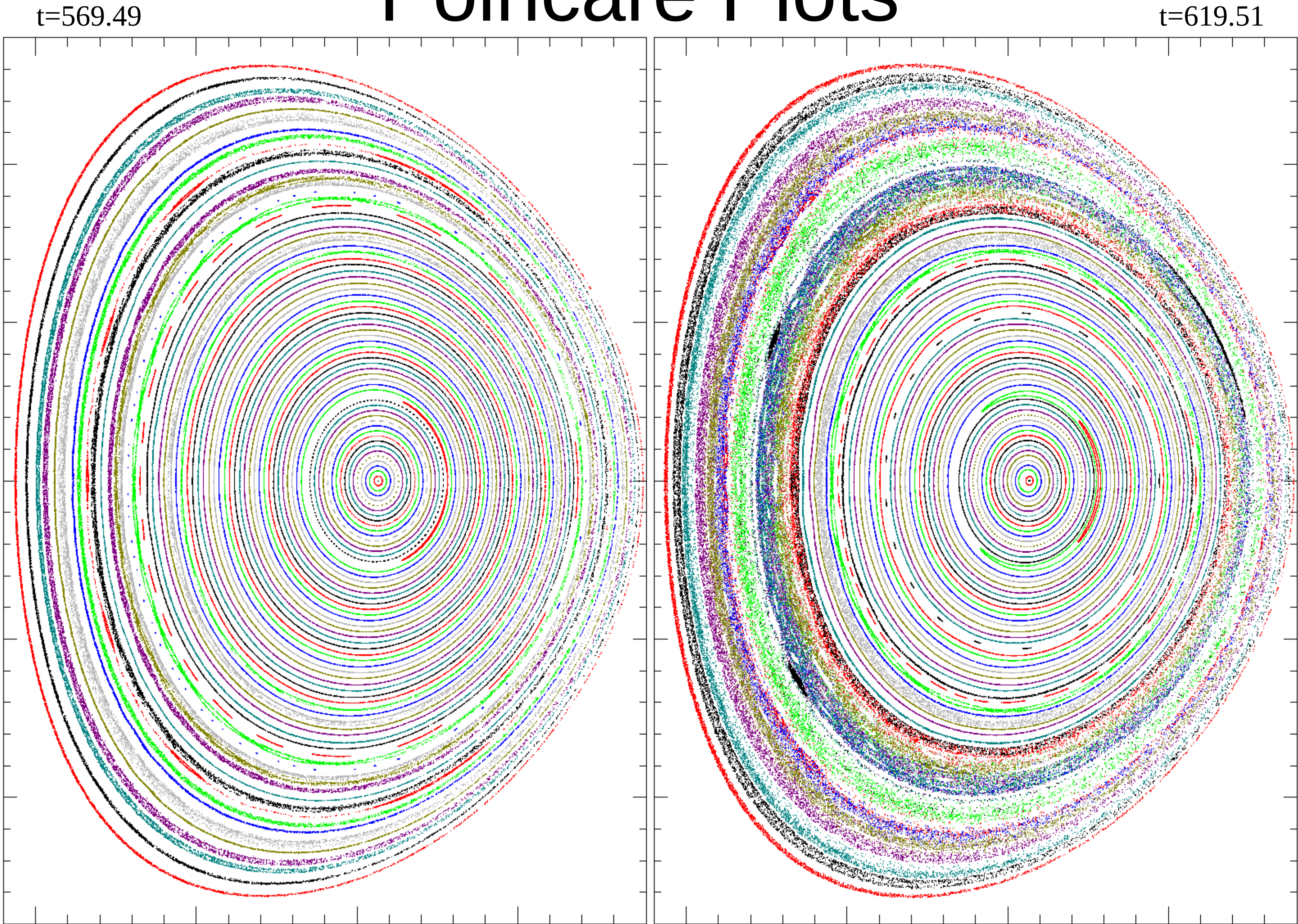
$n = 4$



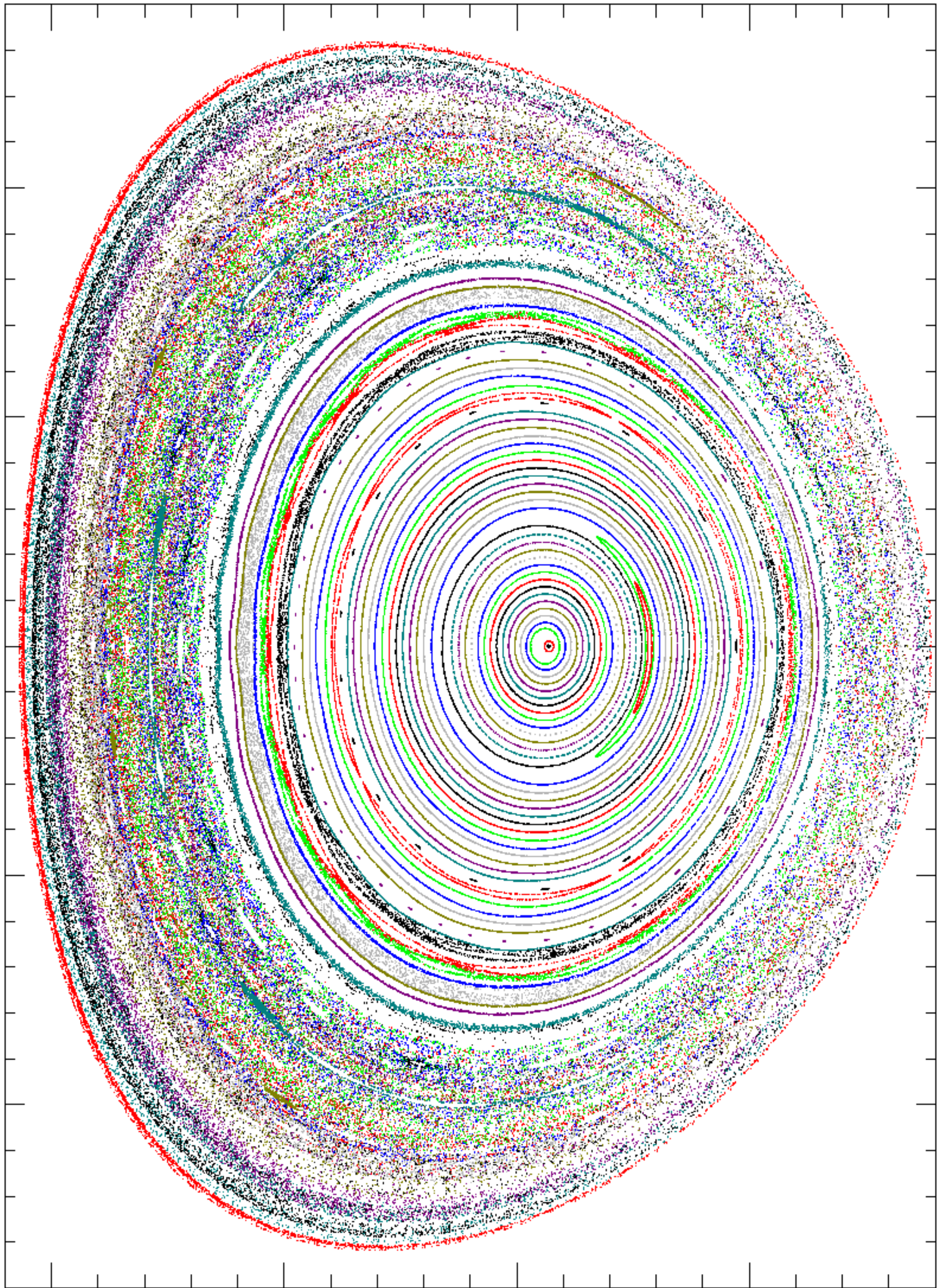
$m \approx 10$

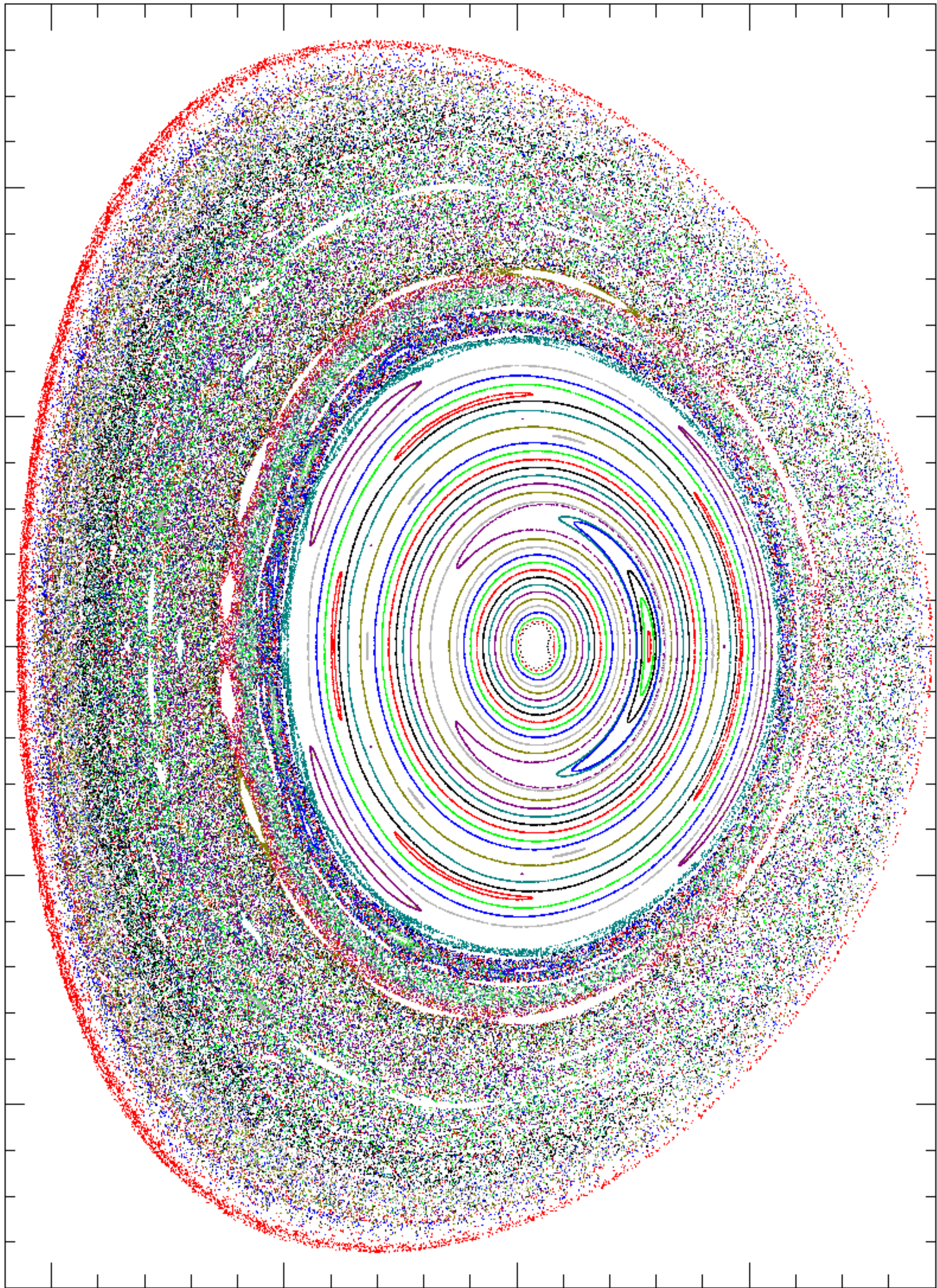
$\gamma \tau_A \approx 1.64 \times 10^{-2}$

Poincaré Plots

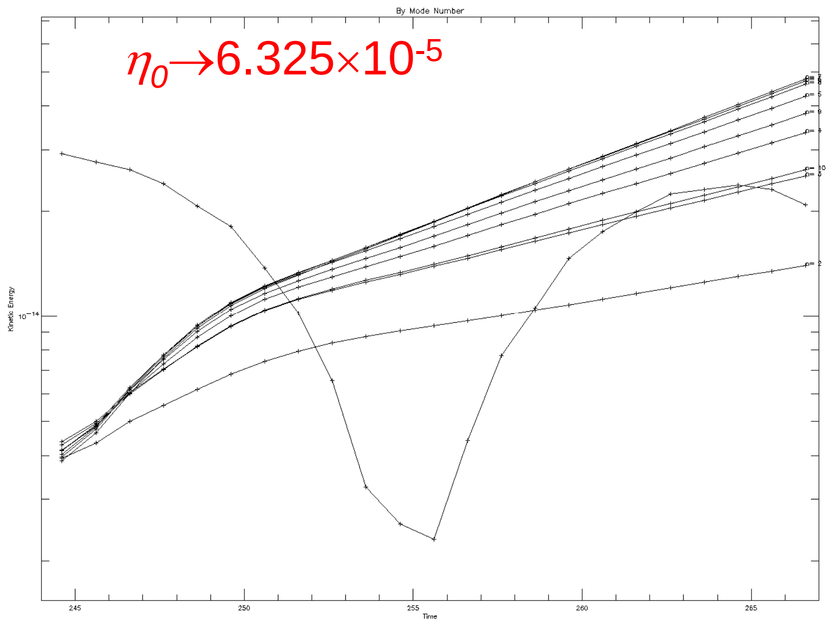


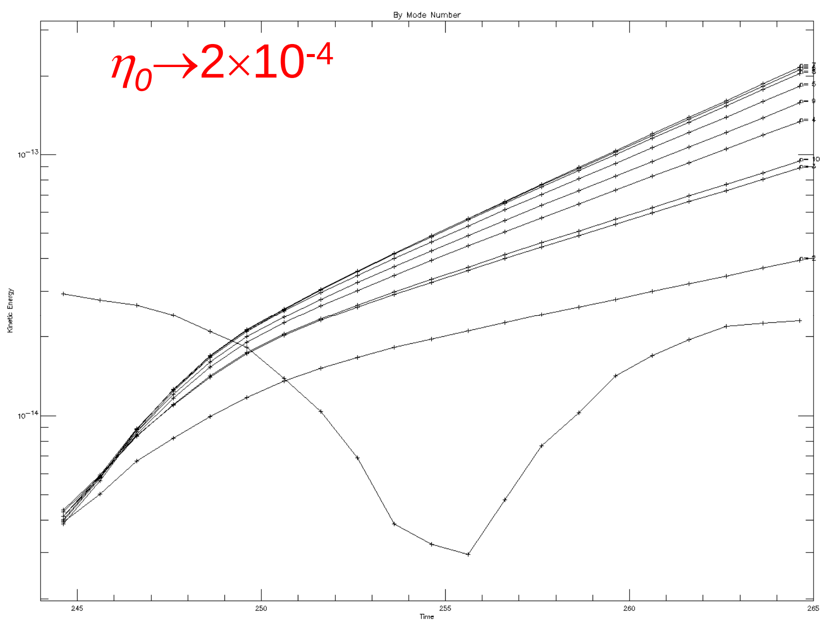
Poincaré Plots, continued





Resistivity Scaling is Consistent with low-*n* Resistive Ballooning Modes





Growth Rates
(`gcut=12`)

n	$\gamma\tau_A$
1	——
2	0.0184
3	0.0270
4	0.0337
5	0.0387
6	0.0416
7	0.0425
8	0.0409
9	0.0363
10	0.0281

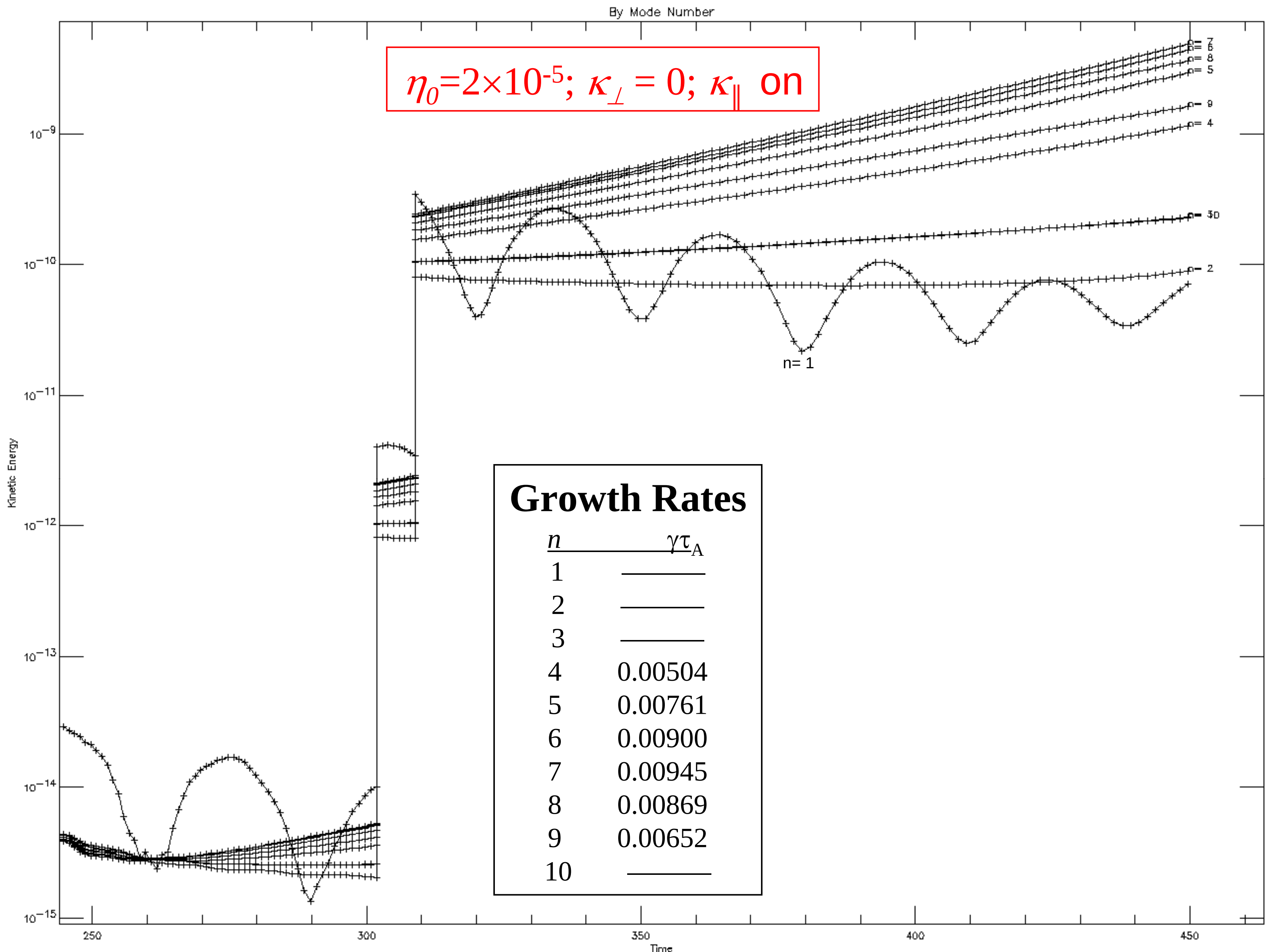
Growth Rates
(`gcut=11`)

n	$\gamma\tau_A$
1	——
2	0.0345
3	0.0498
4	0.0607
5	0.0682
6	0.0726
7	0.0738
8	0.0714
9	0.0646
10	0.0516

For each toroidal mode number *n*, the linear growth rate γ is found to be proportional to η^{α}:

n	α
2	0.597
3	0.590
4	0.568
5	0.553
6	0.543
7	0.542
8	0.546
9	0.560
10	0.586

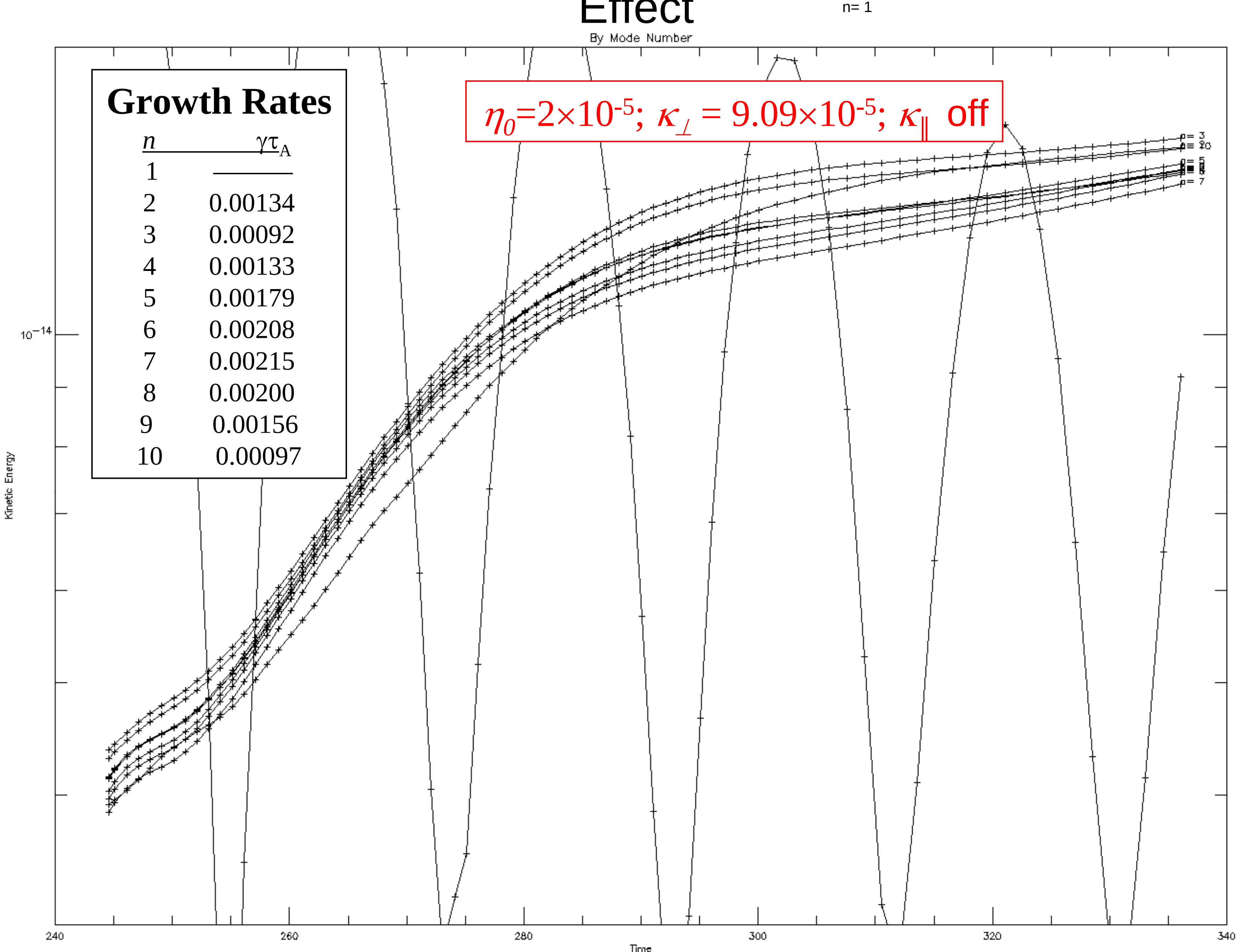
Parallel Heat Conduction Stabilizes Some Modes, But Does Not Appear to Cause Saturation of Unstable Modes



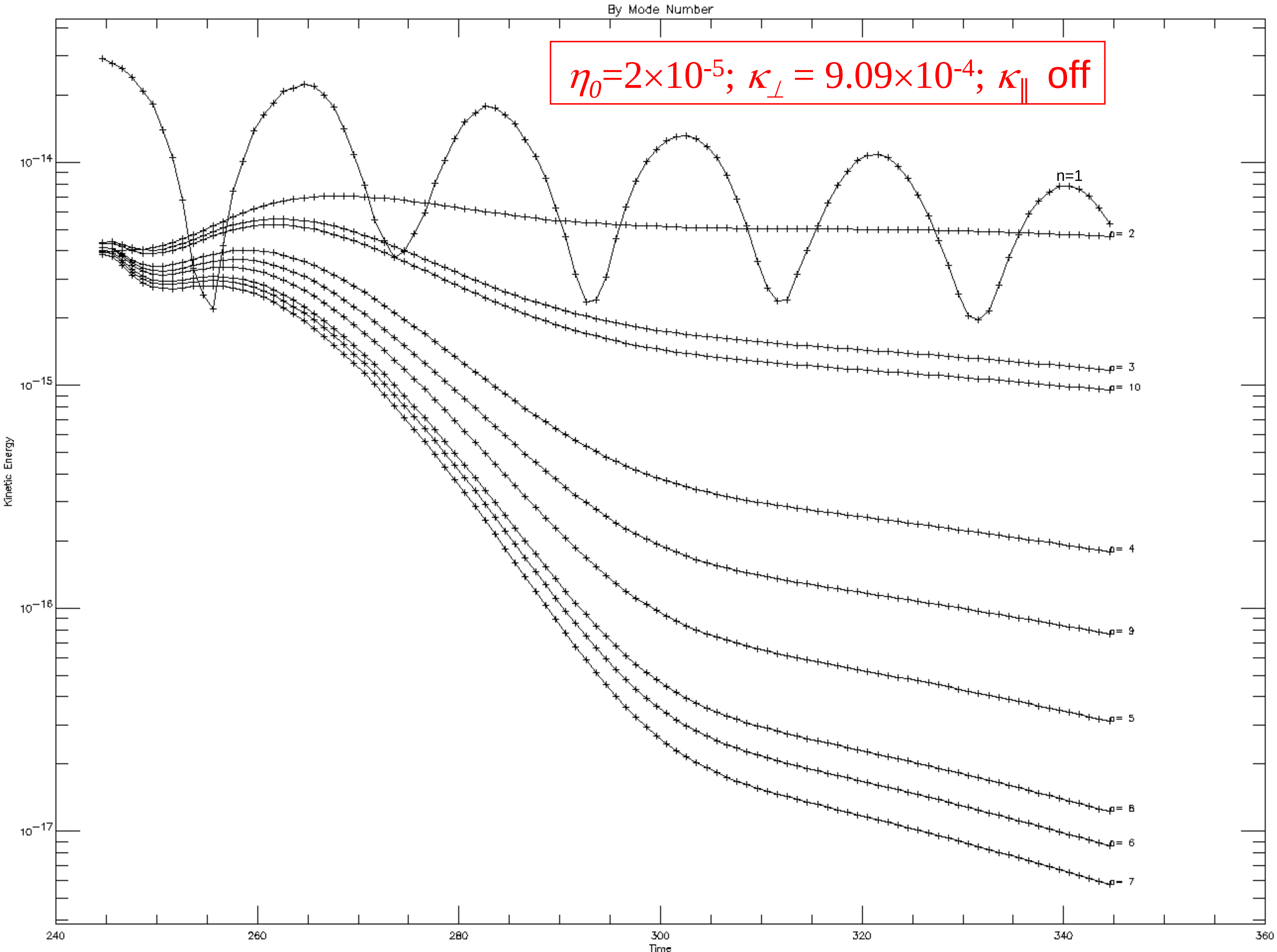
Growth Rates

n	$\gamma\tau_A$
1	——
2	——
3	——
4	0.00504
5	0.00761
6	0.00900
7	0.00945
8	0.00869
9	0.00652
10	——

Moderate Isotropic Heat Conduction Has Stronger Stabilizing Effect



High Perpendicular Heat Conduction Stabilizes All Modes



Conclusions

- The CDX equilibrium is MHD-unstable to resistive ballooning modes in the absence of large perpendicular thermal diffusivity.

- Extended MHD effects may be needed to account for the relative quiescence of the CDX edge.

or

- The modes may be present but saturated as a result of high transport levels arising from stochasticity caused by their nonlinear interaction.

Topics for Further Study

- Determine sensitivity of high-*n* modes to two-fluid parameters.
- Re-run q_0<1 case with realistic heat conduction.