Progress on Integral Parallel Ion Stress in NIMROD

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Momentum diffusion is anisotropic in low collisionality plasmas

- Closure of interest should be derived from full CEL drift kinetic equation.
- For this talk, consider

$$\left(\vec{v}_{\|} \cdot \vec{\nabla}\right) \tilde{F} - \left\langle C(\tilde{F} + \tilde{f}_M) \right\rangle = v_{\|} \left(\hat{\mathbf{b}} \cdot \vec{\nabla} \cdot \tilde{\mathbf{\Pi}}_{\|} - \tilde{R}_{\|}\right) \frac{f_M^0}{p^0}$$
$$- \frac{m}{T^0} (\hat{\mathbf{b}}\hat{\mathbf{b}} - \frac{\mathbf{I}}{\mathbf{3}}) \mathpunct{:} \vec{\nabla}_{\|} \tilde{\vec{V}} \left(v_{\|}^2 - \frac{v_{\perp}^2}{2} \right) f_M^0$$

- Invert free-streaming + momentum-restoring collision operator to solve for $\tilde{F}$ assuming:
  1. sheared slab geometry,
  2. $\partial / \partial t \rightarrow 0$,
  3. no heat flow terms.

Low collisionality parallel stress takes integral form.

- Result is following integral form for $\pi_{||}$:

$$
\begin{aligned}
\mathbf{K}_{21}(U_{||}) \quad & +(1+B_2)\pi_{||} + \mathbf{K}_{2}{}^{2(\pi_{||})} = \\
\int_0^\infty \quad & d\bar{L}\left(V_{||}(L+\bar{L}) - V_{||}(L-\bar{L})\right)\frac{\partial K_2(\bar{L})}{\partial \bar{L}}, \\
\mathbf{K}_{11}(U_{||}) \quad & +\mathbf{K}_{12}(\pi_{||}) = \\
\int_0^\infty \quad & d\bar{L}\left(V_{||}(L+\bar{L}) + V_{||}(L-\bar{L})\right)\frac{\partial K_1(\bar{L})}{\partial \bar{L}} + B_1 V_{||}(L),
\end{aligned}
$$

- $\pi_{||}$ appears on right side of flow evolution equation:

$$
\rho\frac{d\vec{V}}{dt} = \vec{J}\times\vec{B} - \vec{\nabla}p - \vec{\nabla}\cdot(\mathbf{\Pi}_{||} + \mathbf{\Pi}_{gv}),
$$

where $\mathbf{\Pi}_{||} = (\hat{\mathbf{b}}\hat{\mathbf{b}} - \mathbf{I}/3)\pi_{||}$.

Need semi-implicit operator to stabilize explicit integral stress.

- NIMROD has used $\mathbf{\Pi} = -\rho\nu\vec{\nabla}\vec{V}$ in past to introduce viscous dissipation.

- Upgrade of isotropic form to $\mathbf{\Pi} = -\rho\nu(\vec{\nabla}\vec{V} + (\vec{\nabla}\vec{V})^T - \vec{\nabla}\cdot\vec{V}/3)$ in progress. Full Braginskii stress including gyroviscosity in progress.

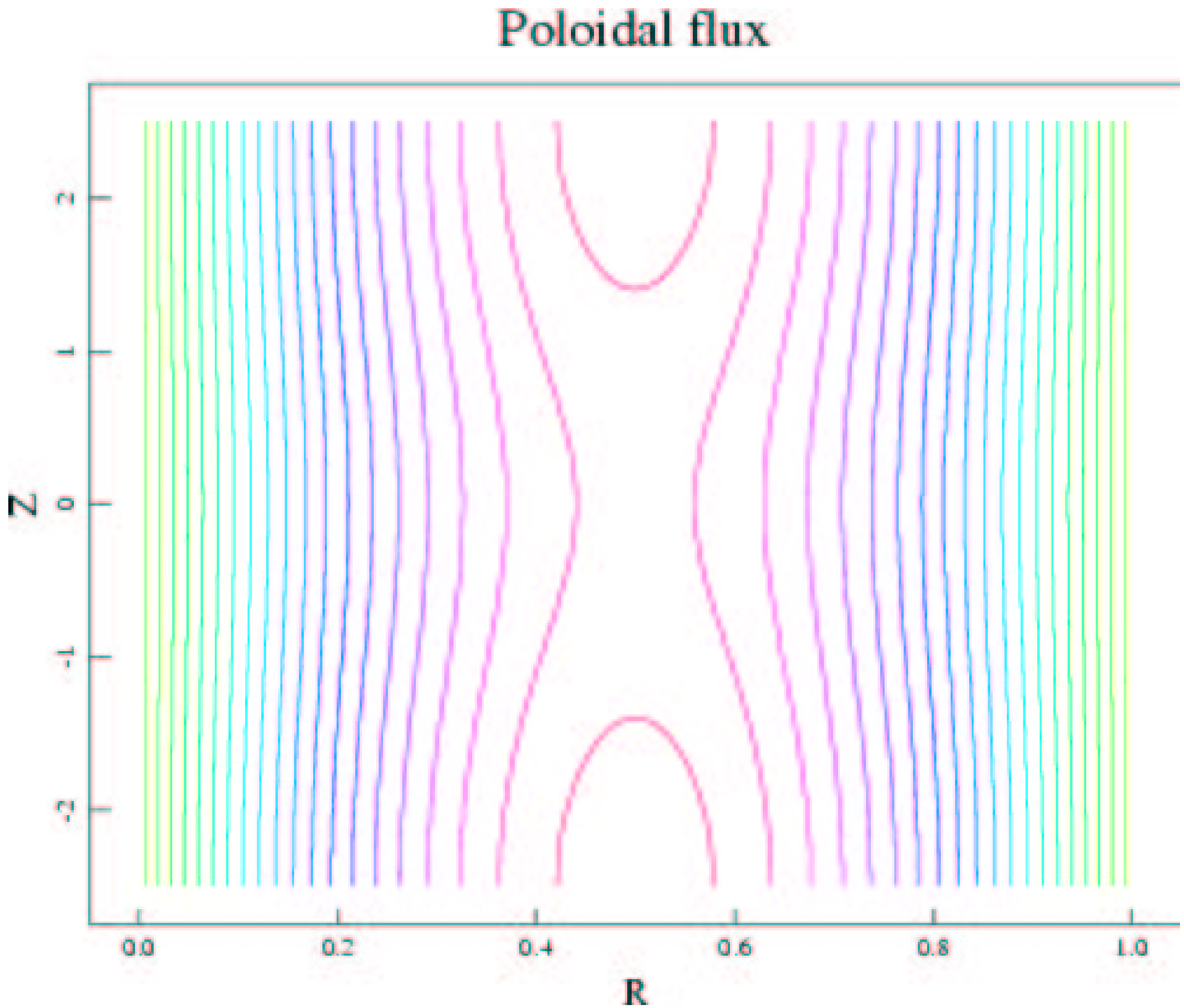
- For the purpose of this talk consider evolving:

$$\left(\rho^n - \Delta t f\vec{\nabla}\cdot(\rho^n\nu\vec{\nabla} - \mathbf{\Pi}_{si})\right)\Delta\vec{V} = \Delta t\vec{\nabla}\cdot\left(\rho^n\nu\vec{\nabla}\vec{V}^n - \mathbf{\Pi}_{||}\right),$$

where $\mathbf{\Pi}_{||} = -\rho\nu_{||}(\hat{\mathbf{b}}\hat{\mathbf{b}} - \mathbf{I}/3)\hat{\mathbf{b}}\hat{\mathbf{b}} : \vec{\nabla}\vec{V}$ and $\mathbf{\Pi}_{si}$ is analogous but acts on $\Delta\vec{V}$.

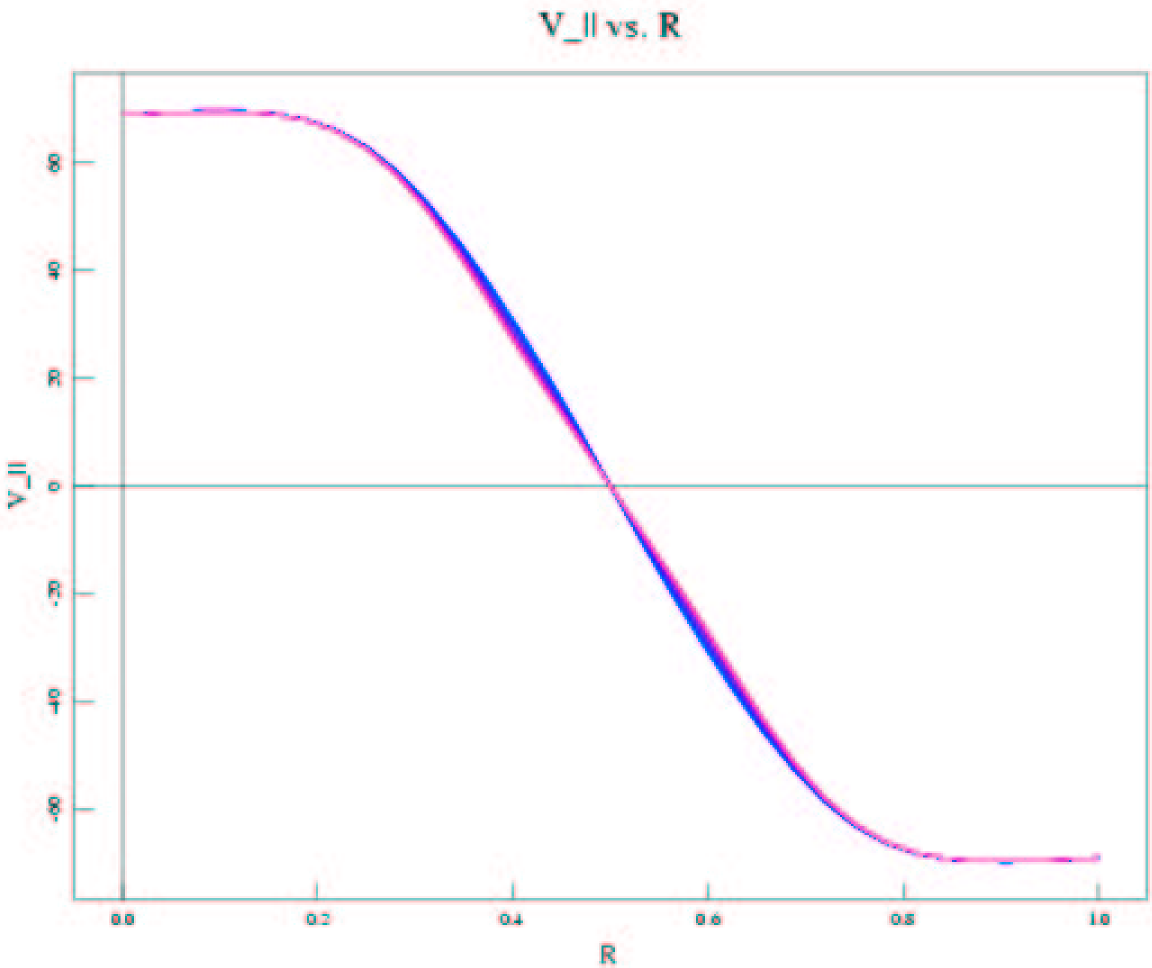
Test fully implicit advance in slab geometry.

- Flux of ϕ momentum (into page) at left boundary in $\vec{\nabla}R$ direction.



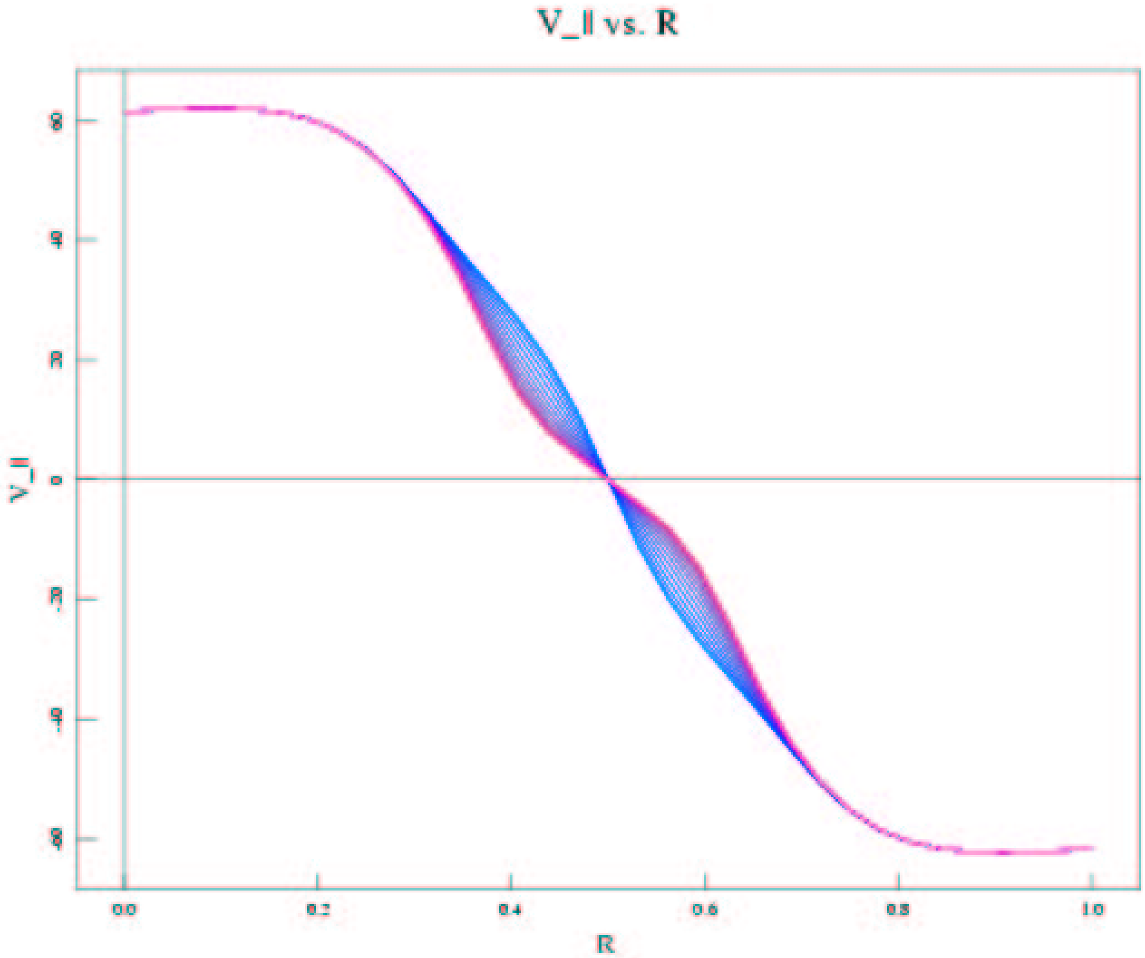
Parallel flow damping depends on ratio $\nu_{\|}/\nu$.

- Slight damping evident for $\nu_{\|}/\nu = 10^4$.



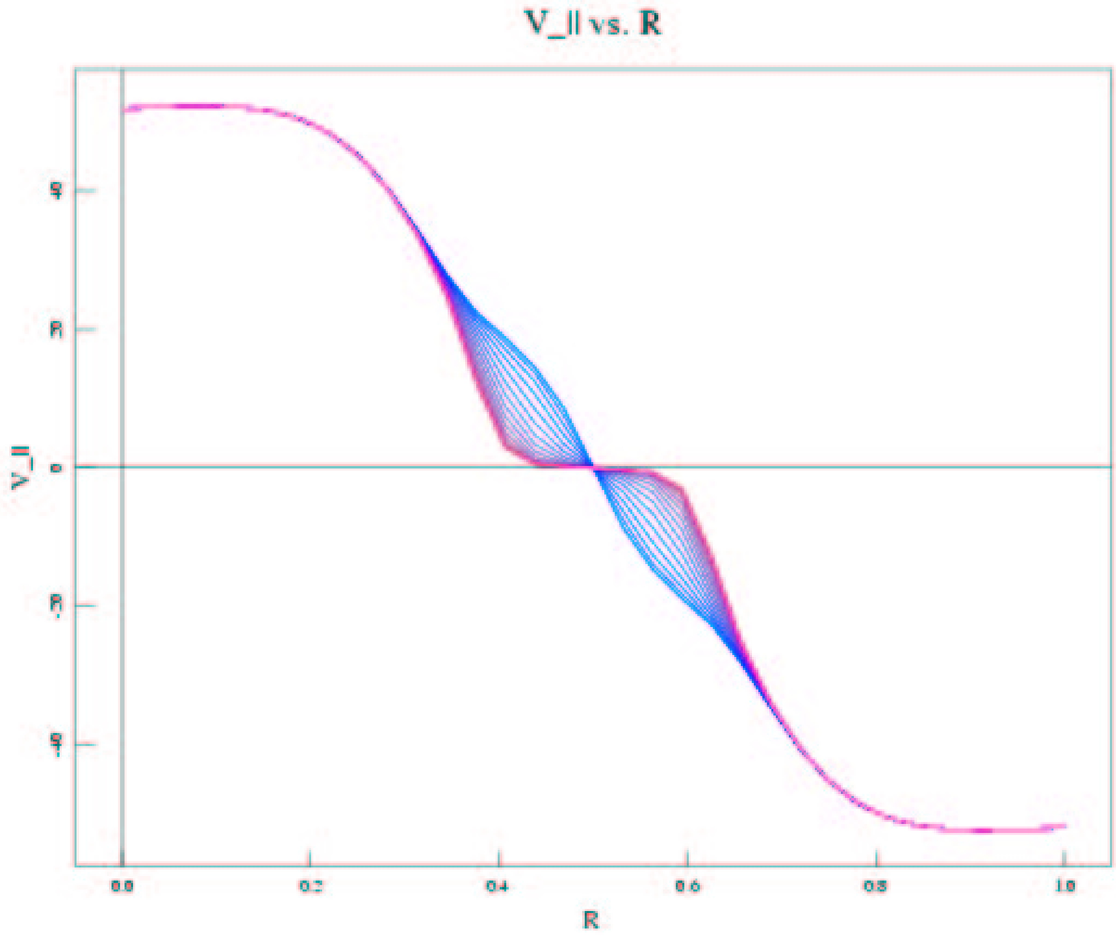
Parallel flow damping depends on ratio $\nu_{\|}/\nu$.

- Damping evident for $\nu_{\|}/\nu = 10^5$.



Parallel flow damping depends on ratio $\nu_{\|}/\nu$.

- Large damping for $\nu_{\|}/\nu = 10^6$.



Incorporate $\mathbf{\Pi}_{\|}$ *into fluid codes, typically* $\nu_{\|}/\nu \geq 10^5$.

- For low collisionality regimes, $\nu_{\|} = v_{thi}/k_{\|} \approx (10^5 - 10^6)/(.01 - 10) \approx 10^4 - 10^8$.
- Comparison of low collisionality $\pi_{\|}$ with gyroviscosity yields:

$$\pi_{\perp}/\pi_{\|} = \frac{\rho_i v_{thi} \nabla_{\perp} V}{v_{thi} {k_{\|}}^{-1} \nabla_{\|} V} = \rho_i \nabla_{\perp}$$

- To do list:
  1. Finish implementation of 2-D preconditioner for anisotropic momentum diffusion operator.
  2. Implement integral closure and test effectiveness of semi-implicit stabilization.
  3. Implement viscous heating using integral $\mathbf{\Pi}_{\|}$ term in temperature evolution.