CEMM Meeting

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MINIMAL MODELING OF TWO-FLUID EFFECTS IN LOW-COLLISIONALITY EXTENDED-MHD

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GOAL:

SIMPLEST TWO-FLUID MODEL THAT COULD BE APPLIED SATISFACTORILY TO EXTENDED-MHD PROBLEMS SUCH AS:

- DRIFT-RESISTIVE BALLOONING MODES
- NEOCLASSICAL TEARING MODES
- RESISTIVE-WALL MODES
- SAWTEETH OSCILLATIONS
- MAGNETIC RECONNECTION
- ...

INCLUDING ALL DIAMAGNETIC EFFECTS AND VALID FOR:

- LOW COLLISIONALITY
- ARBITRARY MAGNETIC GEOMETRY
- ARBITRARY BETA
- ELECTROMAGNETIC PERTURBATIONS

EXACT FLUID MOMENT EQUATIONS

FOR EACH SPECIES α:

$$\frac{d_\alpha n_\alpha}{dt} + n_\alpha \bar{\nabla}\cdot\bar{u}_\alpha = 0 \qquad \left(\frac{d_\alpha}{dt} \equiv \frac{\partial}{\partial t} + \bar{u}_\alpha\cdot\bar{\nabla}\right)$$

$$m_\alpha n_\alpha \frac{d_\alpha \bar{u}_\alpha}{dt} + \bar{\nabla} p_\alpha + \bar{\nabla}\cdot \mathbb{\Pi}_\alpha = e_\alpha n_\alpha (\bar{E} + \bar{u}_\alpha \times \bar{B}) + \bar{F}_\alpha^{coll}$$

$$\frac{3}{2}\frac{d_\alpha p_\alpha}{dt} + \frac{5}{2} p_\alpha \bar{\nabla}\cdot\bar{u}_\alpha + \bar{\nabla}\cdot\bar{q}_\alpha + \mathbb{\Pi}_\alpha : (\bar{\nabla}\bar{u}_\alpha) = H_\alpha^{coll}$$

$$p_\alpha = \frac{1}{3}(p_{\alpha\parallel} + 2 p_{\alpha\perp})$$

$$\mathbb{\Pi}_\alpha = \mathbb{\Pi}_{\alpha\parallel} + \mathbb{\Pi}_\alpha^{GV} + \mathbb{\Pi}_{\alpha\perp}^{coll}$$

$$\mathbb{\Pi}_{\alpha\parallel} = (p_{\alpha\parallel} - p_{\alpha\perp})\left(\bar{b}\bar{b} - \frac{1}{3}\mathbb{1}\right)$$

(I) PRIMARY SIMPLIFICATIONS

* QUASINEUTRALITY WITH SINGLE ION SPECIES OF UNIT CHARGE:

$$n_e = n_i \equiv n$$

$$\bar{u}_e = \bar{u}_i - \frac{1}{ne}\bar{j} \equiv \bar{u} - \frac{1}{ne}\bar{j}$$

$$\bar{j} = \bar{\nabla}\times\bar{B}$$

* NEGLECT m_e

* NEGLECT $\Pi_{e\perp}^{coll}$

$$\frac{\partial \bar{B}}{\partial t} + \bar{\nabla}\times\bar{E} = 0$$

$$\frac{dn}{dt} + n\,\bar{\nabla}\cdot\bar{u} = 0$$

$$\bar{E} = -\bar{u}\times\bar{B} + \frac{1}{ne}\left(\bar{j}\times\bar{B} - \bar{\nabla}p_e - \bar{\nabla}\cdot\Pi_{e\parallel} + \bar{F}_e^{coll}\right)$$

$$m_i n\frac{d\bar{u}}{dt} + \bar{\nabla}(p_i+p_e) + \bar{\nabla}\cdot\left(\Pi_{i\parallel} + \Pi_{e\parallel} + \Pi_i^{GV}\right) = \bar{j}\times\bar{B}$$

$$\frac{3}{2}\frac{dp_i}{dt} + \frac{5}{2}p_i\,\bar{\nabla}\cdot\bar{u} + \bar{\nabla}\cdot\bar{q}_i + \left(\Pi_{i\parallel} + \Pi_i^{GV}\right):(\bar{\nabla}\bar{u}) = H_i^{coll}$$

$$\frac{3}{2}\frac{dp_e}{dt} - \frac{3}{2ne}\bar{j}\cdot\bar{\nabla}p_e + \frac{5}{2}p_e\,\bar{\nabla}\cdot\left(\bar{u} - \frac{1}{ne}\bar{j}\right) + \bar{\nabla}\cdot\bar{q}_e +$$

$$+\Pi_{e\parallel} : \left[\bar{\nabla}\left(\bar{u} - \frac{1}{ne}\bar{j}\right)\right] = H_e^{coll}$$

REGARDLESS OF THE CLOSURE MODEL[*], THE ABOVE SYSTEM POSSESSES AN EXACT ENERGY CONSERVATION LAW:

$$\frac{\partial}{\partial t}\left(\frac{1}{2} m_i n u^2 + \frac{3}{2} p_i + \frac{3}{2} p_e\right) + \bar{\nabla}\cdot\bar{\Phi} =$$
$$= \bar{j}\cdot\bar{E} = -\frac{\partial}{\partial t}\left(\frac{B^2}{2}\right) - \bar{\nabla}\cdot(\bar{E}\times\bar{B}),$$

WHERE $\bar{\Phi}$ IS THE TOTAL PLASMA ENERGY FLUX:

$$\bar{\Phi} = \frac{1}{2} m_i n u^2 \bar{u} + \left(2p_{i\perp} + \frac{1}{2} p_{i\parallel}\right)\bar{u} + (p_{i\parallel} - p_{i\perp}) u_\parallel \bar{b} +$$
$$+ \left(2p_{e\perp} + \frac{1}{2} p_{e\parallel}\right)\left(\bar{u} - \frac{1}{en}\bar{j}\right) + (p_{e\parallel} - p_{e\parallel})\left(u_\parallel - \frac{1}{en} j_\parallel\right)\bar{b} +$$
$$+ \Pi_i^{GV}\cdot\bar{u} + \bar{q}_i + \bar{q}_e$$

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[*] PROVIDED $H_e^{coll} = -H_i^{coll} + \frac{1}{en}\bar{j}\cdot\bar{F}_e^{coll}$

II

ION GYROVISCOSITY AND PERPENDICULAR HEAT FLUX

* STRONG MAGNETIZATION : $\delta = \frac{\rho_i}{L} \ll 1$

* FAST DYNAMICS ORDERING :

$$\frac{\partial}{\partial t} \sim \delta \Omega_{ci} \quad , \quad u \sim v_{thi}$$

* KEEP ION F-L-R TERMS TO $O(\delta)$

* STRONG ANISOTROPY :

$$p_{i\parallel} - p_{i\perp} \sim p_i \quad , \quad q_{i\parallel} \sim p_i v_{thi} \quad (q_{i\perp} \sim \delta p_i v_{thi})$$

$$(\Pi_i^{GV})_{jk} = b_{lj} (h_{i\perp})_{kl} + \varepsilon_{ljlm} b_l (\delta_{nk} - b_n b_k) S_{mn}$$

WHERE

$$\bar{h}_{i\perp} = \frac{m_i}{eB} \bar{b} \times \left[2 p_{i\parallel} (\bar{b} \cdot \bar{\nabla}) \bar{u} + p_{i\perp} \bar{b} \times \bar{\omega} + \bar{\nabla} q_{iT\parallel} + 2 (q_{iB\parallel} - q_{iT\parallel}) \bar{k} \right]$$

$$\bar{\omega} = \bar{\nabla} \times \bar{u} \quad , \quad \bar{k} = (\bar{b} \cdot \bar{\nabla}) \bar{b} \qquad \text{AND}$$

$$S_{mn} = \frac{m_i}{4eB} \left(p_{i\perp} \frac{\partial u_{lm}}{\partial x_{nl}} + q_{iT\parallel} \frac{\partial b_{lm}}{\partial x_{nl}} \right)$$

$$\bar{q}_{i\perp} = \bar{q}_{iB\perp} + \bar{q}_{iT\perp}$$

$$\bar{q}_{iB\perp} = \frac{m_i}{eB}\,\bar{b}\times\left[\frac{p_{i\perp}}{2m_i}\,\bar{\nabla}\left(\frac{p_{i\parallel}}{n}\right) + \frac{p_{i\parallel}(p_{i\parallel}-p_{i\perp})}{m_i\,n}\,\bar{\kappa} + \right.$$
$$\left. + 2\,q_{iB\parallel}(\bar{b}\cdot\bar{\nabla})\bar{u} + q_{iT\parallel}\,\bar{b}\times\bar{\omega}\right] +$$
$$+ \tilde{q}_{iB\perp} + \bar{q}_{iB\perp}^{coll}$$

$$\bar{q}_{iT\perp} = \frac{m_i}{eB}\,\bar{b}\times\left[\frac{2p_{i\perp}}{m_i}\,\bar{\nabla}\left(\frac{p_{i\perp}}{n}\right) + 4\,q_{iT\parallel}(\bar{b}\cdot\bar{\nabla})\bar{u}\right] +$$
$$+ \tilde{q}_{iT\perp} + \bar{q}_{iT\perp}^{coll}$$

$$\tilde{q}_{iB\perp} = \frac{1}{eB}\,\bar{b}\times\left[\bar{\nabla}\,\tilde{r}_{iB\perp}^{(0)} + \left(\tilde{r}_{i\parallel}^{(0)} - 5\,\tilde{r}_{iB\perp}^{(0)}\right)\bar{\kappa}\right]$$

$$\tilde{q}_{iT\perp} = \frac{1}{eB}\,\bar{b}\times\left[\bar{\nabla}\left(\tilde{r}_{i\perp}^{(0)} - \tilde{r}_{iB\perp}^{(0)}\right) + \left(5\,\tilde{r}_{iB\perp}^{(0)} - \tilde{r}_{i\perp}^{(0)}\right)\bar{\kappa}\right]$$

TAKING INTO ACCOUNT THE ABOVE Π_i^{GV} AND $\bar{q}_{i\perp}$ RESULTS, THE F-L-R DYNAMIC EVOLUTION EQUATIONS FOR p_i AND $(p_{i\parallel}-p_{i\perp})$ ARE:

$$\frac{3}{2}\frac{dp_i}{dt}+\frac{5}{2}p_i\,\bar{\nabla}\cdot\bar{u}+(p_{i\parallel}-p_{i\perp})\left\{\bar{b}\cdot[(\bar{b}\cdot\bar{\nabla})\bar{u}]-\frac{1}{3}\bar{\nabla}\cdot\bar{u}\right\}+$$
$$+\bar{\nabla}\cdot(q_{i\parallel}\bar{b}+\bar{q}_{i\perp})+\bar{h}_{i\perp}\cdot[2(\bar{b}\cdot\bar{\nabla})\bar{u}+\bar{b}\times\bar{\omega}]+$$
$$+q_{iT\parallel}\,\sigma_i=H_i^{coll}$$

$$\frac{d}{dt}(p_{i\parallel}-p_{i\perp})+(p_{i\parallel}-p_{i\perp})\left\{\bar{b}\cdot[(\bar{b}\cdot\bar{\nabla})\bar{u}]+\frac{4}{3}\bar{\nabla}\cdot\bar{u}\right\}+$$
$$+p_i\left\{3\bar{b}\cdot[(\bar{b}\cdot\bar{\nabla})\bar{u}]-\bar{\nabla}\cdot\bar{u}\right\}+\bar{\nabla}\cdot[(2q_{iB\parallel}-q_{iT\parallel})\bar{b}+2\bar{q}_{iB\perp}-\bar{q}_{iT\perp}]+$$
$$+3\,q_{iT\parallel}\,\bar{b}\cdot\bar{\nabla}(\ln B)-2\bar{h}_{i\perp}\cdot[(\bar{b}\cdot\bar{\nabla})\bar{u}-\bar{b}\times\bar{\omega}]-$$
$$-6\bar{q}_{iB\perp}\cdot\bar{k}+2q_{iT\parallel}\,\sigma_i=G_i^{coll}$$

WHERE

$$\sigma_i=\frac{m_i}{4eB}\,\varepsilon_{jkl}\,b_j\,\frac{\partial b_{lk}}{\partial x_{mj}}\,(\delta_{mn}-b_m b_n)\,\frac{\partial u_{le}}{\partial x_{nj}}$$

COLLISIONLESS ION CLOSURE VARIABLES

$$q_{i\parallel} = q_{iB\parallel} + q_{iT\parallel}$$

$$q_{iB\parallel} = \frac{m_i}{2} \int d^3\bar{v} \; (\bar{v}' \cdot \bar{b})^3 \; f_i(\bar{v}, \bar{x}, t)$$

$$\bar{v}' = \bar{v} - \bar{u}(\bar{x})$$

$$q_{iT\parallel} = \frac{m_i}{2} \int d^3\bar{v} \; (\bar{v}' \cdot \bar{b}) \left[|\bar{v}'|^2 - (\bar{v}' \cdot \bar{b})^2 \right] f_i(\bar{v}, \bar{x}, t)$$

$$\tilde{r}_{i\parallel}^{(0)} = \frac{m_i^2}{2} \int d^3\bar{v} \; |\bar{v}'|^2 (\bar{v}' \cdot \bar{b})^2 \; (f_i^{(0)} - f_{iM})\left(\frac{m_i v'^2}{2}, \lambda, \bar{x}, t\right)$$

$$\sin\lambda = \frac{\bar{v}' \cdot \bar{b}}{|\bar{v}'|}$$

$$\tilde{r}_{i\perp}^{(0)} = \frac{m_i^2}{4} \int d^3\bar{v} \; |\bar{v}'|^2 \left[|\bar{v}'|^2 - (\bar{v}' \cdot \bar{b})^2 \right] \; (f_i^{(0)} - f_{iM})$$

$$\tilde{r}_{iB\perp}^{(0)} = \frac{m_i^2}{4} \int d^3\bar{v} \; (\bar{v}' \cdot \bar{b})^2 \left[|\bar{v}'|^2 - (\bar{v}' \cdot \bar{b})^2 \right] \; (f_i^{(0)} - f_{iM})$$

$$f_{iM} = \left(\frac{m_i}{2\pi}\right)^{3/2} \frac{n^{5/2}}{p_{i\parallel}^{1/2} \, p_{i\perp}} \; \exp\left[-\frac{m_i n v'^2}{2} \left(\frac{\cos^2\lambda}{p_{i\perp}} + \frac{\sin^2\lambda}{p_{i\parallel}} \right) \right]$$

III

WEAK ANISOTROPY ION REDUCTION

WITHOUT RIGOROUS JUSTIFICATION AT LOW COLLISIONALITY, ASSUME:

$$\Delta = \frac{p_{i\parallel} - p_{i\perp}}{p_i} \ll 1$$

$$q_{iB\parallel} \sim q_{iT\parallel} \sim \Delta\, p_i\, v_{th_i}$$

$$\tilde{r}_{i\parallel}^{(0)} - \tilde{r}_{i\perp}^{(0)} \sim 5\,\tilde{r}_{iB\perp}^{(0)} - \tilde{r}_{i\perp}^{(0)} \sim \Delta\, \tilde{r}_i^{(0)} \equiv \Delta\left(\tilde{r}_{i\parallel}^{(0)} + 2\,\tilde{r}_{i\perp}^{(0)}\right)/3$$

THEN, THE LEADING ORDER GYROVISCOSITY AND PERPENDICULAR HEAT FLUX BECOME:

$$(\Pi_i^{GV})_{jk} = b_{ij}\,(h_{i\perp})_{kj} + \varepsilon_{ijlm}\, b_l\,(\delta_{nkj} - b_n b_{kj})\, S_{mn}$$

$$\bar{h}_{i\perp} = \frac{m_i p_i}{eB}\, \bar{b} \times \left[2\,(\bar{b}\cdot\bar{\nabla})\,\bar{u} + \bar{b}\times\bar{\omega}\right]$$

$$S_{mn} = \frac{m_i p_i}{4eB}\, \frac{\partial u_{lm}}{\partial x_{nl}}$$

SO THAT $\Pi_i^{GV} : (\bar{\nabla}\bar{u}) = 0$,

AND

$$\bar{q}_{i\perp} = \frac{1}{eB}\, \bar{b} \times \left[\frac{5p_i}{2}\, \bar{\nabla}\left(\frac{p_i}{n}\right) + \bar{\nabla}\tilde{r}_i^{(0)}\right] + \bar{q}_{i\perp}^{coll}$$

WITH THE WEAK ANISOTROPY ORDERING, THE ION PRESSURE EQUATIONS BECOME:

$$\frac{3}{2}\frac{dp_i}{dt} + \frac{5}{2} p_i \bar{\nabla}\cdot\bar{u} + (p_{i\parallel} - p_{i\perp})\left\{\bar{b}\cdot[(\bar{b}\cdot\bar{\nabla})\bar{u}] - \frac{1}{3}\bar{\nabla}\cdot\bar{u}\right\} + \bar{\nabla}\cdot(q_{i\parallel}\bar{b} + \bar{q}_{i\perp}) = H_i^{coll}$$

$$\frac{d}{dt}(p_{i\parallel} - p_{i\perp}) + (p_{i\parallel} - p_{i\perp})\left\{\bar{b}\cdot[(\bar{b}\cdot\bar{\nabla})\bar{u}] + \frac{4}{3}\bar{\nabla}\cdot\bar{u}\right\} + p_i\left\{3\bar{b}\cdot[(\bar{b}\cdot\bar{\nabla})\bar{u}] - \bar{\nabla}\cdot\bar{u}\right\} + \bar{\nabla}\cdot\left[(3q_{iB\parallel} - q_{i\parallel})\bar{b} - \frac{2}{5}\bar{q}_{i\perp}\right] + 3(q_{i\parallel} - q_{iB\parallel})\bar{b}\cdot\bar{\nabla}(\ln B) - \frac{6}{5}\bar{q}_{i\perp}\cdot\bar{\kappa} + 3\bar{h}_{i\perp}\cdot(\bar{b}\times\bar{\omega}) = G_i^{coll}$$

CONSISTENCY OF THE ASSUMED WEAK ANISOTROPY ORDERING REQUIRES:

$$p_i\left\{3\bar{b}\cdot[(\bar{b}\cdot\bar{\nabla})\bar{u}] - \bar{\nabla}\cdot\bar{u}\right\} - G_i^{coll} = O(\Delta p_i v_{thi}/L)$$

NOW, THE ONLY COLLISIONLESS CLOSURE VARIABLES NEEDED ARE:

$$q_{i\parallel} = \frac{m_i}{2}\int d^3\bar{v}\, |\bar{v}'|^2 (\bar{v}'\cdot\bar{b})\, f_i(\bar{v}, \bar{x}, t)$$

$$q_{iB\parallel} = \frac{m_i}{2}\int d^3\bar{v}\, (\bar{v}'\cdot\bar{b})^3 f_i(\bar{v}, \bar{x}, t)$$

$$\tilde{r}_i^{(0)} = \frac{m_i^2}{6}\int d^3\bar{v}\, |\bar{v}'|^4 \left(f_i^{(0)} - f_{im}\right)\left(\frac{m_i v'^2}{2}, \lambda, \bar{x}, t\right)$$

IV

PARALLEL HEAT FLUX EVOLUTION EQUATIONS

WITH THE WEAK ANISOTROPY ORDERING, THE F-L-R DYNAMIC EVOLUTION EQUATIONS FOR THE ION PARALLEL HEAT FLUXES ARE:

$$\frac{dq_{i\parallel}}{dt} + 2\, q_{i\parallel}\, \bar{\nabla}\cdot\bar{u} + q_{iB\parallel}\left\{3\bar{b}\cdot\left[(\bar{b}\cdot\bar{\nabla})\bar{u}\right] - \bar{\nabla}\cdot\bar{u}\right\} +$$

$$+ \frac{1}{m_i}\left[p_i + \frac{2}{3}(p_{i\parallel} - p_{i\perp})\right]\bar{b}\cdot\bar{\nabla}\left\{\frac{1}{n}\left[\frac{5}{2}p_i + \frac{2}{3}(p_{i\parallel} - p_{i\perp})\right]\right\} -$$

$$- \frac{1}{m_i n}\left[p_i - \frac{1}{3}(p_{i\parallel} - p_{i\perp})\right](p_{i\parallel} - p_{i\perp})\; \bar{b}\cdot\bar{\nabla}(\ln B) +$$

$$+ \frac{1}{m_i}\bar{h}_{i\perp}\cdot\left[\frac{5}{2}\bar{\nabla}\left(\frac{p_i}{n}\right) - \frac{p_i}{n}\bar{\kappa}\right] + \frac{p_i}{m}\left[\bar{\nabla}\cdot\left(\frac{1}{n}\bar{h}_{i\perp}\right) + \frac{p_i}{n}\sigma_i\right] +$$

$$+ \bar{q}_{i\perp}\cdot\left[\frac{4}{5}(\bar{b}\cdot\bar{\nabla})\bar{u} + \frac{7}{5}\bar{b}\times\bar{\omega}\right] +$$

$$+ \frac{1}{m_i}\left[\bar{b}\cdot\bar{\nabla}\tilde{r}_{i\parallel}^{(0)} + \left(\tilde{r}_{i\perp}^{(0)} - \tilde{r}_{i\parallel}^{(0)}\right)\bar{b}\cdot\bar{\nabla}(\ln B)\right] + \frac{1}{2}\tilde{S}_i^{(1)} = D_i^{coll}$$

AND

$$\frac{dq_{iB\parallel}}{dt} + q_{iB\parallel}\left\{3\bar{b}\cdot[(\bar{b}\cdot\bar{\nabla})\bar{u}] + \bar{\nabla}\cdot\bar{u}\right\} +$$

$$+ \frac{1}{m_i}\left[\frac{3}{2}p_i + (p_{i\parallel} - p_{i\perp})\right]\bar{b}\cdot\bar{\nabla}\left\{\frac{1}{n}\left[p_i + \frac{2}{3}(p_{i\parallel} - p_{i\parallel})\right]\right\} +$$

$$+ \frac{1}{m_i}\bar{h}_{i\perp}\cdot\left[\frac{3}{2}\bar{\nabla}\left(\frac{p_i}{n}\right) - 3\frac{p_i}{n}\bar{k}\right] + \frac{3}{5}\bar{q}_{i\perp}\cdot(\bar{b}\times\bar{\omega}) +$$

$$+ \frac{1}{m_i}\left[\bar{b}\cdot\bar{\nabla}\left(\tilde{r}_{i\parallel}^{(0)} - 2\tilde{r}_{iB\perp}^{(0)}\right) + \left(5\tilde{r}_{iB\perp}^{(0)} - \tilde{r}_{i\parallel}^{(0)}\right)\bar{b}\cdot\bar{\nabla}(\ln B)\right] +$$

$$+ \frac{1}{2}\tilde{S}_{iB}^{(1)} = D_{iB}^{coll}$$

CONSISTENCY OF THE ASSUMED WEAK ANISOTROPY ORDERING REQUIRES:

$$\frac{1}{m_i}\left[\frac{5}{2}p_i\,\bar{b}\cdot\bar{\nabla}\left(\frac{p_i}{n}\right) + \bar{b}\cdot\bar{\nabla}\tilde{r}_i^{(0)}\right] - D_i^{coll} = O(\Delta p_i v_{th_i}^2/L)$$

AND

$$\frac{1}{m_i}\left[\frac{3}{2}p_i\,\bar{b}\cdot\bar{\nabla}\left(\frac{p_i}{n}\right) + \frac{3}{5}\bar{b}\cdot\bar{\nabla}\tilde{r}_i^{(0)}\right] - D_{iB}^{coll} = O(\Delta p_i v_{th_i}^2/L)$$

IF WE ADOPT THE FOLLOWING APPROXIMATION FOR THE ION FOURTH RANK FLUID MOMENT:

$$m_i^2 \int d^3\bar{v}\; v_j' v_k' v_\ell' v_m' \; f_i(\bar{v},\bar{x},t) =$$

$$= \frac{m_i^2}{n}\left[\int d^3\bar{v}\; v_{\{j}' v_k' \; f_i\right]\left[\int d^3\bar{v}\; v_\ell' v_{m\}}' \; f_i\right] +$$

$$+ \frac{1}{2}\Big[(\tilde{r}_{i\perp}^{(0)} - \tilde{r}_{iB\perp}^{(0)})\,\delta_{\{jk}\,\delta_{\ell m\}} + (5\tilde{r}_{iB\perp}^{(0)} - \tilde{r}_{i\perp}^{(0)})\,\delta_{\{jk}\,b_\ell\, b_{m\}} +$$

$$+ (4\tilde{r}_{i\parallel}^{(0)} + 3\tilde{r}_{i\perp}^{(0)} - 35\tilde{r}_{iB\perp}^{(0)})\, b_j b_k b_\ell b_m\Big],$$

THEN $\tilde{S}_i^{(1)} = \tilde{S}_{iB}^{(1)} = 0$,

AND THE ONLY COLLISIONLESS ION CLOSURE VARIABLES NEEDED ARE THE THREE INDEDENDENT EVEN MOMENTS OF $(f_i^{(0)} - f_{im})$: $\tilde{r}_{i\parallel}^{(0)}$, $\tilde{r}_{i\perp}^{(0)}$ AND $\tilde{r}_{iB\perp}^{(0)}$.

ELECTRON REDUCTION

ANALOGOUS TO THE ION REDUCTION, WITH THE FOLLOWING ORDERING ASSUMPTIONS:

$$p_e \sim p_i \sim m_i n v_{thi}^2$$

$$u_e \sim u_i \sim v_{thi}$$

$$p_{e\parallel} - p_{e\perp} \sim p_{i\parallel} - p_{i\perp} \sim \Delta p_i$$

$$q_{e\parallel} \sim q_{i\parallel} \sim \Delta p_i v_{thi}$$

$$q_{e\perp} \sim q_{i\perp} \sim \delta p_i v_{thi}$$

TAKE THE MASSLESS ELECTRON LIMIT: $m_e/m_i \longrightarrow 0$

THE ELECTRON PRESSURE EQUATIONS HAVE A REGULAR $m_e \to 0$ LIMIT. THEY HAVE THE SAME FORM AS THE ION EQUATIONS, ONLY MISSING THE $3\,\bar{b}_\perp \cdot (\bar{b} \times \bar{\omega})$ GYROVISCOUS TERM IN THE $(P_\parallel - P_\perp)$ EVOLUTION EQUATION:

$$\frac{3}{2}\frac{d_e P_e}{dt} + \frac{5}{2} P_e \bar{\nabla}\cdot\bar{u}_e + (P_{e\parallel} - P_{e\perp})\left\{\bar{b}\cdot\left[(\bar{b}\cdot\bar{\nabla})\bar{u}_e\right] - \frac{1}{3}\bar{\nabla}\cdot\bar{u}_e\right\} + \bar{\nabla}\cdot(q_{e\parallel}\bar{b} + \bar{q}_{e\perp}) = H_e^{coll}$$

$$\frac{d_e}{dt}(P_{e\parallel} - P_{e\perp}) + (P_{e\parallel} - P_{e\perp})\left\{\bar{b}\cdot\left[(\bar{b}\cdot\bar{\nabla})\bar{u}_e\right] + \frac{4}{3}\bar{\nabla}\cdot\bar{u}_e\right\} + P_e\left\{3\bar{b}\cdot\left[(\bar{b}\cdot\bar{\nabla})\bar{u}_e\right] - \bar{\nabla}\cdot\bar{u}_e\right\} + \bar{\nabla}\cdot\left[(3q_{eB\parallel} - q_{e\parallel})\bar{b} - \frac{2}{5}\bar{q}_{e\perp}\right] + 3(q_{e\parallel} - q_{iB\parallel})\bar{b}\cdot\bar{\nabla}(\ln B) - \frac{6}{5}\bar{q}_{e\perp}\cdot\bar{k} = G_e^{coll}$$

WHERE

$$\frac{d_e}{dt} = \frac{\partial}{\partial t} + \bar{u}_e\cdot\bar{\nabla}$$

$$\bar{u}_e = \bar{u} - \frac{1}{en}\bar{j}$$

AND

$$\bar{q}_{e\perp} = \frac{-1}{eB}\bar{b}\times\left[\frac{5}{2}P_e\bar{\nabla}\left(\frac{P_e}{n}\right) + \bar{\nabla}\tilde{T}_e^{(0)}\right] + \bar{q}_{e\perp}^{coll}$$

THE ELECTRON PARALLEL HEAT FLUX EQUATIONS HAVE LARGE TERMS PROPORTIONAL TO $1/m_e$ AS $m_e \to 0$.

REQUIRING THESE TERMS TO CANCEL, YIELDS THE FOLLOWING CONSTRAINTS:

$$\left[p_e + \frac{2}{3}(p_{e\parallel} - p_{e\perp})\right] \bar{b}\cdot\bar{\nabla}\left\{\frac{1}{n}\left[p_e + \frac{2}{3}(p_{e\parallel} - p_{e\perp})\right]\right\} +$$
$$+ \bar{b}\cdot\bar{\nabla}\left[\frac{2}{5}\tilde{r}_e^{(0)} + \frac{4}{15}\left(\tilde{r}_{e\parallel}^{(0)} - \tilde{r}_{e\perp}^{(0)}\right) + \frac{4}{15}\left(\tilde{r}_{e\parallel}^{(0)} - 5\,\tilde{r}_{eB\perp}^{(0)}\right)\right] -$$
$$- \frac{2}{3}\left(\tilde{r}_{e\parallel}^{(0)} - 5\,\tilde{r}_{eB\perp}^{(0)}\right) \bar{b}\cdot\bar{\nabla}(\ln B) = 0$$

$$\frac{p_e}{B}\,\bar{b}\cdot\bar{\nabla}\left[\frac{B}{n}(p_{e\parallel} - p_{e\perp})\right] + \frac{nB}{3}\,\bar{b}\cdot\bar{\nabla}\left[\frac{1}{Bn^2}(p_{e\parallel} - p_{e\perp})^2\right] +$$
$$+ \left[\left(\tilde{r}_{e\parallel}^{(0)} - \tilde{r}_{e\perp}^{(0)}\right) - \frac{5}{3}\left(\tilde{r}_{e\parallel}^{(0)} - 5\,\tilde{r}_{eB\perp}^{(0)}\right)\right] \bar{b}\cdot\bar{\nabla}(\ln B) +$$
$$+ \frac{2}{3}\,\bar{b}\cdot\bar{\nabla}\left(\tilde{r}_{e\parallel}^{(0)} - 5\,\tilde{r}_{eB\perp}^{(0)}\right) = 0$$

SUMMARY OF TWO-FLUID MODEL

$$n_e = n_i \equiv n$$

$$\bar{u}_i \equiv \bar{u}$$

$$\bar{u}_e = \bar{u} - \frac{1}{en}\bar{j}$$

$$\bar{j} = \bar{\nabla} \times \bar{B}$$

$$\frac{\partial \bar{B}}{\partial t} + \bar{\nabla} \times \bar{E} = 0$$

$$\frac{dn}{dt} + n\,\bar{\nabla}\cdot\bar{u} = 0$$

$$\bar{E} = -\bar{u}\times\bar{B} + \frac{1}{en}\left(\bar{j}\times\bar{B} - \bar{\nabla}p_e - \bar{\nabla}\cdot\Pi_{e\parallel} + \bar{F}_e^{coll}\right)$$

$$m_i n \frac{d\bar{u}}{dt} + \bar{\nabla}(p_i + p_e) + \bar{\nabla}\cdot\left(\Pi_{i\parallel} + \Pi_{e\parallel} + \Pi_i^{GV}\right) = \bar{j}\times\bar{B}$$

$$\left(\Pi_{\alpha\parallel}\right)_{jk} = \left(p_{\alpha\parallel} - p_{\alpha\perp}\right)\left(b_j b_k - \frac{1}{3}\delta_{jk}\right) \qquad \alpha = i, e$$

$$\left(\Pi_i^{GV}\right)_{jk} = \frac{m_i p_i}{4eB}\,\varepsilon_{ljem}\, b_e\left(\delta_{nk}\delta_{lj} + 3 b_n b_k \delta_{lj}\right)\frac{\partial u_{lm}}{\partial x_{nj}}$$

ION AND ELECTRON PRESSURE EQUATIONS:

$$\frac{3}{2}\frac{d_\alpha P_\alpha}{dt} + \frac{5}{2} P_\alpha \bar{\nabla}\cdot\bar{u}_\alpha + (P_{\alpha\parallel} - P_{\alpha\perp})\left\{\bar{b}\cdot[(\bar{b}\cdot\bar{\nabla})\bar{u}_\alpha] - \frac{1}{3}\bar{\nabla}\cdot\bar{u}_\alpha\right\} +$$
$$+ \bar{\nabla}\cdot(q_{\alpha\parallel}\bar{b} + \bar{q}_{\alpha\perp}) = H_\alpha^{coll}$$

$$\frac{d_\alpha}{dt}(P_{\alpha\parallel} - P_{\alpha\perp}) + (P_{\alpha\parallel} - P_{\alpha\perp})\left\{\bar{b}\cdot[(\bar{b}\cdot\bar{\nabla})\bar{u}_\alpha] + \frac{4}{3}\bar{\nabla}\cdot\bar{u}_\alpha\right\} +$$
$$+ P_\alpha\left\{3\bar{b}\cdot[(\bar{b}\cdot\bar{\nabla})\bar{u}_\alpha] - \bar{\nabla}\cdot\bar{u}_\alpha\right\} + \bar{\nabla}\cdot\left[(3q_{\alpha B\parallel} - q_{\alpha\parallel})\bar{b} - \frac{2}{5}\bar{q}_{\alpha\perp}\right] +$$
$$+ 3(q_{\alpha\parallel} - q_{\alpha B\parallel})\bar{b}\cdot\bar{\nabla}(\ln B) - \frac{6}{5}\bar{q}_{\alpha\perp}\cdot\bar{\kappa} +$$
$$+ 3\,\bar{h}_{\alpha\perp}\cdot(\bar{b}\times\bar{\omega}) = G_\alpha^{coll}$$

$$\frac{d_\alpha}{dt} = \frac{\partial}{\partial t} + \bar{u}_\alpha\cdot\bar{\nabla}$$
$$\bar{\kappa} = (\bar{b}\cdot\bar{\nabla})\bar{b}$$
$$\bar{\omega} = \bar{\nabla}\times\bar{u}$$
$$\bar{h}_{i\perp} = \frac{m_i P_i}{eB}\,\bar{b}\times\left[2(\bar{b}\cdot\bar{\nabla})\bar{u} + \bar{b}\times\bar{\omega}\right]$$
$$\bar{h}_{e\perp} = 0$$
$$\bar{q}_{\alpha\perp} = \frac{1}{e_\alpha B}\,\bar{b}\times\left[\frac{5}{2}P_\alpha\bar{\nabla}\left(\frac{P_\alpha}{n}\right) + \bar{\nabla}\tilde{r}_\alpha^{(0)}\right] + \bar{q}_{\alpha\perp}^{coll}$$

$$e_i = -e_e = e$$

ION PARALLEL HEAT FLUX EQUATIONS:

$$\frac{dq_{i\parallel}}{dt} + 2q_{i\parallel}\,\bar{\nabla}\cdot\bar{u} + q_{iB\parallel}\left\{3\bar{b}\cdot\left[(\bar{b}\cdot\bar{\nabla})\bar{u}\right] - \bar{\nabla}\cdot\bar{u}\right\} +$$
$$+ \frac{1}{m_i}\left[p_i + \frac{2}{3}(p_{i\parallel}-p_{i\perp})\right]\bar{b}\cdot\bar{\nabla}\left\{\frac{1}{n}\left[\frac{5}{2}p_i + \frac{2}{3}(p_{i\parallel}-p_{i\perp})\right]\right\} -$$
$$- \frac{1}{m_i n}\left[p_i - \frac{1}{3}(p_{i\parallel}-p_{i\perp})\right](p_{i\parallel}-p_{i\perp})\,\bar{b}\cdot\bar{\nabla}(\ln B) +$$
$$+ \frac{1}{m_i}\bar{h}_{i\perp}\cdot\left[\frac{5}{2}\bar{\nabla}\left(\frac{p_i}{n}\right) - \frac{p_i}{n}\bar{k}\right] + \frac{p_i}{m_i}\left[\bar{\nabla}\cdot\left(\frac{1}{n}\bar{h}_{i\perp}\right) + \frac{p_i}{n}\sigma_i\right] +$$
$$+ \bar{q}_{i\perp}\cdot\left[\frac{4}{5}(\bar{b}\cdot\bar{\nabla})\bar{u} + \frac{7}{5}\bar{b}\times\bar{\omega}\right] +$$
$$+ \frac{1}{m_i}\left\{\bar{b}\cdot\bar{\nabla}\left[\tilde{r}_i^{(0)} + \frac{2}{3}\left(\tilde{r}_{i\parallel}^{(0)} - \tilde{r}_{i\perp}^{(0)}\right)\right] - \left(\tilde{r}_{i\parallel}^{(0)} - \tilde{r}_{i\perp}^{(0)}\right)\bar{b}\cdot\bar{\nabla}(\ln B)\right\} = D_i^{coll}$$

$$\frac{dq_{iB\parallel}}{dt} + q_{iB\parallel}\left\{3\bar{b}\cdot\left[(\bar{b}\cdot\bar{\nabla})\bar{u}\right] + \bar{\nabla}\cdot\bar{u}\right\} +$$
$$+ \frac{1}{m_i}\left[\frac{3}{2}p_i + (p_{i\parallel}-p_{i\perp})\right]\bar{b}\cdot\bar{\nabla}\left\{\frac{1}{n}\left[p_i + \frac{2}{3}(p_{i\parallel}-p_{i\perp})\right]\right\} +$$
$$+ \frac{1}{m_i}\bar{h}_{i\perp}\cdot\left[\frac{3}{2}\bar{\nabla}\left(\frac{p_i}{n}\right) - 3\frac{p_i}{n}\bar{k}\right] + \frac{3}{5}\bar{q}_{i\perp}\cdot(\bar{b}\times\bar{\omega}) +$$
$$+ \frac{1}{m_i}\left\{\bar{b}\cdot\bar{\nabla}\left[\frac{3}{5}\tilde{r}_i^{(0)} + \frac{2}{5}\left(\tilde{r}_{i\parallel}^{(0)} - \tilde{r}_{i\perp}^{(0)}\right) + \frac{2}{5}\left(\tilde{r}_{i\parallel}^{(0)} - 5\tilde{r}_{iB\perp}^{(0)}\right)\right] -\right.$$
$$\left. - \left(\tilde{r}_{i\parallel}^{(0)} - 5\tilde{r}_{iB\perp}^{(0)}\right)\bar{b}\cdot\bar{\nabla}(\ln B)\right\} = D_{iB}^{coll}$$

$$\sigma_i = \frac{m_i}{4eB}\,\varepsilon_{jkl}\,b_j\,\frac{\partial b_{lk}}{\partial x_{mq}}\left(\delta_{mn} - b_m b_n\right)\frac{\partial u_{lq}}{\partial x_{nq}}$$

ELECTRON PARALLEL HEAT FLUXES DETERMINED IMPLICITLY BY:

$$\left[p_e+\frac{2}{3}(p_{e\parallel}-p_{e\perp})\right]\bar{b}\cdot\bar{\nabla}\left\{\frac{1}{n}\left[p_e+\frac{2}{3}(p_{e\parallel}-p_{e\perp})\right]\right\}+$$
$$+\bar{b}\cdot\bar{\nabla}\left[\frac{2}{5}\tilde{r}_e^{(0)}+\frac{4}{15}(\tilde{r}_{e\parallel}^{(0)}-\tilde{r}_{e\perp}^{(0)})+\frac{4}{15}(\tilde{r}_{e\parallel}^{(0)}-5\tilde{r}_{eB\perp}^{(0)})\right]-$$
$$-\frac{2}{3}(\tilde{r}_{e\parallel}^{(0)}-5\tilde{r}_{eB\perp}^{(0)})\,\bar{b}\cdot\bar{\nabla}(\ln B)=0$$

$$\frac{p_e}{B}\bar{b}\cdot\bar{\nabla}\left[\frac{B}{n}(p_{e\parallel}-p_{e\perp})\right]+\frac{nB}{3}\bar{b}\cdot\bar{\nabla}\left[\frac{1}{Bn^2}(p_{e\parallel}-p_{e\perp})^2\right]+$$
$$+\left[(\tilde{r}_{e\parallel}^{(0)}-\tilde{r}_{e\perp}^{(0)})-\frac{5}{3}(\tilde{r}_{e\parallel}^{(0)}-5\tilde{r}_{eB\perp}^{(0)})\right]\bar{b}\cdot\bar{\nabla}(\ln B)+$$
$$+\frac{2}{3}\bar{b}\cdot\bar{\nabla}(\tilde{r}_{e\parallel}^{(0)}-5\tilde{r}_{eB\perp}^{(0)})=0$$

COLLISIONLESS ("LANDAU") CLOSURE VARIABLES:

$$\tilde{r}_\alpha^{(0)} = \frac{m_\alpha^2}{6}\int d^3\bar{v}\ |\bar{v}_\alpha'|^4 \left(f_\alpha^{(0)} - f_{\alpha M}\right)\left(\frac{m_\alpha v_\alpha'^2}{2}, \lambda_\alpha, \bar{x}, t\right)$$

$$\tilde{r}_{\alpha\parallel}^{(0)} - \tilde{r}_{\alpha\perp}^{(0)} = \frac{m_\alpha^2}{4}\int d^3\bar{v}\ |\bar{v}_\alpha'|^4 \left(3\sin^2\lambda_\alpha - 1\right)\left(f_\alpha^{(0)} - f_{\alpha M}\right)$$

$$\tilde{r}_{\alpha\parallel}^{(0)} - 5\tilde{r}_{\alpha B\perp}^{(0)} = \frac{m_\alpha^2}{4}\int d^3\bar{v}\ |\bar{v}_\alpha'|^4 \left(5\sin^4\lambda_\alpha - 3\sin^2\lambda_\alpha\right)\left(f_\alpha^{(0)} - f_{\alpha M}\right)$$

$$\bar{v}_\alpha' = \bar{v} - \bar{u}_\alpha(x)$$

$$\sin\lambda_\alpha = \frac{\bar{v}_\alpha' \cdot \bar{b}}{|\bar{v}_\alpha'|}$$

$$f_{\alpha M} = \left(\frac{m_\alpha}{2\pi}\right)^{3/2} \frac{n^{5/2}}{P_{\alpha\parallel}^{1/2} P_{\alpha\perp}} \exp\left[-\frac{m_\alpha n v_\alpha'^2}{2}\left(\frac{\cos^2\lambda_\alpha}{P_{\alpha\perp}} + \frac{\sin^2\lambda_\alpha}{P_{\alpha\parallel}}\right)\right]$$

COLLISIONAL CLOSURE VARIABLES

$$\bar{F}_e^{coll} = -\bar{F}_i^{coll}$$

$$H_e^{coll} = -H_i^{coll} + \frac{1}{en}\,\bar{j}\cdot\bar{F}_e^{coll}$$

$$G_\alpha^{coll} \qquad (\alpha = i, e)$$

$$\bar{q}_{\alpha\perp}^{coll}$$

$$D_i^{coll}$$

$$D_{iB}^{coll}$$

PROPOSE TO EVALUATE THEM FOLLOWING A CHAPMAN-ENSKOG-LIKE PROCEDURE, WITH TRIAL DISTRIBUTION FUNCTIONS THAT YIELD IDENTICALLY THE KNOWN FLUID MOMENTS AND REPRODUCE THE EXACT RESULTS IN THE HIGH COLLISIONALITY LIMIT.