MHD-Like Kinetic Simulation and Semi-Collisonal Tearing Modes

S.E. Parker, W. Wan, J. Peoble and Y. Chen
University of Colorado, Boulder

Outline

- MHD-like kinetic simulation
  - Electron inertia
  - Pressure vs. stress

- Direct gyrokinetic simulation of tearing modes

MHD-like Kinetic Simulation

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\mathbf{E} = -\mathbf{u} \times \mathbf{B} + \frac{\eta}{\mu_0} \nabla \times \mathbf{B} + \frac{1}{\mu_0 e n} (\nabla \times \mathbf{B}) \times \mathbf{B}$$

$$-\frac{1}{en} \nabla \cdot \mathbf{\Pi}_e - \frac{m_e}{ne} \frac{\partial (n \mathbf{u}_e)}{\partial t}$$

$$n = n_0 + \int \delta f d^3 v, \qquad \mathbf{\Pi}_e \equiv \int \mathbf{v}\mathbf{v} \delta f_e d^3 v$$

$$\mathbf{u} = \mathbf{u}_i + \frac{m_e}{m_i} \mathbf{u}_e, \qquad \mathbf{u}_\alpha = \frac{1}{n} \int \mathbf{v} \delta f_\alpha d^3 v$$

Neglect displacement current and assume quasi-neutrality

Including Electron Inertia

$$\nabla \times (\nabla \times \mathbf{E}) = -\frac{\partial\,(\nabla \times \mathbf{B})}{\partial t} = \mu_0 e \left[\frac{\partial(n\mathbf{u}_e)}{\partial t} - \frac{\partial(n\mathbf{u}_i)}{\partial t}\right]$$

$$-\frac{m_e}{ne}\frac{\partial(n\mathbf{u}_e)}{\partial t} = -\frac{m_e}{\mu_0 n e^2}\nabla \times (\nabla \times \mathbf{E}) - \frac{m_e}{ne}\frac{\partial(n\mathbf{u}_i)}{\partial t}$$

$$\frac{\partial(n\mathbf{u}_i)}{\partial t} = \frac{en}{m_i}(\mathbf{E} + \mathbf{u}_i \times \mathbf{B}) - \frac{1}{m_i}\nabla \cdot \mathbf{\Pi}_i$$

$$-\frac{m_e}{ne}\frac{\partial(n\mathbf{u}_e)}{\partial t} = \frac{m_e}{\mu_0 n e^2}\nabla^2\mathbf{E} - \frac{m_e}{m_i}(\mathbf{E}+\mathbf{u}_i\times\mathbf{B}) + \frac{m_e}{m_i n e}\nabla\cdot\mathbf{\Pi}_i$$

$\frac{m_e}{m_i} \ll 1 \Rightarrow$

$$\left[1 - \frac{c^2}{\omega_{pe}^2}\nabla^2\right]\mathbf{E} = -\mathbf{u} \times \mathbf{B} + \frac{\eta}{\mu_0}\nabla \times \mathbf{B}$$

$$+\frac{1}{\mu_0 e n}(\nabla \times \mathbf{B}) \times \mathbf{B} - \frac{1}{en}\nabla \cdot \mathbf{\Pi}_e$$

See: Jones and Parker, *J. Comput. Phys.* **191** 322 (2003)

Various Choices

- **Direct**:Calculate flow (typically ion) directly from δf

- **Hybrid**: Advance a fluid momentum equation, close with pressure

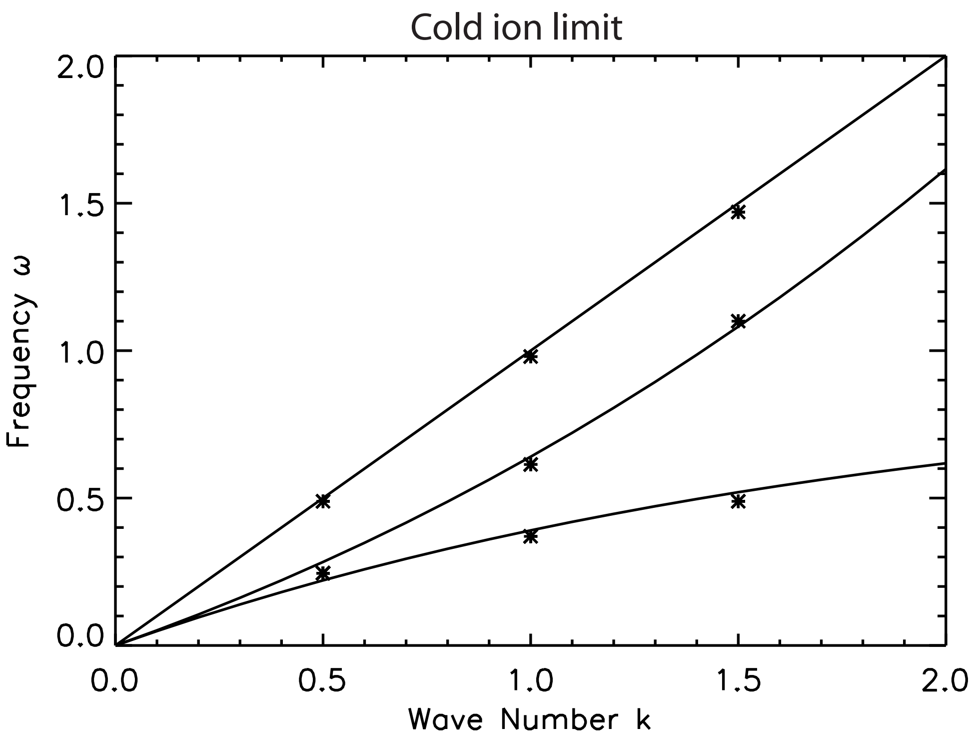
$$m_i \frac{\partial (n\mathbf{u})}{\partial t} = \frac{1}{\mu_0}(\nabla \times \mathbf{B}) \times \mathbf{B} - \sum_\alpha \nabla \cdot \mathbf{\Pi}_\alpha$$

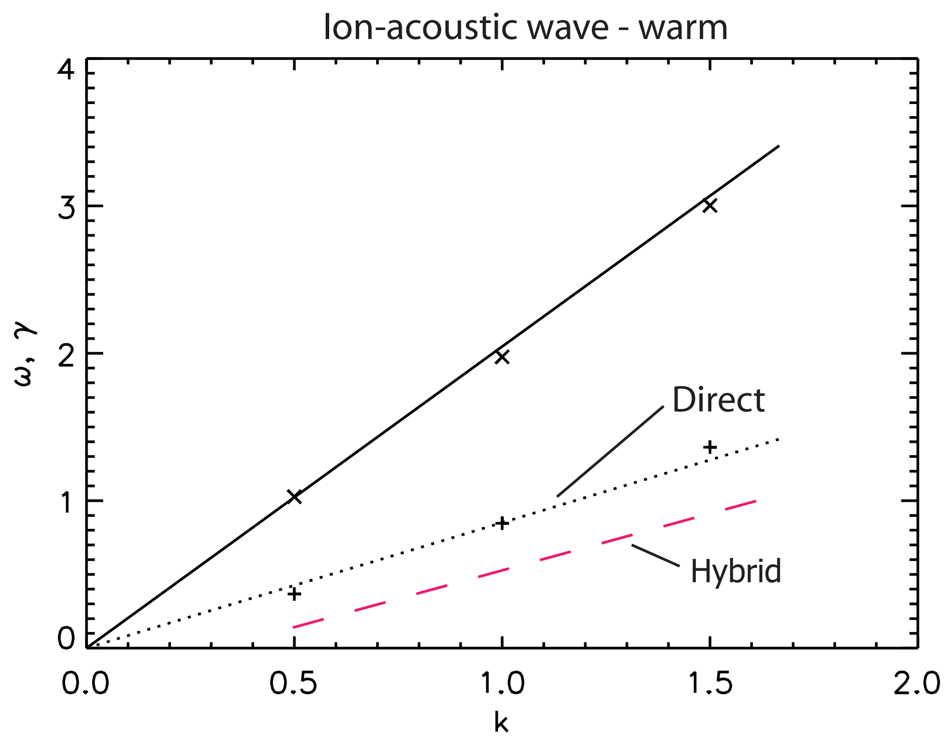
- Use a simpler closure for electron pressure, e.g.

$$\nabla \cdot \mathbf{\Pi}_e = T_{e0}\hat{b}\nabla_{||}\delta n, \quad \text{(isothermal)}$$

or $\nabla \cdot \mathbf{\Pi}_e = -\frac{\sqrt{\pi}}{2}\frac{m_e v_{te}}{e}|k_{||}|u_{||e}$ a la Hammett-Perkins

Direct Kinetic MHD works well - Alfven, Whistler, Ion-acoustic waves





Hess and Winske (1994) - massless electrons, anisotropic pressure
Shay et al. (1998) - massless electrons, isotropic pressure
Belova et al. (2000) - cold massless electrons, δf
Yin et al. (2002) - ions used to calculate ion pressure trensor

Pressure Tensor vs. Stress Tensor

$$\underline{\underline{P}} = \int [\underline{v} - u(\underline{x},t)] [\underline{v} - u(\underline{x},t)] \, f(\underline{x},\underline{v},t) \, d^3v$$

$$\rho \left(\frac{\partial u}{\partial t} + \underline{u} \cdot \nabla \underline{u} \right) = \cdots$$

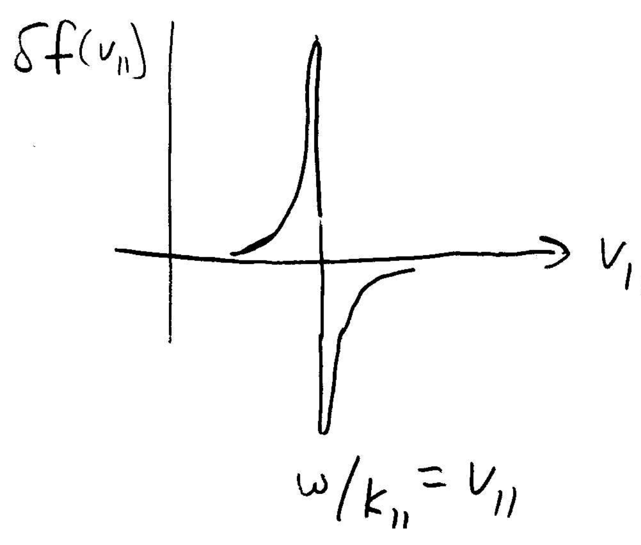
$$\underline{\underline{\tilde{P}}} = \int \underline{v}\,\underline{v} \, f \, d^3v$$

$$\frac{\partial (\rho \underline{u})}{\partial t} = \cdots$$

Problems with Pressure / Stress Closure

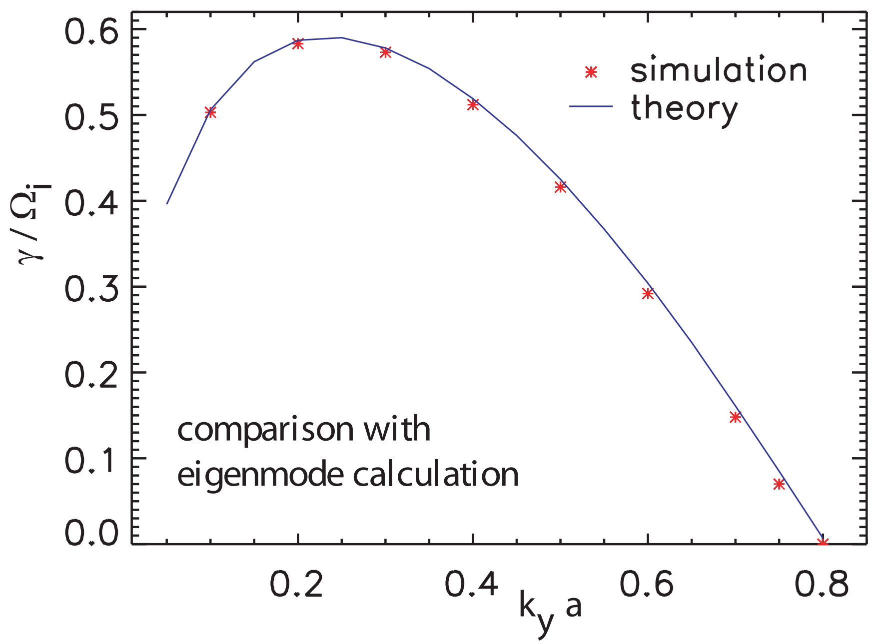
$$\delta P_{\parallel} = \int v_{\parallel}^2 \, \delta f \, d^3v$$

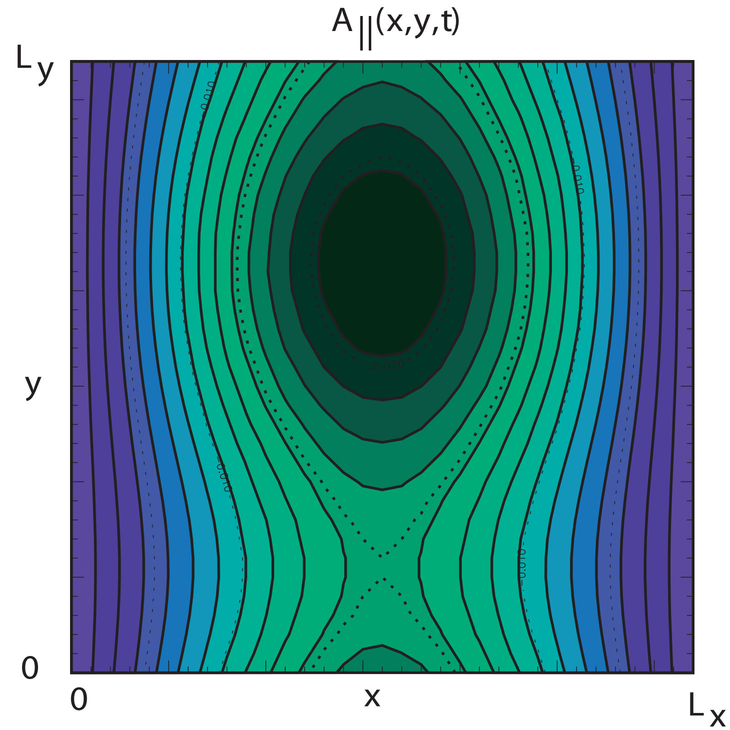
$$\sim \left(\frac{\omega}{k_{\parallel}} \right)^2$$

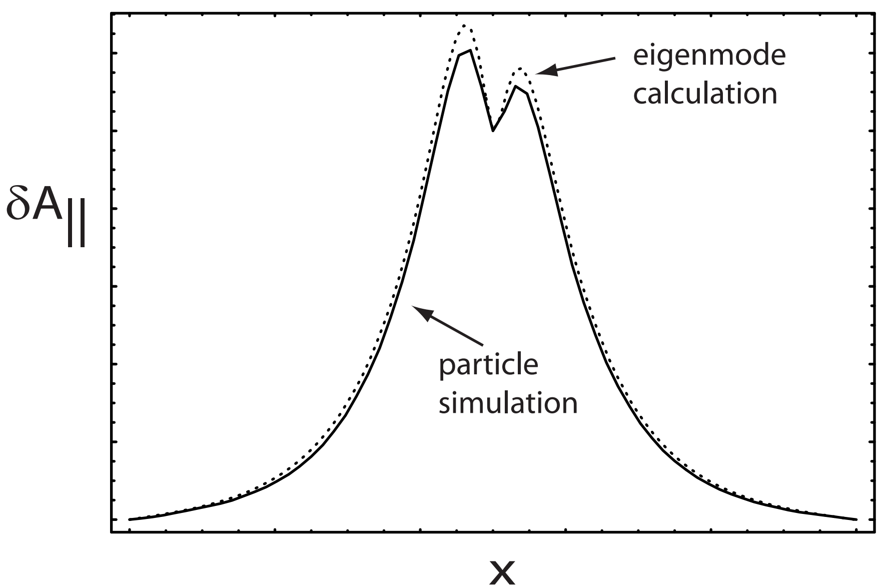


Cohen (2002)

Gyrokinetic Simulation of Tearing Modes





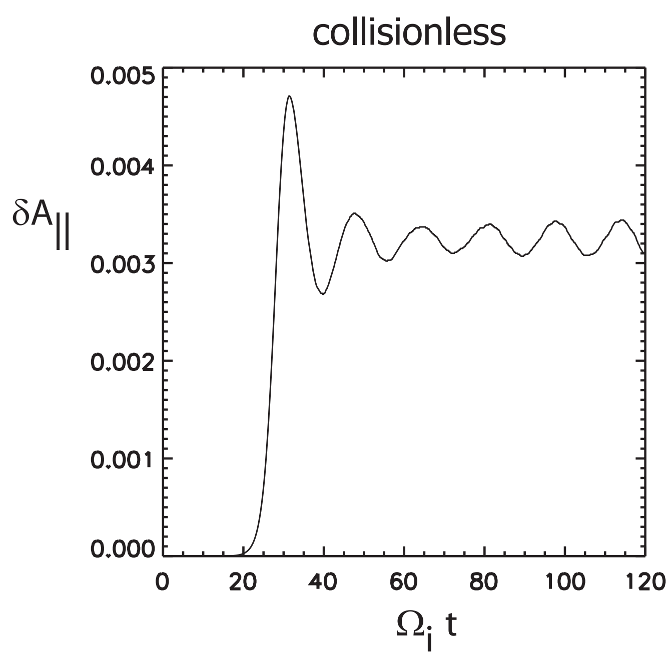


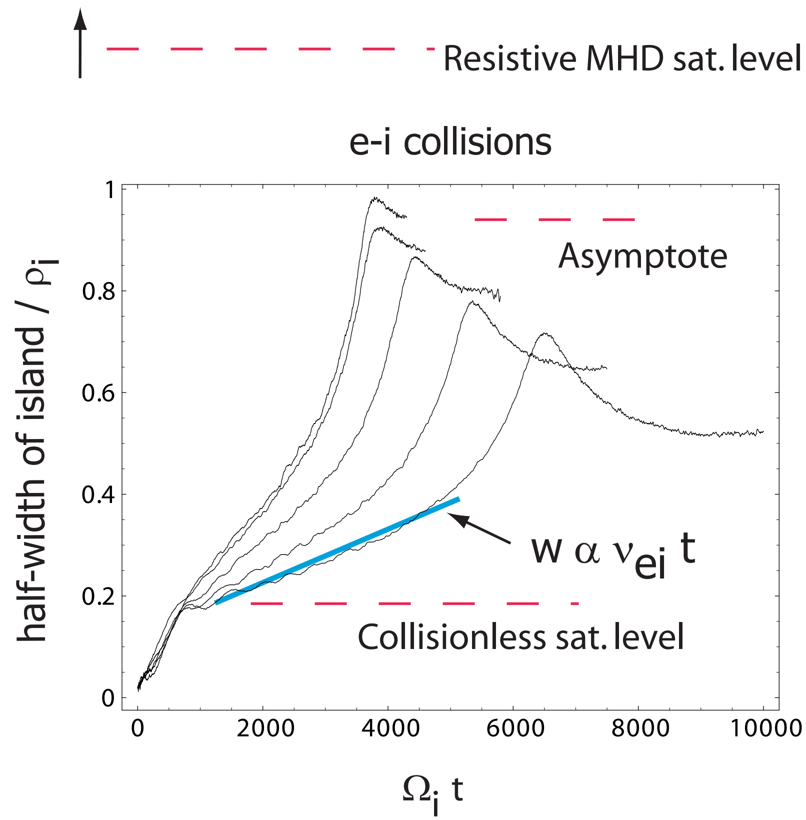
Katanuma '80
Sydora '01

Wan et al., to appear in Phys. Plasmas (2004)

GEM - gyrokinetic, electromagnetic,
general geom. Chen and Parker JCP (2003)

Nonlinear saturation of collisionless and semi-collisional tearing modes





Drake '77

Wan et al., to appear in Phys. Plasmas (2004).