Progress on Modeling the CDX-U Sawtooth with M3D

Josh Breslau

Princeton Plasma Physics Laboratory

The M3D Group: J. Chen, G. Fu, W. Park,

H. Strauss, L. Sugiyama

Denver, Colorado

October 22, 2005

Outline

I. Motivation

II. The CDX Device

  A. Machine parameters

  B. The q_0<1 equilibrium

III. Benchmark parameters

IV. Original Case: $\kappa = 0$

  A. Linear eigenmodes

  B. Stabilizing ballooning modes with high $\kappa_\perp$

V. Followup Studies: $\kappa_\perp$ = 200 m^2/s, $\kappa_\parallel$ on

  A. Original low-resolution run

  B. High-resolution followup

  C. Two-fluid study

VI. Conclusions

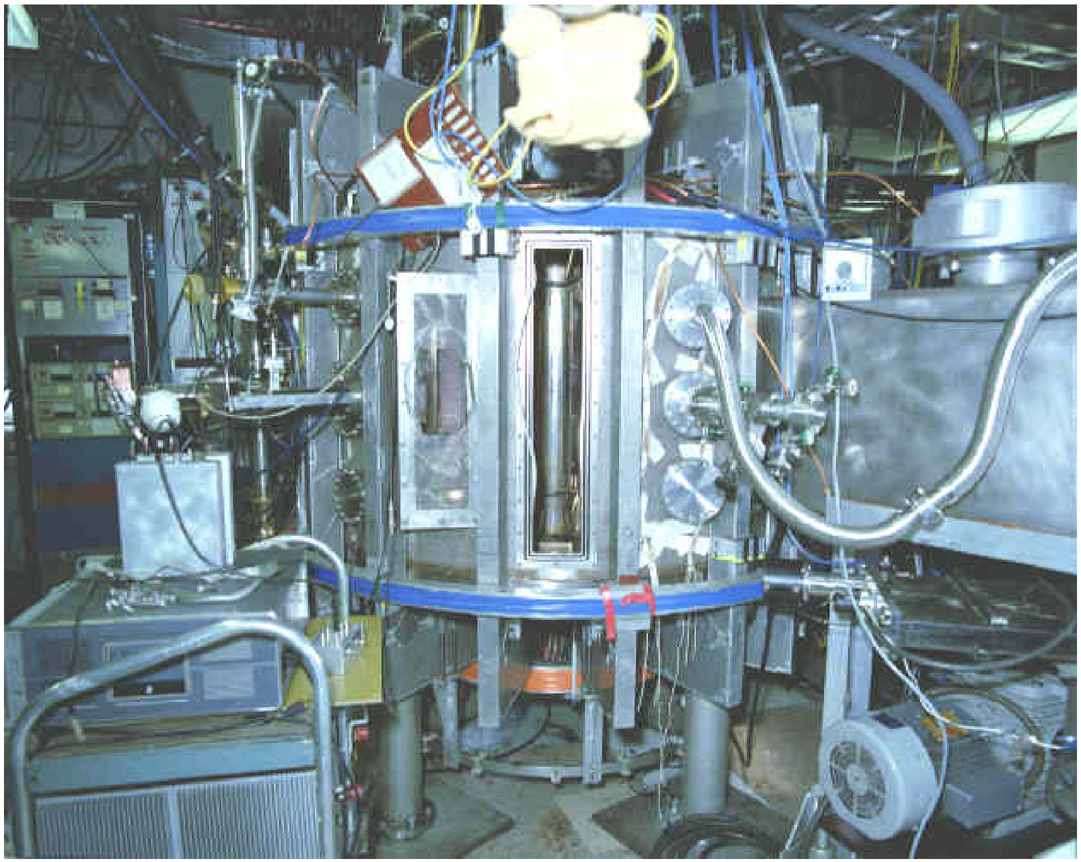
Motivation

- As a nonlinear MHD code benchmark, model resistive MHD events in a tokamak plasma using realistic physical values to make quantitative predictions.

  - Large tokamaks have large disparities in spatial and temporal scales to be resolved.
    - Resistive MHD: Current sheet thickness ~ $S^{-1/2}$
    - Two-fluid MHD: ion skin depth ~ $n^{-1/2}$

  - Small tokamaks operate in regimes accessible to present-day codes.

Characteristics of the Current Drive Experiment Upgrade (CDX-U)

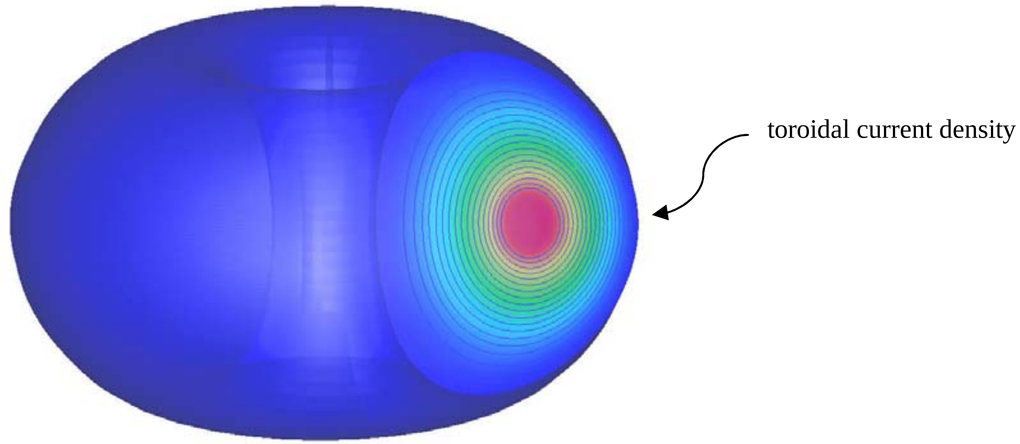


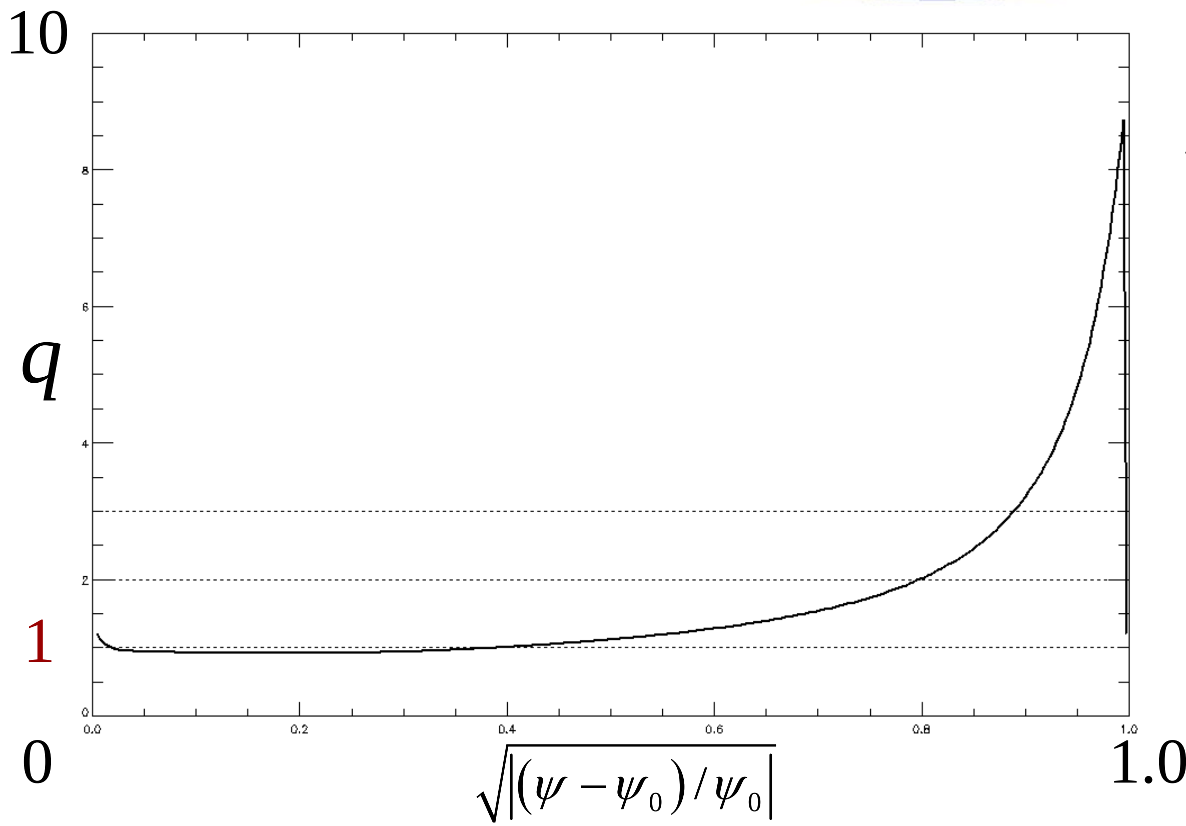
- Low aspect ratio tokamak (R_0/a = 1.4 – 1.5)
- Small (R_0 = 33.5 cm)
- Elongation $\kappa \sim 1.6$
- $B_T \sim$ 2300 gauss
- $I_p \sim$ 70 kA
- $n_e \sim 4\times10^{13}$ cm^{-3}
- $T_e \sim$ 100 eV $\rightarrow$ S ~ 10^4
- Discharge time ~ 12 ms

- Soft X-ray signals from typical discharges indicate two predominant types of low-*n* MHD activity:
  - sawteeth
  - “snakes”

Equilibrium: $q_0 < 1$

- Equilibrium taken from a TSC sequence (Jsolver file).
- $q_{min} \approx 0.922$
- $q(a) \sim 9$





Questions to investigate:

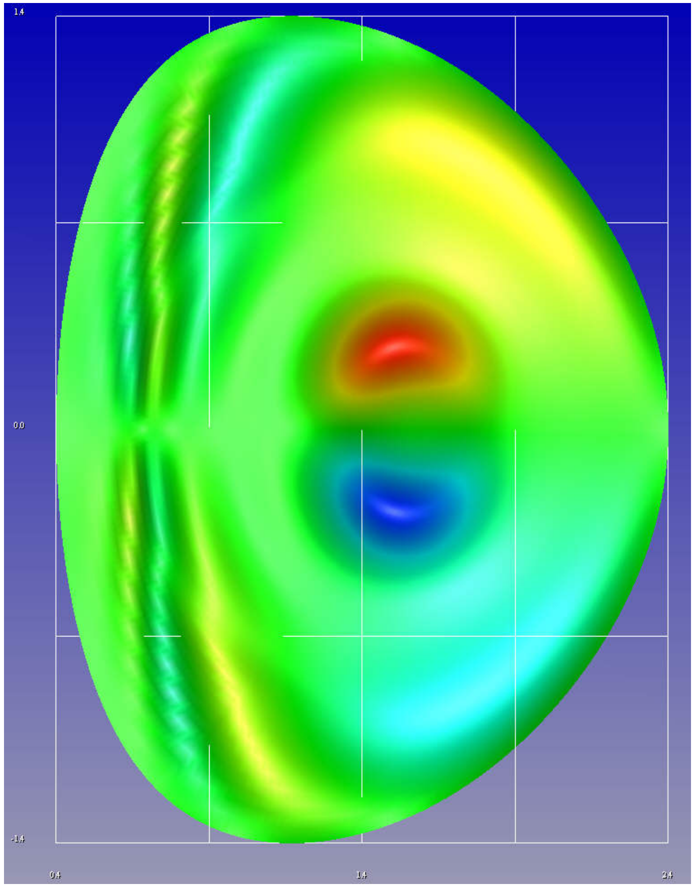
- Linear growth rate and eigenfunctions
- Nonlinear evolution
  - disruption?
  - stagnation?
  - repeated reconnections?

Baseline Parameters for CDX

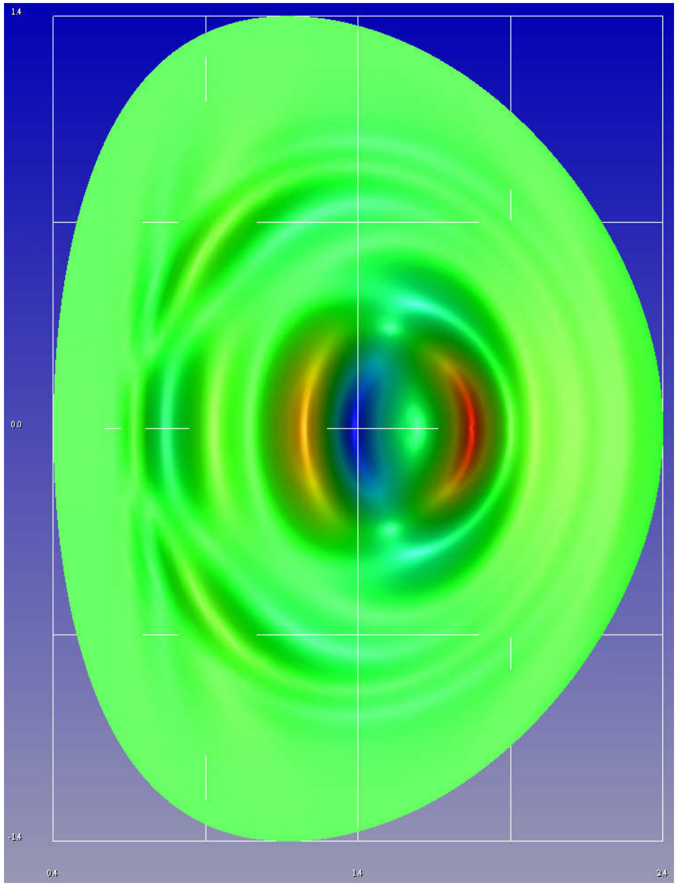
Lundquist Number S	~2×10^4 on axis.
Resistivity η	Spitzer profile $\propto T_{eq}^{-3/2}$, cut off at $100\times\ \eta_0$
Prandtl Number Pr	10 on axis.
Viscosity μ	Constant in space and time.
Perpendicular thermal conduction $\kappa_\perp$	Original study: 0 Followup studies: 200 m^2/s (measured value)
Parallel thermal conduction (sound wave)	Original study: 0 Followup studies: $V_{Te} = 6\ V_A$
Peak Plasma β	~ 3×10^{-2} (low-beta).
Density Evolution	Turned on for nonlinear phase.
Nonlinear initialization	Pure n=1 perturbation such that $\frac{\max(B^1_{\mathrm{pol}})}{\max(B^0_\phi)}=10^{-4}$

n=1 Eigenmode

Incompressible velocity
stream function U



Toroidal current density
J_ϕ

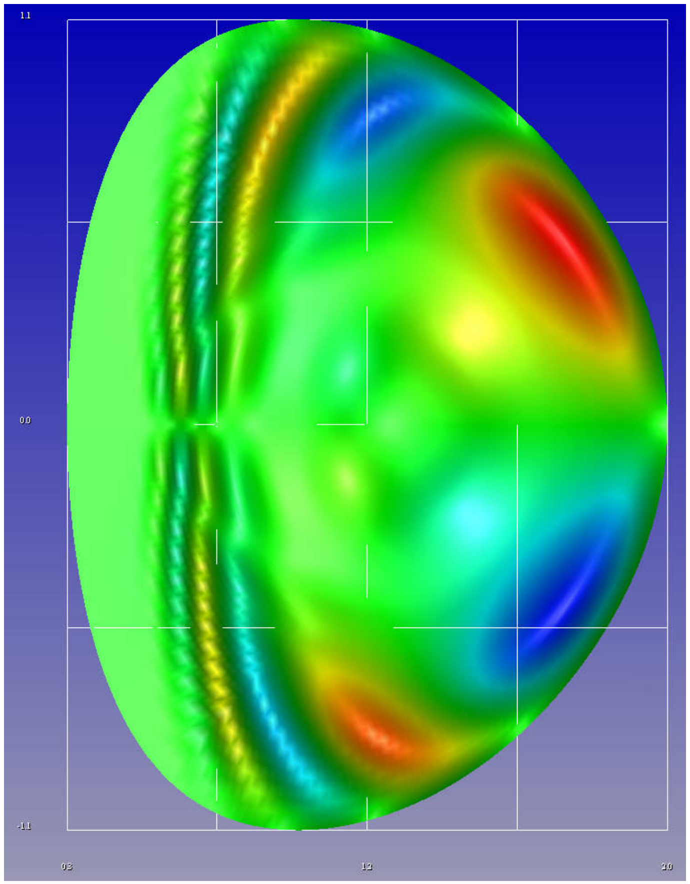


$\gamma\tau_A = 8.61 \times 10^{-3} \rightarrow$ growth time = 116 τ_A

Higher n Eigenmodes

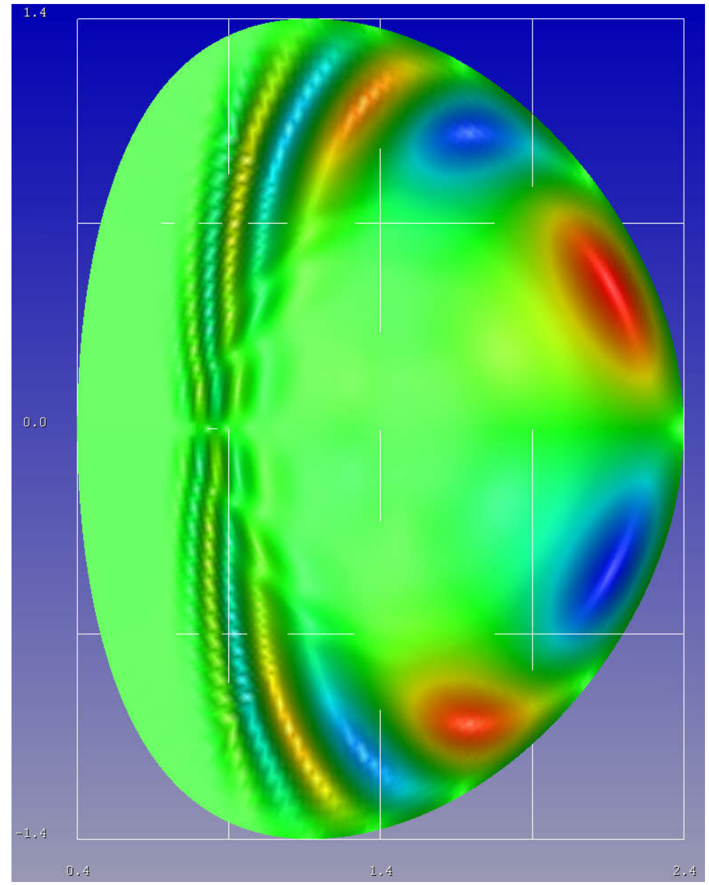
Incompressible velocity
stream function U

$n = 2$



$m \geq 5$
$\gamma \tau_A = 1.28 \times 10^{-2}$

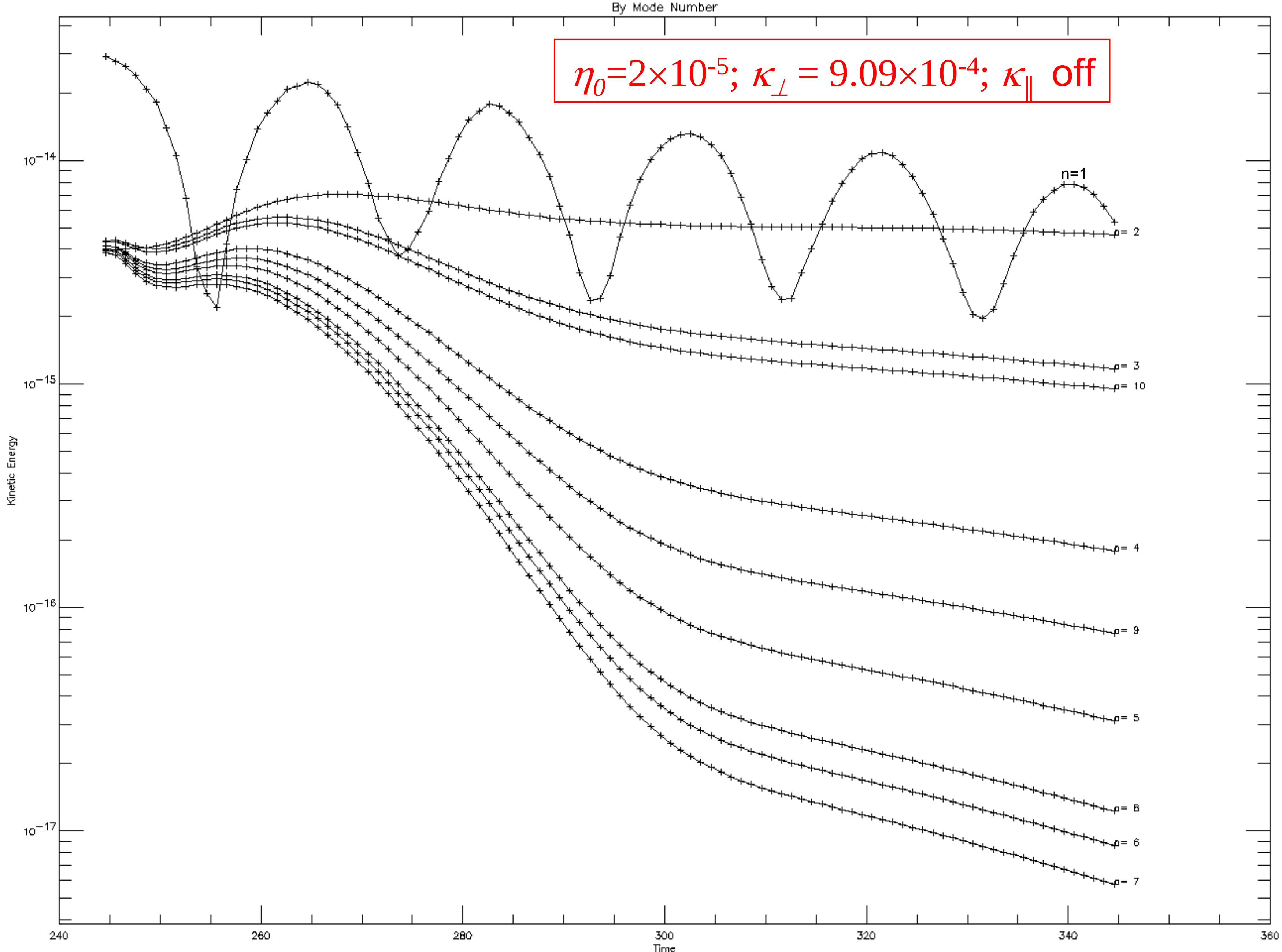
$n = 3$



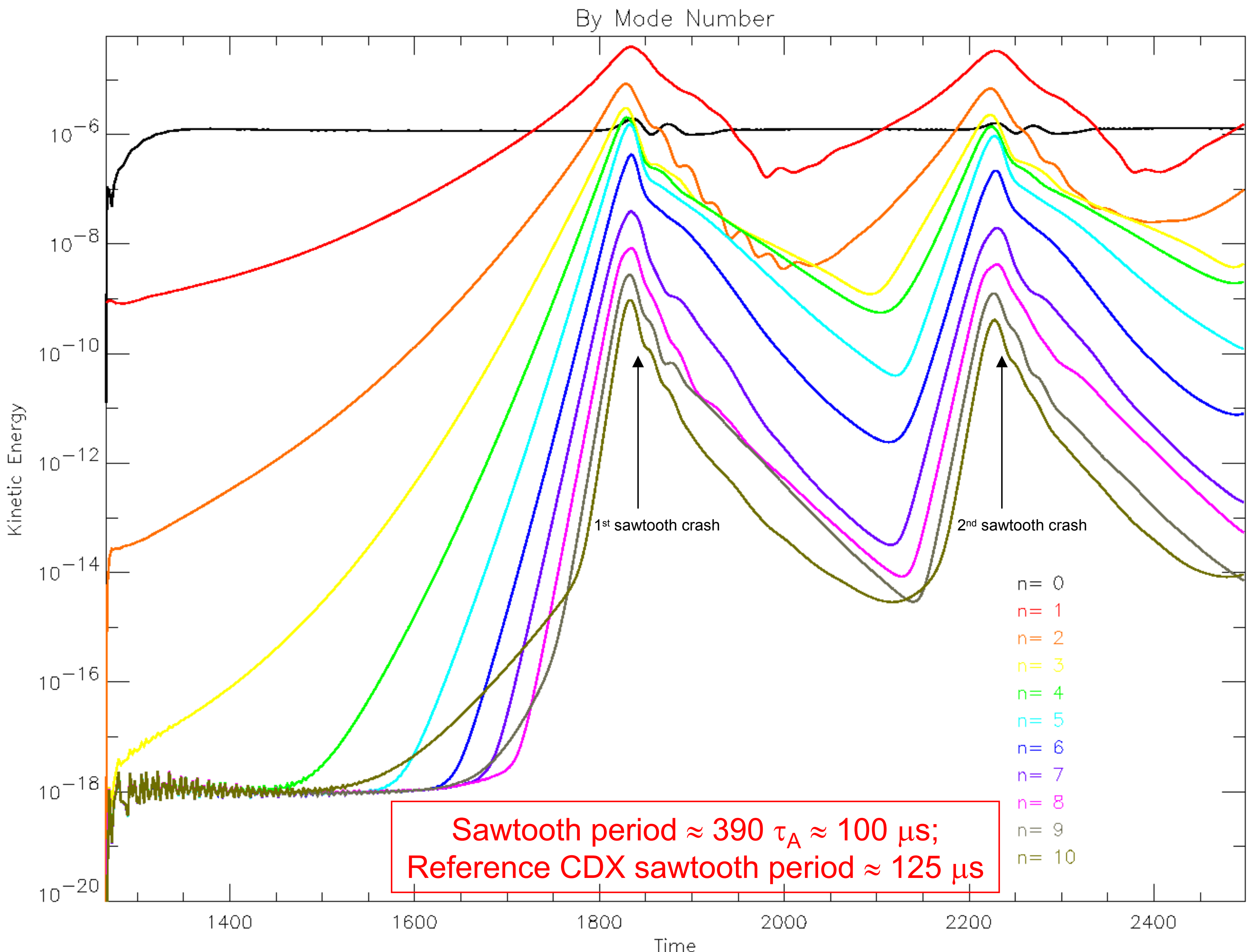
$m \geq 7$
$\gamma \tau_A = 1.71 \times 10^{-2}$

• • •

High Perpendicular Heat Conduction Stabilizes All Ballooning Modes

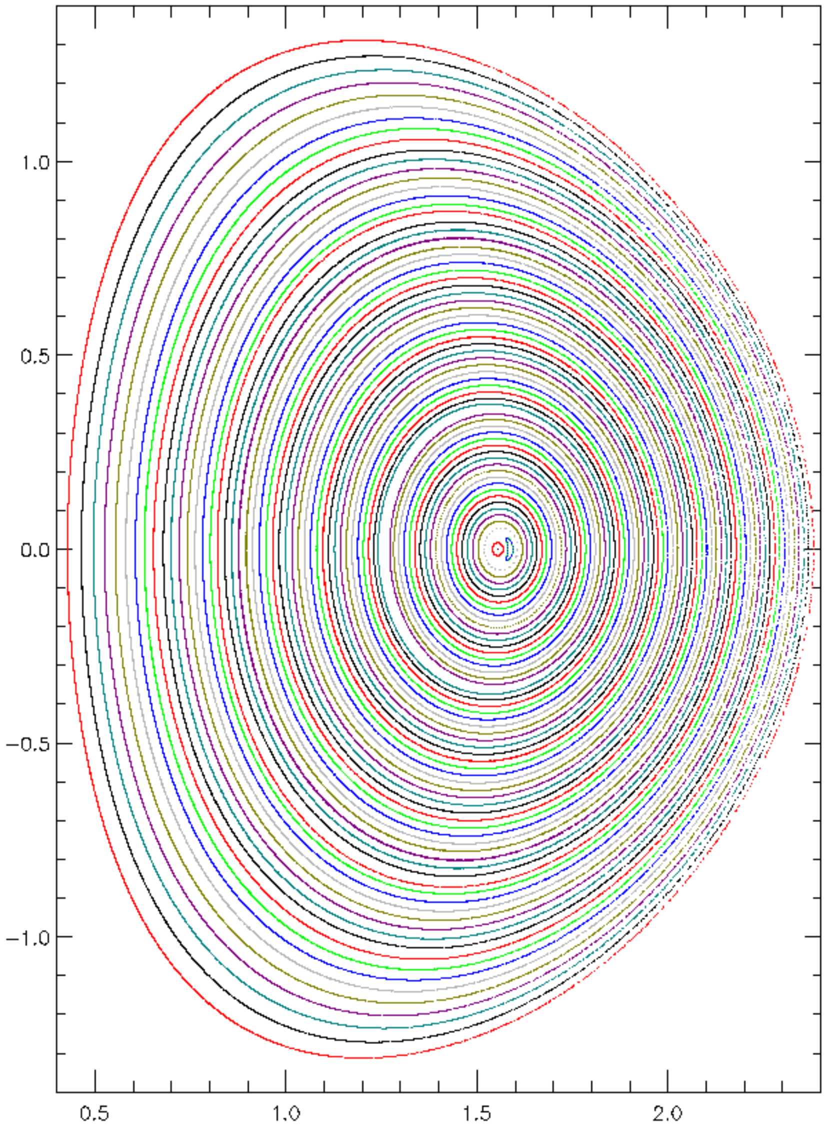


Nonlinear Evolution, Heat Conduction On

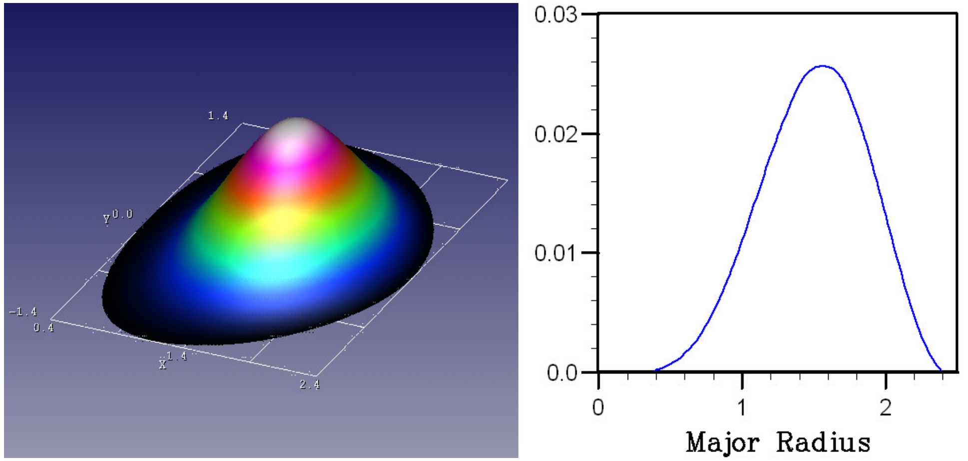


Initial state: t = 1266.17

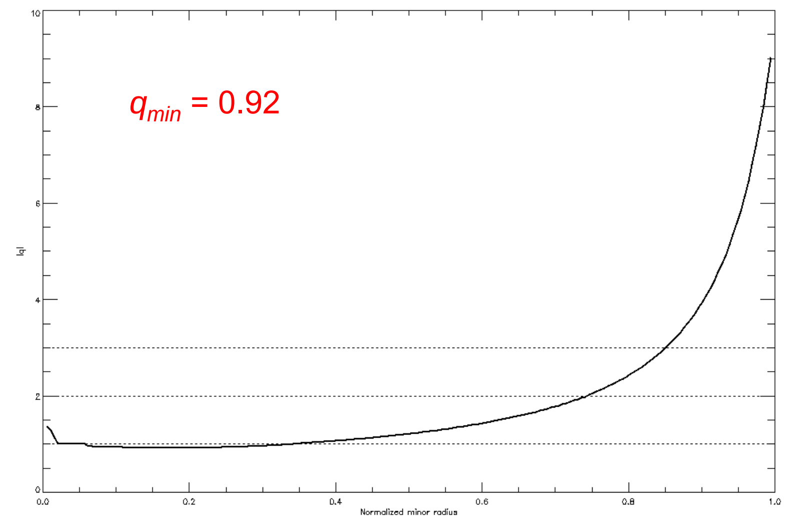
Poincaré plot



Temperature profile

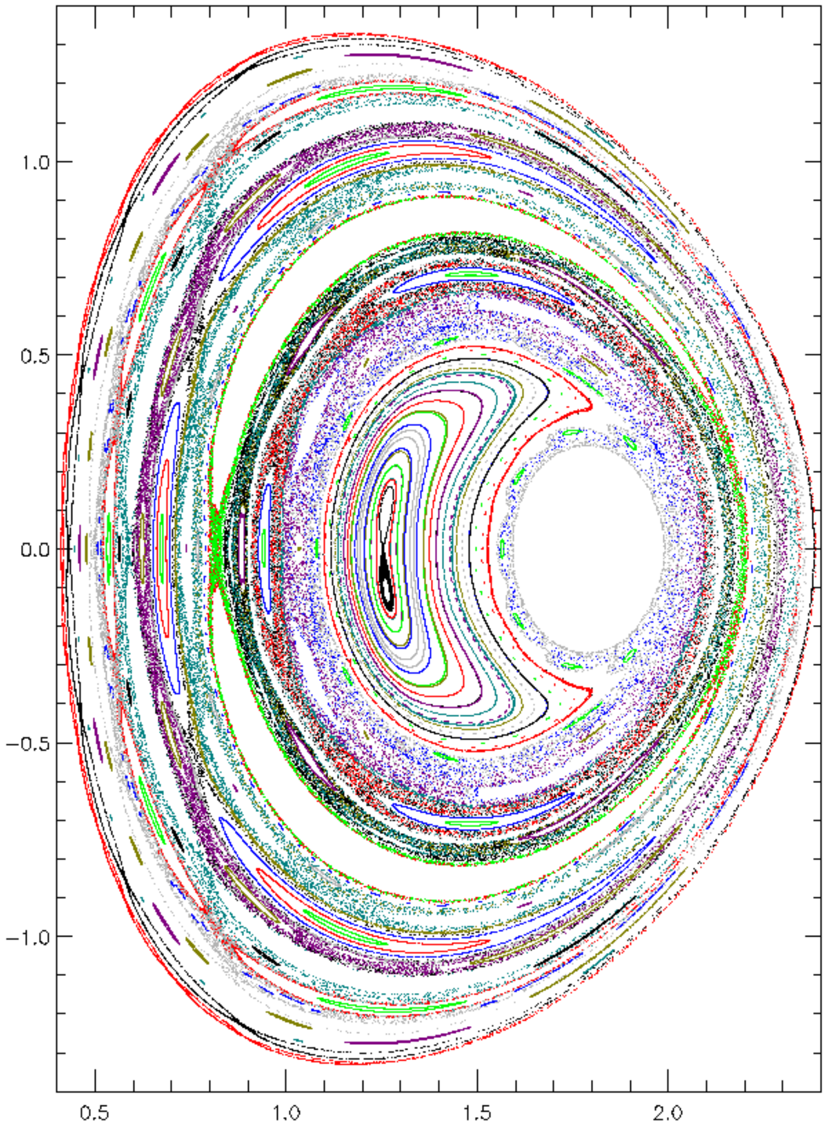


q profile

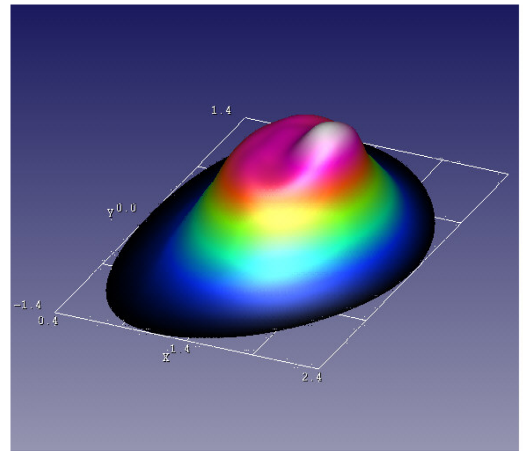


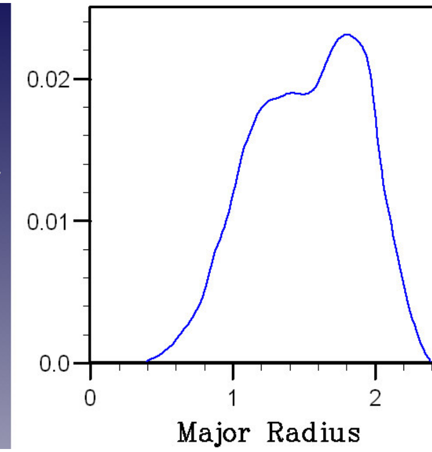
During 1st Crash: t = 1810.51

Poincaré plot

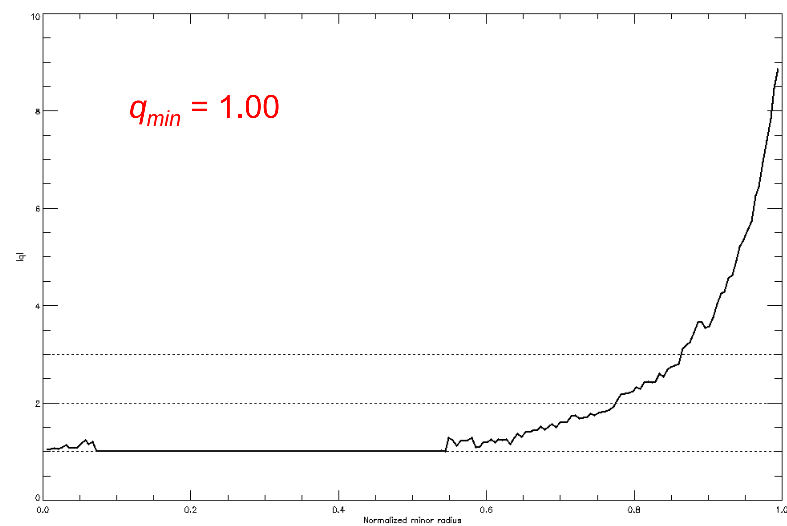


Temperature profile



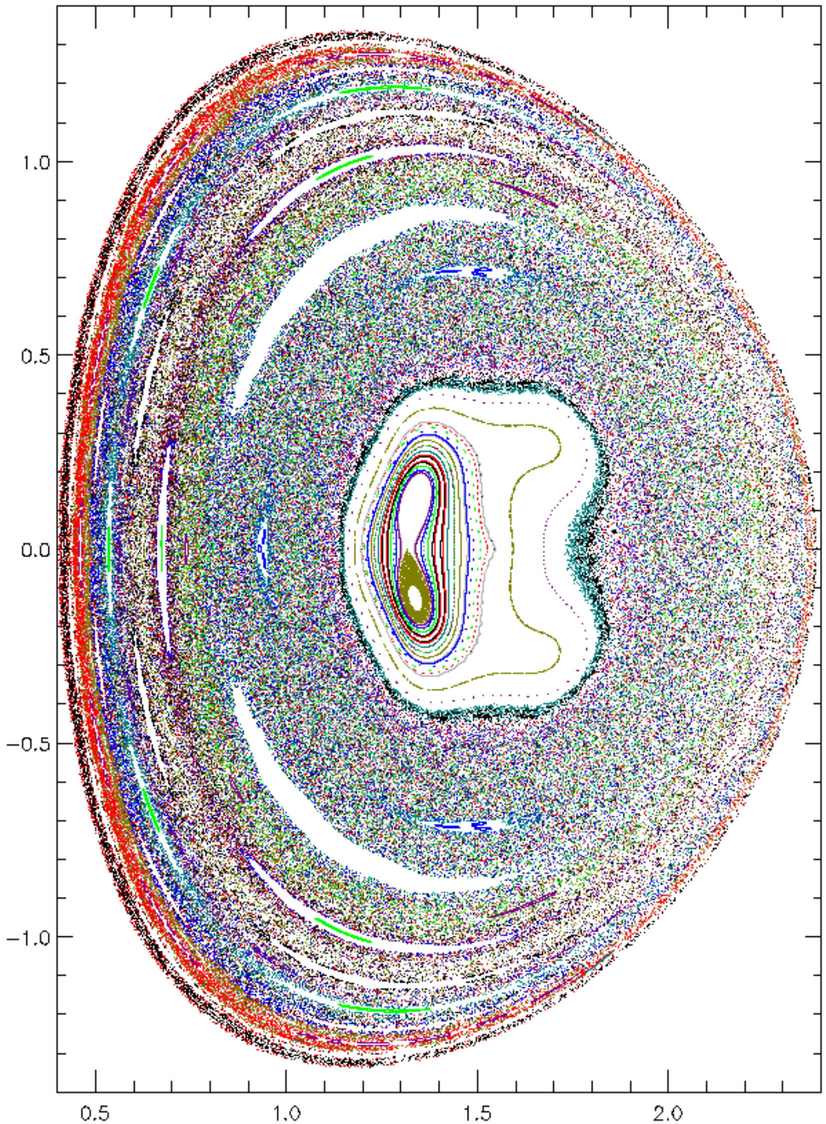


q profile

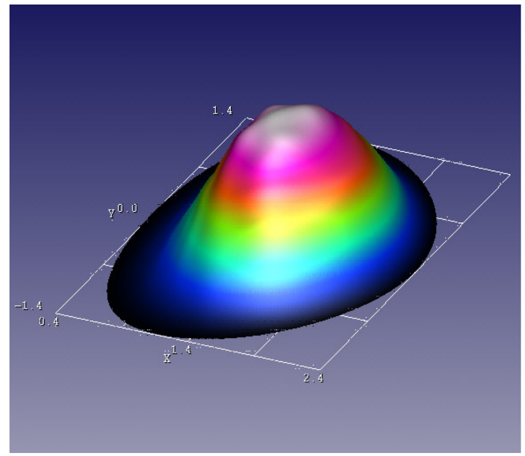


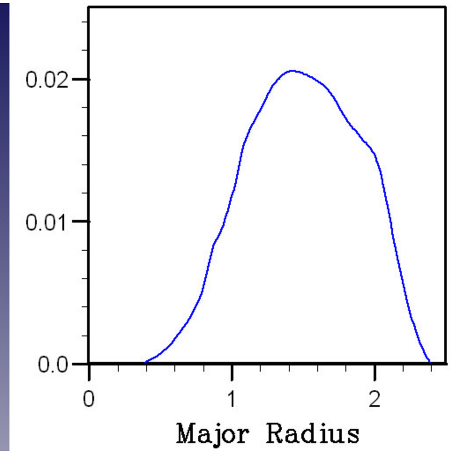
After 1st Crash: t = 1839.86

Poincaré plot

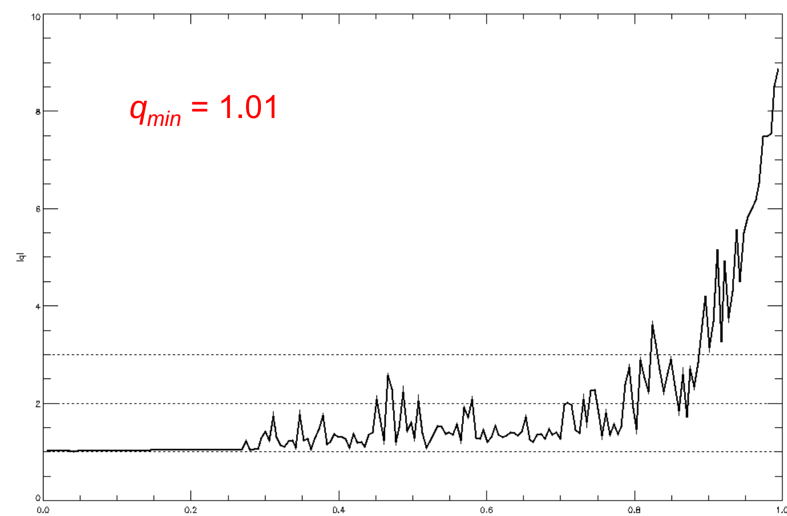


Temperature profile



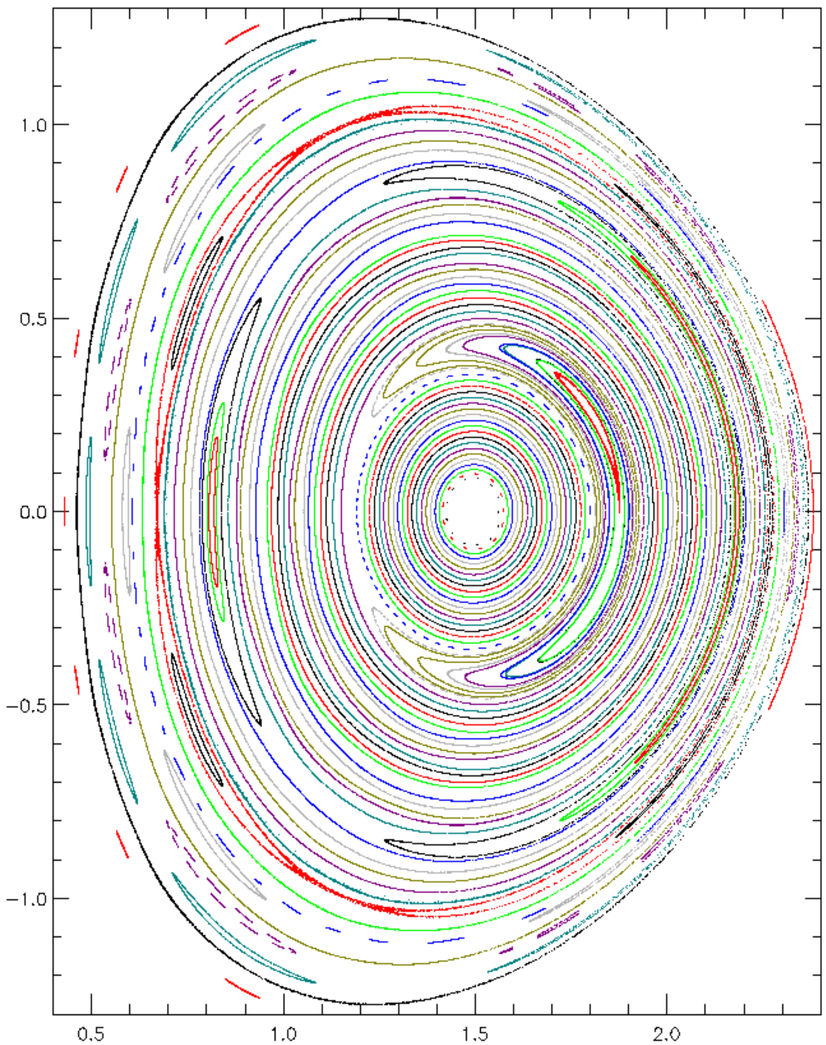


q profile

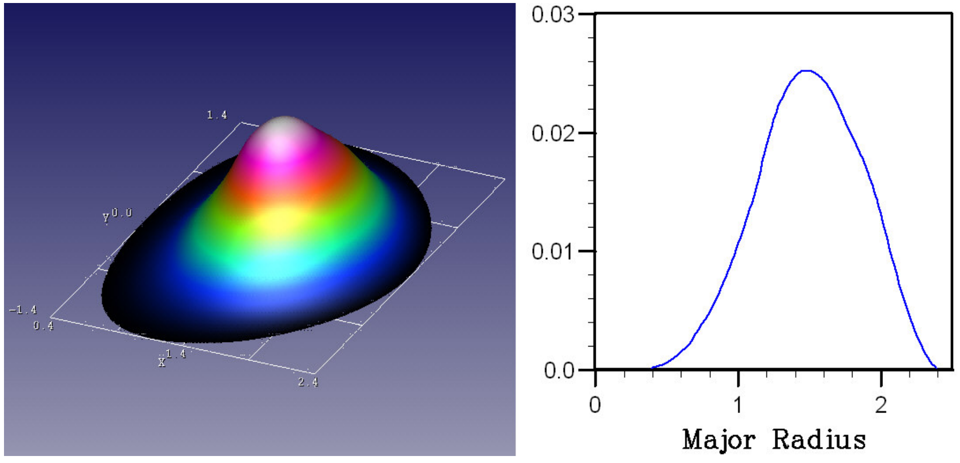


Flux surfaces recovered: t = 2094.08

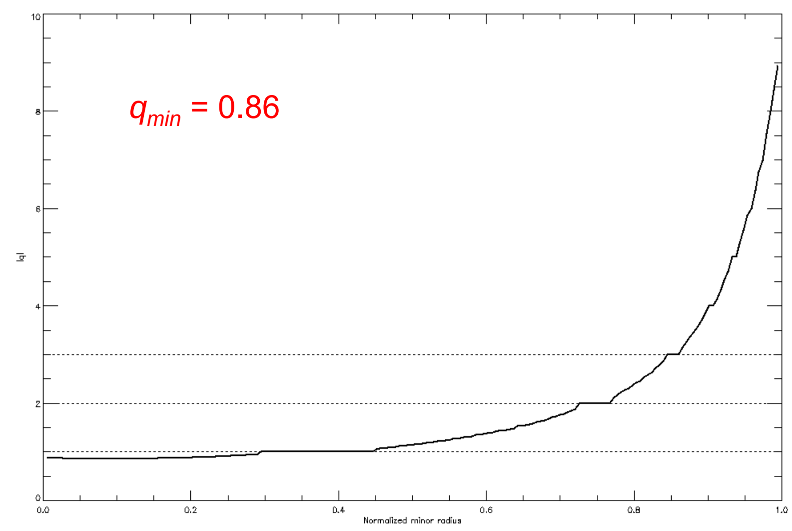
Poincaré plot



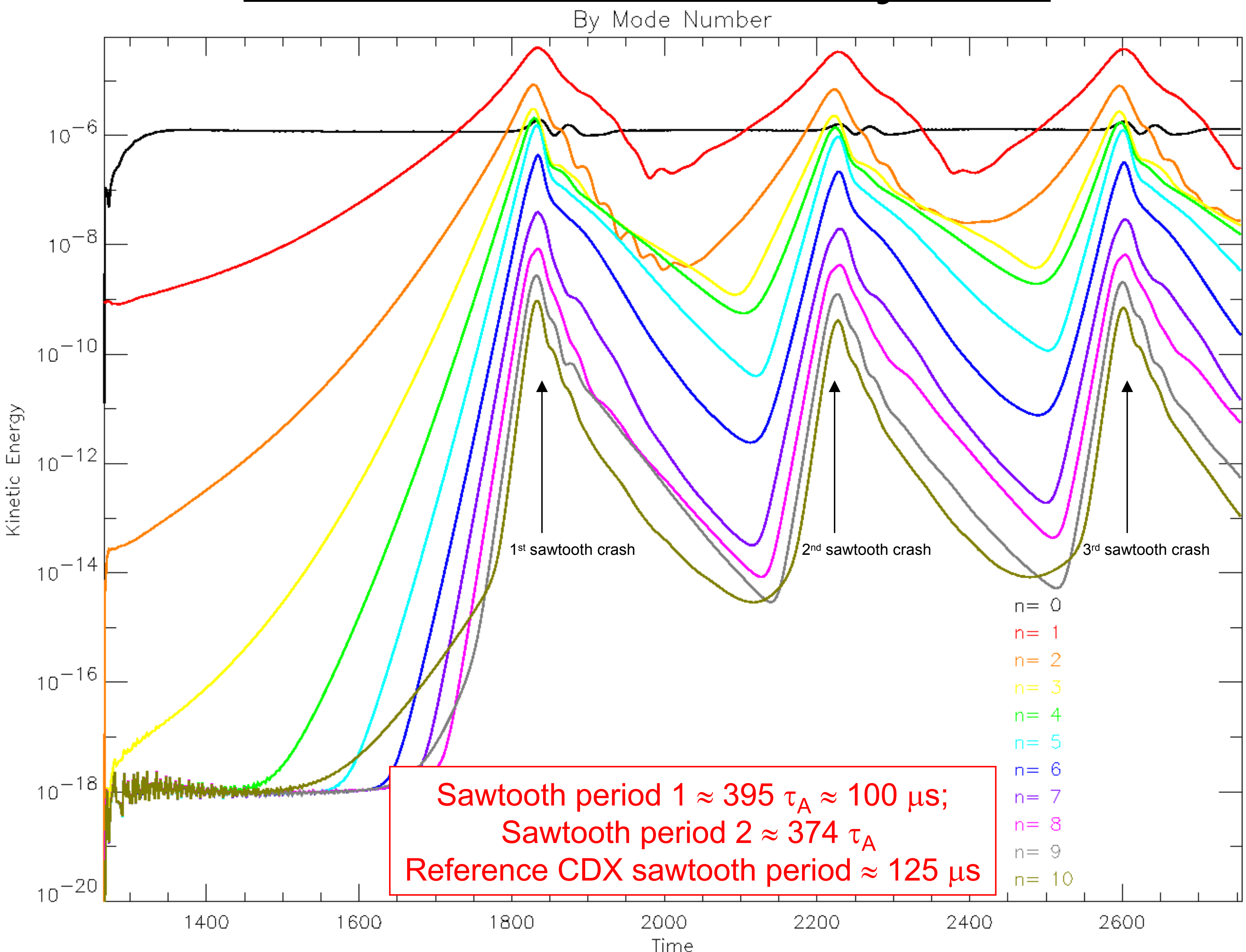
Temperature profile



q profile

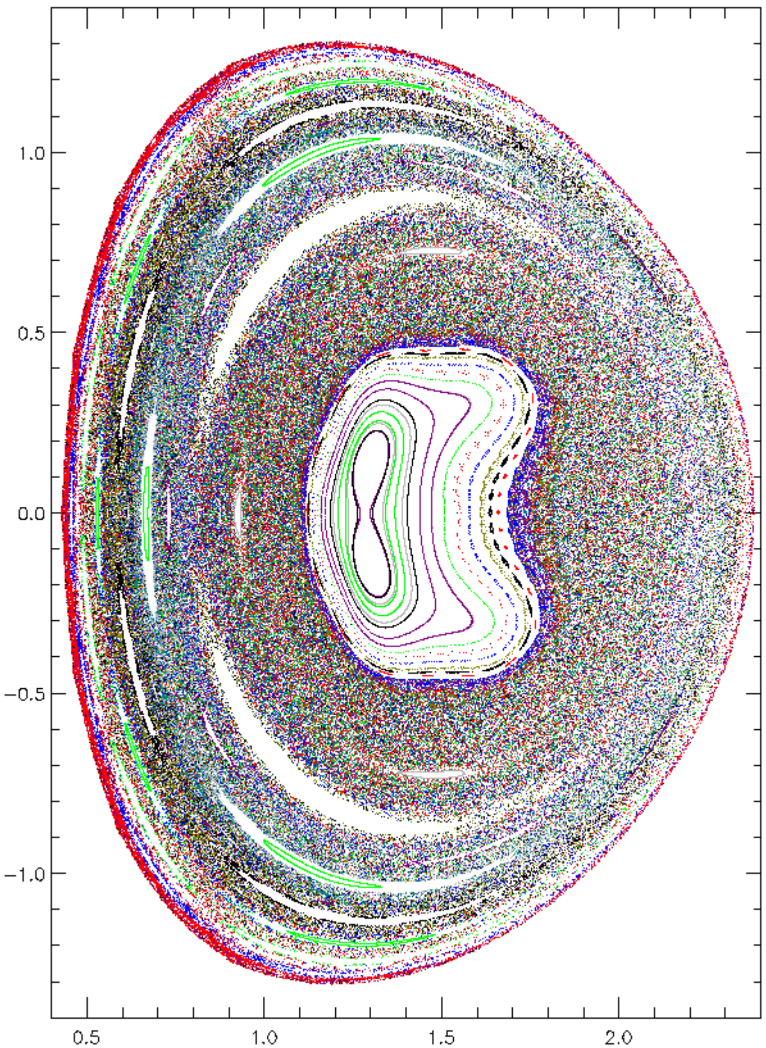


Three Sawtooth Cycles

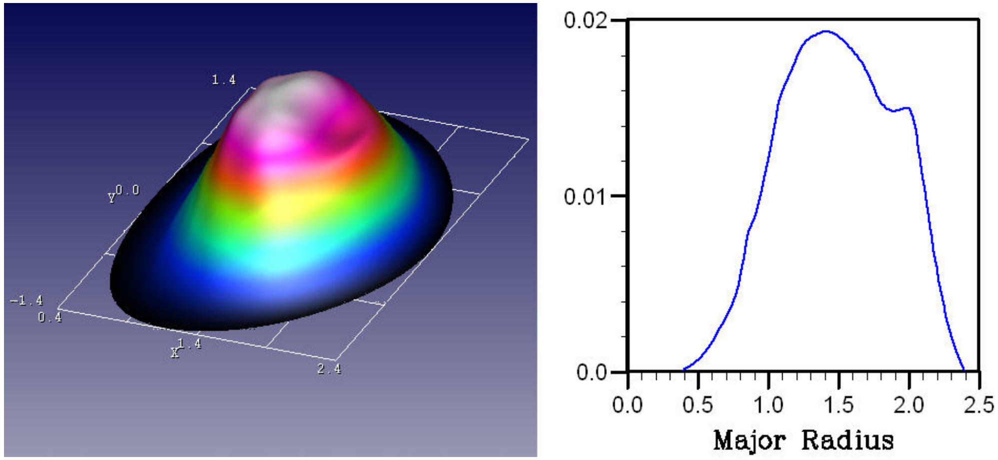


After 3rd Crash: t = 2602.50

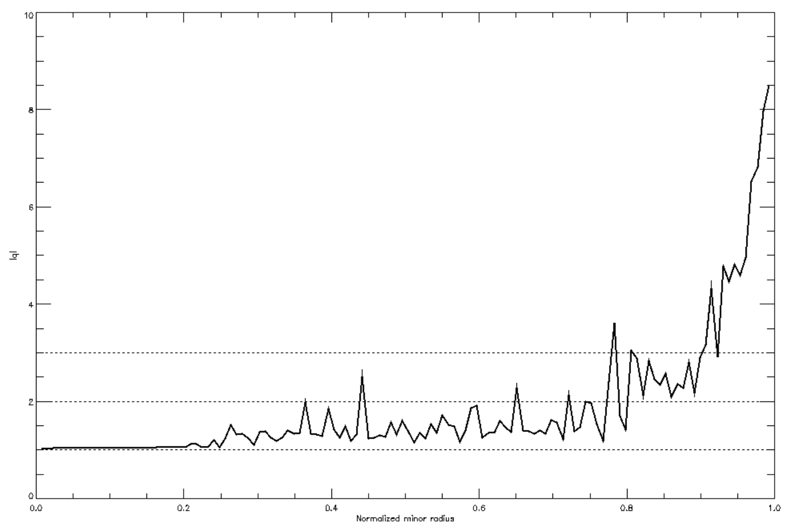
Poincaré plot



Temperature profile

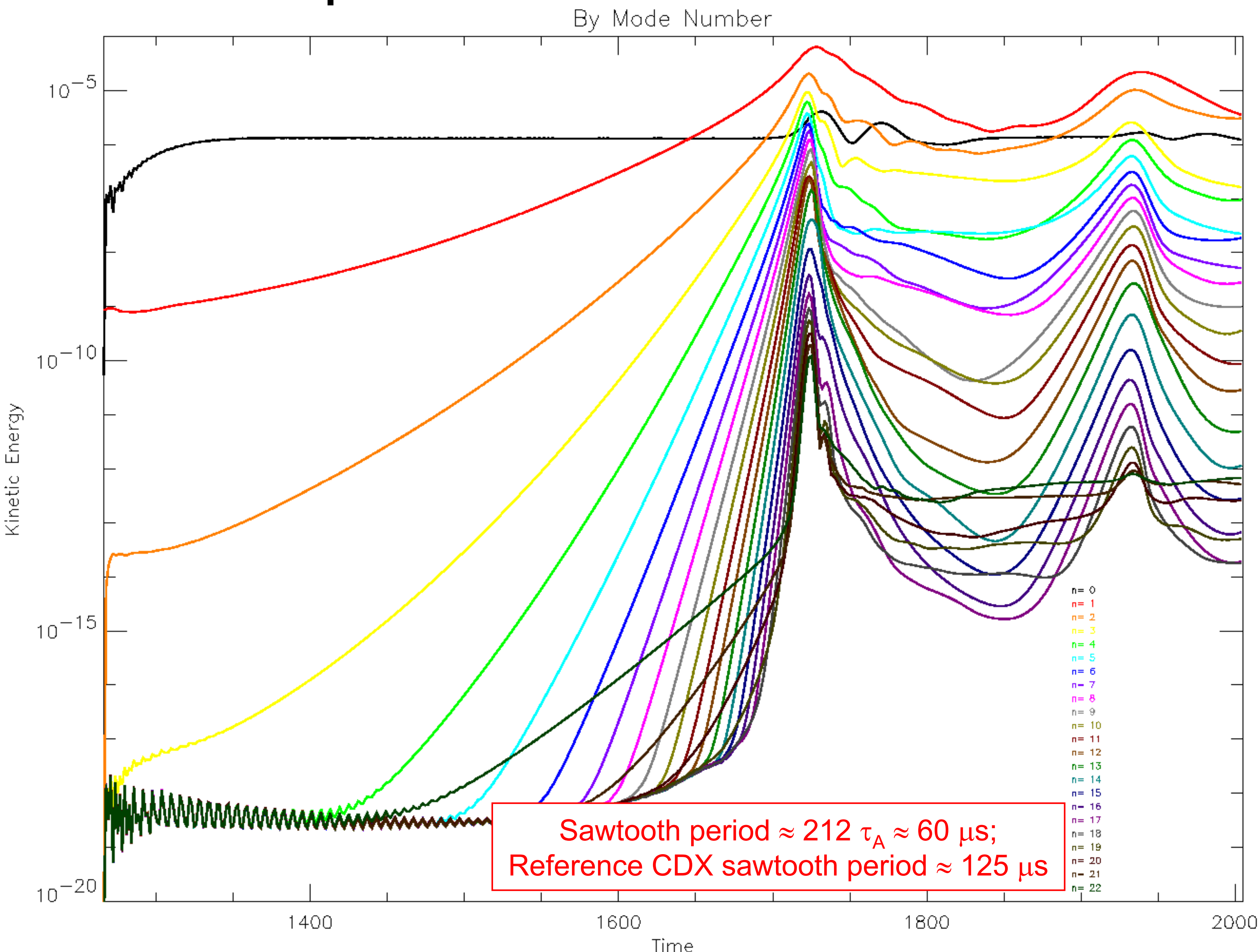


q profile



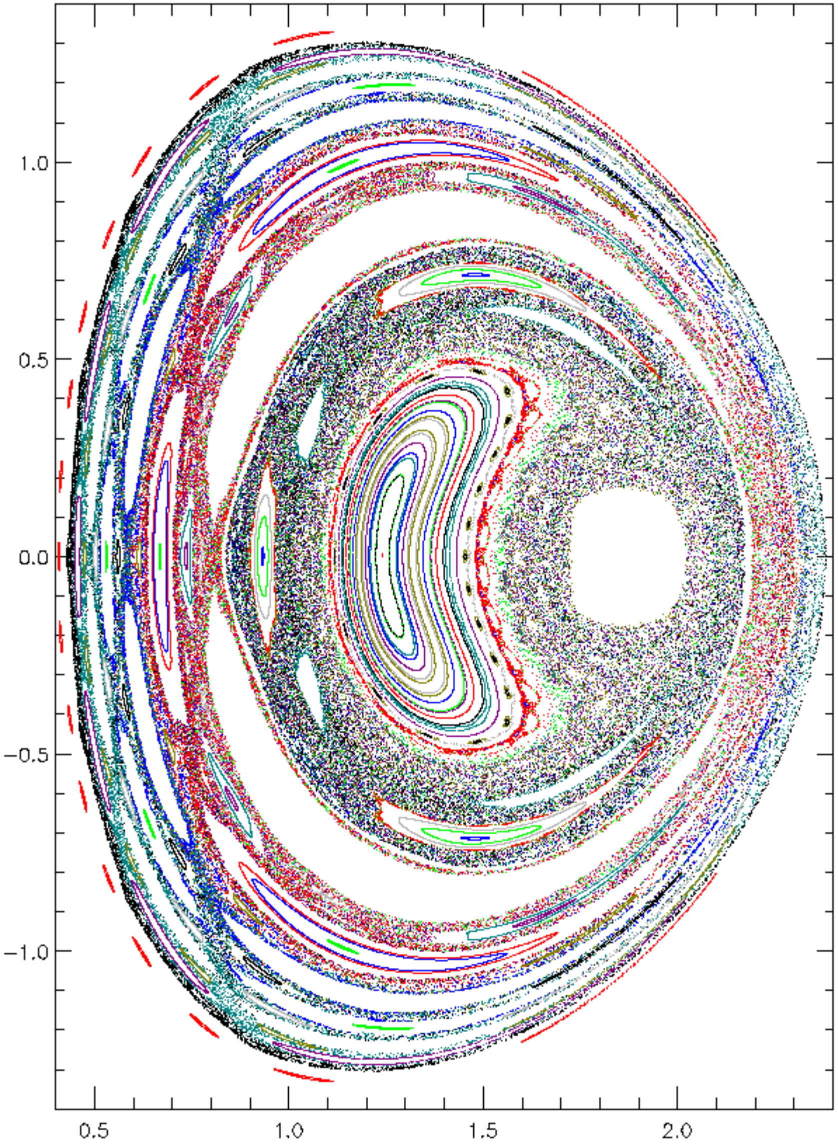
High-Resolution Study

48 planes → 22 toroidal modes

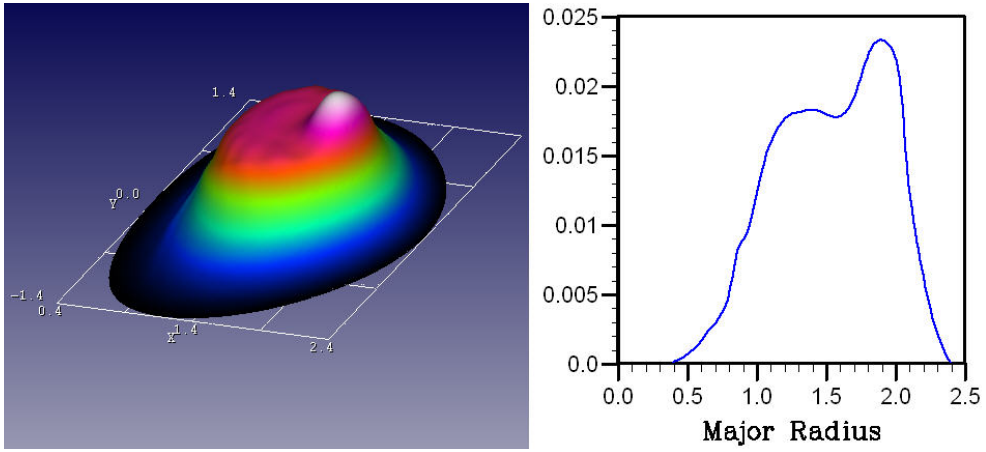


During 1st Crash: *t* = 1717.08

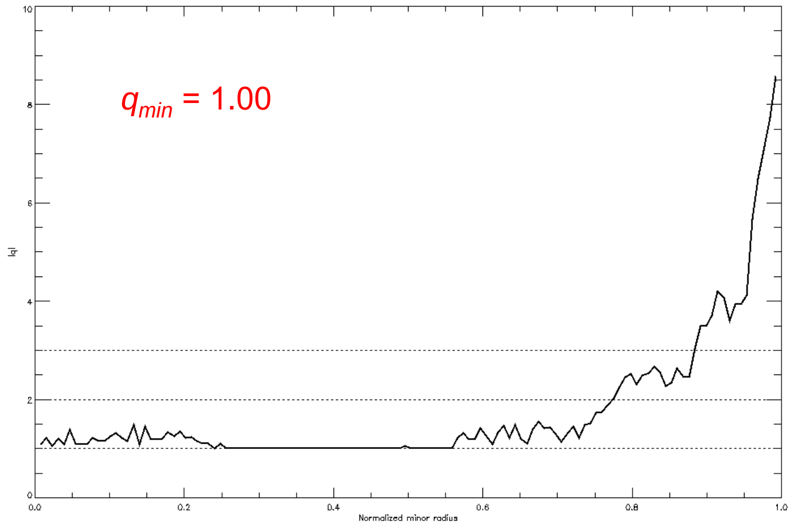
Poincaré plot



Temperature profile

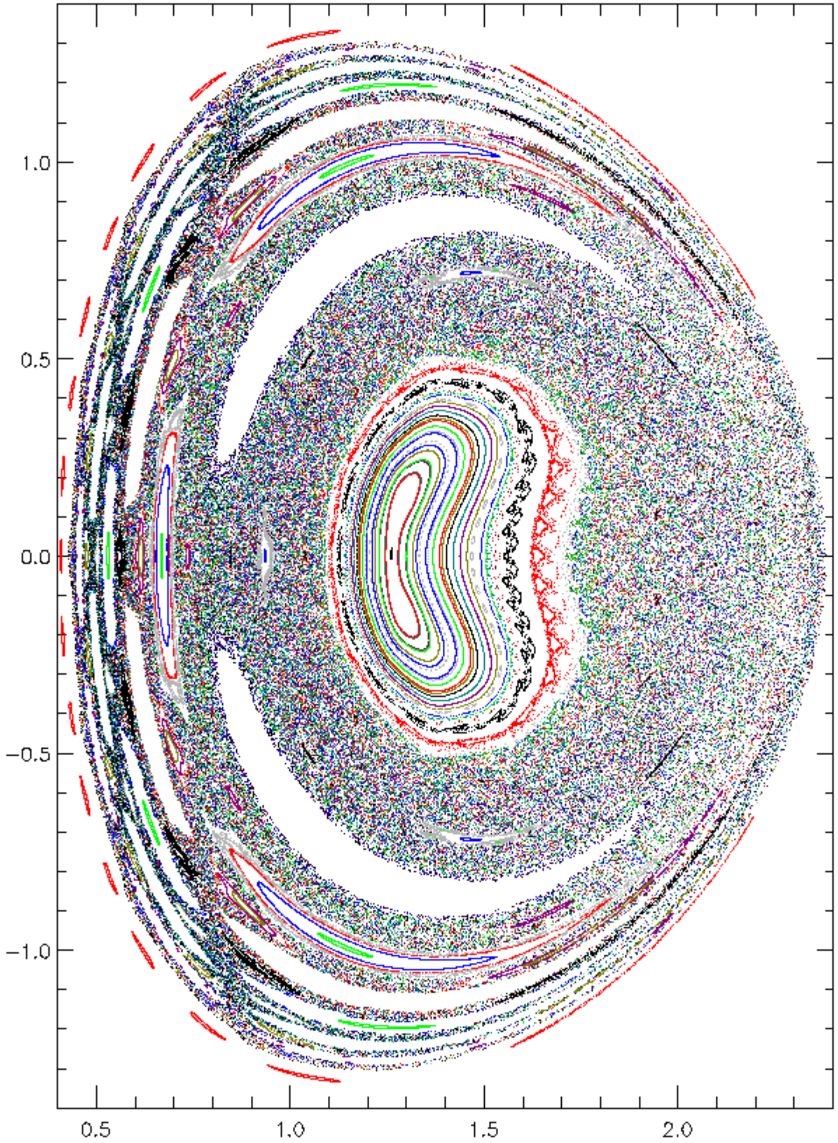


q profile

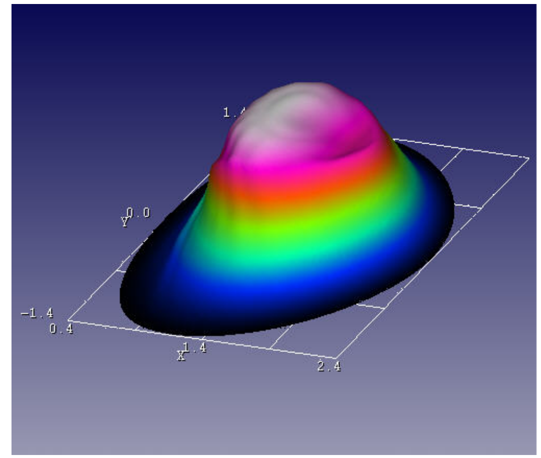


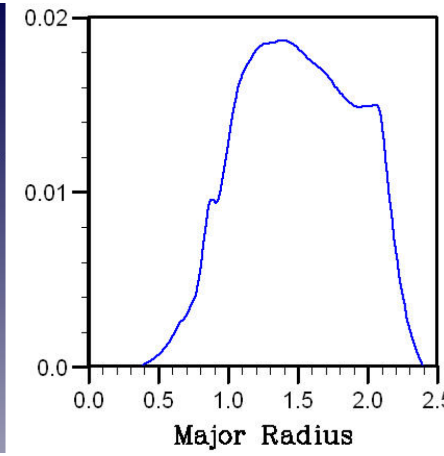
After 1st Crash: t = 1725.34

Poincaré plot

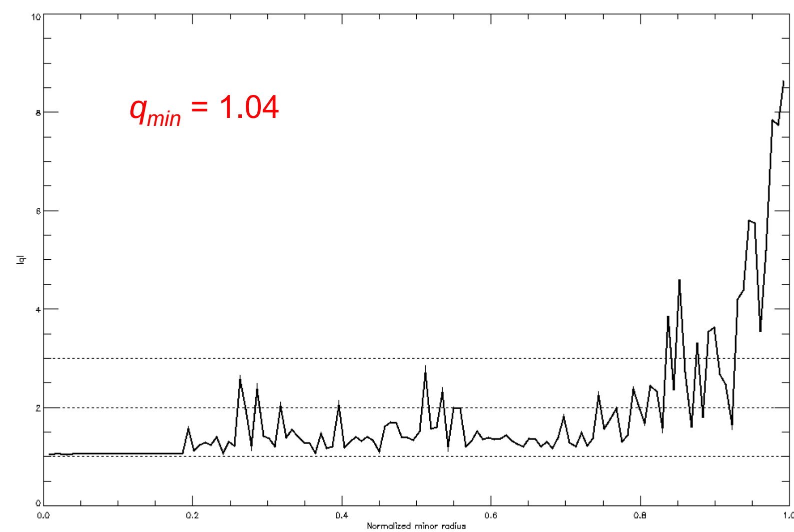


Temperature profile



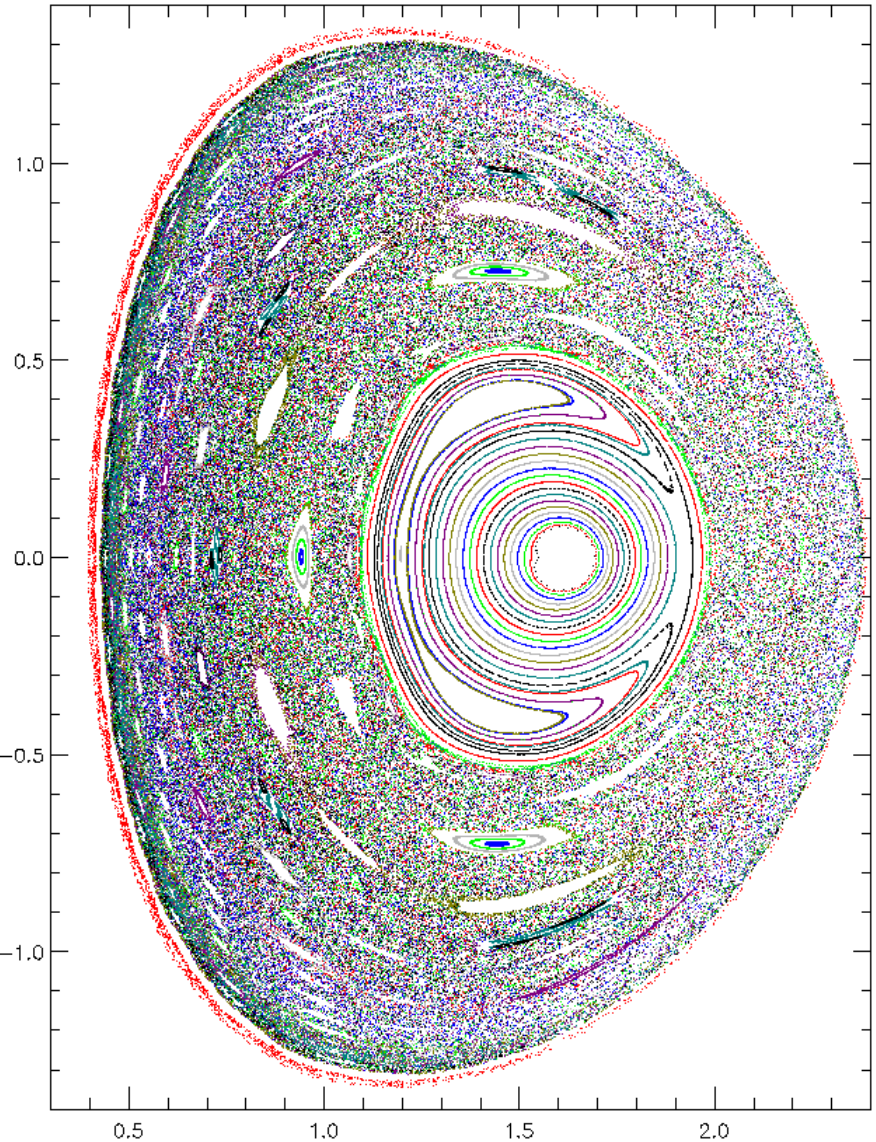


q profile

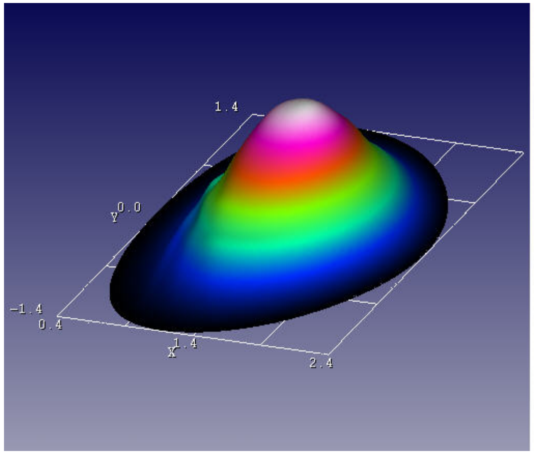


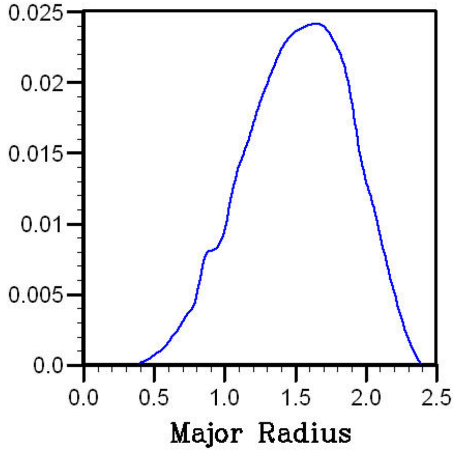
After Incomplete Recovery: t = 1848.30

Poincaré plot

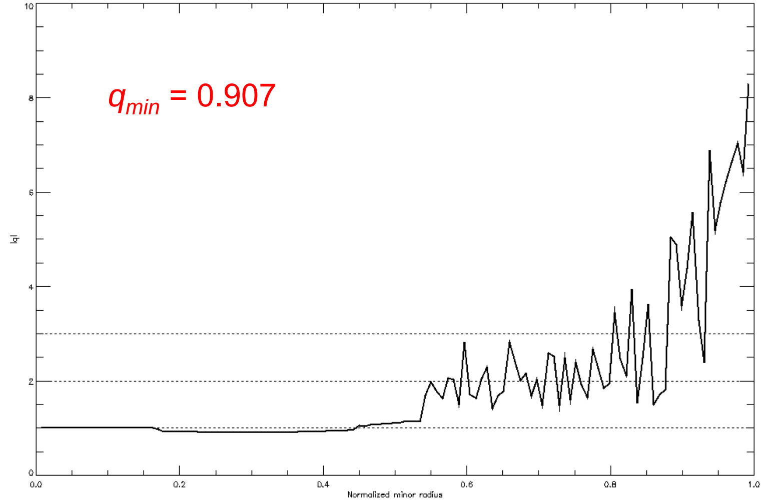


Temperature profile

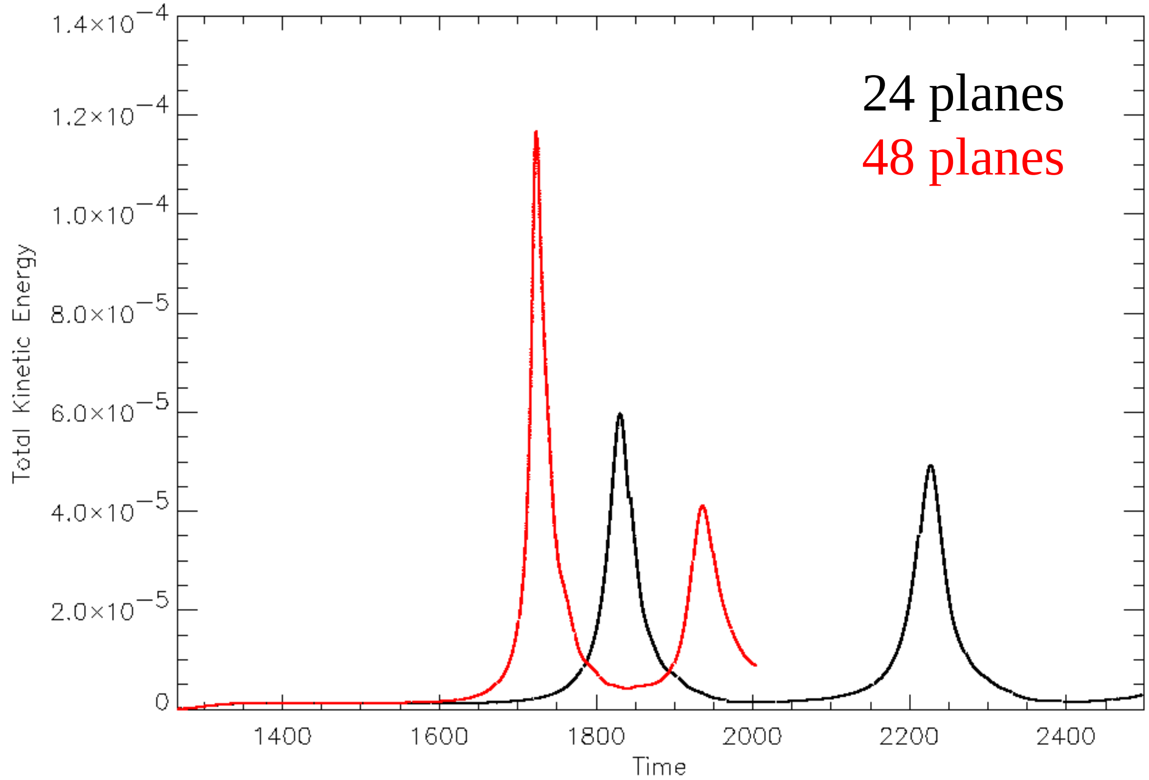




q profile



Disagreement Between Low-res & High-res Runs



- Both timing and energy of peaks are different.
- Outer flux surfaces do not heal in high-res case.

- Energy in higher-*n* modes significantly affects sawtooth evolution.
- Further study is needed to assess convergence on this case.

Two-Fluid Study

- Same parameters as single-fluid, but ω^*_i term on.
- Ion skin depth = 0.05 minor radii.
- Pressure divided evenly between electrons, ions.
- Modest increase in poloidal resolution relative to 1st single-fluid study (89 *vs*. 79 radial grids); same toroidal resolution (24 planes).
- Start nonlinear run with MHD n=1 (1,1) eigenmode, $\gamma\tau_A=5.1\times10^{-3}$ as small initial perturbation.

The Two-Fluid Model

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}_i) = 0$$

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} + (\mathbf{v}_i^* \cdot \nabla)\mathbf{v}_\perp\right] = -\nabla \mathbf{p} + \mathbf{J}\times\mathbf{B} + \mu\nabla^2\mathbf{v}$$

$$\mathbf{E} + \mathbf{v}\times\mathbf{B} = \eta\mathbf{J} - \frac{\nabla_{||} p_e}{ne}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla\times\mathbf{E}$$

$$\mathbf{J} = \nabla\times\mathbf{B}$$

$$\frac{\partial p}{\partial t} + \mathbf{v}\cdot\nabla p = -\gamma p \nabla\cdot\mathbf{v} + \rho\nabla\cdot\kappa_{||}\nabla_{||}\left(\frac{p}{\rho}\right) - \mathbf{v}_i^*\cdot\nabla p - \gamma p\nabla\cdot\mathbf{v}_i^* + \frac{\mathbf{J}\cdot\nabla p_e}{ne} + \gamma p_e \mathbf{J}\cdot\nabla\left(\frac{1}{ne}\right)$$

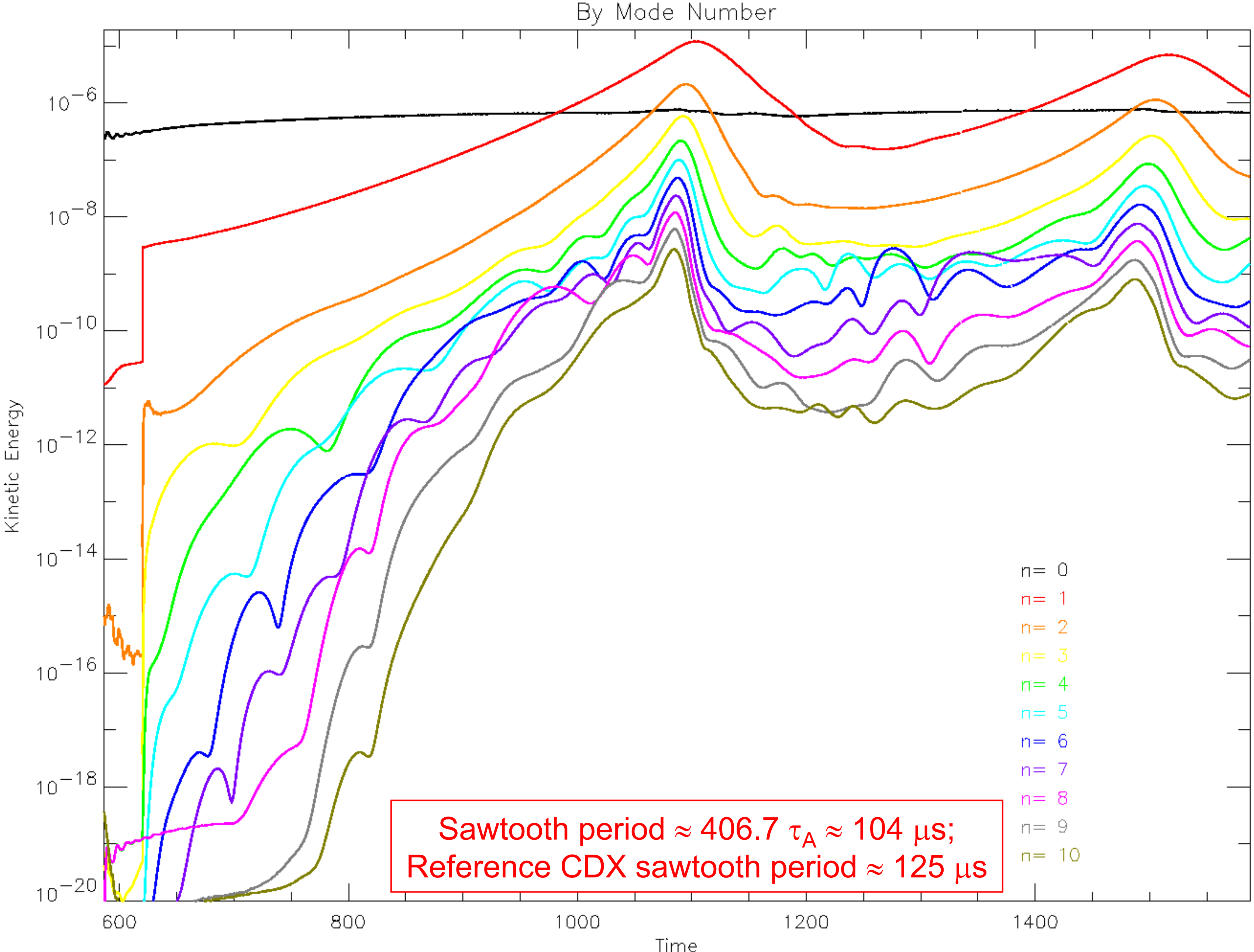
$$\frac{\partial p_e}{\partial t} + \mathbf{v}\cdot\nabla p_e = -\gamma p_e \nabla\cdot\mathbf{v} + \rho\nabla\cdot\kappa_{||}\nabla_{||}\left(\frac{p_e}{\rho}\right) + \frac{\mathbf{J}_{||}\cdot\nabla p_e}{ne} - \gamma p_e \nabla\cdot\left(\mathbf{v}_e^* - \frac{\mathbf{J}_{||}}{ne}\right)$$

where

$$\mathbf{v}_i^* \equiv \mathbf{v}_e^* + \frac{\mathbf{J}_\perp}{ne}, \qquad \mathbf{v}_i \equiv \mathbf{v} + \mathbf{v}_i^*,$$

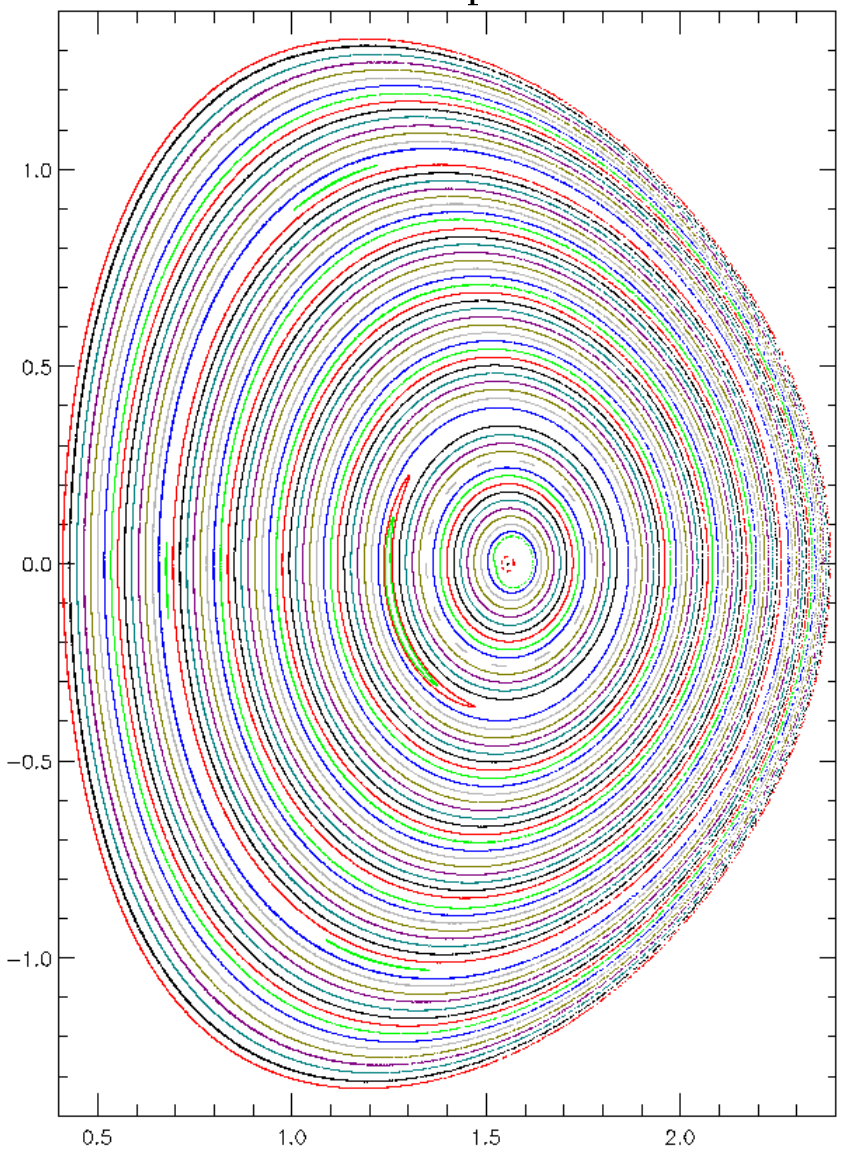
$$\mathbf{v}_e^* \equiv -\frac{\mathbf{B}\times\nabla p_e}{neB^2}, \quad \mathbf{v}_e \equiv \mathbf{v} + \mathbf{v}_e^* - \frac{J_{||}}{ne}$$

Two-Fluid Sawooth Energy History

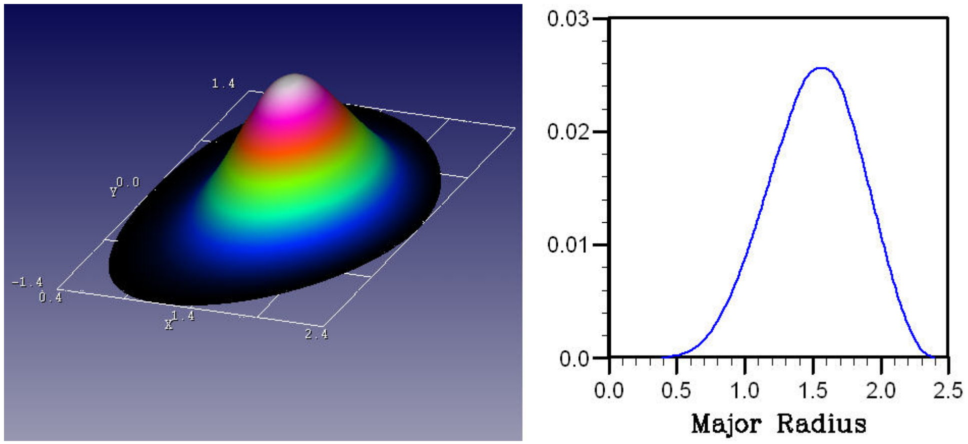


Early state: t = 653.95

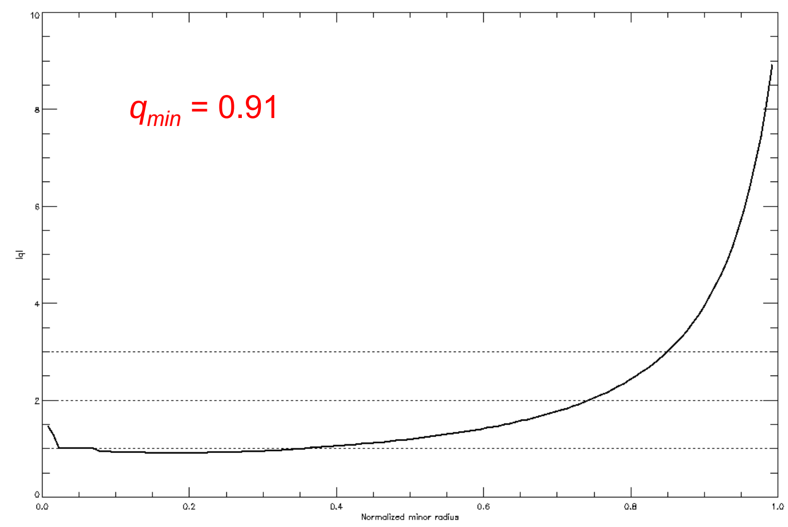
Poincaré plot



Temperature profile

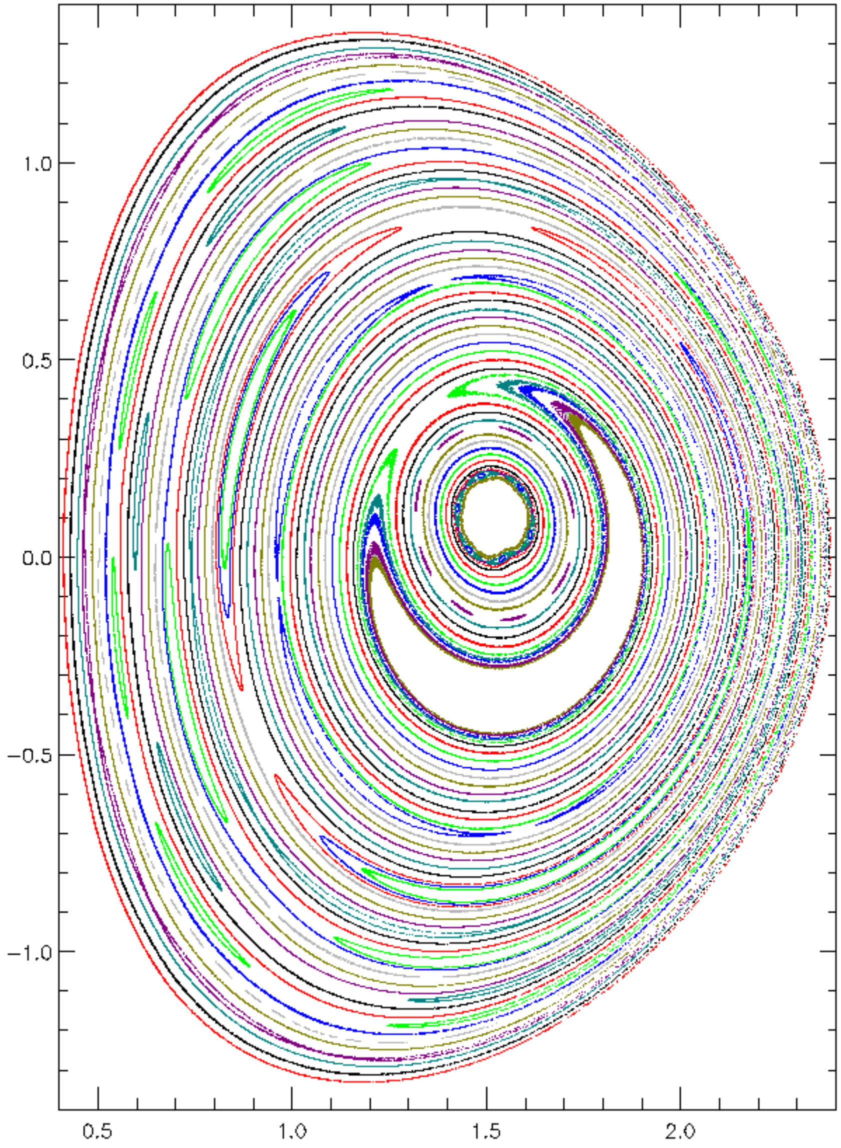


q profile

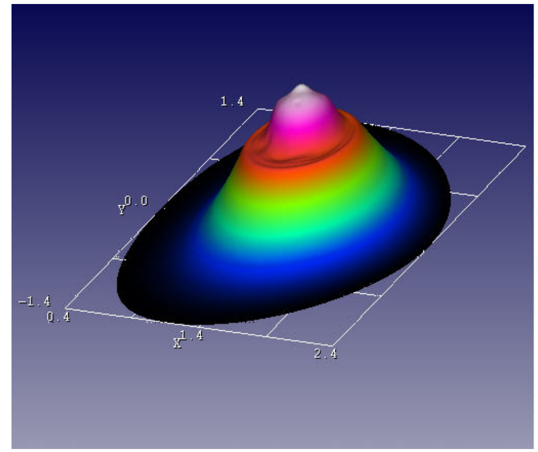


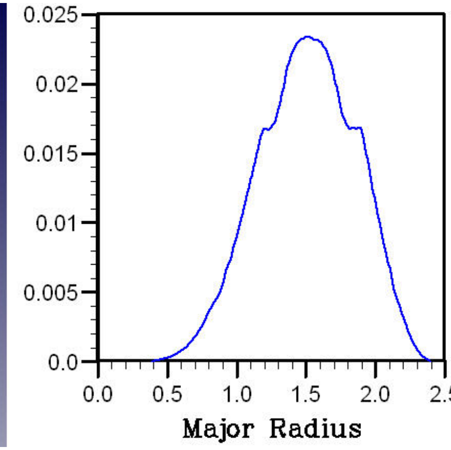
Nonlinear phase: t = 1008.38

Poincaré plot

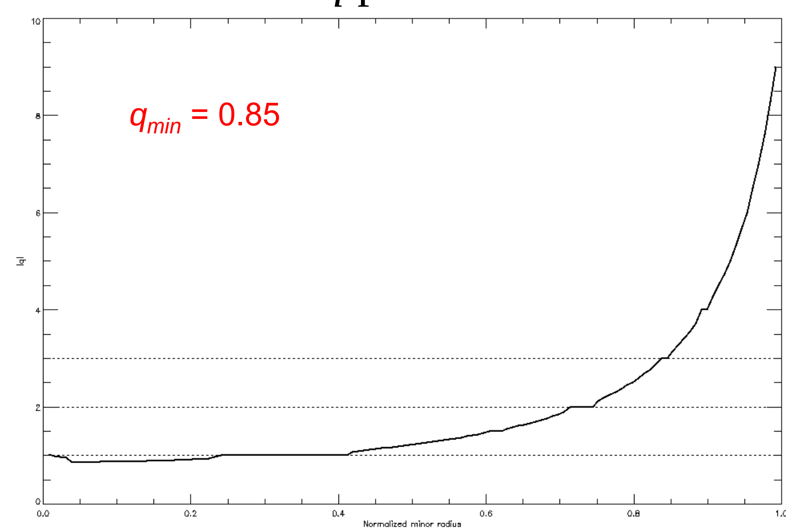


Temperature profile



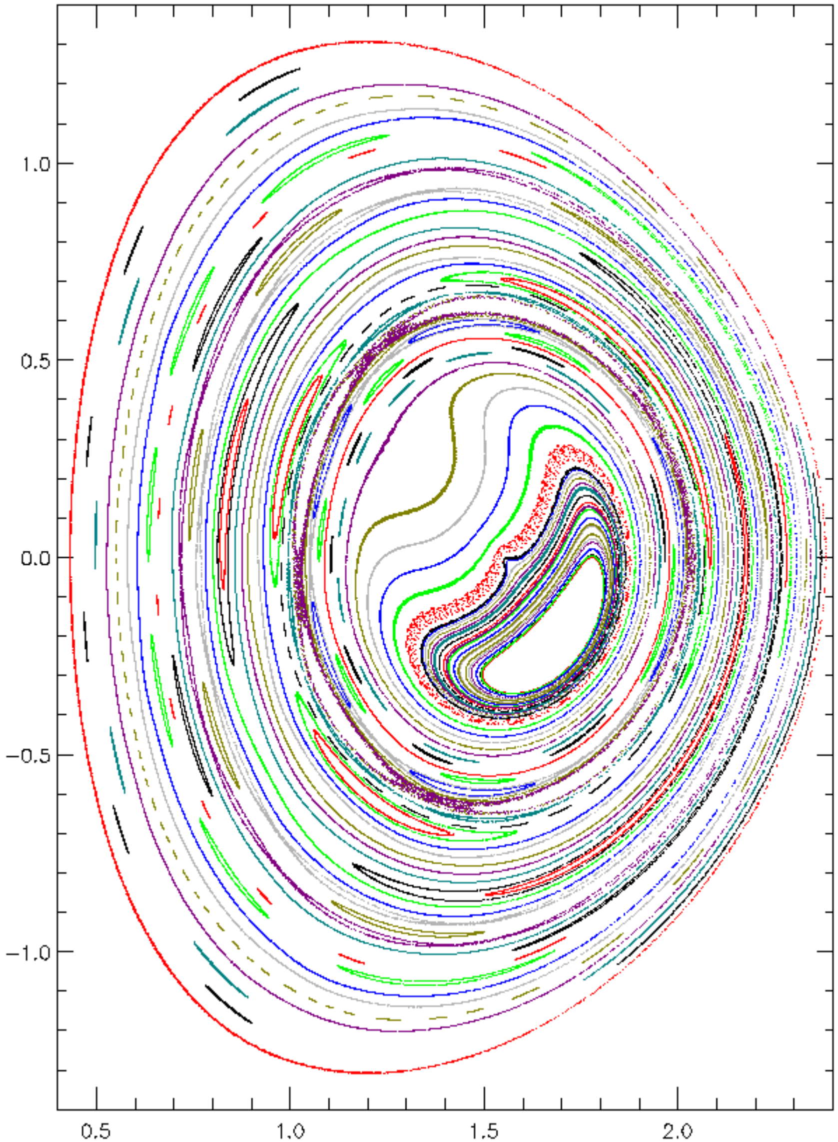


q profile

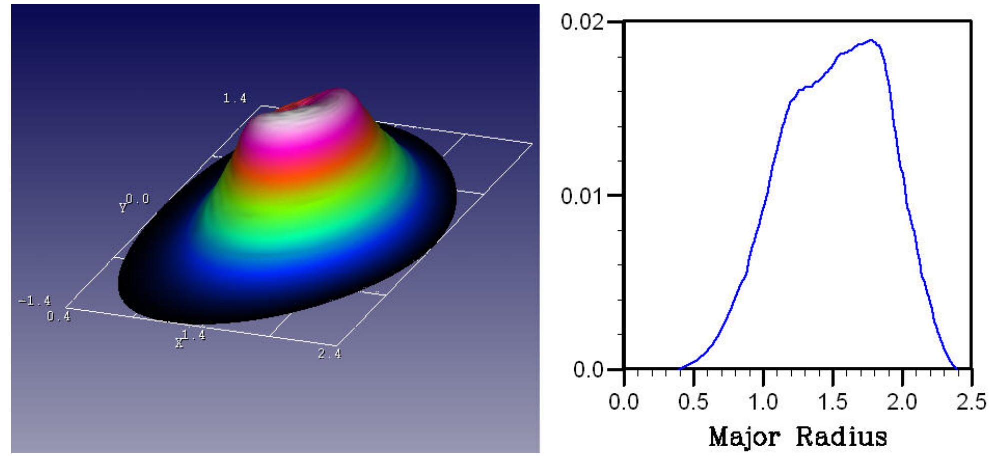


After 1st Crash: t = 1118.44

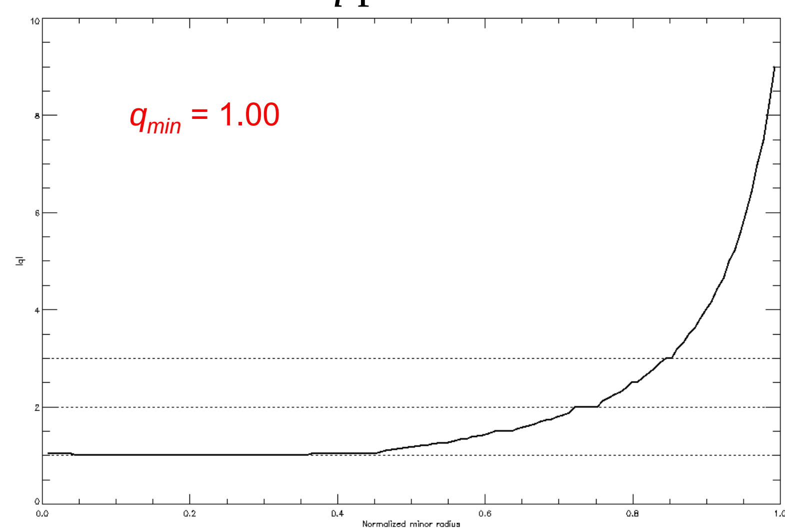
Poincaré plot



Temperature profile

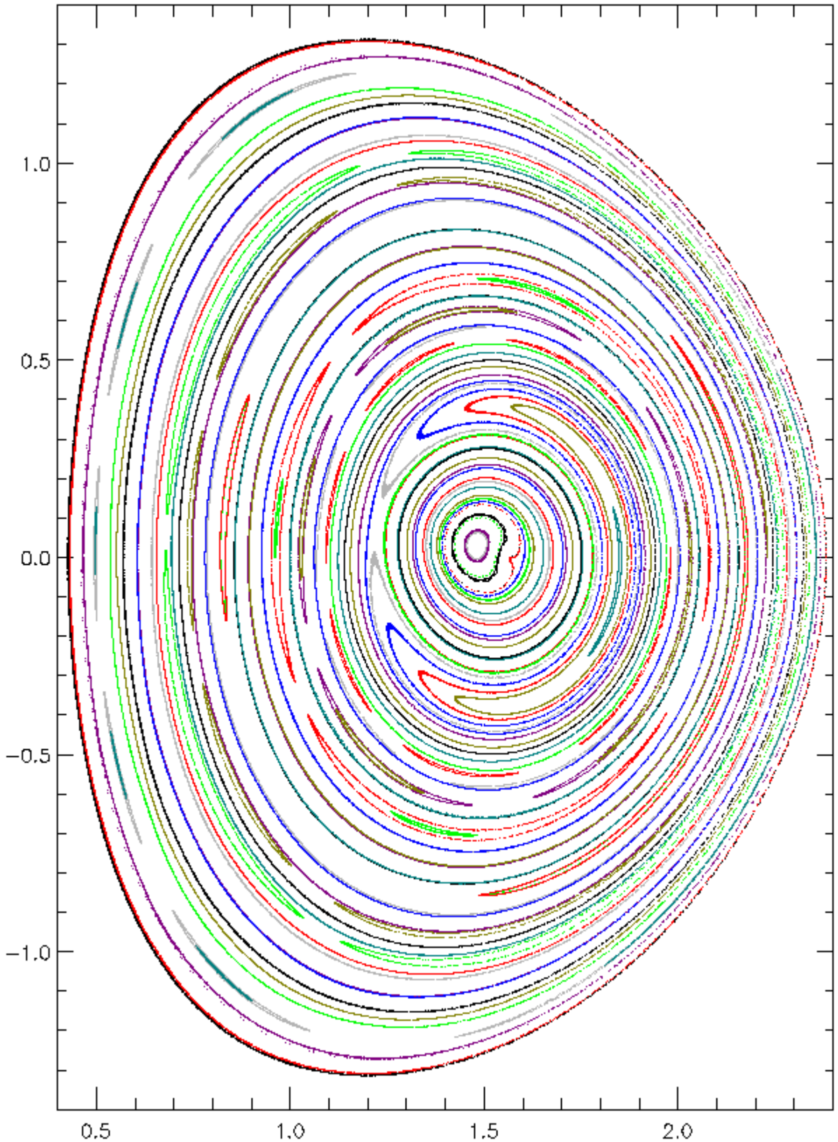


q profile

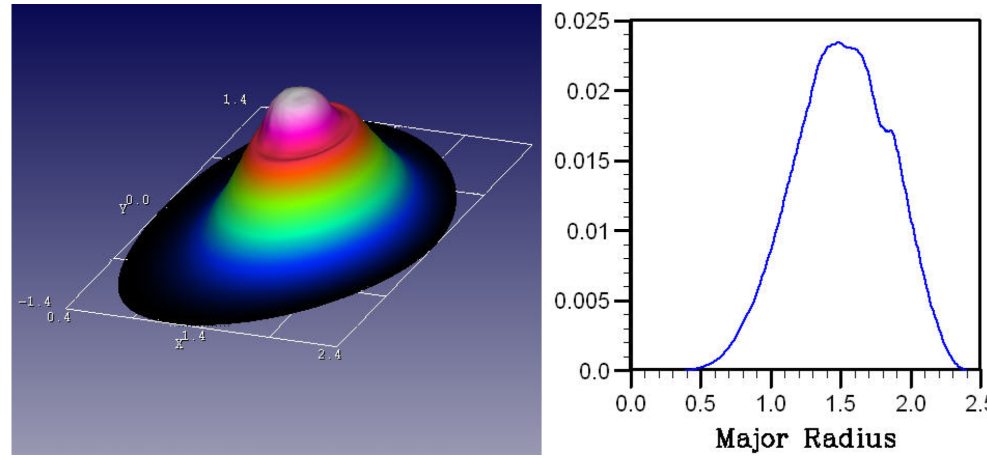


Between Crashes: t = 1316.83

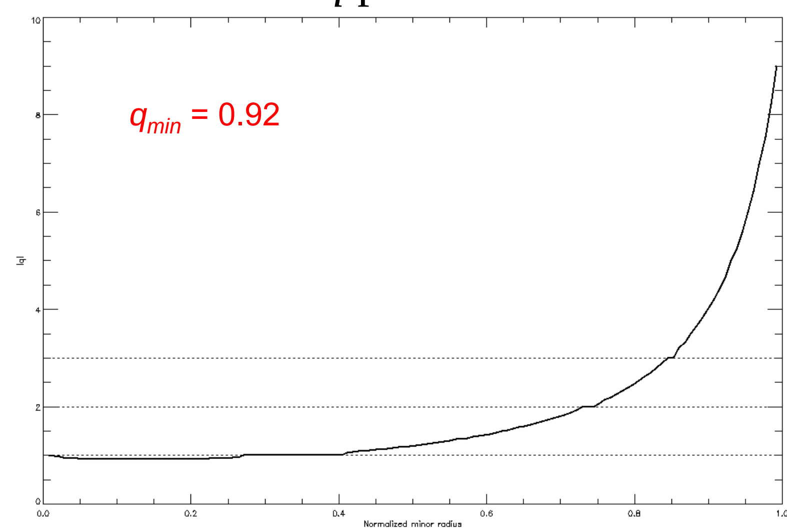
Poincaré plot



Temperature profile

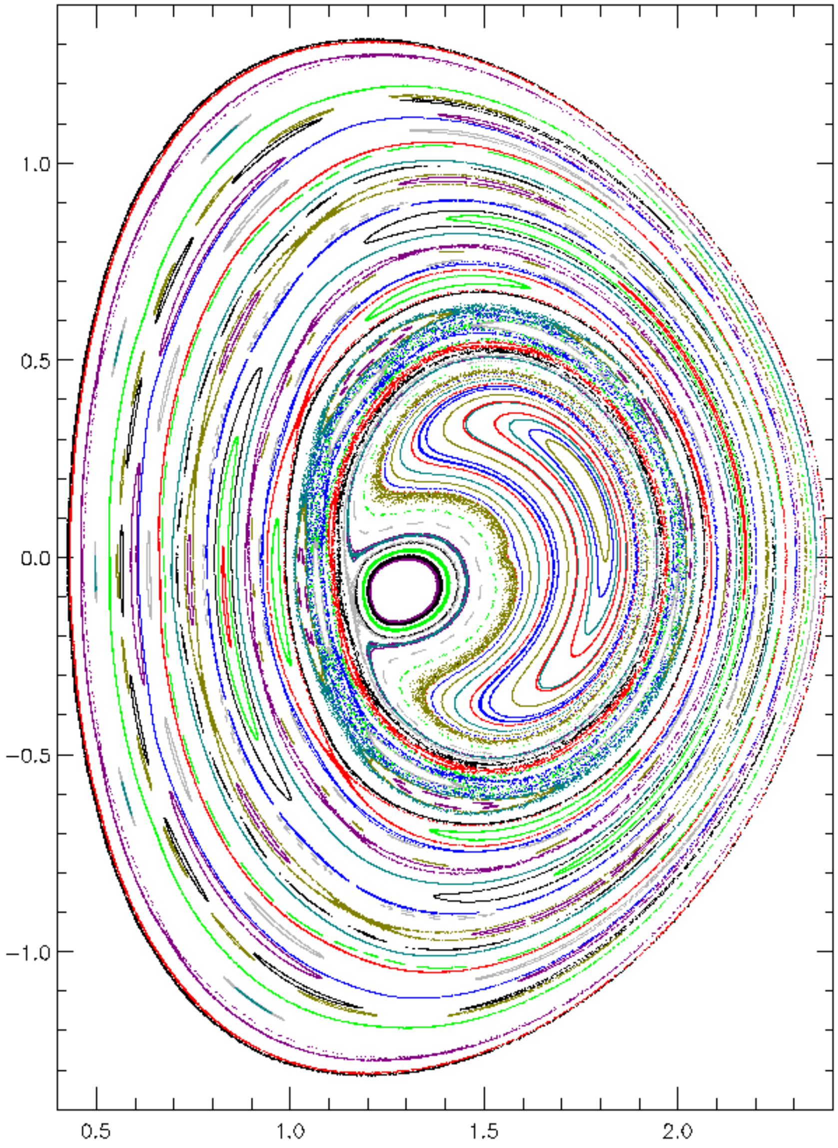


q profile

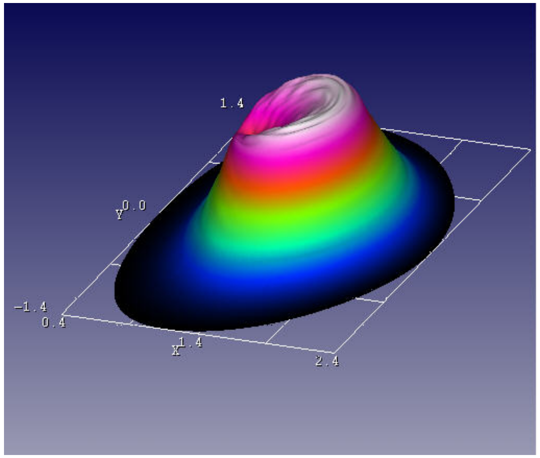


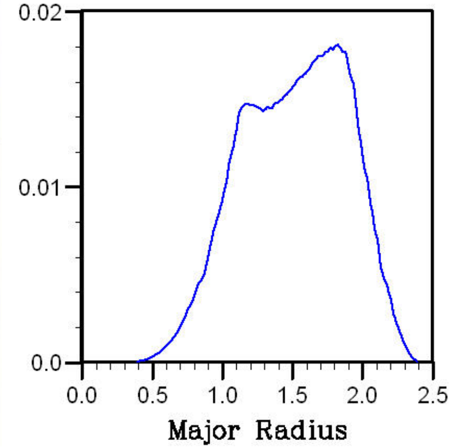
Second Crash: t = 1502.53

Poincaré plot

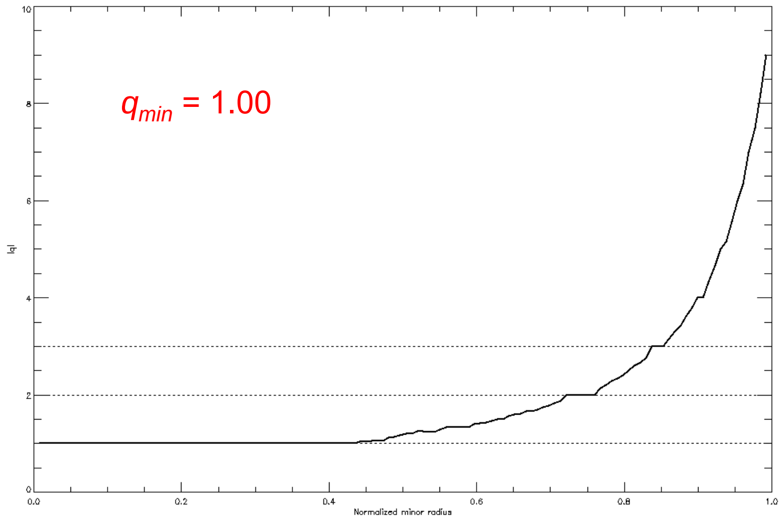


Temperature profile





q profile



Summary of Observed Two-Fluid Effects

- Plasma rotation.
- Oscillations in energy of higher-*n* modes.
- Sawtooth period increases slightly.
- Magnetic field does not become stochastic over most of plasma cross-section.
- Reconnection is incomplete in second crash.

Conclusions

- Nonlinear MHD simulation with actual device parameters is capable of tracking evolution through repeated sawtooth reconnection cycles.
  - Dynamics depend on number of modes retained, implying high energy in high-*n* modes.
  - Failure of convergence in number of modes kept would suggest inadequacy of the MHD model for this problem.
- Two-fluid simulation has now also been applied to the problem.
  - Qualitatively similar to MHD predictions, <u>but</u>
  - Sheared rotation inhibits island growth, reducing stochasticity of field.
  - Incomplete reconnection in second cycle suggests possibility of saturated islands. To be investigated.