On neoclassical and rf closures

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Acknowledgements:
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CEMM pre-APS meeting
Oct. 23, 2005
Denver, CO

Thesis

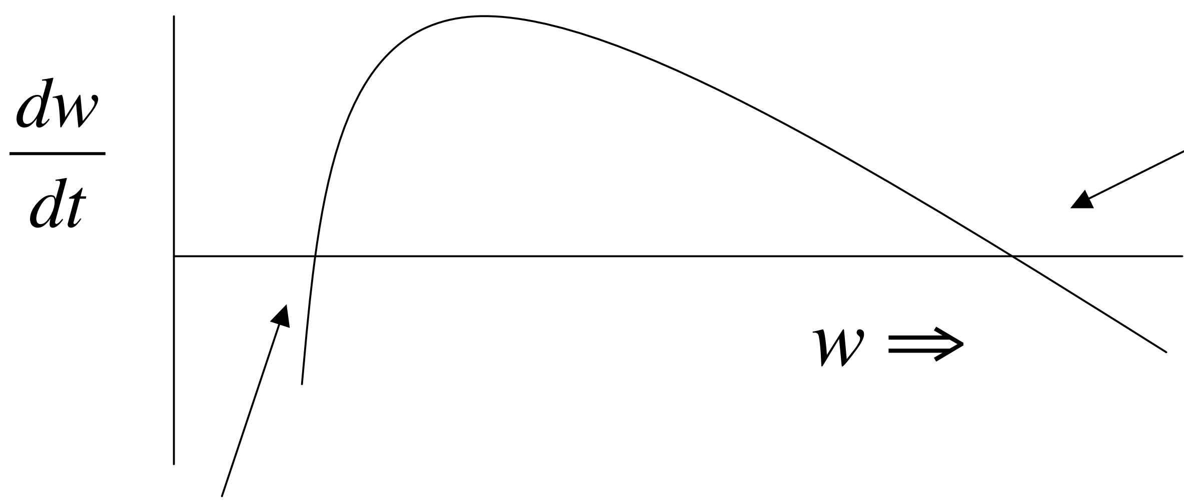
- Accurate predictions of “slow-MHD” island physics in high temperature plasmas requires the inclusion of a number of non ideal-MHD physical effects.
  - Neoclassical-like viscous forces
    - parallel viscous forces
    - gyro-viscosities
  - Anisotropic transport in helical geometry
  - Rf-source effects - ECCD stabilization

Outline

- NTM basics
- Neoclassical related closures
  - Bootstrap current drive - electron viscosity
  - Threshold physics
    - Neoclassical polarization currents - ion viscosity, gyro-viscosity, natural island frequency
    - Anisotropic heat conduction - heat flux
- ECCD stabilization - RF closures

Elements of NTM physics

- Phase diagram illustrates differ aspects of NTM evolution



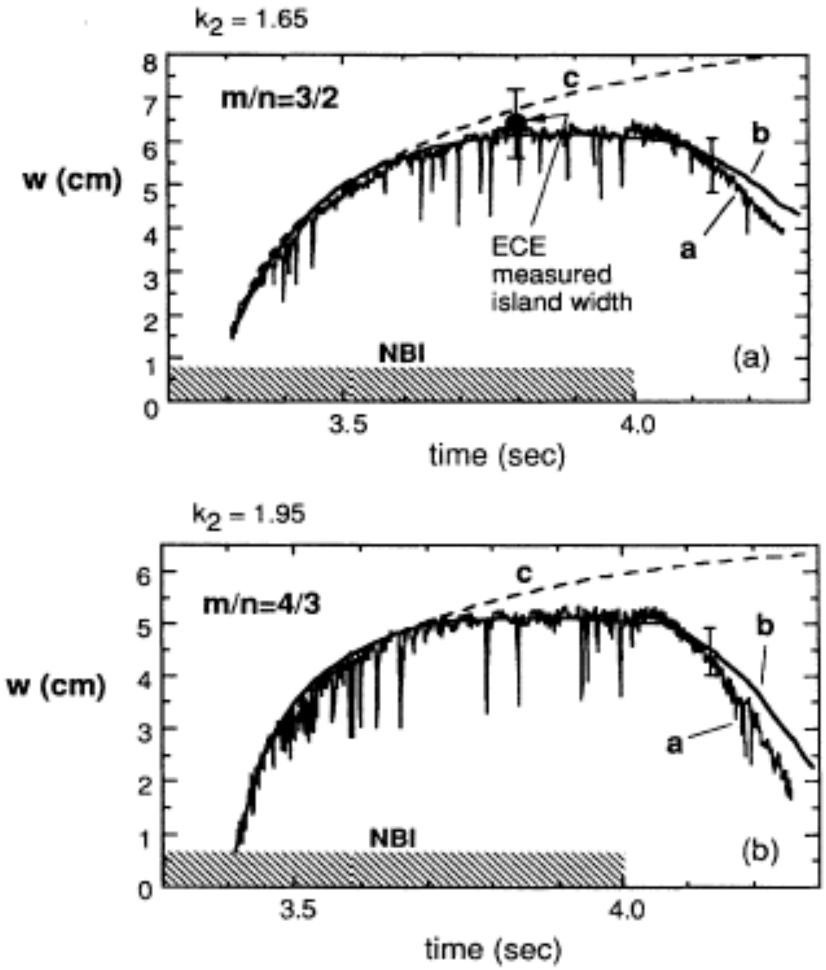
$w = w_{sat}$
$\sim D_{nc} / (-\Delta'))$
$\sim \varepsilon^{0.5} \beta_\theta$

<u>Issues</u>
Saturated state
Temporal evolution
Threshold physics
Seeding

$w = w_{threshold} \sim \varepsilon^{0.5} \rho_{\theta i}$ or $w_d \sim (\chi_{perp} / \chi_{||})^{0.25}$
$W > \delta_{res} \sim S^{-2/5}$

NTM physics has been binned into different parts of the problem problems

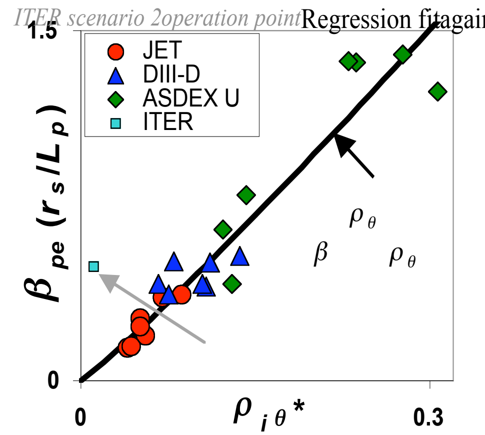
- Some elements of theory are in relatively good shape
  - Saturated state
    $w_{sat} \sim \varepsilon^{0.5}\beta_\theta$ ~ local bootstrap current density ---> in good agreement with experiment
  - Timescale of island evolution ~ resistive diffusion time through the island region ---> in good agreement w/expt
  - Observed on TFTR, JET, DIII-D, JT60U, AUG, etc



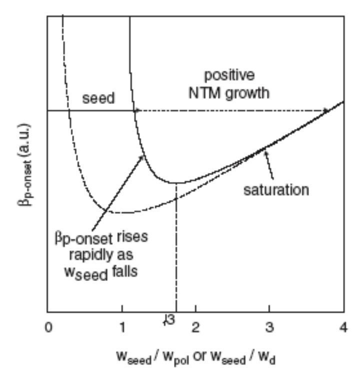
Chang, et al, PRL '95

Other elements of NTM physics are less well understood

- Threshold physics - NTM excitation requires $w > w_{threh}$
  - Empirically, critical β for NTM onset scales with ρ^* ---> very bad for ITER
  - somewhat consistent with theory, but work remains
- Seeding processes
  - How to initiate $w > w_{thresh}$?
  - Some ideas, no single explanation for all observations - transient evolution of Δ', electromagnetic coupling to outside MHD, transport changes, sawteeth period?



Buttery, et al, IAEA '04



Modified Rutherford equation provides a useful model to interpret NTM dynamics

- Nonlinear tearing mode dynamics initially worked out for resistive MHD theory - Rutherford '73
  - Slowly evolving helical MHD "equilibrium"

$$\vec{J} \times \vec{B} = \nabla p, \quad \vec{B} \cdot \nabla \frac{J_{\parallel}}{B} = -\nabla \cdot \frac{\vec{B} \times \nabla p}{B^2}$$

$$\Rightarrow \frac{J_{\parallel}}{B} = \frac{< \vec{J} \cdot \vec{B} >_{\Psi*}}{< B^2 >_{\Psi*}} + helical\ Pfirsch - Schluter\ currents$$

Ψ* =helical magnetic surface label

  - Resistive Ohm's law

$$< \vec{E} \cdot \vec{B} >_{\Psi*} = \eta < \vec{J} \cdot \vec{B} >_{\Psi*} \Rightarrow - < \frac{\partial \tilde{\tilde{\vec{A}}}}{\partial t} \cdot \vec{B} >_{\Psi*} = \eta < \vec{J} \cdot \vec{B} >_{\Psi*}$$

  - Matched asymptotics - Rutherford equation - with Pfirsch-Schluter effects (Kotschenreuther et al '86; CCH '99)

$$k_1 \frac{dw}{dt} = \frac{\eta}{\mu_o}[\Delta' + \frac{D_R^*}{w}]$$

$D_R^* = (E + F + H^2)/(\alpha_s - H)$

(Glasser, et al '75)

Neoclassical electron viscosity brings in bootstrap current effects

- Bootstrap currents - viscous damping of poloidal electron diamagnetic flow
  - Toroidal equilibrium - relevant neoclassical closures are the flux-surface average of parallel viscous forces

$$\begin{pmatrix} < \vec{B}\cdot\nabla\cdot\pi_{\|} > \\ < \vec{B}\cdot\nabla\cdot\Theta_{\|} > \end{pmatrix} = mn < B^2 > \begin{pmatrix} \mu_{00} & \mu_{01} \\ \mu_{10} & \mu_{11} \end{pmatrix} \begin{pmatrix} U_\theta \\ q_\theta \end{pmatrix}, \quad \mu \cong \sqrt{\varepsilon}\nu, \quad U_\theta = \frac{v_{\|}}{B} + \frac{\vec{v}_\perp\cdot\nabla\theta}{\vec{B}\cdot\theta}$$

    - For electrons, $U_{\theta e} = U_{\theta i} - J_\theta/ne \cong - J_{||}/(Bne) - F(dp/d\psi)/(neB^2)$

    Alters electron force balance - Ohm's law

  - For islands, the helical current is produced from the helical deformation of plasma profiles (Carrera, et al '85, …

$$< \vec{B}\cdot\nabla\cdot\vec{\pi}_{\|e} >_{\Psi*} \cong < \vec{B}\cdot\nabla\cdot\vec{\pi}_{\|e} >_{axi} \frac{1}{p_o^{'}}\frac{dp}{d\Psi*} < \partial_x\Psi* >_{\Psi*} + (\ldots)J_{\|}$$

$$\cong \mu_e \frac{m_e}{e} F < B^2 > \frac{dp}{d\Psi*} < \partial_x\Psi* >_{\Psi*} + (\ldots)J_{\|}$$

Inclusion of neoclassical viscosity leads to neoclassical tearing mode instabilities

- Modified Ohm's law

$$< \vec{E} \cdot \vec{B} >_{\Psi *} = \eta < \vec{J} \cdot \vec{B} >_{\Psi *} - \frac{1}{ne} < \vec{B} \cdot \nabla \cdot \vec{\pi}_{\|e} >_{\Psi *}$$

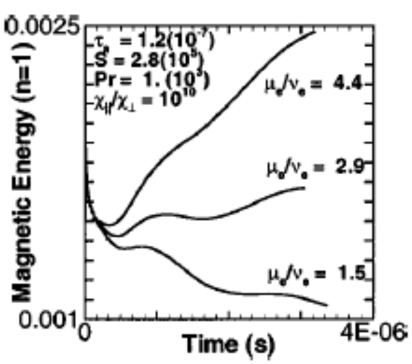
  - Modified Rutherford equation (Carrera et al, '85, Qu and Callen '86)

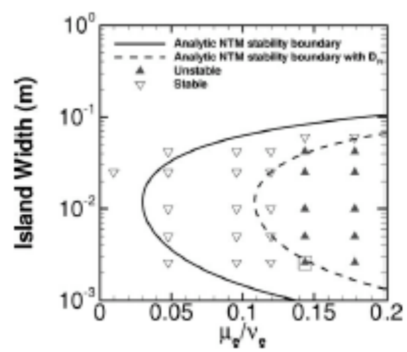
$$k_1 \frac{dw}{dt} = \frac{\eta}{\mu_o} [\Delta' + \frac{D_{nc} + D_R^*}{w}]$$

$D_{nc} > 0$ for $dq/dr > 0$, generally $D_{nc} > |D_R^*|$

  - Heuristic model suggested to facilitate NIMROD simulations of NTMs (Gianakon, et al '02)

$$\nabla \cdot \vec{\pi}_{\|e} = \mu_e m_e n_e < B^2 >_{axi} \frac{\vec{v}_e \cdot \vec{e}_\theta}{(\vec{B} \cdot \vec{e}_\theta)^2} \vec{e}_\theta \qquad \vec{e}_\theta = \sqrt{g} \nabla \zeta \times \nabla \psi = \frac{\nabla \zeta \times \nabla \psi}{\vec{B} \cdot \nabla \theta}$$





The heuristic model has a number of positive features

- Properties of heuristic model
  - Manifestly dissipative, as it should be
  - Yields correct perturbed bootstrap current for sufficiently small islands
  - No toroidal flow damping; damps poloidal flows

$$\nabla \zeta \cdot \nabla \cdot \overleftrightarrow{\pi} = 0$$

  - Gives neoclassical particle transport

$$\Gamma \cdot \nabla \psi = \frac{\vec{B} \times \nabla \cdot \pi_{\| e}}{B^2} \cdot \nabla \psi \neq 0$$

  - Relies on coordinate system of the initial axisymmetric equilibrium - with larger island - deformed equilibrium? - neoclassical toroidal viscosity? (Shaing '02)

Polarization currents alter island stability

- In resistive MHD, rotating islands add polarization effects which alters island evolution

$$\vec{B}\cdot\nabla\frac{J_{\parallel}}{B} = -\nabla\cdot\frac{\rho\vec{B}}{B^2}\times\frac{d\vec{v}}{dt}$$

  - In sheared slabs with cold ions, modified Rutherford equation (w > ρ_L) - Smolyakov '89

$$k_1\frac{dw}{dt} = \frac{\eta}{\mu_o}[\Delta' + \frac{d_{pol}}{w^3}] \quad d_{pol} \sim \rho_i^2\beta_\theta$$

    - A number of folks are looking at various 2-fluid, cold ion, sheared slab modificiations of this (Waelbroeck, et al '03; Ottaviani et al '05, etc.)

Neoclassical physics "enhance" polarization currents

- Neoclassical physics dominate the MHD polarization currents

$$\vec{B}\cdot\nabla\frac{J_{\parallel}}{B} = -\nabla\cdot\frac{\vec{B}\times\nabla\cdot\vec{\pi}_i}{B^2} \cong \frac{\partial}{\partial\psi}\frac{F}{B^2}\vec{B}\cdot\nabla\cdot\pi_{\parallel i}, \quad \vec{B}\cdot\nabla\cdot\pi_{\parallel i} \cong -m_i n_i \vec{B}\cdot\frac{d\vec{v}}{dt} \cong m_i n_i F\frac{d}{dt}\left(\frac{\partial\phi}{\partial\psi}+\frac{1}{ne}\frac{\partial p_i}{\partial\psi}\right)$$

$$\Rightarrow \nabla\cdot\frac{\vec{B}\times\nabla\cdot\vec{\pi}_i}{B^2} \cong \frac{B_\zeta^2}{B_\theta^2}\nabla\cdot\frac{\rho\vec{B}}{B^2}\times\frac{d\vec{v}_\perp}{dt}$$

  - The q^2/ε^2 enhancement of the dielectric tensor - neoclassical MHD (Callen, et al '86)
  - For lower collisionality regimes n < w/e, the enhancement is down by $\varepsilon^{3/2}$ (Wilson, et al '96)

$$\nabla\cdot\frac{\vec{B}\times\nabla\cdot\vec{\pi}_i}{B^2} \cong \frac{q^2}{\sqrt{\varepsilon}}\nabla\cdot\frac{\rho\vec{B}}{B^2}\times\frac{d\vec{v}}{dt}$$

Physics - at low collisionality, infrequent collisions across trapped/passing boundary eliminates the passing particle enhancement of the banana's "diamagnetic" current

Strength of neoclassical enhancement depends on collisionality

- Modified Rutherford equation with neoclassical polarization currents

$$k_1 \frac{dw}{dt} = \frac{\eta}{\mu_o}[\Delta' + \frac{D_{nc} + D_R^*}{w} + \frac{D_{pol}}{w^3} g(\varepsilon, \frac{\nu_i}{\varepsilon\omega}) F(\omega)]$$

  - Valid for w > $\varepsilon^{0.5}\rho_{\theta i}$

$$D_{pol} \cong \varepsilon \rho_{\theta i}^2 \beta_\theta (\frac{d \ln p}{d \ln q})^2, \quad g(\varepsilon, \frac{\nu_i}{\varepsilon\omega}) \begin{array}{ll} \to 1 & \nu_i >> \varepsilon\omega \\ \to \varepsilon^{1.5} & \nu_i << \varepsilon\omega \end{array}$$

  - For F(ω) < 0, neoclassical polarization currents are stabilizing and provides a threshold island $w_{thresh} \sim \varepsilon^{0.5}\rho_{\theta i}$ ---- crudely consistent with $\beta_{onset} \sim \rho_i^*$
    - The "collisional" version of this model should be described by the heuristic closure scheme

$$\nabla \cdot \vec{\pi}_{\parallel i} = \mu_i m_i n_i < B^2 >_{axi} \frac{\vec{v}_i \cdot \vec{e}_\theta}{(\vec{B} \cdot \vec{e}_\theta)^2} \vec{e}_\theta$$

Transition formula for different collisionality regimes requires evaluation of a temporally varying viscous force

- Recent calculation has accounted for temporal variation of parallel viscous force (Garcia-Perciante '05). For small ε

$$< \vec{B} \cdot \nabla \cdot \vec{\pi} > \cong mn < B^2 > [U_\theta(t) + \frac{1}{\nu} \frac{\partial U}{\partial t} + \sum_n c_n \int_0^t d\tau e^{-\bar{\nu}\kappa_n (t-\tau)} \frac{\partial U_\theta}{\partial t}],$$

  - Long time behavior reproduces correct asymptotic value
  - Application to the NTM problem hasn't been addressed yet.
  - Not clear how to implement this formula into initial value codes

--- require evaluation of a "time-history" integral

Stability properties of polarization effects depend on frequency: gyroviscosity is important

- Two-fluid effects are important in evaluating the neoclassical polarization effect

$$D_{pol} \sim \omega(\omega - \omega_i^*)$$

  - Origin of this dependence is two-fluid ion flows + gyroviscous cancellation

$$D_{pol} \sim \omega(\omega - \omega_i^*)$$

$$\vec{v}_{i\perp} = \frac{\vec{E} \times \vec{B}}{B^2} + \frac{\vec{B} \times \nabla p_i}{neB^2},$$

$$\frac{d}{dt}\vec{v} + \nabla \cdot \pi_\wedge = (\frac{\partial}{\partial t} + \vec{v}_i \cdot \nabla)\vec{v} + \nabla \cdot \pi_\wedge = (\frac{\partial}{\partial t} + \vec{v}_{E\times B} \cdot \nabla)\vec{v}_i$$

  - Neoclassical polarization currents are stabilizing for $\omega(\omega-\omega_i^*) < 0$ --- threshold physics requires finite T_i and island rotation frequency in the range $0 < \omega < \omega_i^*$

Predictions of natural island rotation frequencies are thought to depend on relative strength of ion to electron viscosities

- "Dissipative" contributions to the island region parallel current determine the natural island frequency. (Connor et al '02, …)
  - Frequency is determined from the relative contributions of electron and ion viscosities (Shaing PoP '02; CCH '03; Fitzpatrick and Waelbroeck PoP '05)
    - In sheared slabs, phenomenological cross-field viscosities are included in many calculations

$$m_s n_s \frac{\partial \vec{v}_s}{\partial t} = \ldots \nu_{\perp s} \nabla^2 \vec{v}_s \qquad \Rightarrow \omega = \frac{\nu_{\perp e}\omega_e^* + \nu_{\perp i}\omega_i^*}{\nu_{\perp e} + \nu_{\perp i}} \cong \omega_i^* \ \ for\ \nu_{\perp i} >> \nu_{\perp e}$$

    - Alternative model - see CCH APS '05 poster

Anisotropic transport properties can also produce a threshold island

- NTM instability relies on self-consistent deformation of plasma profiles -
  - rapid equilibration along field lines relative to cross-field transport (Fitzpatrick, '95; Gorelenkov et al '96)

$$\frac{\partial p}{\partial t} = \ldots \nabla \cdot [\hat{b}\hat{b}\chi_{\parallel} + \chi_{\perp}]\nabla p$$

  - Characteristic length scale $w_d \sim (\chi_{perp}/x_{||})^{0.25}$

$$k_1 \frac{dw}{dt} = \frac{\eta}{\mu_o}[\Delta' + \frac{D_{nc} w}{w^2 + w_d^2} + \frac{D_{pol}}{w^3} \frac{\omega(\omega - \omega_i^*)}{|\omega_i^*|^2}]$$

  - Parallel heat flux in island-like geometry has non-local feature (Held et al, '03)

$$q_{\parallel} = \int dL' \, \kappa(L, L') \, T(L')$$

RF physics can be treated as a closure problem

- Inclusion of RF operator modifies kinetic theory

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f + \frac{q}{m}(\vec{E} + \vec{v} \times \vec{B}) \cdot \frac{\partial f}{\partial \vec{v}} = C\{f\} + Q_{rf}\{f\},$$

  - Moment equations are modified

$$mn\frac{d\vec{v}}{dt} = nq(\vec{E} + \vec{v} \times \vec{B}) - \nabla p - \nabla \cdot \vec{\pi} + \vec{R} + \vec{F}_{rf}, \quad \vec{F}_{rf} = \int d\vec{v}\ m\vec{v}Q_{rf}\{f\}$$

  - Ohm's Law ($m_e \cong 0$)

$$\vec{E} + \vec{v} \times \vec{B} = \left(\frac{\vec{J} \times \vec{B} - \nabla p_e}{ne}\right) - \frac{1}{ne}\nabla \cdot \vec{\pi} + \eta \vec{J} + \vec{F}_{rf}$$

    - Relevant quantity for modified Rutherford equation is the helical flux surface average of Ohm's law (CCH and JDC '97; Zohm '97; Sauter '04)

$$< \vec{E} \cdot \vec{B} >_{\Psi *} = \eta < \vec{J} \cdot \vec{B} >_{\Psi *} - \frac{1}{ne} < \vec{B} \cdot \nabla \cdot \vec{\pi}_{\parallel e} >_{\Psi *} + < \vec{F}_{rf} \cdot \vec{B} >_{\Psi *}$$

Theoretical modeling includes an ECCD contribution to modified Rutherford equation

- Modified Rutherford equation with rf-current drive

$$k_1 \frac{dw}{dt} = \frac{\eta}{\mu_o}[\Delta' + \frac{D_{nc} w}{w^2 + w_d^2} + \frac{D_{pol}}{w^3}\frac{\omega(\omega - \omega_i^*)}{|\omega_i^*|^2} - \Delta_{cd}(\frac{w}{\delta_{CD}})]$$

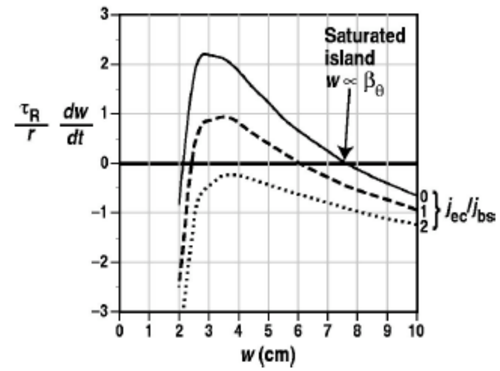
$$r_s \Delta_{cd} = -\frac{16\mu_o L_q}{\pi B_\theta}\frac{I_{cd}}{\delta_{cd}^2}\eta(\frac{w}{\delta_{cd}})$$

$$\eta(\frac{w}{w_{cd}}) = \frac{\delta_{cd}^2}{w^2}\frac{\int_{-1}^{\infty} d\psi J(\psi) W(\psi)}{\int_{-1}^{\infty} d\psi J(\psi) V(\psi)}$$

$$J(\psi) = \frac{1}{V(\psi)}\int d\alpha \frac{J_{cd}(\psi,\alpha)}{\sqrt{\psi + \cos^2(m\alpha/2)}}$$

$$W(\psi) = \int d\alpha \frac{\cos(m\alpha)}{\sqrt{\psi + \cos^2(m\alpha/2)}}$$

$$V(\psi) = \int d\alpha \frac{1}{\sqrt{\psi + \cos^2(m\alpha/2)}}$$



- ECCD stabilization has been successful in stabilizing NTMs (LaHaye '02)

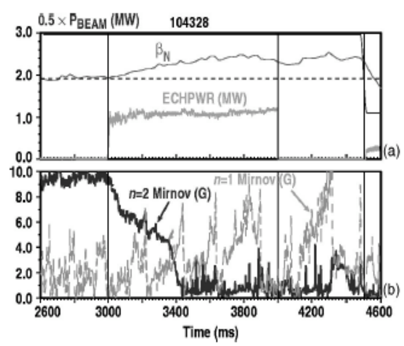


FIG. 5. Complete suppression of an $m/n=3/2$ NTM by ECCD in the presence of continued uncoupled sawteeth (2 gyrotrons, $P_{rf}/P_{beams}\approx 1$ MW/6 MW, fixed B_T and R_{surf}). (a) The β_N and ECH power, (b) $n=2$ (solid) and $n=1$ (dashed) integrated Mirnov amplitudes.

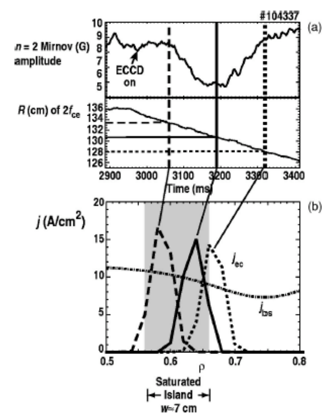


FIG. 4. (a) A B_T sweep to move the $2f_{ce}$ location past the island. ECCD on at 3000 ms (2 gyrotrons for ≈ 1 MW injected), (b) calculated j_{ec} from TORAY-GA [at the three different times indicated in (a)] and j_{bs} from ONETWO code reconstruction [at the middle time indicated in (a)]. The island width and location from the ECE radiometer at the first time indicated in (a).

There are a number of modeling issues associated with rf-modeling

- Logic from a MHD perspective - turn the rf-term as a closure problem. Issues/opportunities arise:
  - With lots of rf/beams, etc - non-Maxwellian distribution - viability of expansion - Laguerre energy Polynomials?
  - RF modeling is generally 2D - how do we account for 3D with island induced $E_{||}$ and helical magnetic fluctuations?
  - Self-consistent evolution on transport timescale with particle, momentum, energy inputs?
  - Numerical instabilities from energetic particle components that significantly modify Ohm's law (Jardin and Ignat '95)
  - ECCD efficiency requires an accurate temporal evolution of electron slowing-down etc. Particularly an issue for phasing current drive (Giruzzi, et al)

Summary

- Heuristic model for parallel viscosity has a number of desirable features
  - Correctly reproduces perturbed bootstrap current and polarization currents
  - Manifestly dissipative, damps poloidal flows, no toroidal flow damping, gives neoclassical cross-field particle transport
- To properly describe the neoclassical polarization current effect, it requires
  - Neoclassical ion poloidal flow damping
  - Two-fluid physics - Hall-MHD terms in Ohm's law
  - Gyroviscosity - gyroviscous cancellation to yield proper frequency dependence
- The inclusion of RF-forces on the electron momentum balance equation can be treated as a closure problem, but a number of issues need to be resolved