Parallel kinetic closures for NTM studies

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Outline

- Approximate CEL drift kinetic equation
- Collision operator and moment expansion
- Test problem for Sptizer resistivity
- NTM issues

Close fluid equations with kinetically derived $\vec{q}$ and $\mathbf{\Pi}$.

- Species evolution equations and closure moments for five moment model:

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot n\vec{u} = 0 \quad \rightarrow \text{density}$$

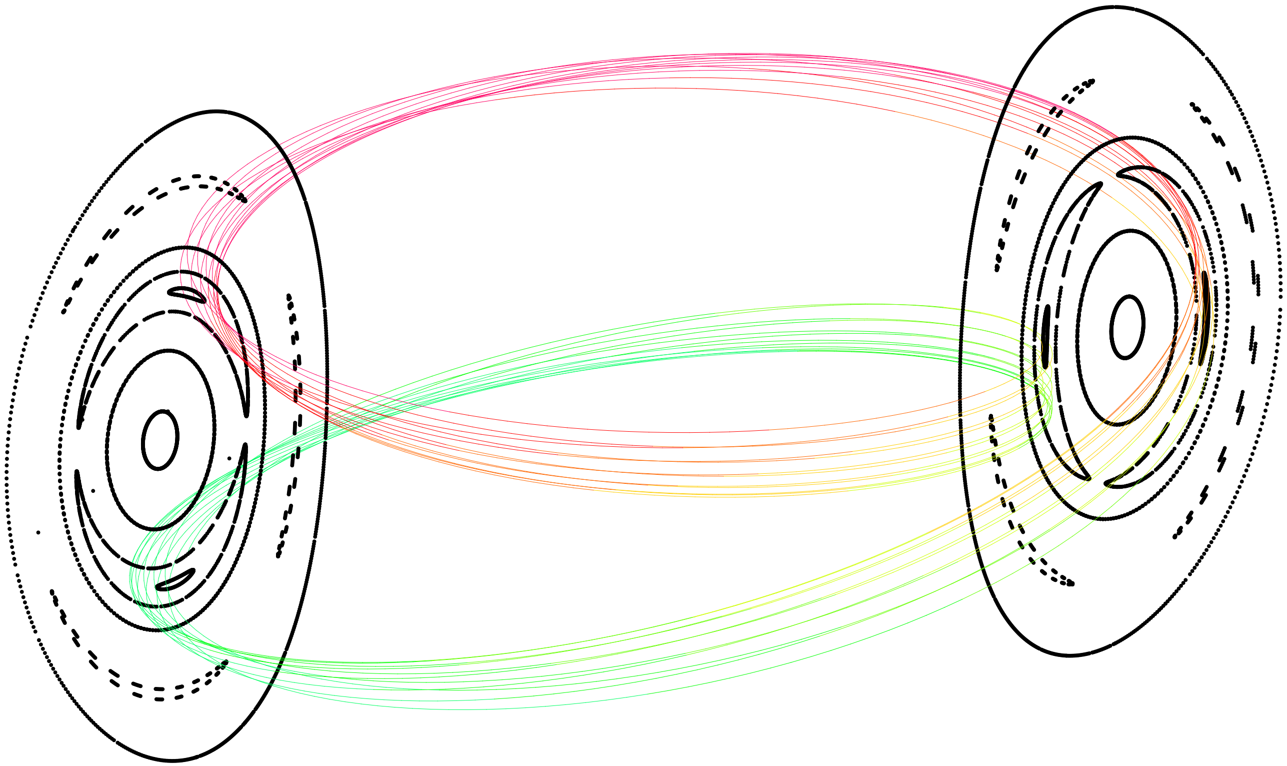
$$mn\left(\frac{\partial}{\partial t} + \vec{u} \cdot \vec{\nabla}\right)\vec{u} = en(\vec{E} + \frac{1}{c}\vec{u} \times \vec{B}) - \vec{\nabla}p - \underline{\vec{\nabla} \cdot \mathbf{\Pi}} + \vec{R} \quad \rightarrow \text{flow}$$

$$\frac{3}{2}n\left(\frac{\partial}{\partial t} + \vec{u} \cdot \vec{\nabla}\right)T = -p\vec{\nabla} \cdot \vec{u} - \underline{\mathbf{\Pi}{:}\vec{\nabla}\vec{u}} - \underline{\vec{\nabla} \cdot \vec{q}} + Q \quad \rightarrow \text{temperature}$$

$$\underbrace{\vec{q} \equiv \int d^3v' \frac{1}{2}mv'^2\vec{v}' f,}_{\text{heat flow}} \qquad \underbrace{\mathbf{\Pi} \equiv \int d^3v' m[\vec{v}'\vec{v}' - \frac{v'^2}{3}\mathbf{I}]f}_{\text{stress tensor}}.$$

Changing magnetic topology results in large $q_\parallel$.

- Particles see T perturbations of scale length, L_T, which is comparable to the collision length, L_ν.

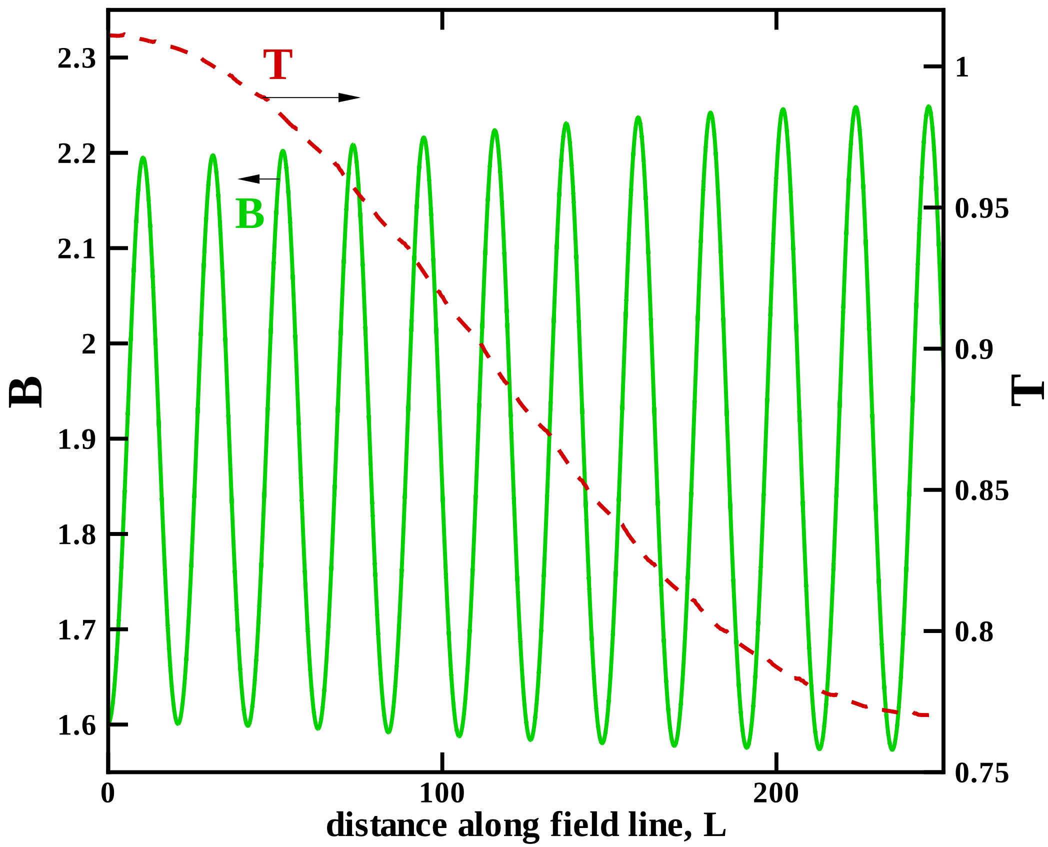


Nonlocal closures involve multiple parallel scale lengths.

- $L_T \equiv (\nabla_\| \ln T)^{-1}$

  $L_\nu \equiv v_{th}/\nu_0 \quad \Rightarrow \quad \underbrace{L_T \sim L_\nu \sim 100m >> l}_{\text{moderately collisional}} \quad (T = 1$ keV, $n = 10^{20} m^{-3})$

  $l \equiv (\nabla_\| \ln B)^{-1}$



Take Chapman-Enskog-like approach to derive closures.

- Chapman and Enskog proposed following form for f: [a]

$$f = f_M + F = \underbrace{n\left(\frac{m}{2\pi T}\right)^{\frac{3}{2}} \exp\left(-\frac{m{v'}^2}{2T}\right)}_{\text{dynamic Maxwellian}} + \underbrace{F(\vec{x}, \vec{v}, t)}_{\text{kinetic distortion}}.$$

- Use fluid moment equations to rewrite df_M/dt in full kinetic equation

$$\frac{dF}{dt} - C(F + f_M) = -\frac{df_M}{dt} = -(\mathrm{CEL}).$$

- Assume gyrofrequency, Ω, greater than other frequencies $\partial/\partial t/\Omega \sim \delta$.
- Gyro-average using $(\Omega \oint d\gamma/2\pi)$ to derive order δ constraint equation:

$$\left\langle L(\bar{F} + \tilde{F}) \right\rangle - \left\langle C(\bar{F} + \tilde{F} + f_M) \right\rangle = -\langle \mathrm{CEL} \rangle,$$

where $L = d/dt - \Omega\partial/\partial\gamma$.

[a] S. Chapman and T. G. Cowling, *The Mathematical Theory of Non-Uniform Gases* (Cambridge University Press, Cambridge, 1939).

Approximate $\bar{F} + \tilde{F}$ with $\bar{F}$ in constraint equation.

- Order $\vec{u}/v_{th}$ small so

$$f_M \approx f_M(n(\vec{x},t), T(\vec{x},t)) \left[1 + \frac{2}{v_{th}^2}\vec{v}\cdot\vec{u}\right]$$

- Resultant approximate $O(\delta)$ equation is:

$$\left[\frac{\partial}{\partial t} + \vec{v}_{\|}\cdot\vec{\nabla} - \frac{\mu}{B}\frac{\partial B}{\partial t}\frac{\partial}{\partial \mu} + q\vec{v}_{\|}\cdot\vec{E}\frac{\partial}{\partial \epsilon}\right]\bar{F} - C(f_M + \bar{F}) =$$
$$-\frac{m}{T}(v_{\|}^2 - \frac{v_{\perp}^2}{2})(\hat{\mathbf{b}}\hat{\mathbf{b}} - \frac{\mathbf{I}}{3}):\vec{\nabla}\vec{u}f_M$$
$$+\vec{v}_{\|}\cdot\left(\vec{\nabla}\cdot\mathbf{\Pi} - \vec{R}\right)\frac{f_M}{p}$$
$$+L_1^{1/2}(\vec{\nabla}\cdot\vec{q} - \tilde{Q})f_M - L_1^{3/2}\vec{v}_{\|}\cdot\vec{\nabla}T\frac{f_M}{T}.$$

Use novel treatment for linearized collision operators.

- Plan to invert Lorentz scattering terms but use moment approach for the remainder of the operator.

$$C_{\mathrm{e}} = C_{\mathrm{e}}^{-} + (L_{\mathrm{ee}} + L_{\mathrm{ei}})F_{\mathrm{e}},\ C_{\mathrm{e}}^{-} \approx C_{\mathrm{Mei}}^{(1)} + C_{\mathrm{Fee}}^{-(1)}$$

where

$$L_s{}^{s'} = \frac{\Gamma_{ss'}}{2}\frac{G_{s',v}^{(0)}}{v^3}\frac{\partial}{\partial(\frac{v_\|}{v})}(1-(\frac{v_\|}{v})^2)\frac{\partial}{\partial(\frac{v_\|}{v})}$$

with

$$G_{s',v}^{(0)} = n_{0s'}[(\eta - \frac{1}{2\eta})\frac{E}{\eta} + \frac{1}{2\eta}E']$$

and using a small mass ratio approximation

$$C_{\mathrm{Mei}}^{(1)} \approx 2f_{\mathrm{Me}}^{(0)}\frac{\Gamma_{\mathrm{ei}}n_{0\mathrm{i}}C_1(\xi)}{v_{\mathrm{te}}^4\zeta^2}(u_{0\|\mathrm{i}} - u_{0\|\mathrm{e}}),$$

where $C_1(\xi)$ is an eigenfunction of $L_{ss'}$.

Use novel treatment for linearized collision operators..

- Speed diffusion and drag terms handled with moment expansion to provide accuracy in the collisional limit.

$$\bar{F} \quad = \quad f_M \sum_{n=1}^{N} \sum_{m=0}^{M} \mathbf{a}_{nm} C_n(\xi) L_m^{((n+2)/2)}(v/v_{th}),$$

where C_n's and $L_m^{((n+2)/2)}$ are pitch-angle eigenfunctions and Laguerre polynomials, respectively.

- Resultant operator looks like:

$$C_{\mathrm{Fee}}^{-(1)} = f_{\mathrm{Me}}^{(0)} \frac{\Gamma_{\mathrm{ee}} n_{0\mathrm{e}}}{v_{\mathrm{te}}^3} [\frac{2C_1(\xi)}{v_{\mathrm{te}}} (u_{1\|\mathrm{e}} \nu_{u_1}^{-} + u_{2\|\mathrm{e}} \nu_{u_2}^{-} + \cdots)$$
$$+ \frac{2C_2(\xi)}{3n_{0\mathrm{e}} T_{0\mathrm{e}}} (\pi_{0\|\mathrm{e}} \nu_{\pi_0}^{-} + \pi_{1\|\mathrm{e}} \nu_{\pi_1}^{-} + \cdots) + \cdots],$$

where, for example, when $v_{\|}/v = \xi$

$$\nu_{u_1}^{-}(\zeta) = (-\frac{5}{4\zeta^3} - \frac{7}{\zeta} + 3\zeta)\frac{E}{\zeta} + (\frac{5}{4\zeta^3} + \frac{19}{2\zeta})E'.$$

Simple" ion operator in large mass ratio approximation.@

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Apply collision operator in calculation of Sptizer resistivity.

- For Spitzer problem can replace electron collisional friction force, $R_{||}$, with $en_eE_{||}$ in CEL drives.
- Results for 1, 2, and 3-moment model approach:

moments $\rightarrow$ coefficient $\downarrow$	1	2	3	Spitzer
flow	2.326	1.992	1.986	1.96
heat flow		-.544	-.545	
energy-weighted heat flow			-.0114	

Solve system of hyperbolic equations.

- Final form of the equations is

$$\mathbf{I}\frac{\partial \vec{F}}{\partial t} + \mathbf{A}(v\frac{\partial}{\partial L} + \frac{qE_{\|}}{m}\frac{\partial}{\partial v})\vec{F} + \mathbf{B}\nu\vec{F} = \vec{g}.$$

- Find eigenvectors of $\mathbf{A}$ and expand $\vec{F}$ in this basis to identify characteristics, $\vec{F} = \mathbf{W}\vec{f}$.
- Diagonalize by approximating coupled terms in $\mathbf{W}^{-1}\mathbf{B}\mathbf{W}\vec{f}$ again with moment expansion.
- Integrate separated PDE's along charactersitics to determine f_i's.
- Take desired closure moments and write as coupled system of integral equations or solve equations for f_i's via "particle" approach.

Remaining Issues

- Effect of axisymmetric toroidal geometry

  Cordey eigenfunctions.

  Trapped and passing particle distributions.

  Form of coupled hyperbolic equations.

- Numerical implementation

  Premliminary form exists in NIMROD.