CDX-U Sawtooth Update

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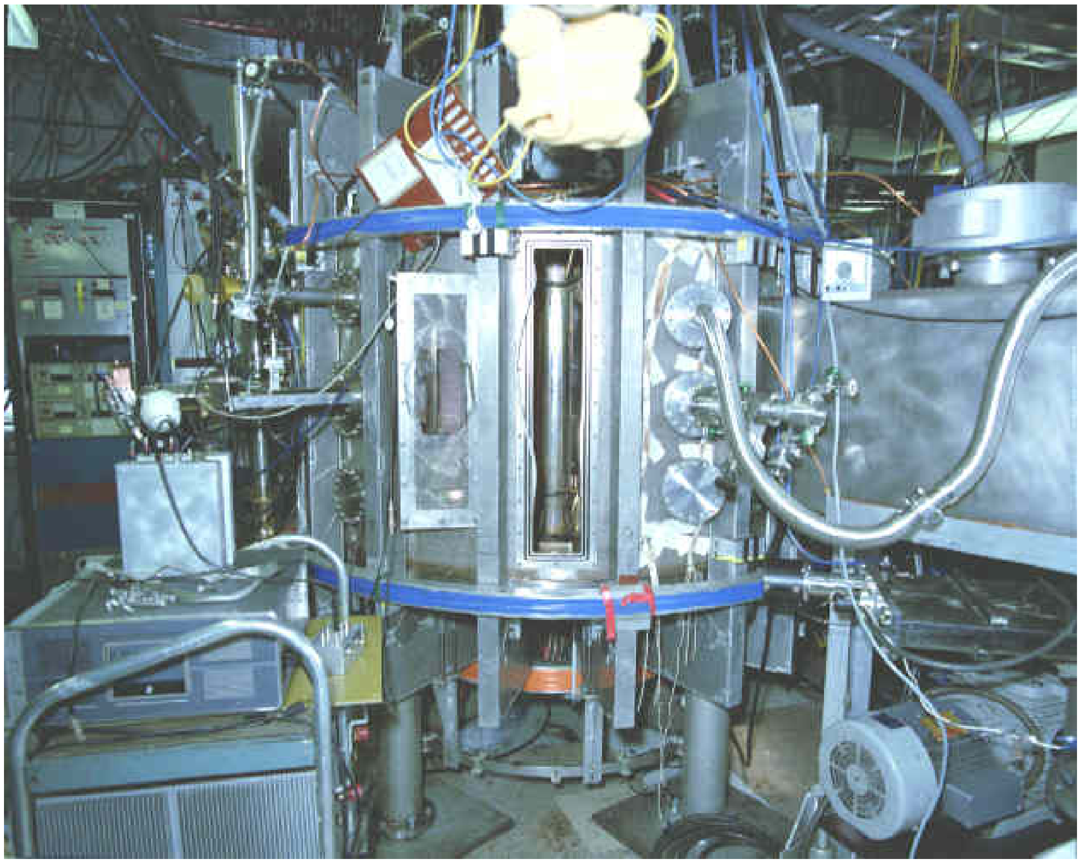
Characteristics of the Current Drive Experiment Upgrade (CDX-U)

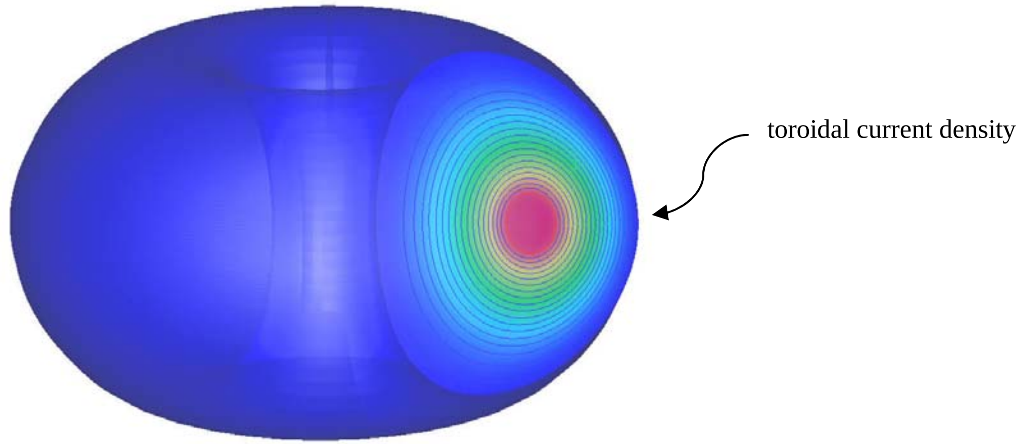


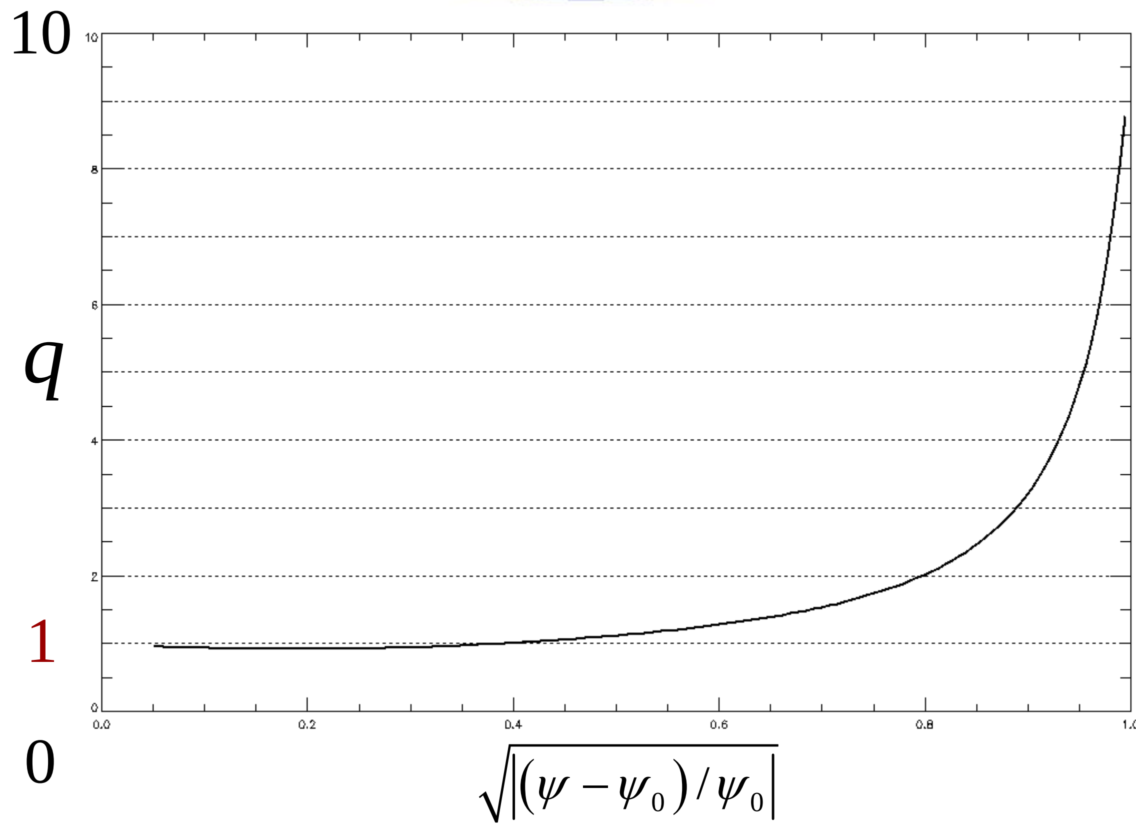
- Low aspect ratio tokamak (R_0/a = 1.4 – 1.5)
- Small (R_0 = 33.5 cm)
- Elongation $\kappa \sim 1.6$
- $B_T \sim 2300$ gauss
- $I_p \sim 70$ kA
- $n_e \sim 4\times10^{13}$ cm^{-3}
- $T_e \sim 100$ eV $\rightarrow$ S ~ 10^4
- Discharge time ~ 12 ms

- Soft X-ray signals from typical discharges indicate two predominant types of low-*n* MHD activity:
  - sawteeth
  - “snakes”

Equilibrium: TSC run06, time11

- Equilibrium taken from a TSC sequence (Jsolver file).
- $\beta \approx 3\%$
- $q_{\min} \approx 0.922$
- $q(a) \sim 9$



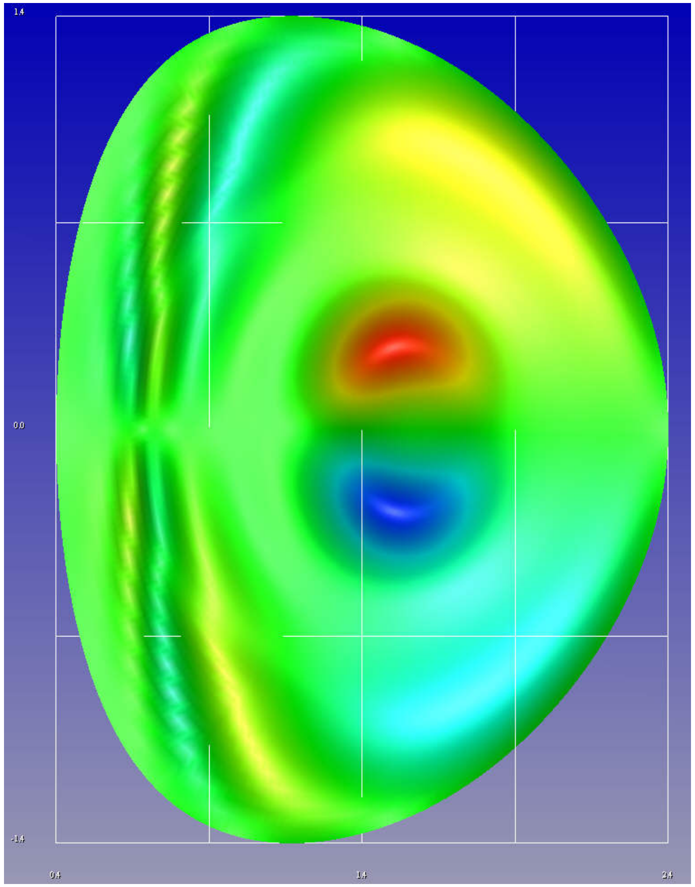


Baseline Parameters for CDX

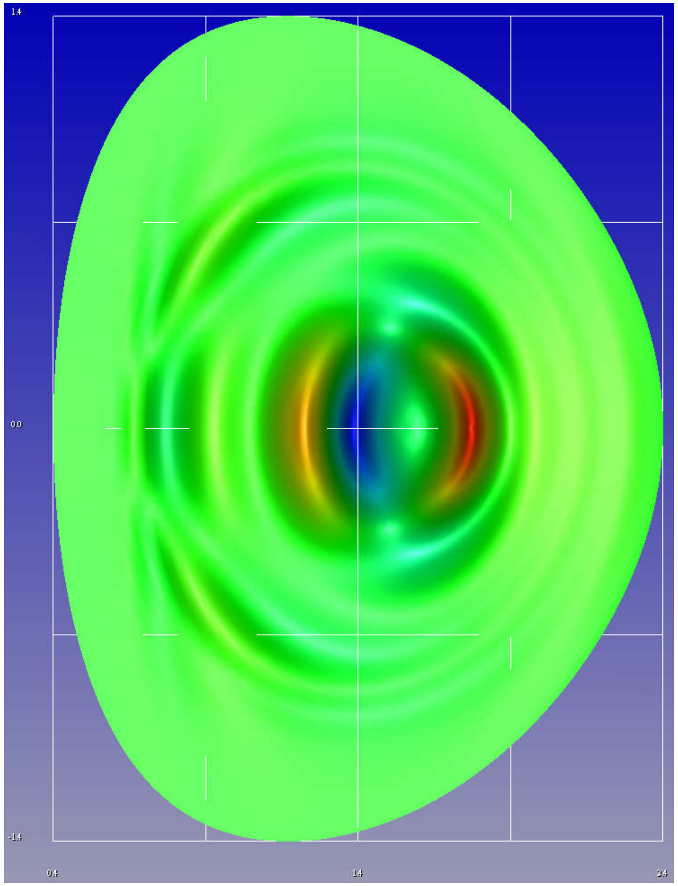
Lundquist Number S	$\sim 2\times 10^4$ on axis.
Resistivity η	Spitzer profile $\propto T_{eq}^{-3/2}$, cut off at $100\times\ \eta_0$
Prandtl Number Pr	10 on axis.
Viscosity μ	Constant in space and time.
Perpendicular thermal conduction $\kappa_\perp$	200 m^2/s (measured value at edge)
Parallel thermal conduction (sound wave)	$V_{Te} = 6\ V_A$
Peak Plasma β	$\sim 3\times 10^{-2}$ (low-beta).
Density Evolution	Turned on for nonlinear phase.
Nonlinear initialization	Pure n=1 perturbation such that $\frac{\max(B^1_{pol})}{\max(B^0_\phi)} = 10^{-4}$

n=1 Eigenmode

Incompressible velocity
stream function *U*



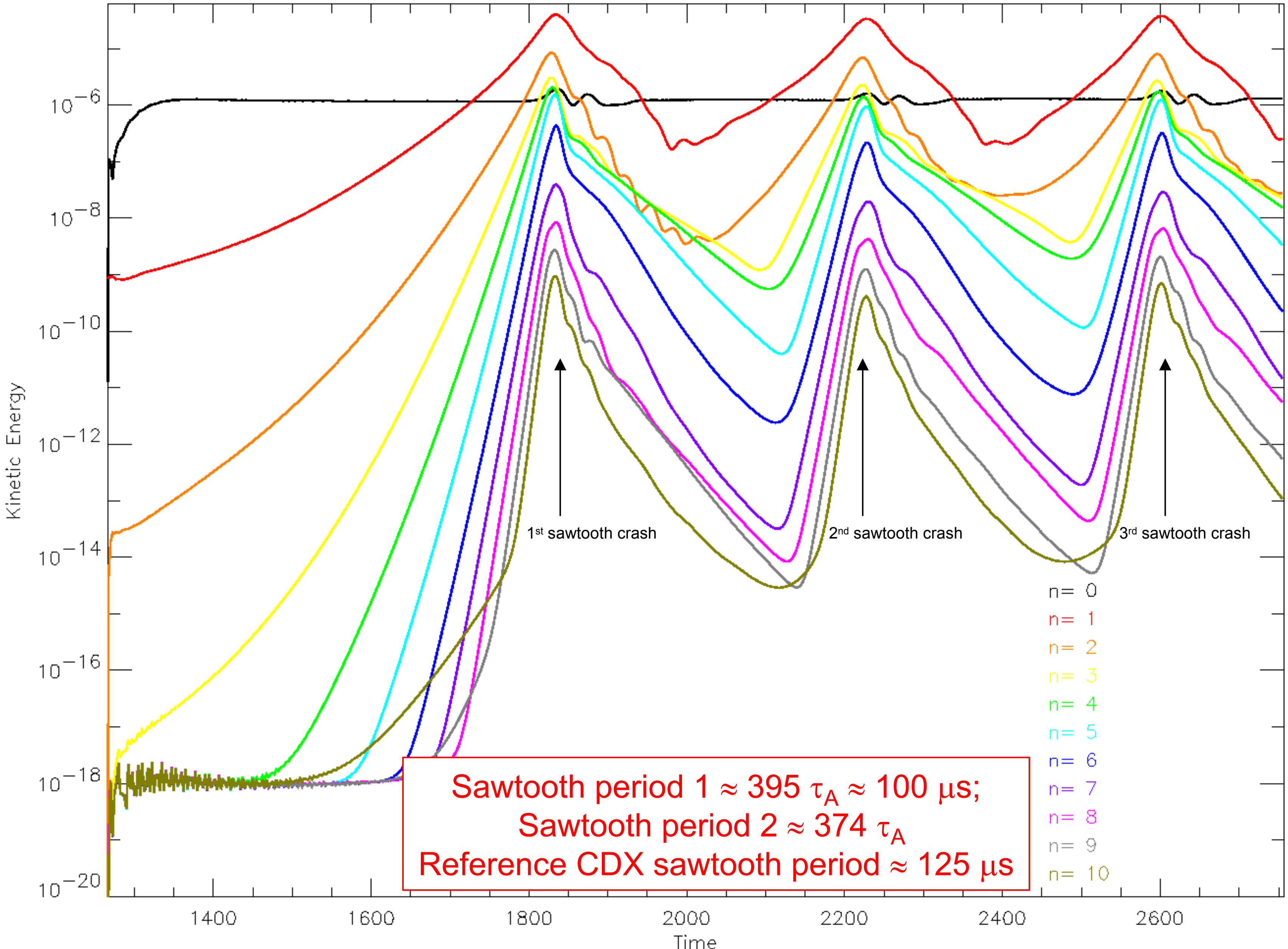
Toroidal current density
J_ϕ



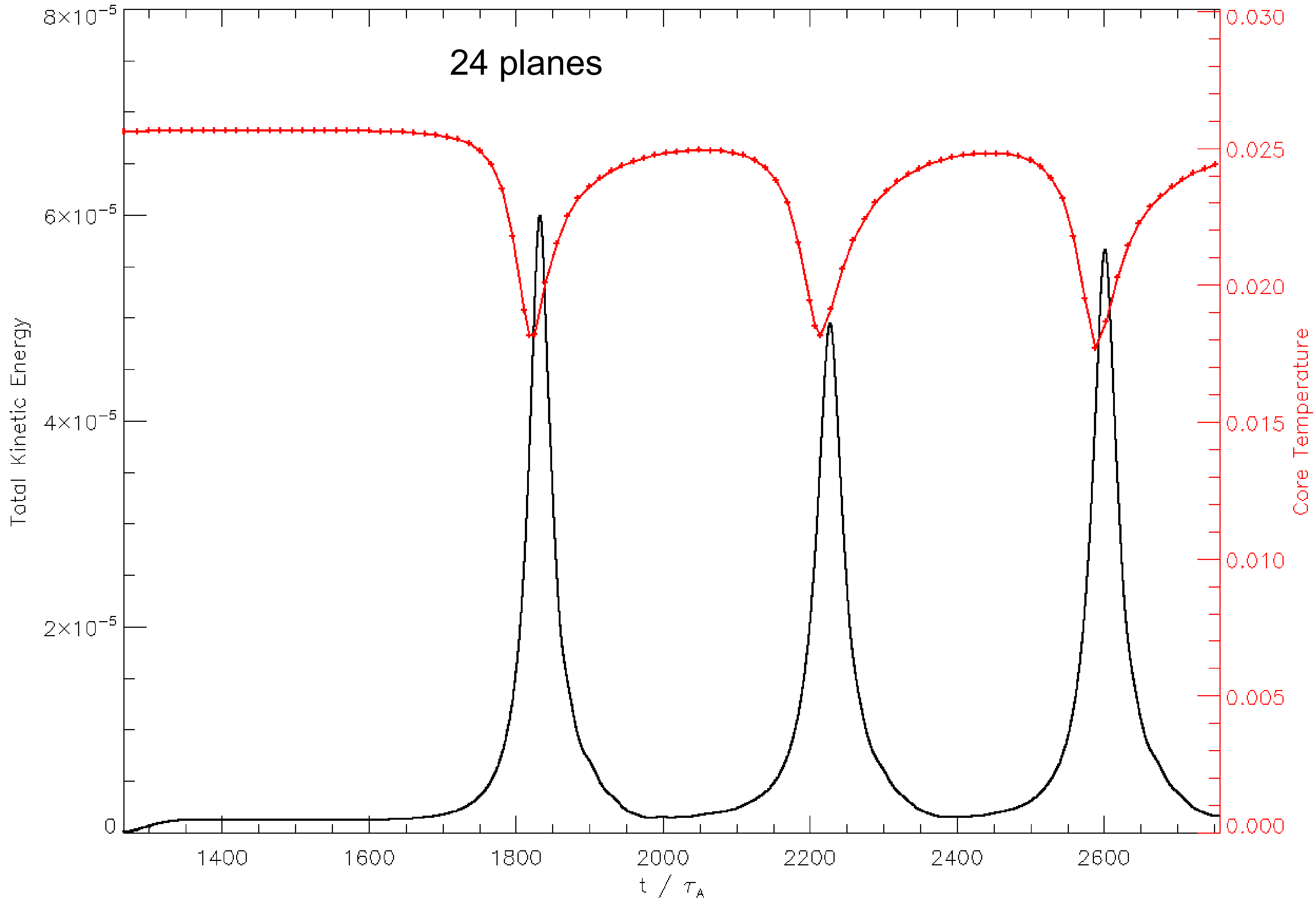
$\gamma \tau_A = 5.1 \times 10^{-3} \rightarrow$ growth time = 196 τ_A

Nonlinear Sawtooth History

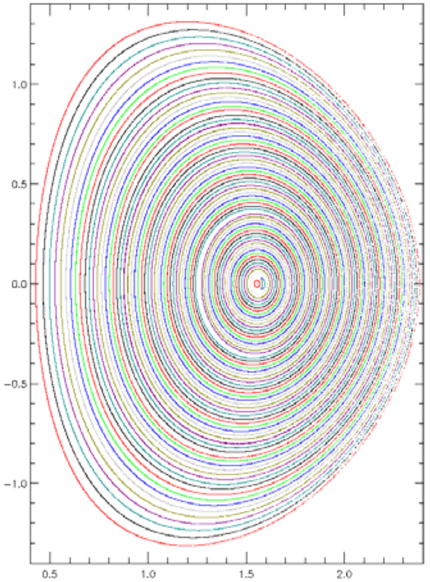
10 Modes Retained



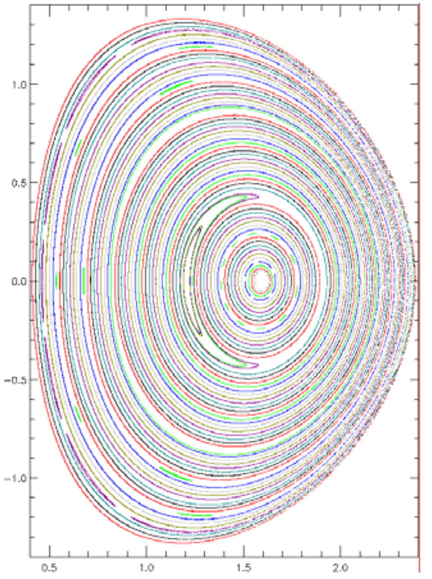
Total Energy and Core Temperature



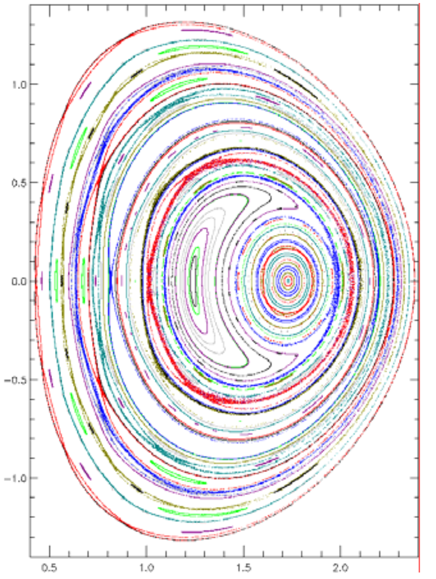
Poincaré Plots



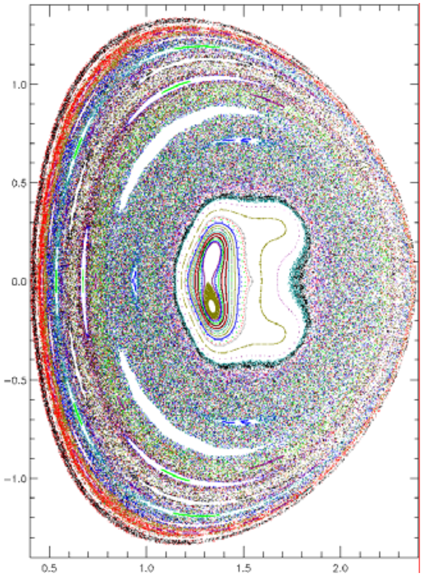
t = 1266.17



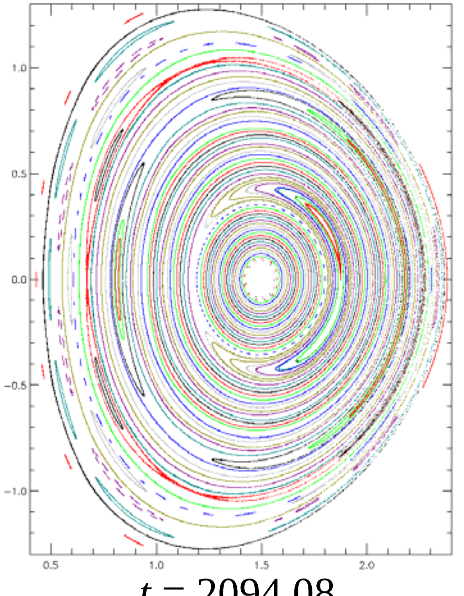
t = 1660.70



t = 1795.61

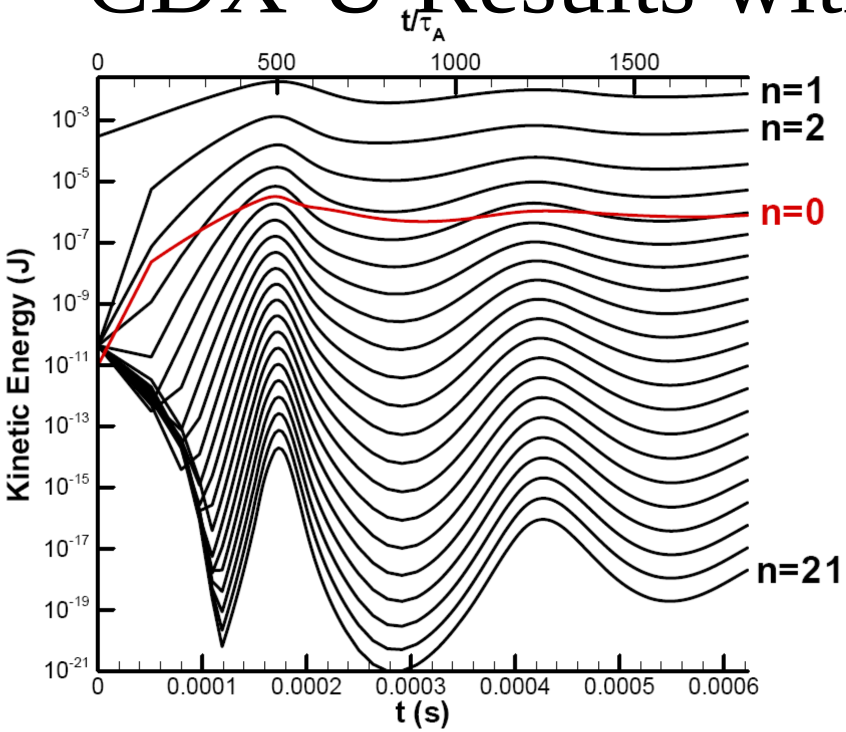


t = 1839.86

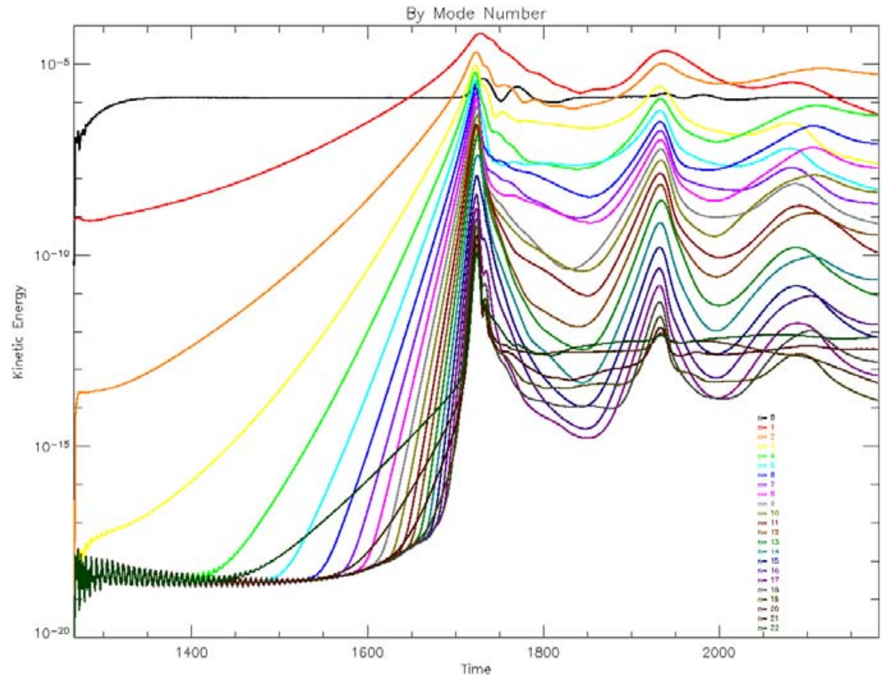


t = 2094.08

Differences Between NIMROD and M3D CDX-U Results with 22 Toroidal Modes



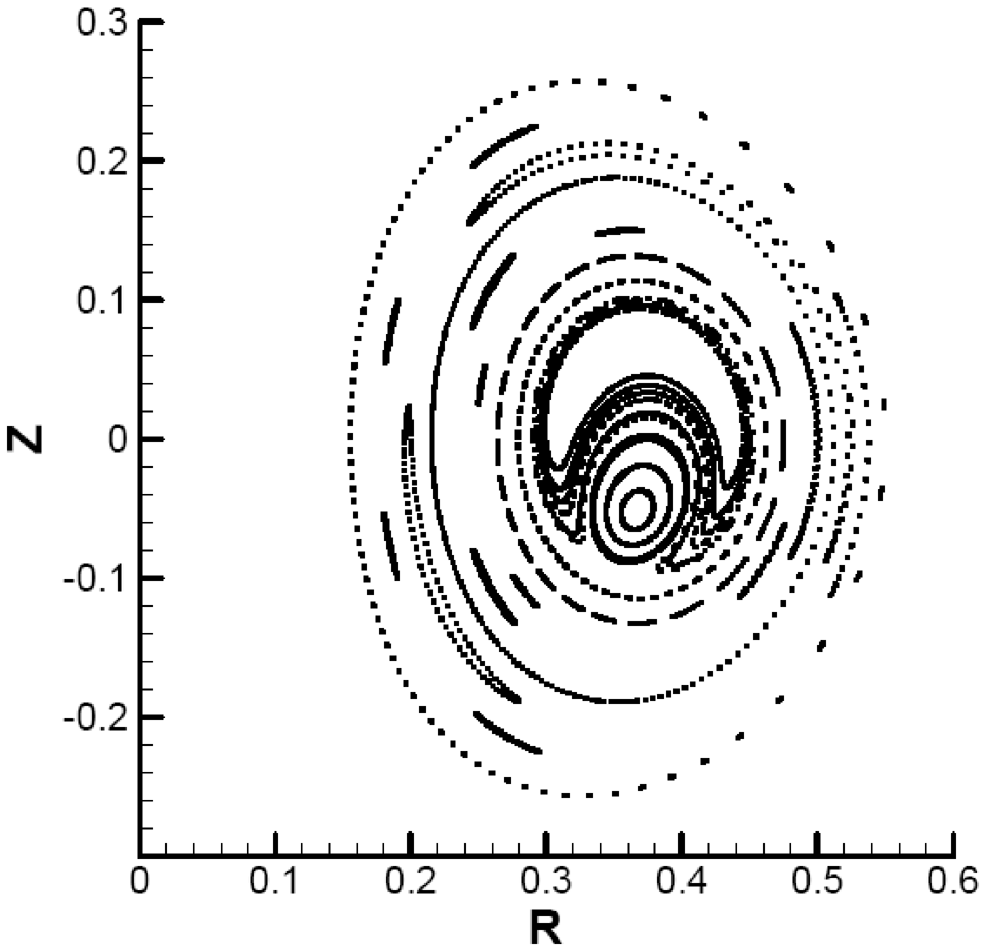
NIMROD



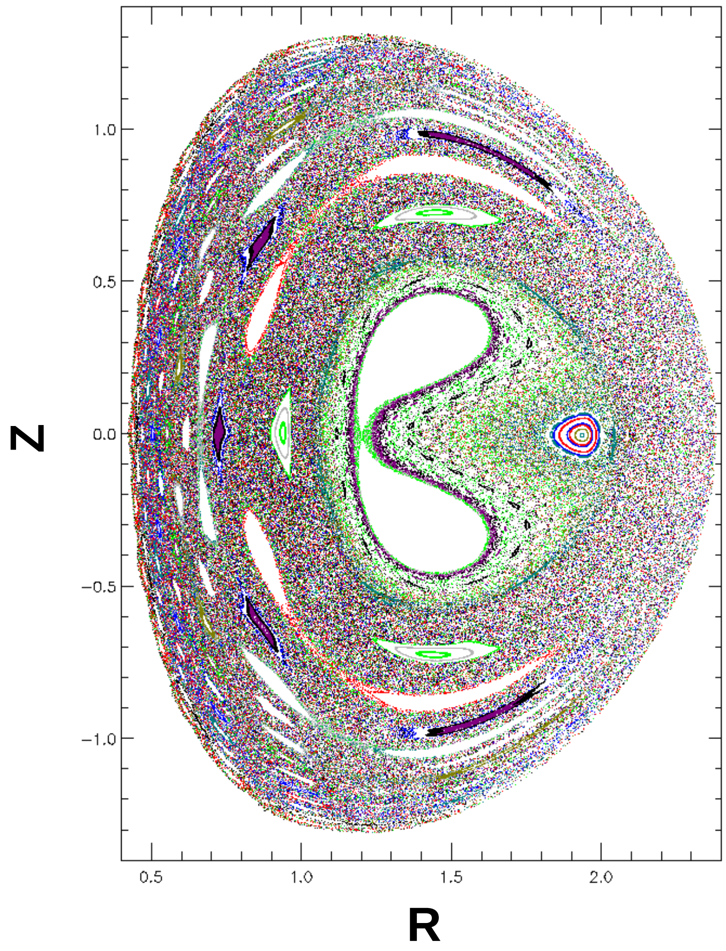
M3D

- Roughly 500 τ_A from initialization to first crash in both cases.
- Kinetic energies of successive modes show greater separation in NIMROD run than in M3D run; $E_{n=1}/E_{n=4}$ at
1[st] peak in NIMROD is ~2000; in M3D, $E_{n=1}/E_{n=4} \sim 6$.
- Periods between "crashes" differ: ~710 τ_A for NIMROD vs. 212 τ_A for M3D.
- Crash time in M3D appears much more rapid than in NIMROD.
- Magnetic field in NIMROD does not become stochastic during crash.

Poincaré plots at peak of second crash



NIMROD, t=433 μs



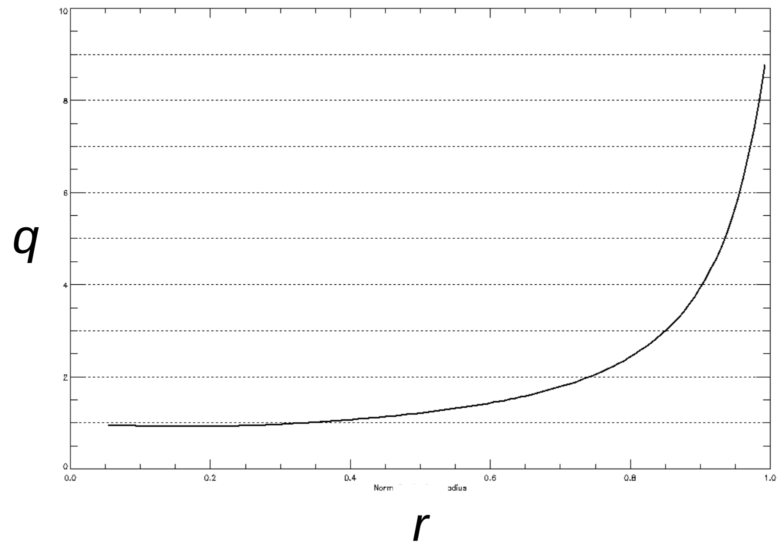
M3D, t = 1936 τ_A

Assigned Tasks

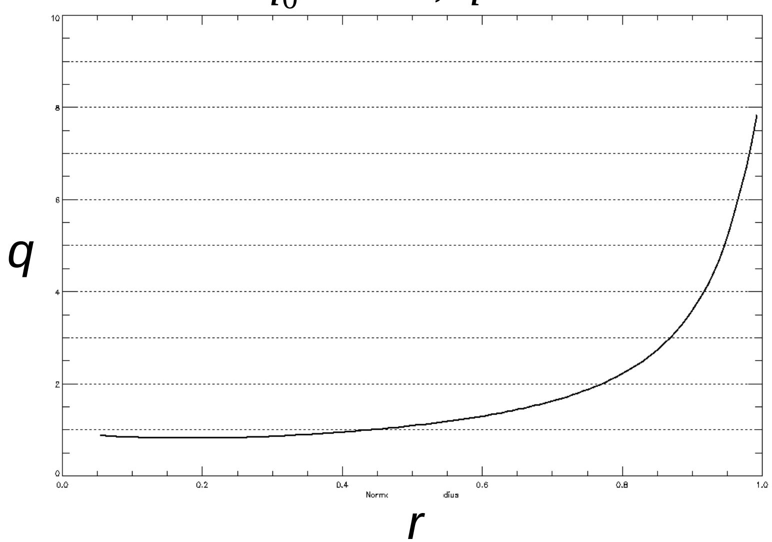
- M3D
  - Run an isotropic nonlinear case.
  - Show convergence information on M3D linear results with isotropic & anisotropic heat transport.
- NIMROD
  - Using new code version, initialize with smaller n=1 eigenmode, zeroing n>1 modes.
  - Run an isotropic case.

New Equilibria

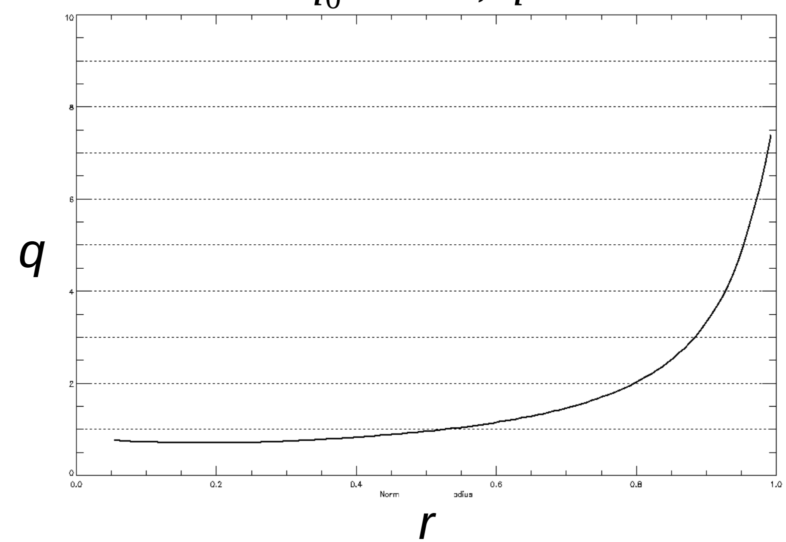
Original: time 11: q_0 = 0.92; q=1 at r=0.33



time 19: q_0 = 0.82; q=1 at r=0.44

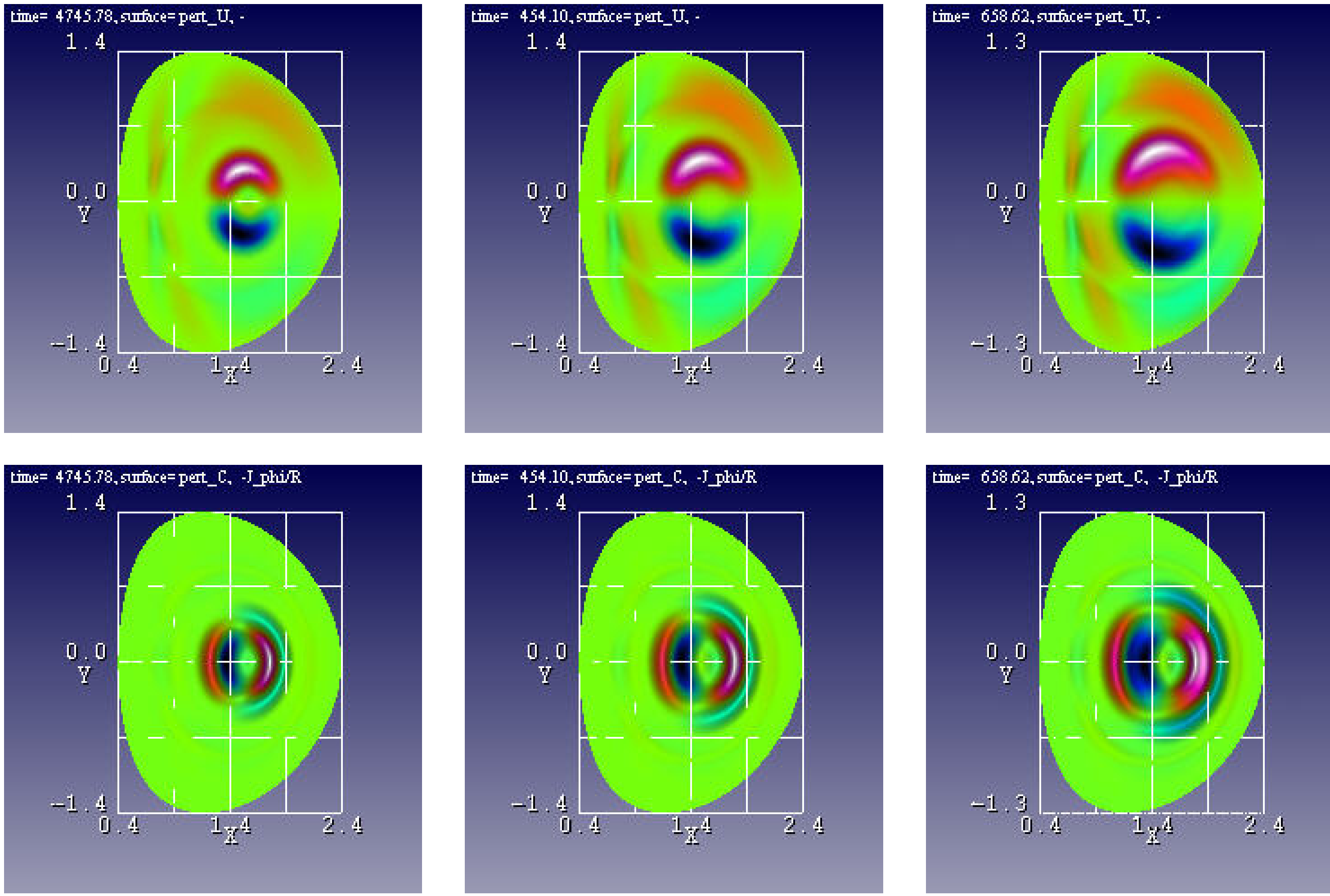


time 29: q_0 = 0.71; q=1 at r=0.53

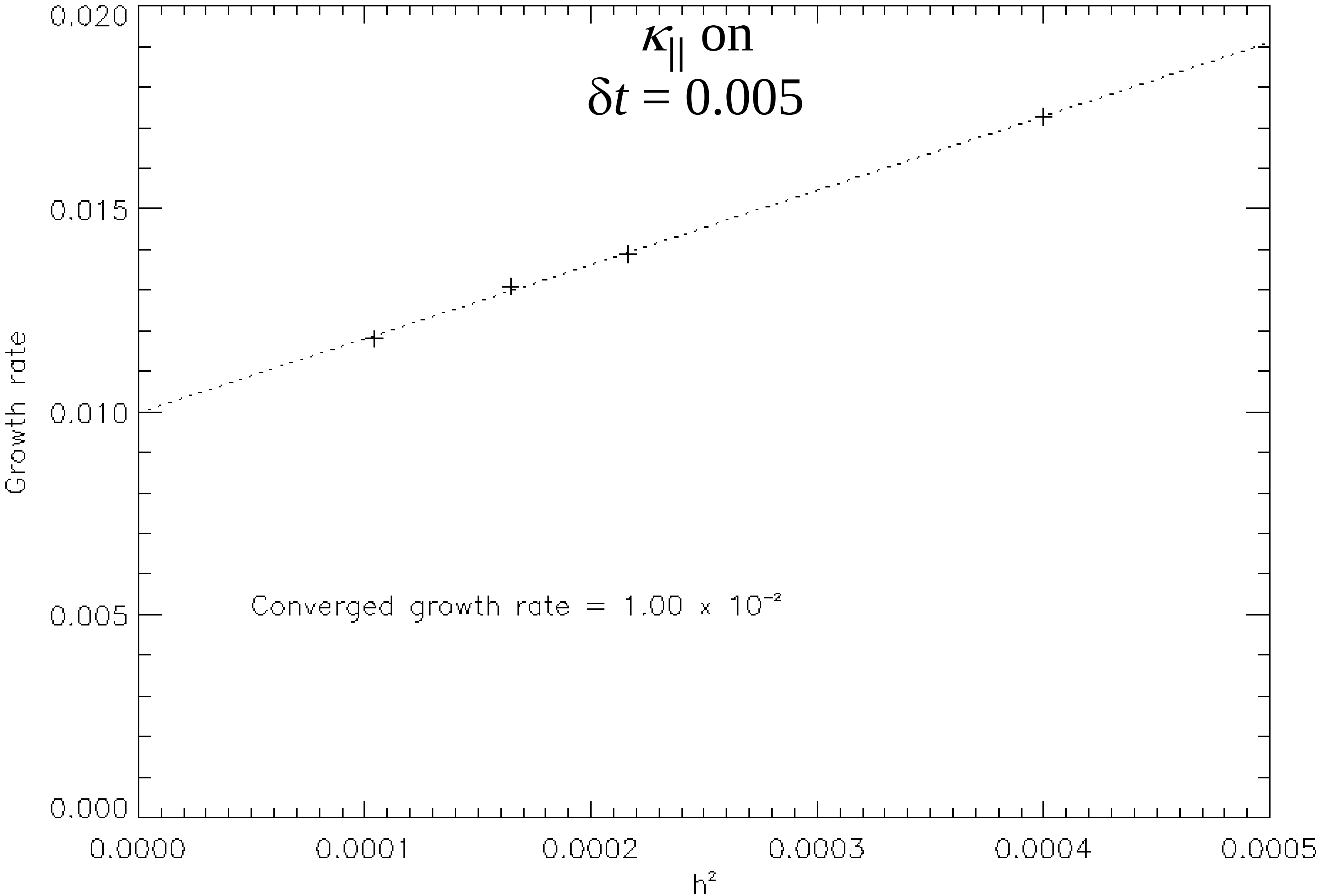


n=1 Eigenmodes

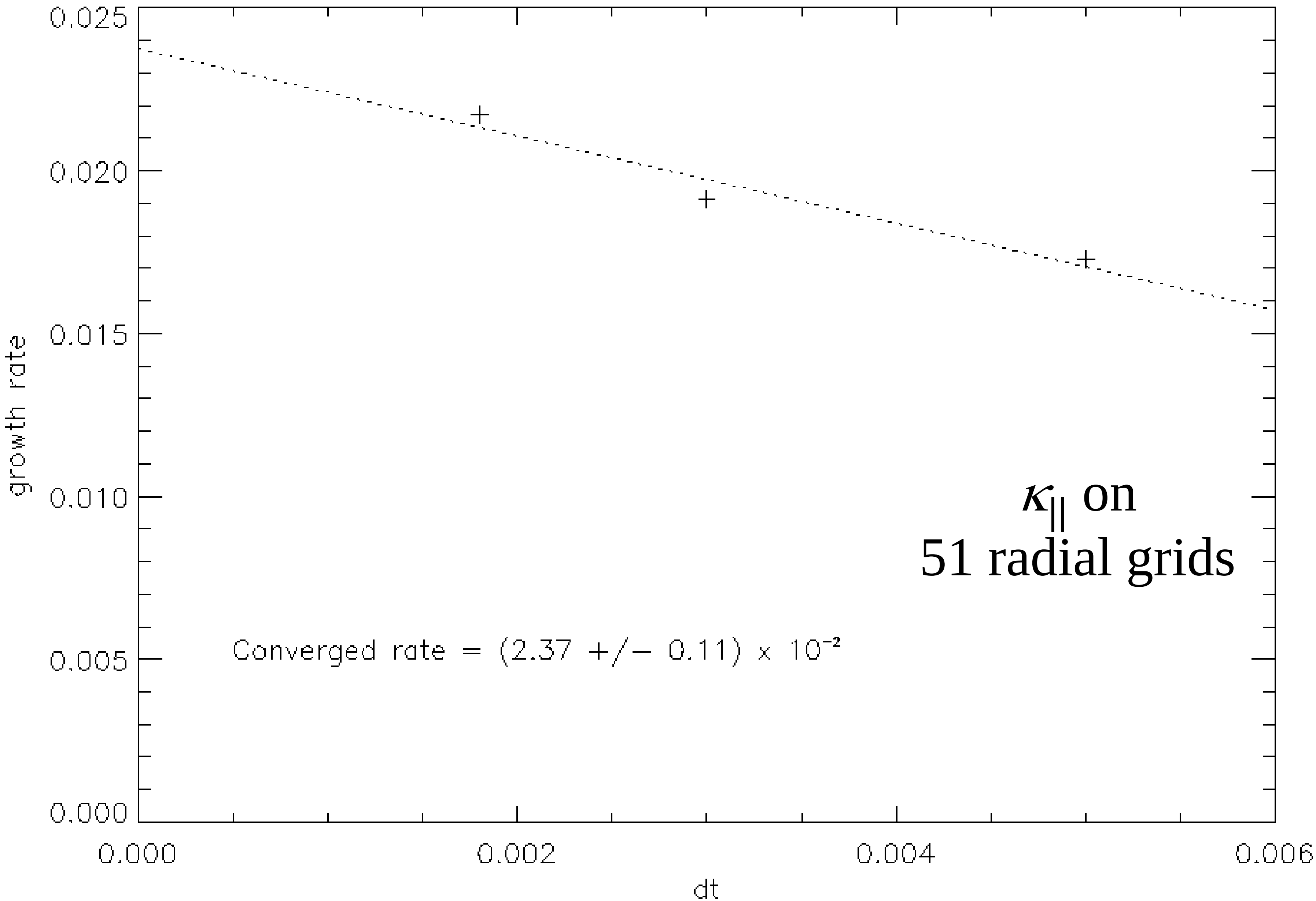
	time 11	time 19	time 29
	$q_0 = 0.92$	$q_0 = 0.82$	$q_0 = 0.71$
Poloidal velocity stream function			
Toroidal current density			



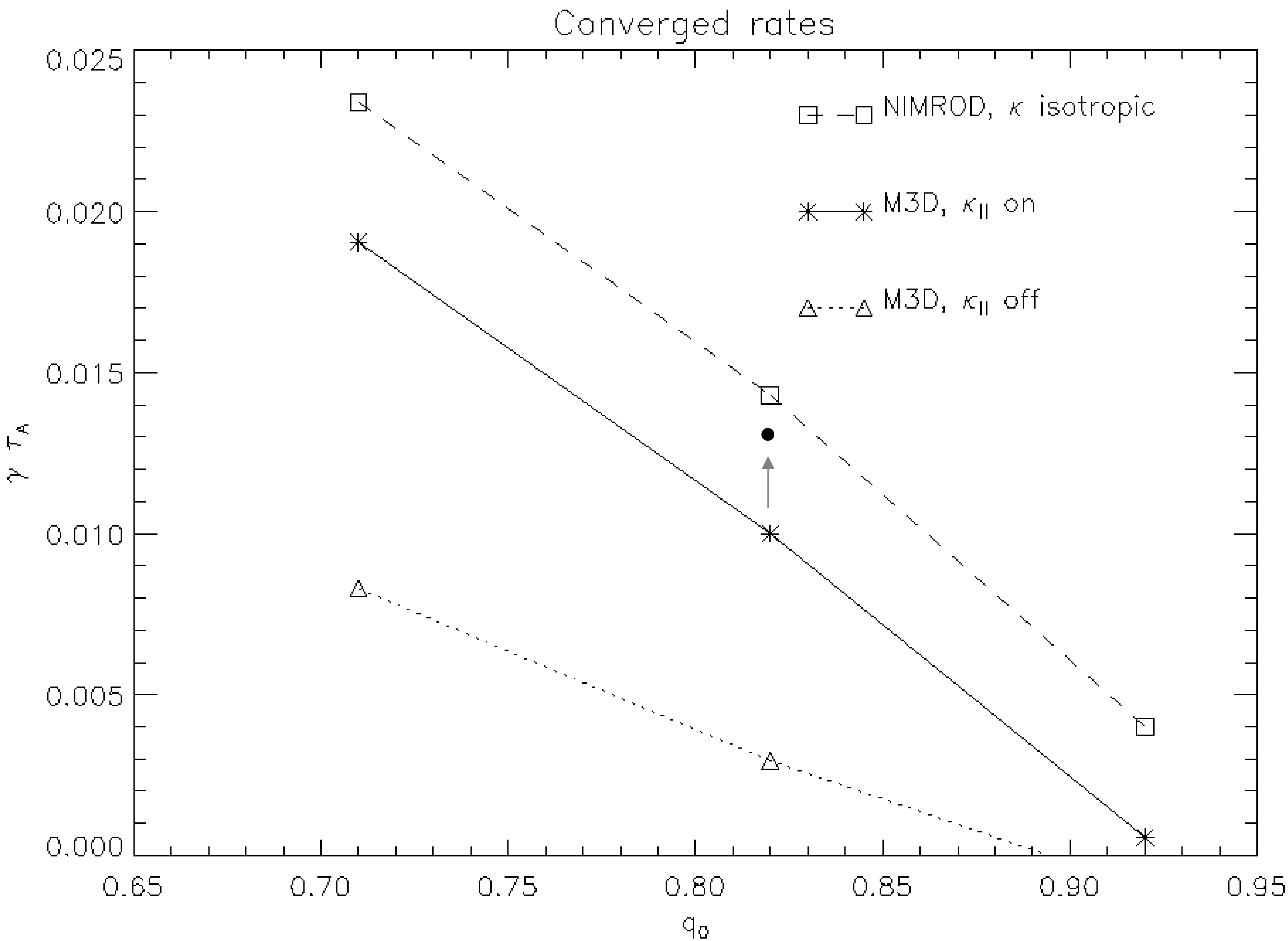
Convergence Study in *h* (time 19)



Convergence Study in dt (time 19)



Linear n=1 Growth Rates

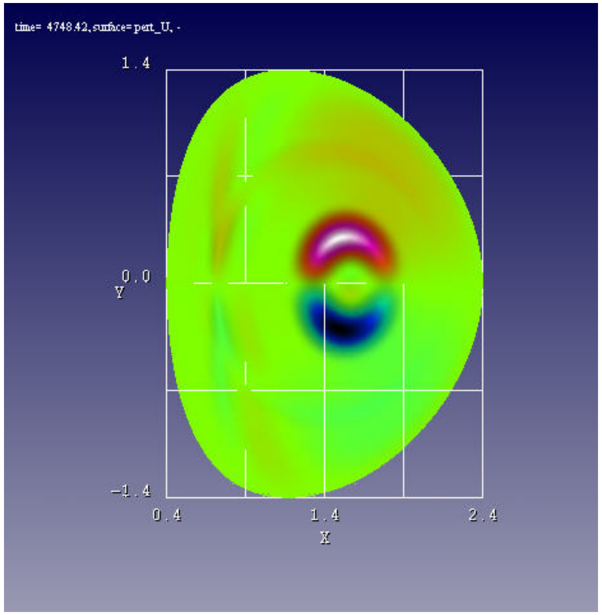


run 06, time11 with lower μ

n=1 eigenmode

Reduce $\mu \times$ ¼, from 5.15×10^{-4} to 1.2875×10^{-4}; $\kappa_{||}$ on

U



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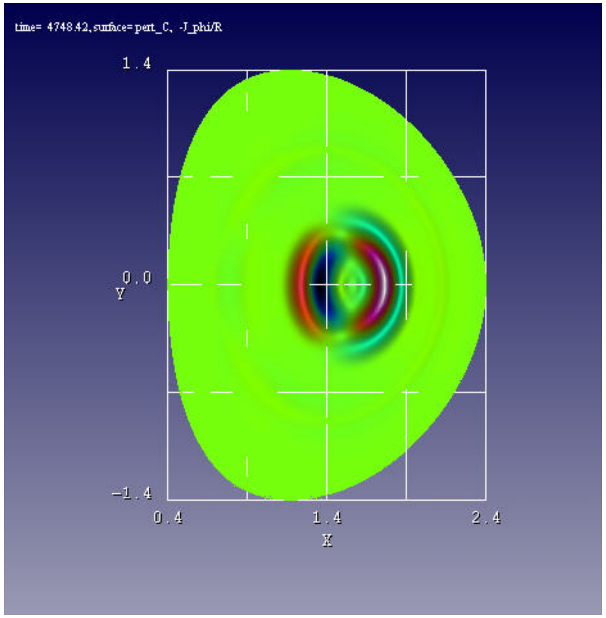
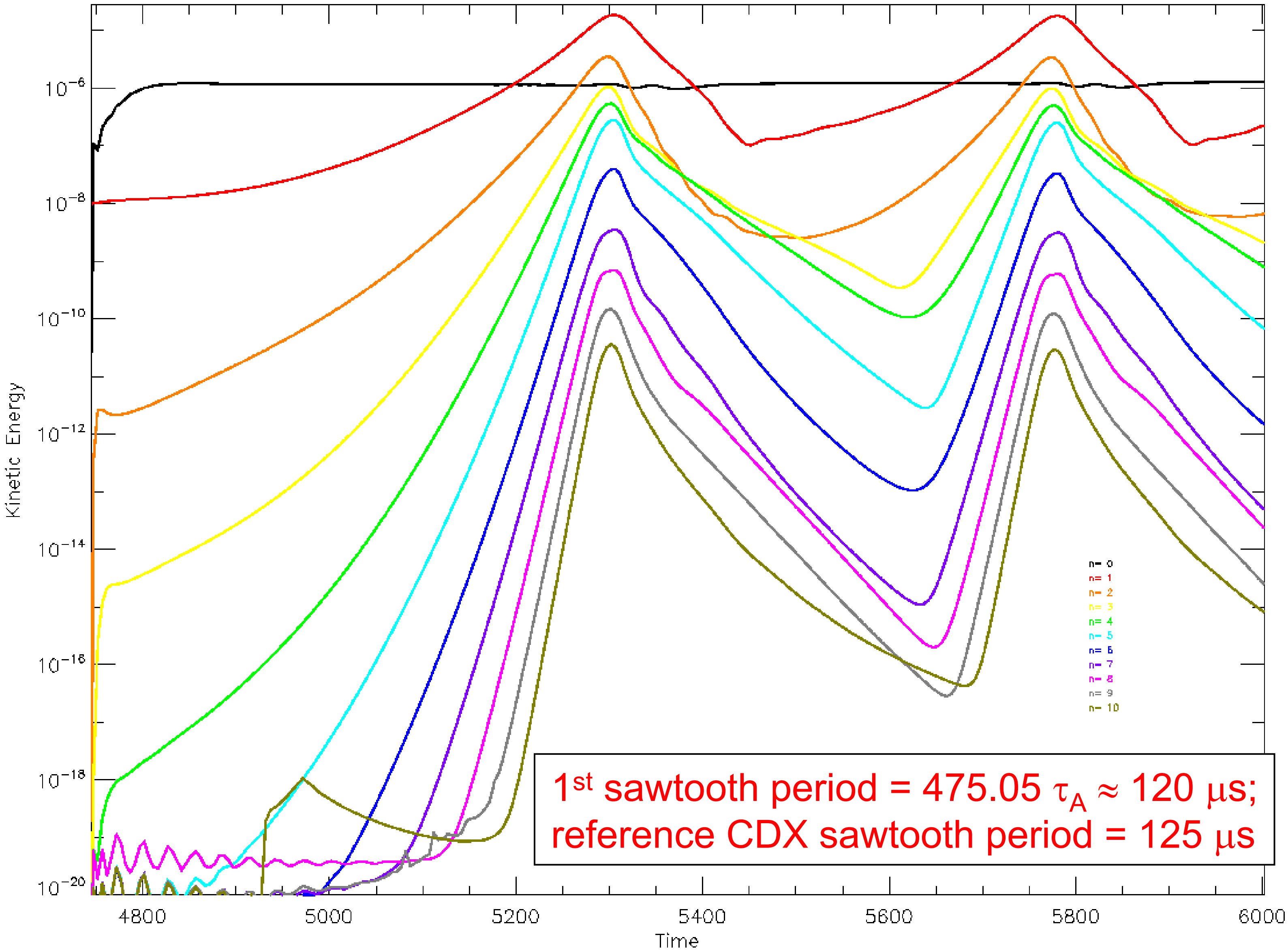
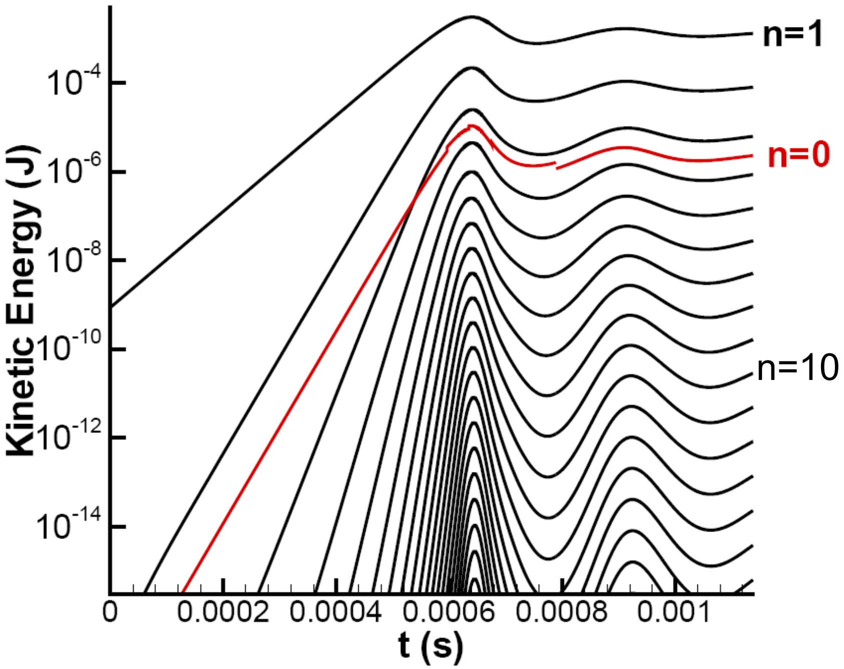


Converged growth rate: $\gamma\tau_A = 7.1\times10^{-3}$

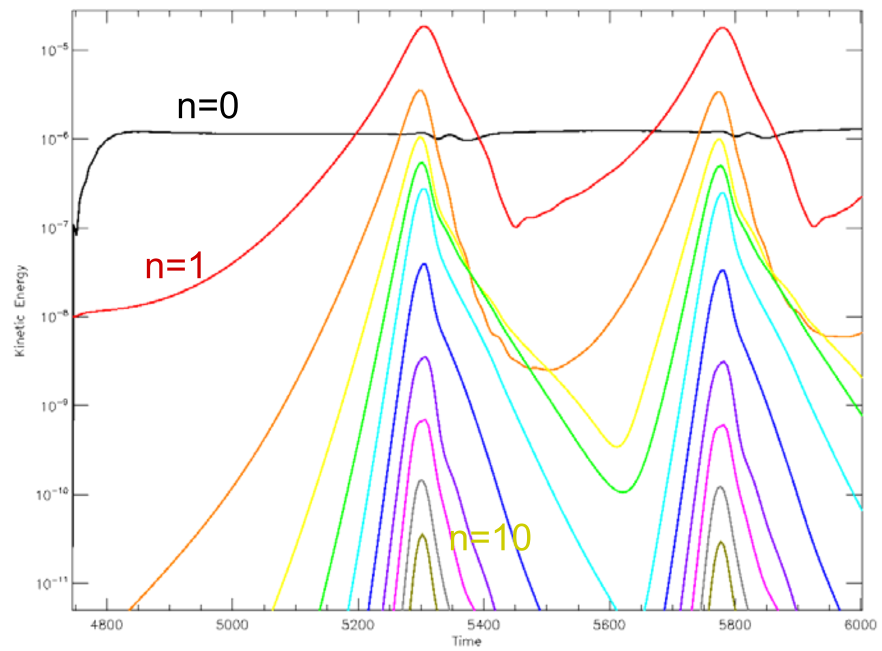
Re-run Nonlinear time11 with New Version, Ĩ Source



Differences Between NIMROD and M3D CDX-U Nonlinear Results



NIMROD



M3D

- NIMROD n=1 growth rate never exceeds linear value.
- Periods between crashes differ: ~800 τ_A for NIMROD vs. 480 τ_A for M3D.
- 2nd crash energy is diminished more in NIMROD than in M3D.

Viscosity in M3D

v_ϕ equation: Advance ϕ component of ideal momentum equation explicitly to get v_ϕ*; then advance

$$\frac{\partial v_\phi}{\partial t} = \mu \nabla_\perp^2 \left(v_\phi - v_\phi^0 \right)$$

implicitly by solving

$$\left(\nabla_\perp^2 - \frac{1}{\mu \delta t} \right) \left(v_\phi^{n+1} - v_\phi^0 \right) = -\frac{\left(v_\phi^* - v_\phi^0 \right)}{\mu \delta t}$$

Here $\nabla_\perp^2 \equiv \frac{\partial^2}{\partial R^2} + \frac{\partial^2}{\partial z^2},$

and the source term v_ϕ^0 is zero except in cases with equilibrium flow. Dirichlet (no-slip) boundary conditions are being used for the elliptic solve in these cases.

Viscosity in M3D, continued

w equation ($\Delta^{\dagger}U$): Advance *w* in ideal momentum equation explicitly to get w*; then advance

$$\frac{\partial w}{\partial t} = \mu \nabla_{\perp}^{2} w$$

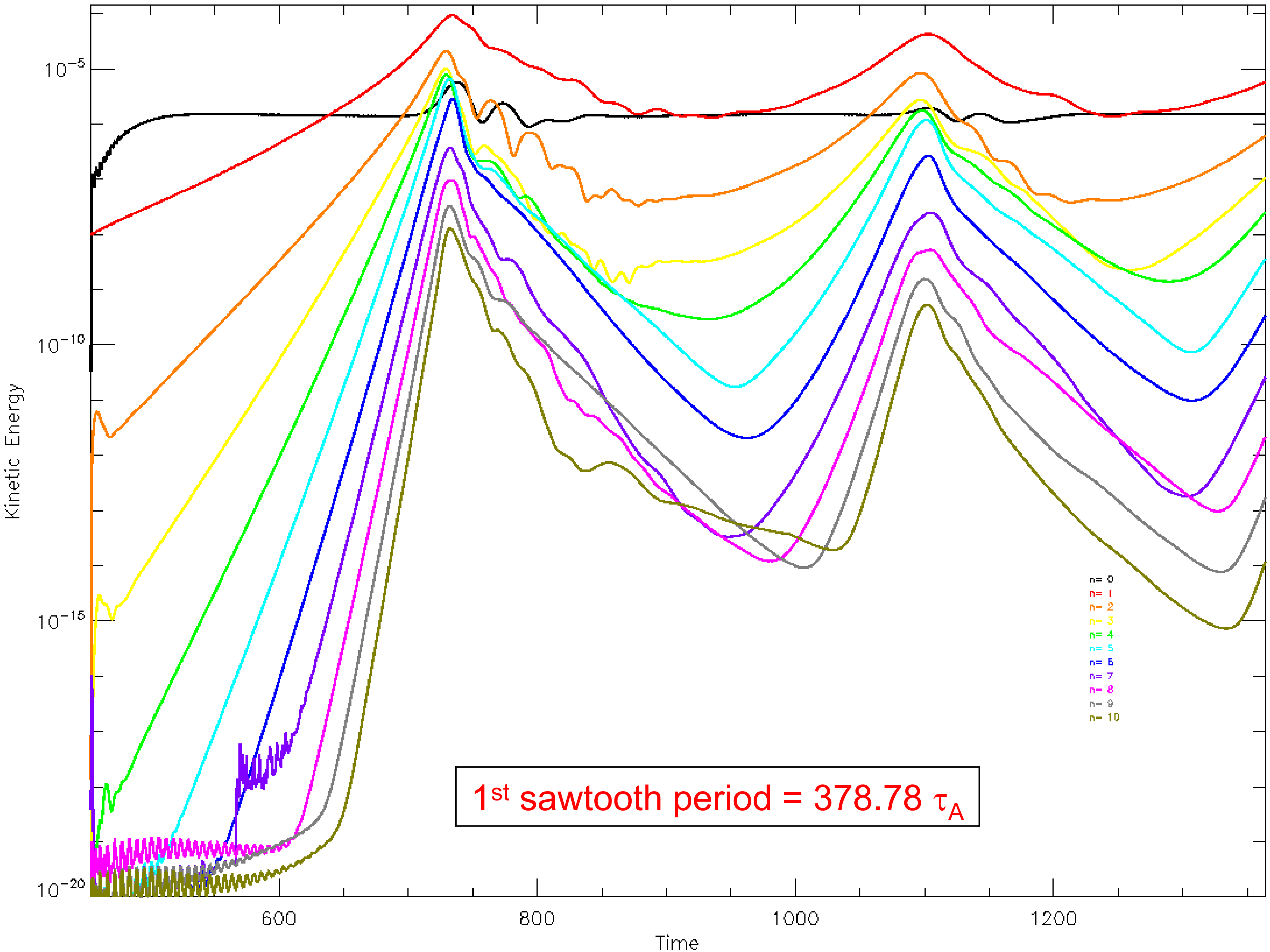
implicitly by solving

$$\left(\nabla_{\perp}^{2} - \frac{1}{\mu \delta t} \right) w_{\phi}^{n+1} = -\frac{w_{\phi}^{*}}{\mu \delta t}$$

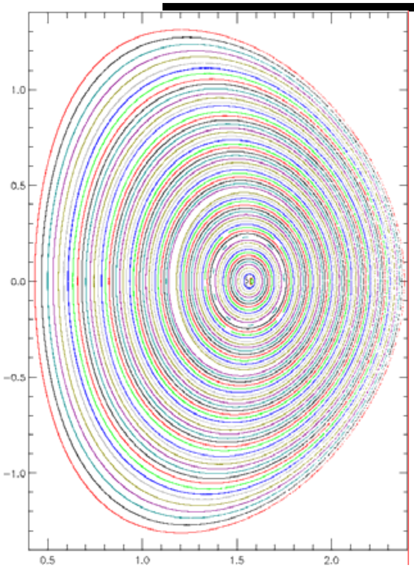
subject to Neumann boundary conditions $(\hat{n} \cdot \nabla w = 0)$.

Then do a second elliptic solve to find *U*, using Dirichlet b.c.s.

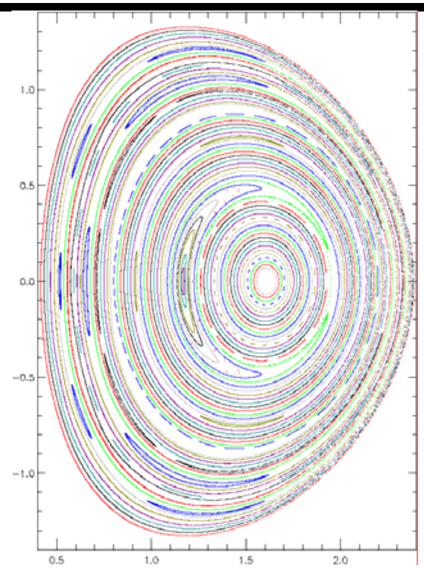
Nonlinear time 19



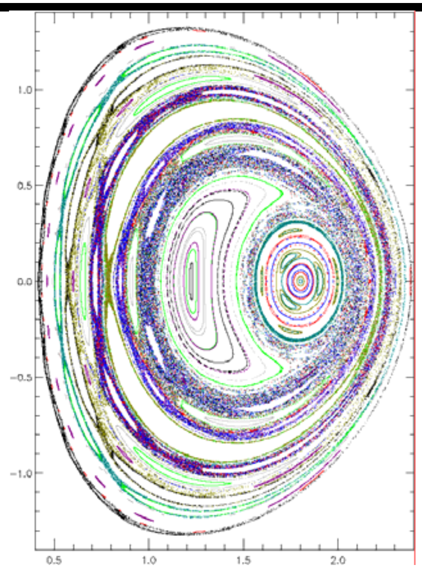
Poincaré Plots for time 19



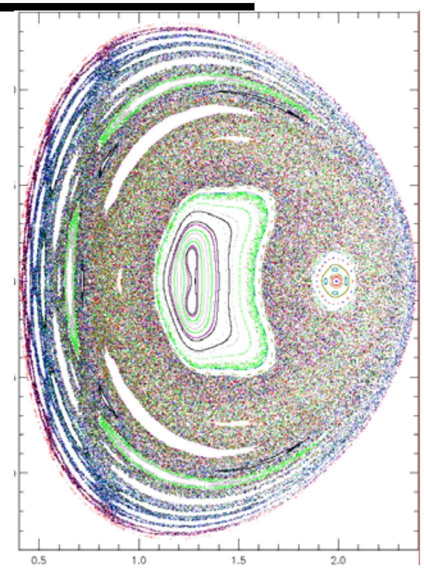
t = 454.100



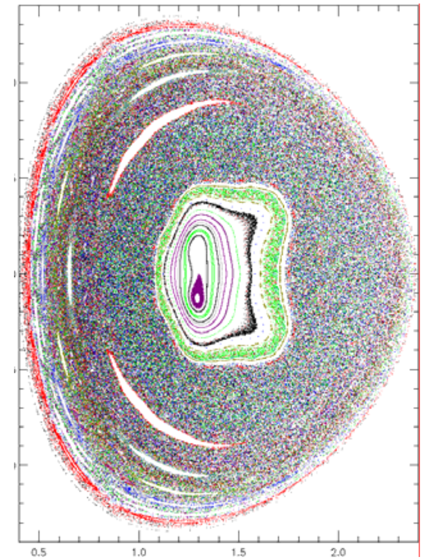
t = 630.900



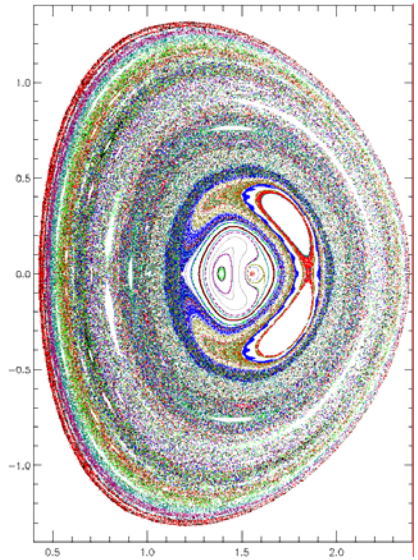
t = 712.500



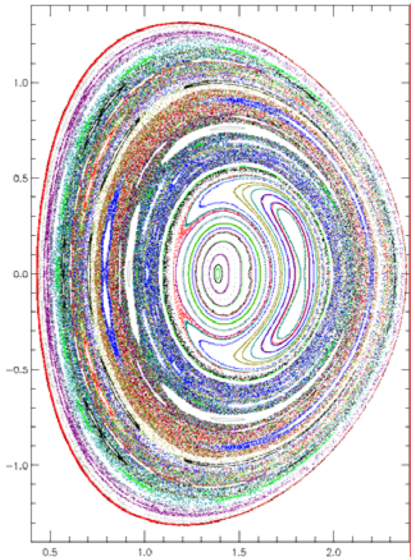
t = 726.100



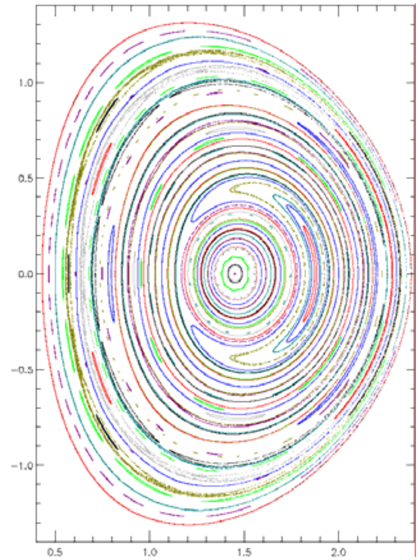
t = 731.540



t = 794.100



t = 821.300



t = 892.700

Outstanding Questions

- Why are growth rates inconsistent between versions? (Why is perpendicular heat conduction case 11 now stable?)
- Why is $\kappa_{||}$ destabilizing?
- Why does the M3D equilibrium evolve (q_0 decreasing) during the nonlinear run?
- How will these new cases converge toroidally?