M3D-C^1 Update, GEM benchmark, and moving to 3D

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The Extended MHD equations for a magnetized (fusion) plasma are a high-order system of 8 scalar variables that are characterized by a wide range of space and timescales.

M3D-C^1 approach is as follows:

Multiple space scales	→	unstructured adaptive elements
Multiple time scales	→	implicit time differencing
High order derivatives	→	C^1 continuity elements (up to 4^{th} order)
8 scalar variables	→	split implicit time advance & compact rep.
Strong magnetic field	→	stream function/potential representation

Extended MHD Equations:

Resistive MHD

2-fluid Extended MHD terms

$$\vec{V} = \nabla U \times \hat{z} + \nabla_\perp \chi + V_z$$

$$\vec{B} = \nabla \psi \times \hat{z} + I \hat{z}$$

$$\frac{\partial n}{\partial t} + \nabla \bullet (n\vec{V}) = 0$$

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} \qquad \vec{J} = \nabla \times \vec{B}$$

$$nM_i\left(\frac{\partial \vec{V}}{\partial t} + \vec{V} \bullet \nabla \vec{V}\right) + \nabla p = \vec{J} \times \vec{B} - \nabla \bullet \Pi_i$$

$$\vec{E} + \vec{V} \times \vec{B} = \eta \vec{J} + \frac{1}{ne}\left(\vec{J} \times \vec{B} - \nabla p_e\right) - \lambda_H (\Delta x)^2 \nabla^2 \vec{J}$$

$$\frac{3}{2}\frac{\partial p_e}{\partial t} + \nabla \bullet \left(\frac{3}{2} p_e \vec{V}\right) = -p_e \nabla \bullet \vec{V} + \eta J^2 + \frac{\vec{J}}{ne} \bullet \left[\frac{3}{2}\nabla p_e - \frac{5}{2}\frac{p_e}{n}\nabla n\right] - \nabla \bullet \vec{q}_e + Q_\Delta$$

$$\frac{3}{2}\frac{\partial p_i}{\partial t} + \nabla \bullet \left(\frac{3}{2} p_i \vec{V}\right) = -p_i \nabla \bullet \vec{V} + \overleftrightarrow{\Pi}_i : \nabla \vec{V} - \nabla \bullet \vec{q}_i - Q_\Delta$$

$$\Pi_i = \mu\left[\nabla \vec{V} + \nabla \vec{V}^\dagger - \tfrac{2}{3}\nabla \bullet \vec{V}\, \overleftrightarrow{I}\right] + \left(2\mu_C - \tfrac{4}{3}\mu\right)\nabla \bullet \vec{V}\, \overleftrightarrow{I} + \overleftrightarrow{\Pi}_i^{GV}$$

$$\nabla \bullet \Pi_i = \mu \nabla^2 \vec{V} + \left(2\mu_C - \mu\right)\nabla(\nabla \bullet \vec{V}) + \nabla \bullet \Pi_{GV}$$

8 scalar variables: $\psi, I, U, \chi, V_z, n, p_e, p_i$

Δx is typical zone (element) size

Fully implicit Extended MHD (2-fluid) equations

-- time step determined by accuracy only:

$$\begin{bmatrix} S_{11}^{v} & S_{12}^{v} & S_{13}^{v} \\ S_{21}^{v} & S_{22}^{v} & S_{23}^{v} \\ S_{31}^{v} & S_{32}^{v} & S_{33}^{v} \end{bmatrix} \bullet \begin{bmatrix} U \\ V_z \\ \chi \end{bmatrix}^{n+1} = \begin{bmatrix} D_{11}^{v} & D_{12}^{v} & D_{13}^{v} \\ D_{21}^{v} & D_{22}^{v} & D_{23}^{v} \\ D_{31}^{v} & D_{32}^{v} & D_{33}^{v} \end{bmatrix} \bullet \begin{bmatrix} U \\ V_z \\ \chi \end{bmatrix}^{n} + \begin{bmatrix} R_{11}^{v} & R_{12}^{v} & R_{13}^{v} \\ R_{21}^{v} & R_{22}^{v} & R_{23}^{v} \\ R_{31}^{v} & R_{32}^{v} & R_{33}^{v} \end{bmatrix} \bullet \begin{bmatrix} \psi \\ I \\ P_e \end{bmatrix}^{n}$$

$$\vec{V} = \nabla U \times \hat{z} + \nabla_{\perp}\chi + V_z$$
$$\vec{B} = \nabla \psi \times \hat{z} + I\hat{z}$$

Alfven Wave physics

$$S_{11}^{n} \bullet N^{n+1} = D_{11}^{n} \bullet N^{n} + \begin{bmatrix} R_{11}^{n} & R_{12}^{n} & R_{13}^{n} \end{bmatrix} \bullet \begin{bmatrix} U \\ V_z \\ \mathrm{X} \end{bmatrix}^{n+1} + \begin{bmatrix} Q_{11}^{n} & Q_{12}^{n} & Q_{13}^{n} \end{bmatrix} \bullet \begin{bmatrix} U \\ V_z \\ \mathrm{X} \end{bmatrix}^{n} + Q_{14}^{n}$$

density

$$S_{11}^{p} \bullet P^{n+1} = D_{11}^{p} \bullet P^{n} + \begin{bmatrix} R_{11}^{p} & R_{12}^{p} & R_{13}^{p} \end{bmatrix} \bullet \begin{bmatrix} U \\ V_z \\ \mathrm{X} \end{bmatrix}^{n+1} + \begin{bmatrix} Q_{11}^{p} & Q_{12}^{p} & Q_{13}^{p} \end{bmatrix} \bullet \begin{bmatrix} U \\ V_z \\ \mathrm{X} \end{bmatrix}^{n} + Q_{14}^{p}$$

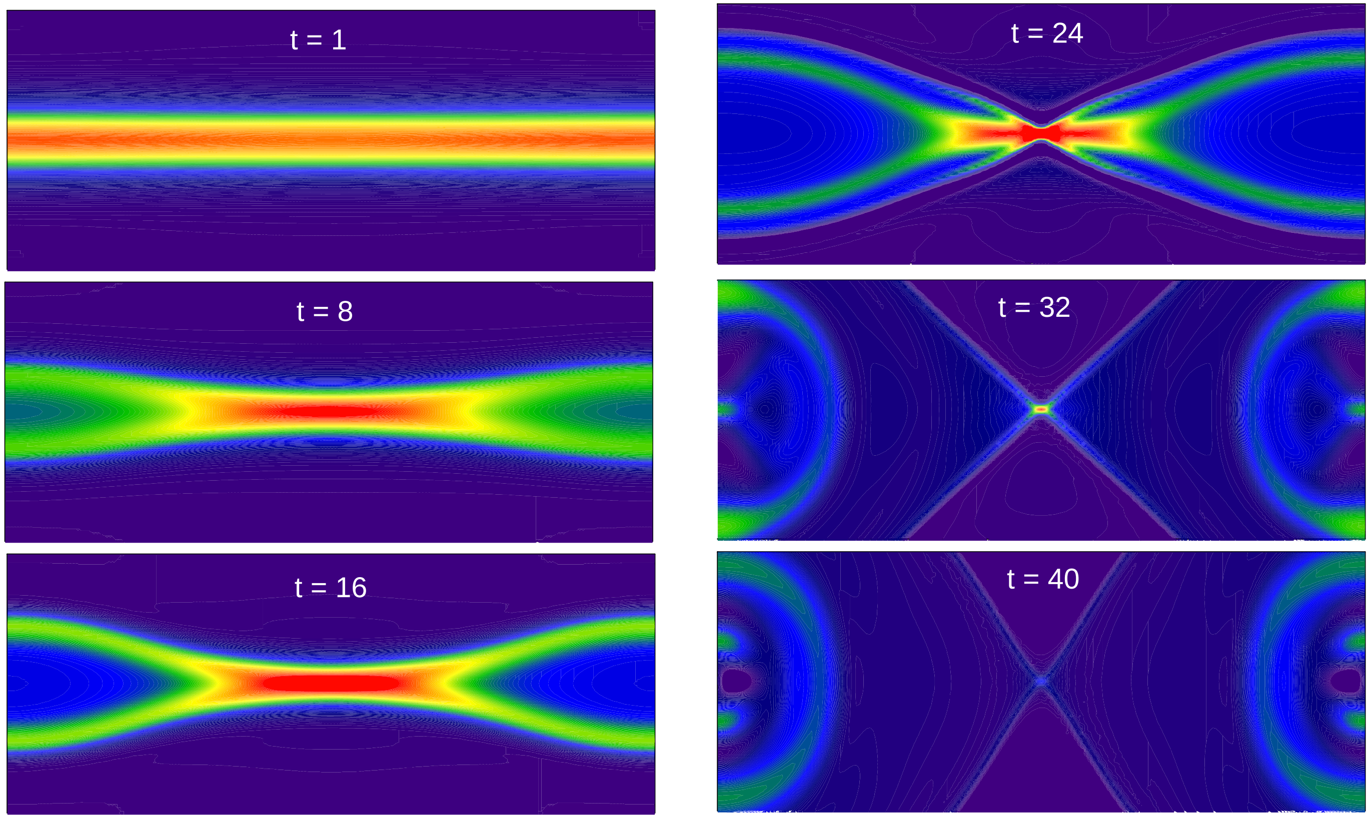
pressure

$$\begin{bmatrix} S_{11}^{p} & S_{12}^{p} & S_{13}^{p} \\ S_{21}^{p} & S_{22}^{p} & S_{23}^{p} \\ S_{31}^{p} & S_{32}^{p} & S_{33}^{p} \end{bmatrix} \bullet \begin{bmatrix} \psi \\ I \\ P_e \end{bmatrix}^{n+1} = \begin{bmatrix} D_{11}^{p} & D_{12}^{p} & D_{13}^{p} \\ D_{21}^{p} & D_{22}^{p} & D_{23}^{p} \\ D_{31}^{p} & D_{32}^{p} & D_{33}^{p} \end{bmatrix} \bullet \begin{bmatrix} \psi \\ I \\ P_e \end{bmatrix}^{n} + \begin{bmatrix} R_{11}^{p} & R_{12}^{p} & R_{13}^{p} \\ R_{21}^{p} & R_{22}^{p} & R_{23}^{p} \\ R_{31}^{p} & R_{32}^{p} & R_{33}^{p} \end{bmatrix} \bullet \begin{bmatrix} U \\ V_z \\ \chi \end{bmatrix}^{n+1} + \begin{bmatrix} Q_{11}^{p} & Q_{12}^{p} & Q_{13}^{p} \\ Q_{21}^{p} & Q_{22}^{p} & Q_{23}^{p} \\ Q_{31}^{p} & Q_{32}^{p} & Q_{33}^{p} \end{bmatrix} \bullet \begin{bmatrix} U \\ V_z \\ \chi \end{bmatrix}^{n}$$

Whistler, KAW, field diffusion physics

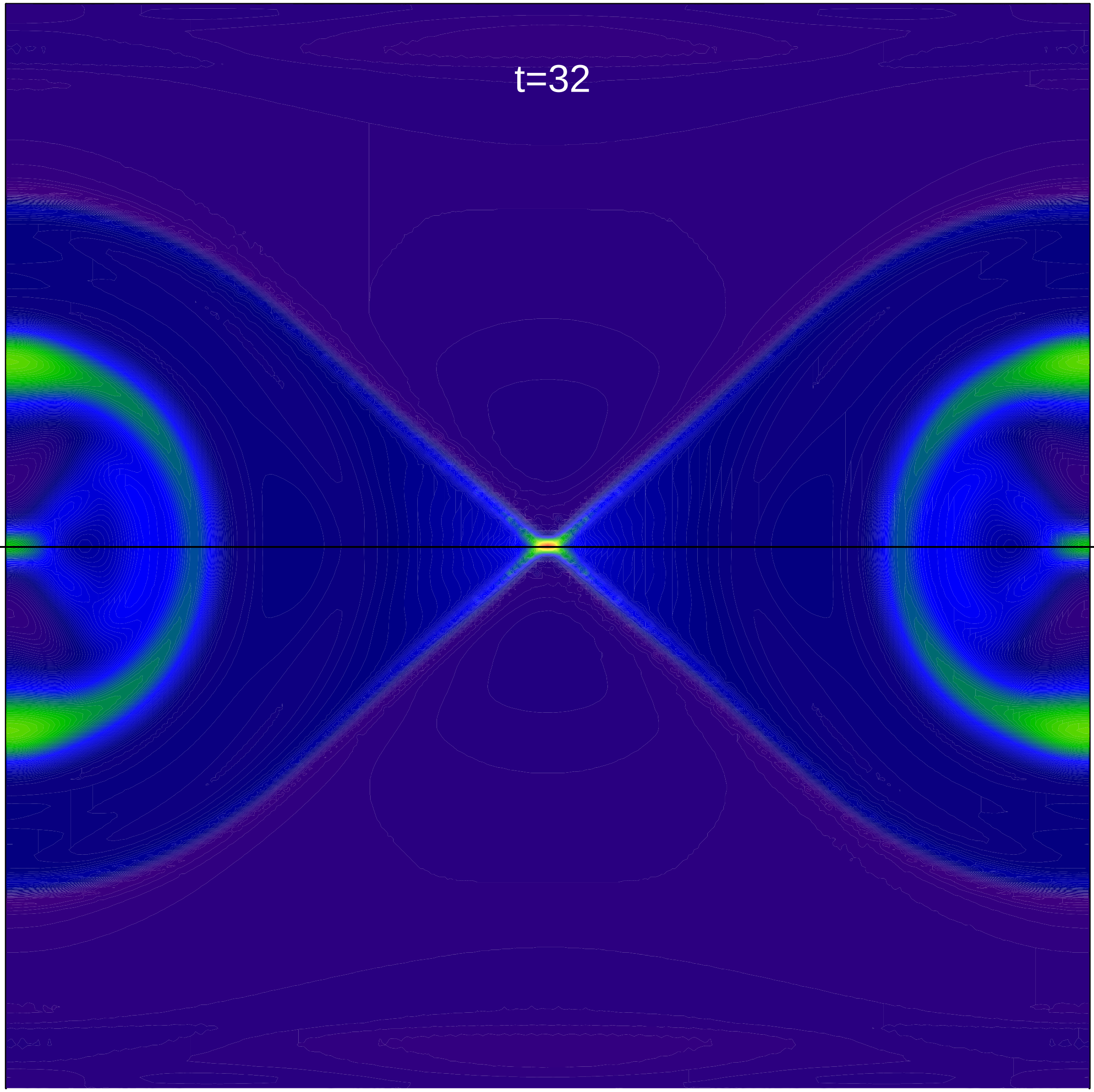
- 4 sequential matrix solves per time step
- 3 non-trivial subsets with 6,4,2 variables

Current Density contours for GEM Nonlinear Benchmark



- Starts like resistive MHD
- Dramatic change in configuration for $t > 20$

Close-up of 2-fluid current density at t=32

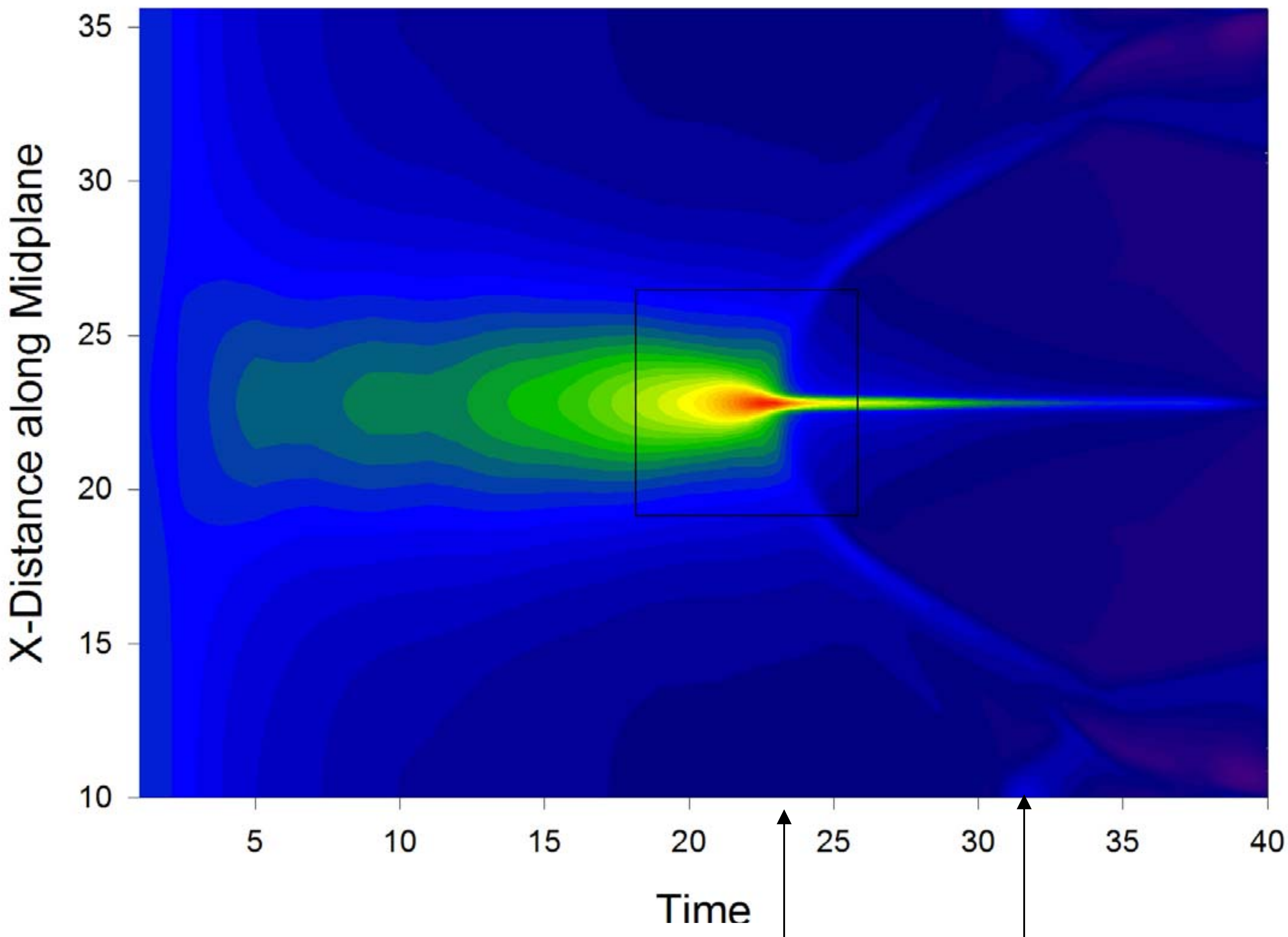


midplane

- Note very localized region of high current density in center

These calculations did not assume any symmetry, except for initial and boundary conditions

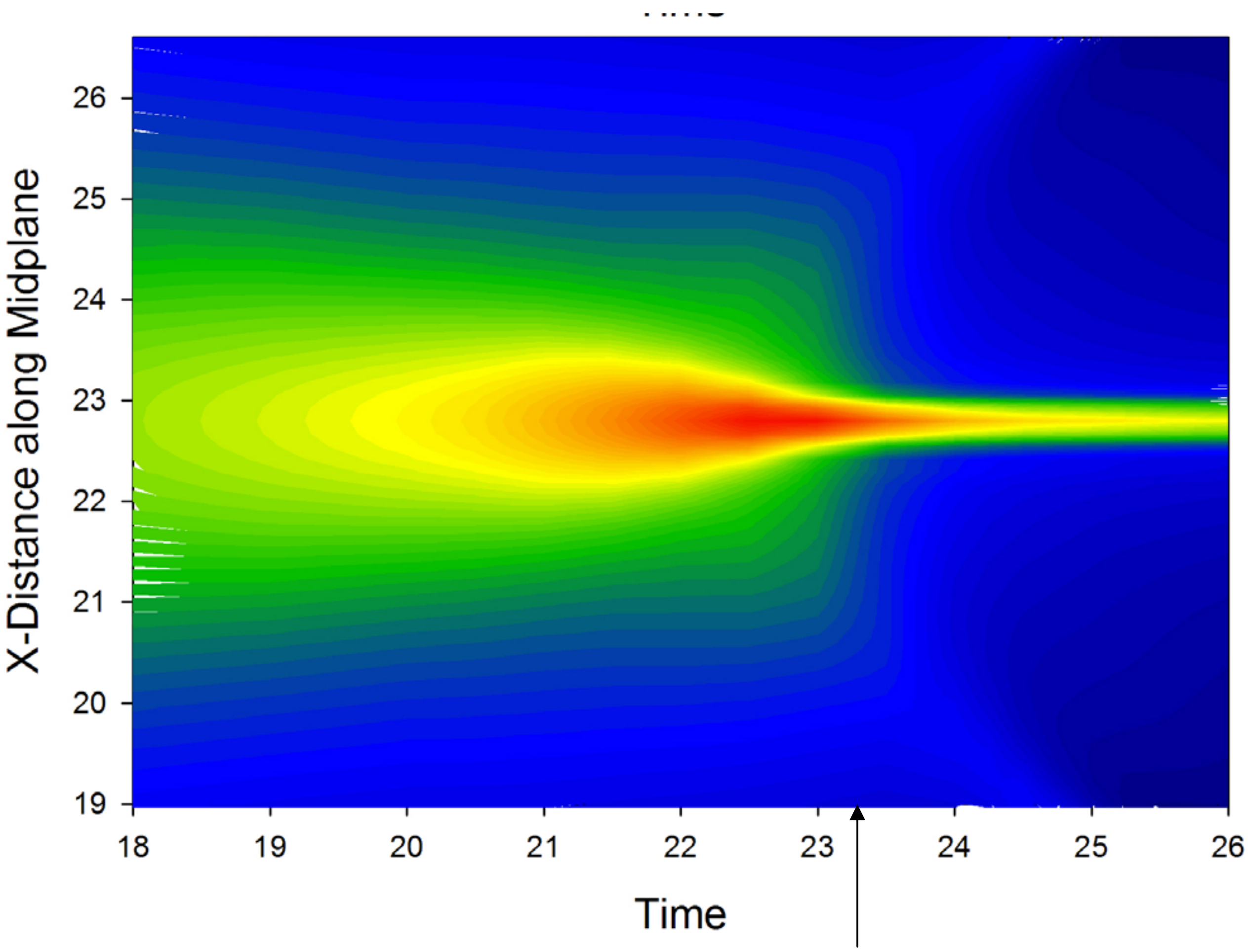
Midplane Current density collapses to the width of 1-3 triangular elements



t=32 time of previous contour plot

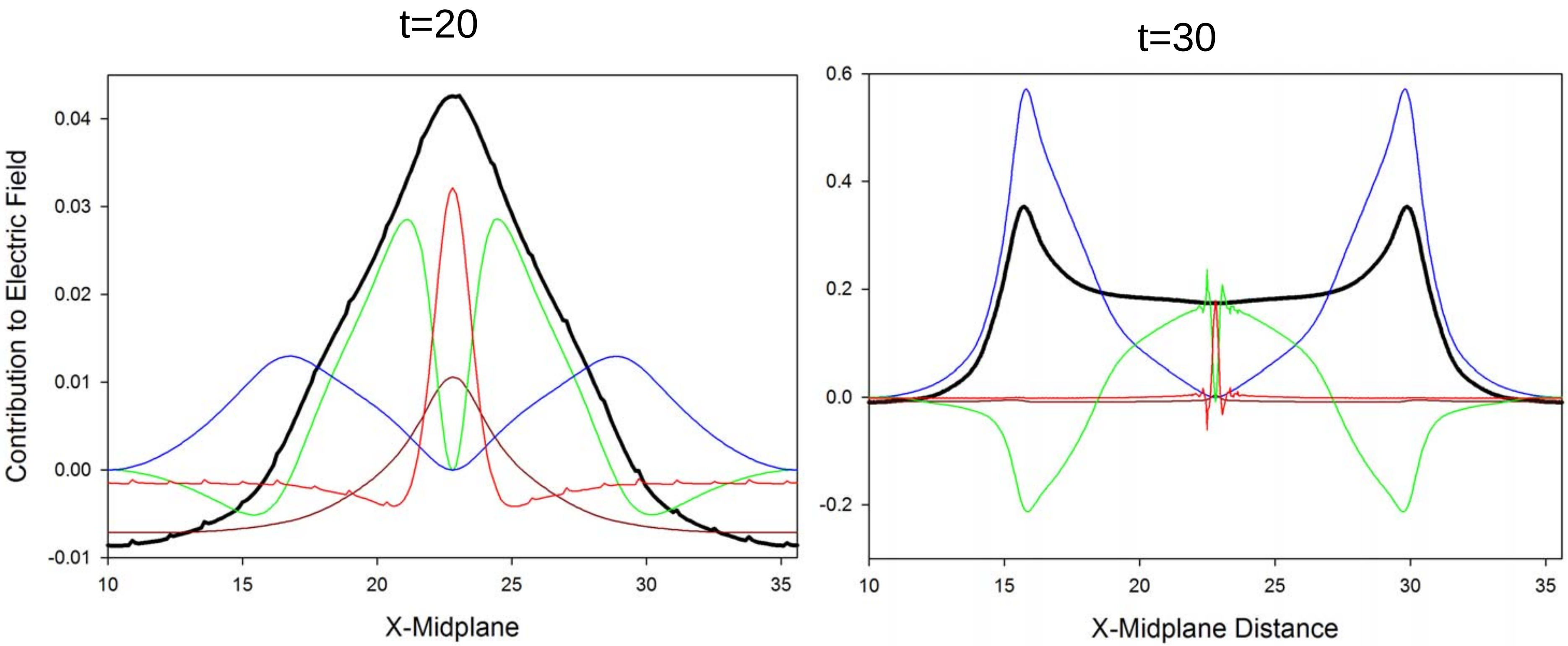
(note sudden collapse at t=23+)

Blowup of Midplane Current density vs time



(note sudden collapse at t~23+)

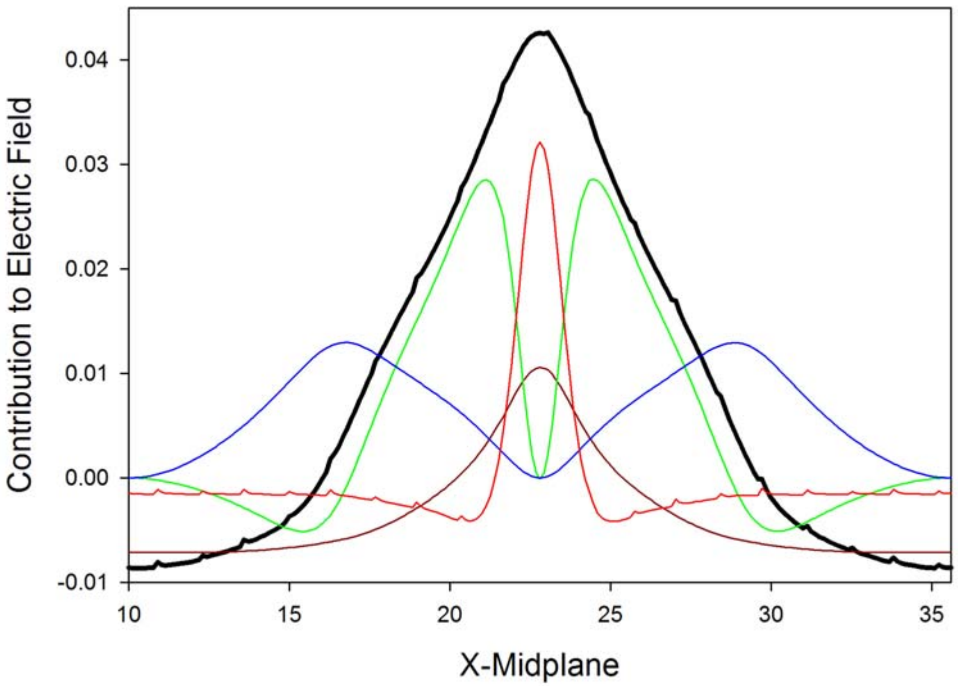
Midplane electric field before and after transition



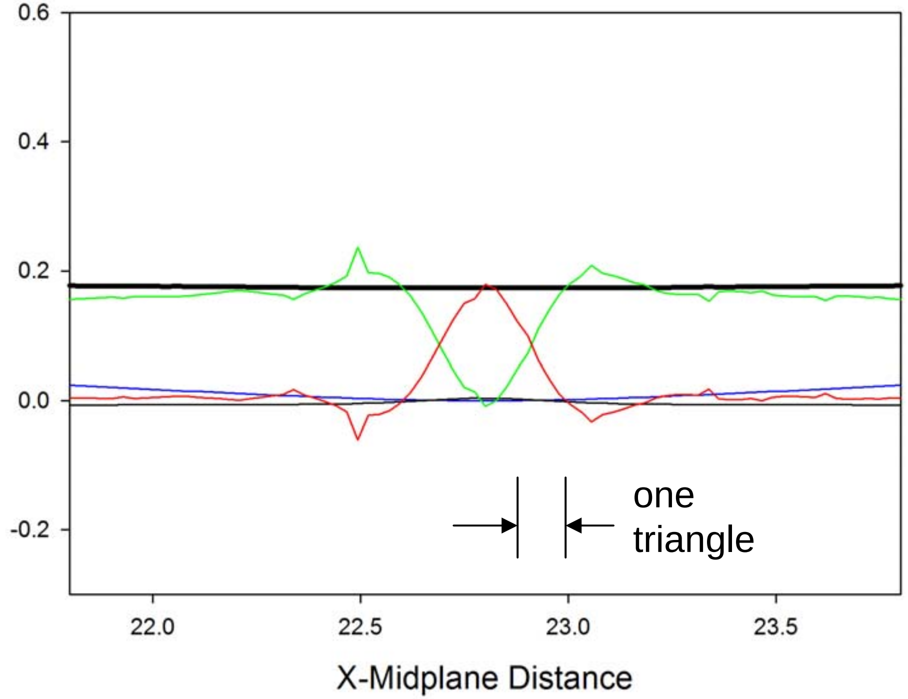
Reconnection rate: $\hat{z}\bullet\left[\vec{E} = -\vec{V}\times\vec{B} + \eta\vec{J} + \frac{1}{ne}\left(\vec{J}\times\vec{B} - \nabla p_e\right) - \lambda_H(\Delta x)^2\nabla^2\vec{J}\right]$

Midplane electric field before and after transition

t=20



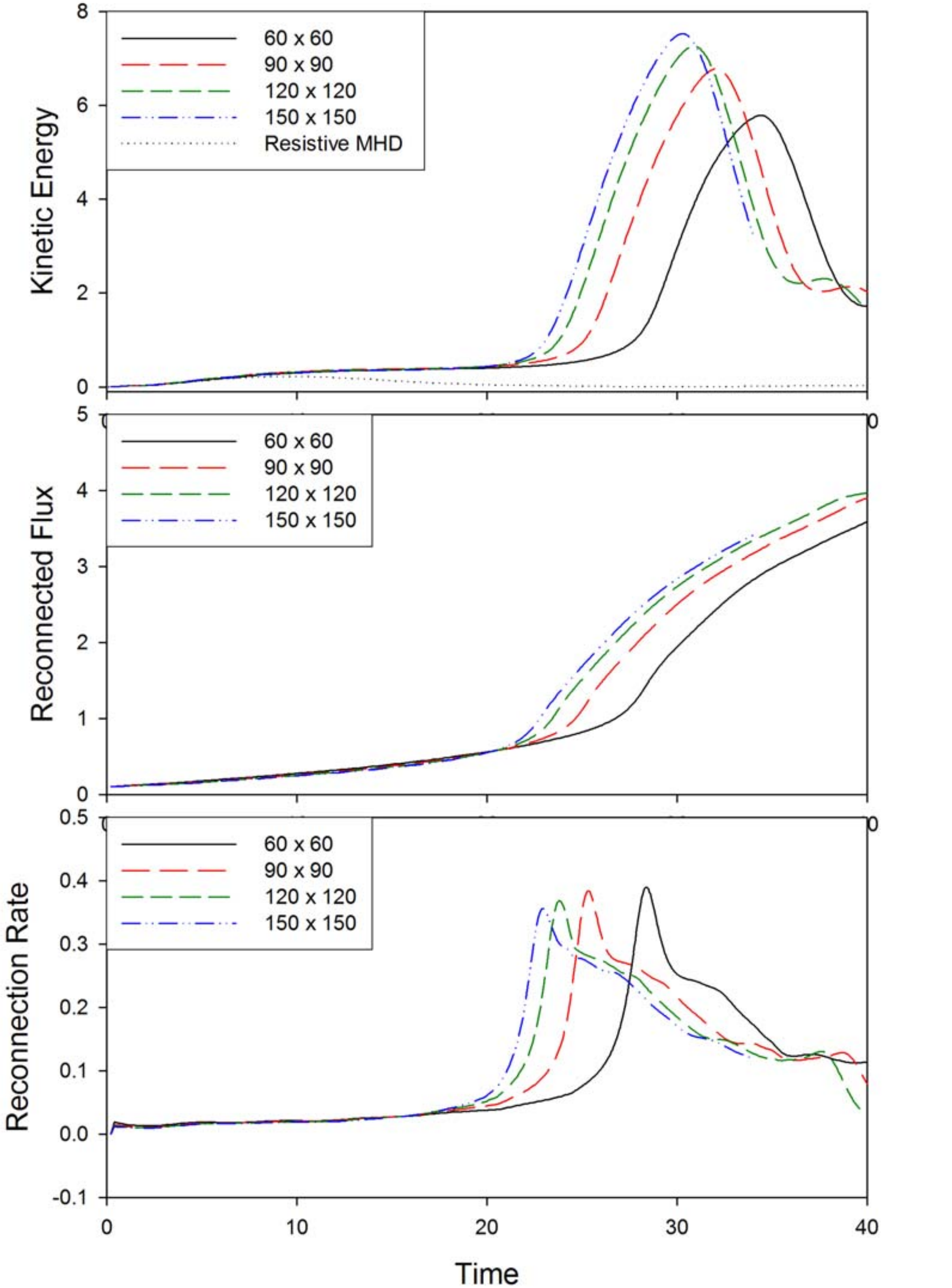
t=30



$$\hat{z}\bullet\left[\vec{E} = -\vec{V}\times\vec{B} + \eta\vec{J} + \frac{1}{ne}\left(\vec{J}\times\vec{B} - \nabla p_e\right) - \lambda_H (\Delta x)^2 \nabla^2 \vec{J}\right]$$

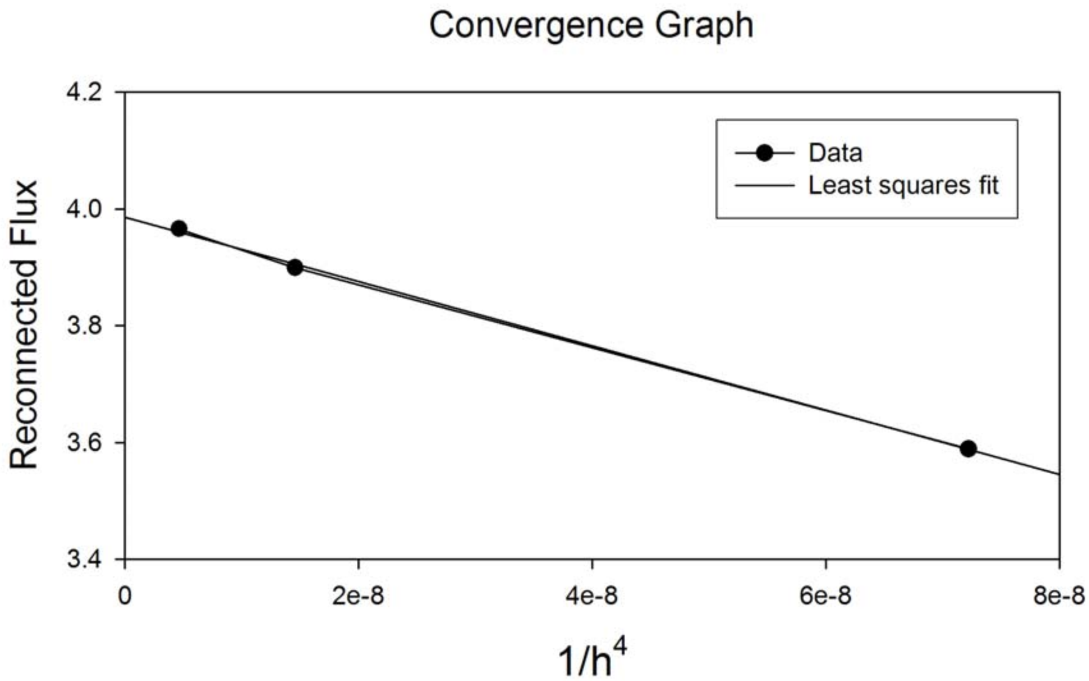
Hyper-resistivity coefficient must be large enough that current density collapse is limited to 1-2 triangles: *reason for factor* $(\Delta x)^2$

2-fluid reconnection requires high resolution for convergent results



- Note sudden transition where velocity abruptly increases
- These calculations used a hyperviscosity term in Ohm's law proportional to $(\Delta x)^2 \cdots$ required for a stable calculation

Results appear to be converging



Now working on 180 x 180 and higher: Details for bassi.nersc.gov:

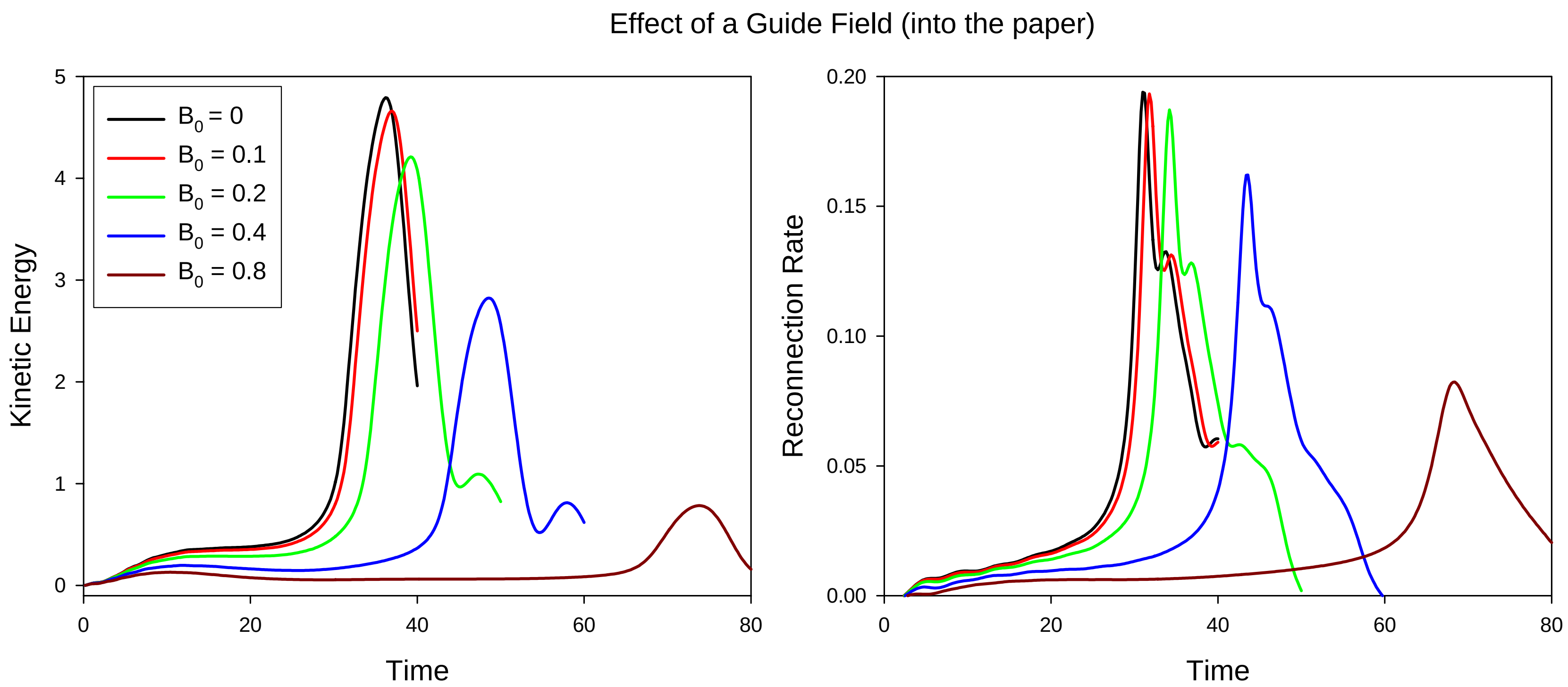
Mesh points	180 x 180
Matrix Rank	5.9×10^5
# Non-zeros	9.5×10^7
# NZ in L/U	8.8×10^8

# processors	8	32	128
Factor (s)	69.5	38.1	16.9
Gflop/s	27.2	50.1	112.8

Total problem time (8 processors) = 208 s x 400 cycles x 8p = 185 p-hrs

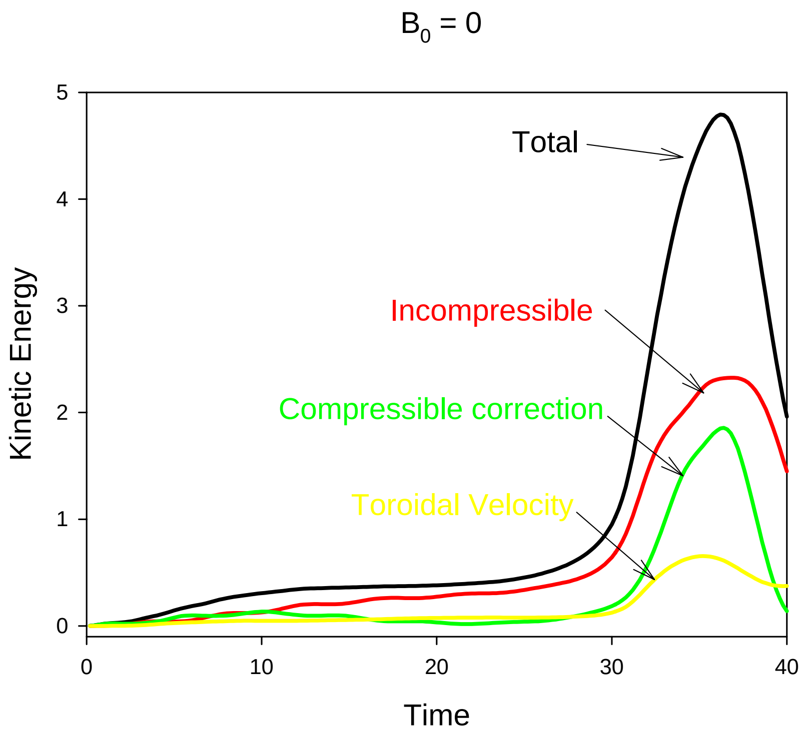
NOTE: mesh adaptation (almost implemented) should bring this down sharply

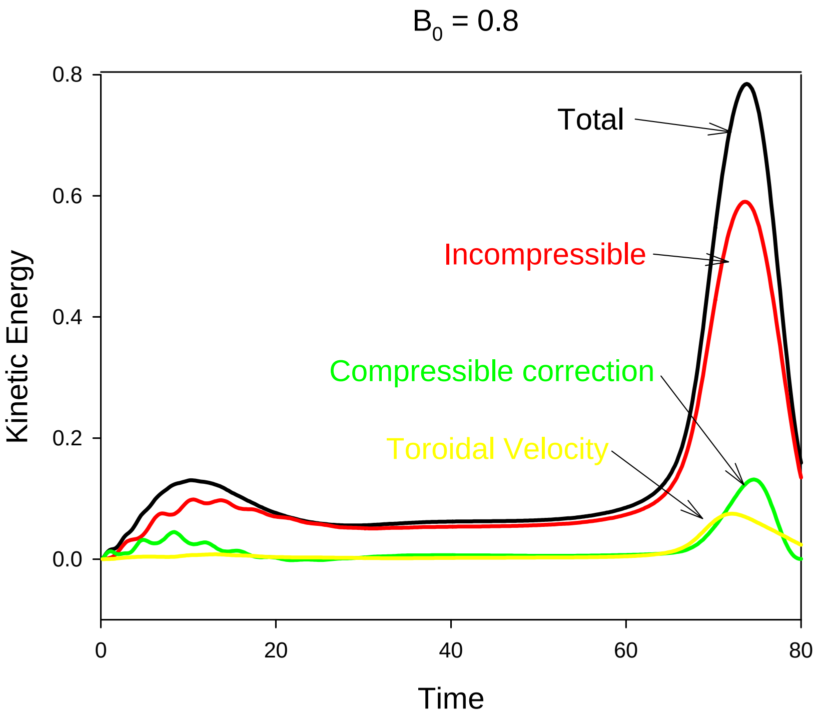
Effect of adding a guide field on 2-fluid reconnection



- Adding a guide field delays transition time and reduces maximum reconnection rate
- Small effect for $B_0 < 0.2$
- To be studied further

Change in velocity field with toroidal field strength





- Velocity field becomes more like incompressible flow as toroidal field strength increases

Developmental Plans

- 2D Slab, 2-fluid, Fully Unstructured, adaptive Parallel Code near completion (A. Bauer, RPI to deliver within weeks)

- 2D Toroidal Code, with 2-fluid and Gyroviscosity, will provide accurate 2-fluid equilibrium with flow and neoclassical drive terms (N. Ferraro to discuss)

- Linear, toroidal 3D code is logical next step. Will be an improvement on MARS code (full 2-fluid physics)

- 3D nonlinear code will initially have explicit finite differencing in toroidal direction.
  - Should scale to a teraflop (actual) for 20 planes and 640 p

An Implicit Method for Magnetic Fusion MHD Calculations using Adaptive, High-Order, High-Continuity, Finite Elements

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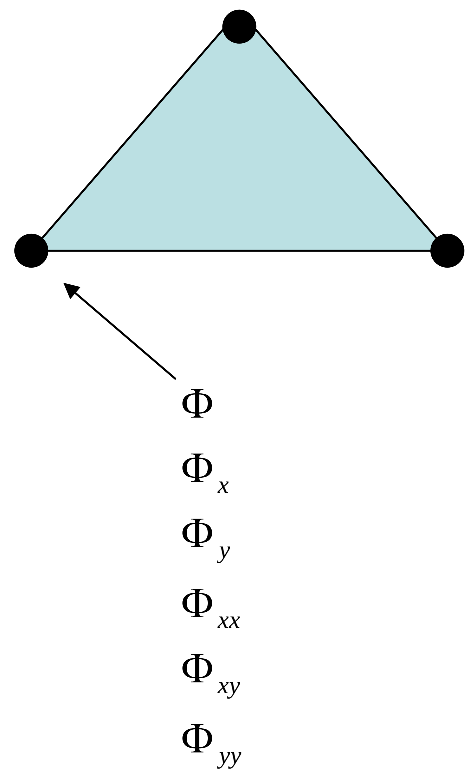
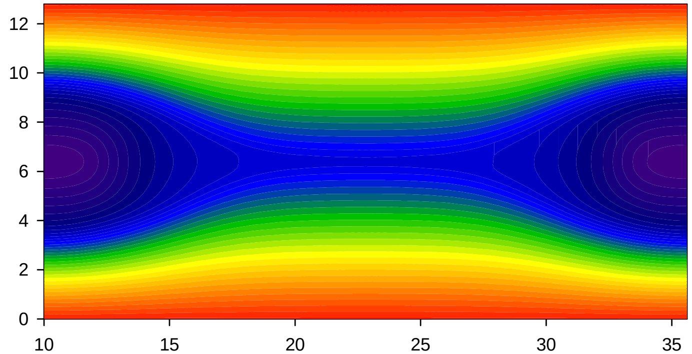
Scalar data is represented using 18 degree of freedom quintic triangular finite elements Q_{18}

- All data is at nodes: function + first 5 derivatives (6 dof)
- Complete quintic polynomial has 21 coefficients
  - 18 values come from the 3 nodes (3 x 6)
  - 3 values come from requirement that the normal derivative along each edge be only a (univariate) cubic….leads to C^1 continuity
- Contains a complete Taylor series through 4th order…error ~ h^5
- Compact representation … only 3 dof/triangle
- C^1 continuity allows up to 4th derivatives in space without introducing auxiliary variables
- Unstructured triangular mesh allows adaptive zoning

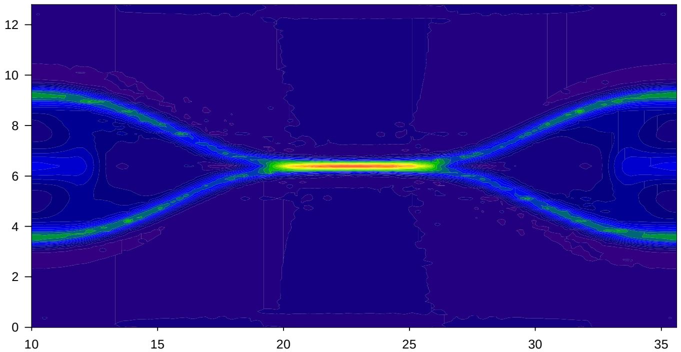


GEM Nonlinear Benchmark

Poloidal Flux Ψ



Current Density J



GEM Reconnection Problem

$$\psi^0(x,y) = \frac{1}{2}\ln(\cosh 2y)$$

$$P^0(x,y) = \left[\operatorname{sech}^2 2y + 0.2\right]$$

$$\tilde{\psi}(x,y) = \varepsilon \cos k_x x \cos k_y y$$

1. Resistive MHD
   High and Low Viscosity
   ($\mu = 10\ \eta$, $\mu = 0.1\ \eta$)
2. Two-Fluid

- Provides a non-trivial, convenient test problem for code verification and validation and cross-code comparison
- Also, extending this by adding an equilibrium magnetic field into the plane (guide field)

J.Birn, J.F.Drake, M.A. Shay *et al.*,J. Geophys. Res. **106** 3715 (2001)