Sawtooth Studies and Plans

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CEMM Meeting

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Specification of Analytic Equilibrium

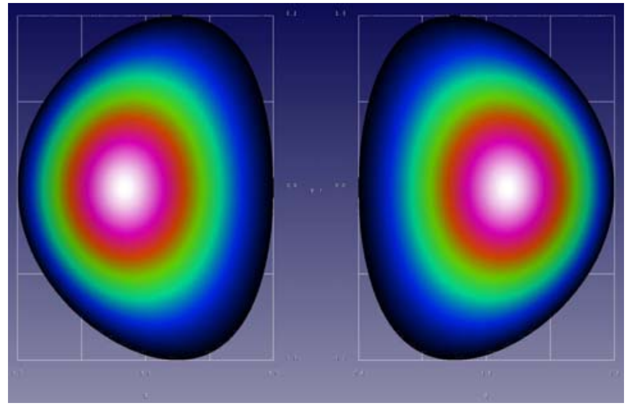
Quantity	Value
Major radius R_0	0.341 m
Minor radius a	0.247 m (aspect ratio = 1.38)
Ellipticity κ	1.35
Triangularity δ	0.25
Central temperature (T_e=T_i)	100 eV
Normalized central pressure $\mu_0 p_0$	7.5×10^{-4} (implies $n_0 = 1.86 \times 10^{19}$ m^{-3})
α Parameter in pressure equation*	0.1
Vacuum value g_0 of $R \cdot B_T$	0.04252 T·m
Effective ion charge Z_{EFF}	2.0
Loop voltage V_L	3.1741 V (implies $q_0 \approx 0.82$)

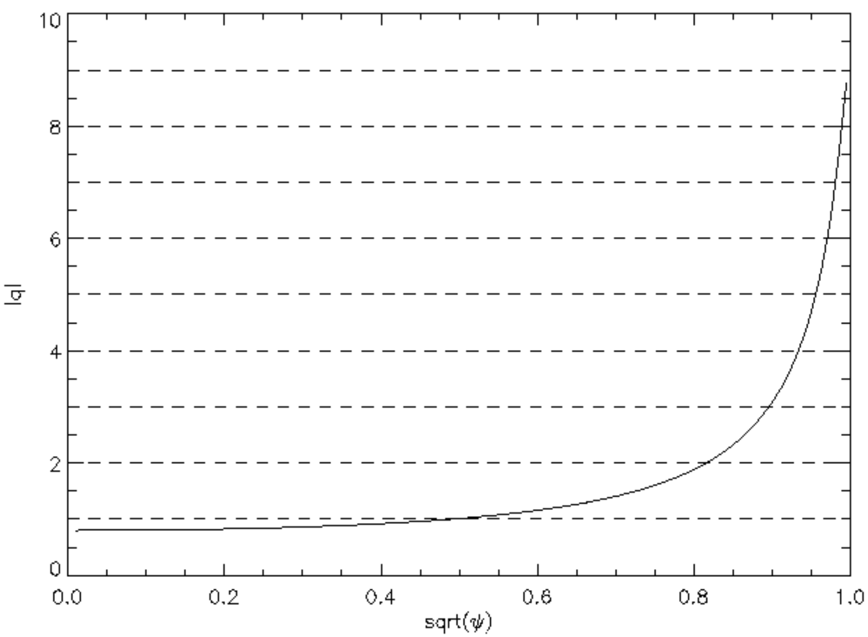
$$*p(\psi) = p_0\left[\alpha\tilde{\psi} + (1-\alpha)\tilde{\psi}^2\right], \text{ where } \tilde{\psi} \equiv \frac{\psi - \psi_{\text{limiter}}}{\psi_{\text{axis}} - \psi_{\text{limiter}}}.$$

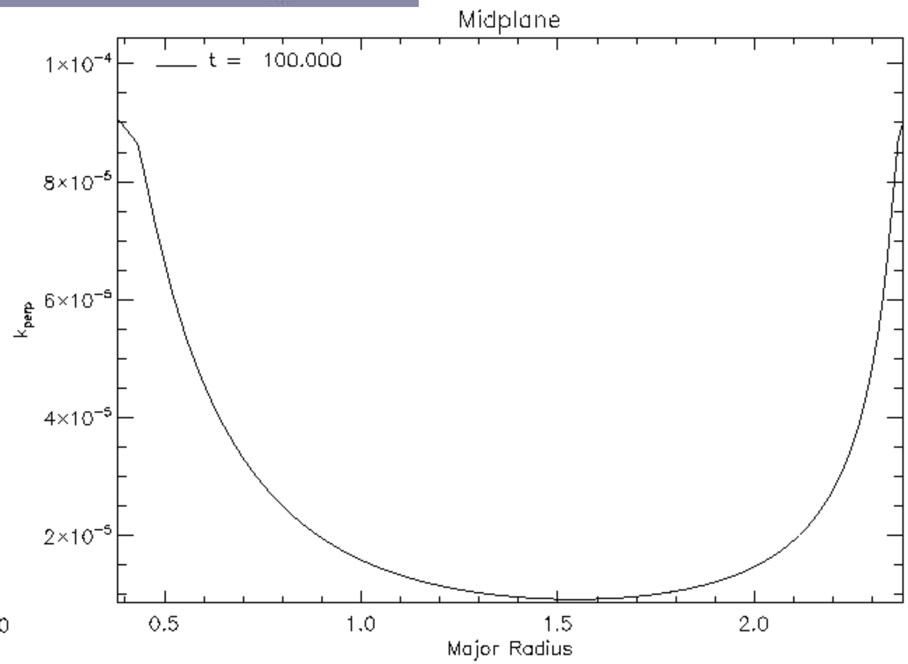
Use equilibrium code to solve Grad-Shafranov equation, with profile of heat conduction coefficient χ computed self-consistently to keep temperature constant given profile, energy supplied by applied V_L.

Form of Analytic Equilibrium

$$R(\theta) = R_0 + a\cos\left[\theta + \delta\sin(\theta)\right]$$
$$z(\theta) = a\kappa\sin(\theta)$$
$$T(\psi) = T_0\tilde{\psi},$$
$$n(\psi) = \frac{p}{2k_B T} = \frac{p_0}{2k_B T_0}\left[\alpha + (1-\alpha)\tilde{\psi}\right]$$







q_{min} = 0.8023

Minimum value: 9.21 × 10^{-6}
Old case: pkkk ≡ 9.09 × 10^{-4}

Transport Coefficients

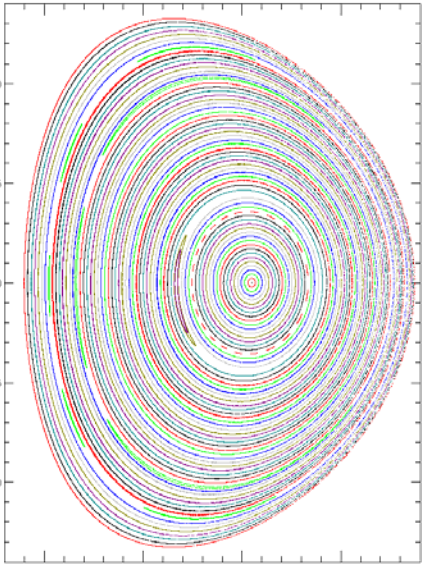
- Evolving Spitzer resistivity $\eta(\boldsymbol{x},t) \propto T^{-3/2}$ with cutoff 100x initial central value; initial central $S = 1.94 \times 10^4$.

- Constant Prandtl number 10 (evolving axisymmetric viscosity).

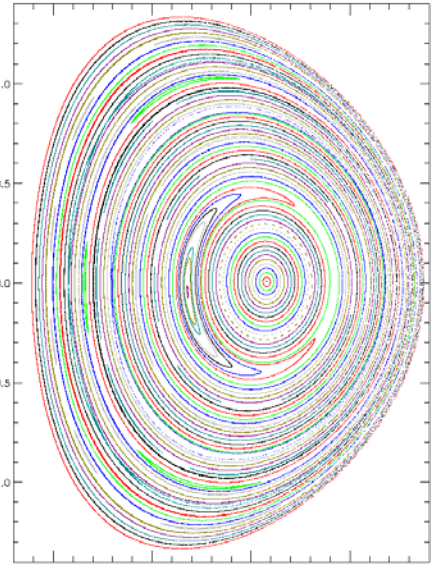
- Perpendicular heat diffusivity $\kappa_\perp$ read from self-consistent steady state computed with equilibrium code; central value renormalized to about 2.03 m^2/s to maintain steady-state.

- Parallel heat conduction as in previous case ($v_{Te} = 6\ v_A$).

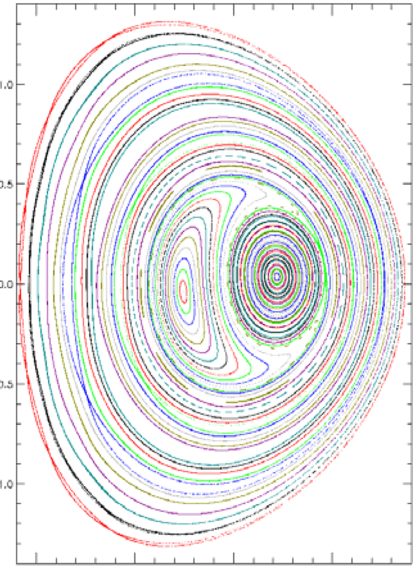
Poincaré Plots



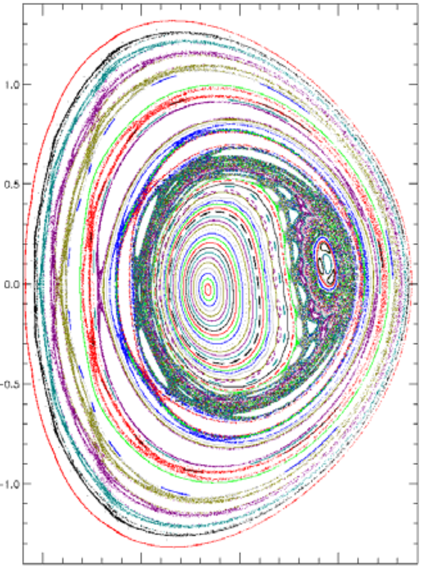
t = 987.80; q_{min} = 0.8022



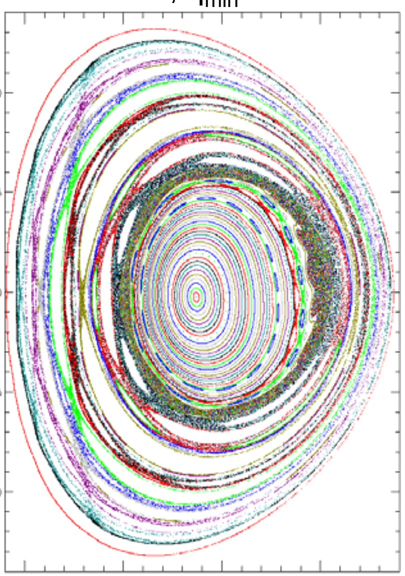
t = 1109.55; q_{min} = 0.7965



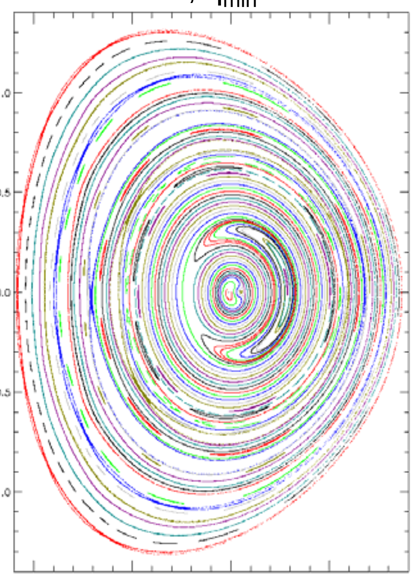
t = 1207.05; q_{min} = 0.7955



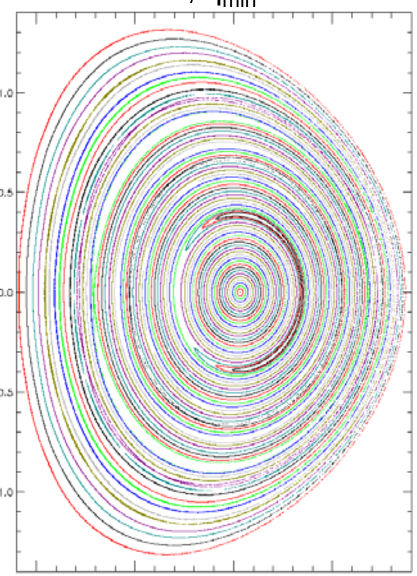
t = 1244.55



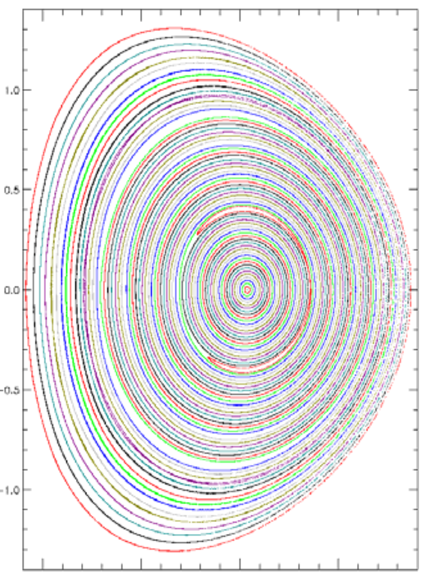
t = 1252.05; q_{min} = 1.0329



t = 1357.05; q_{min} = 0.9953

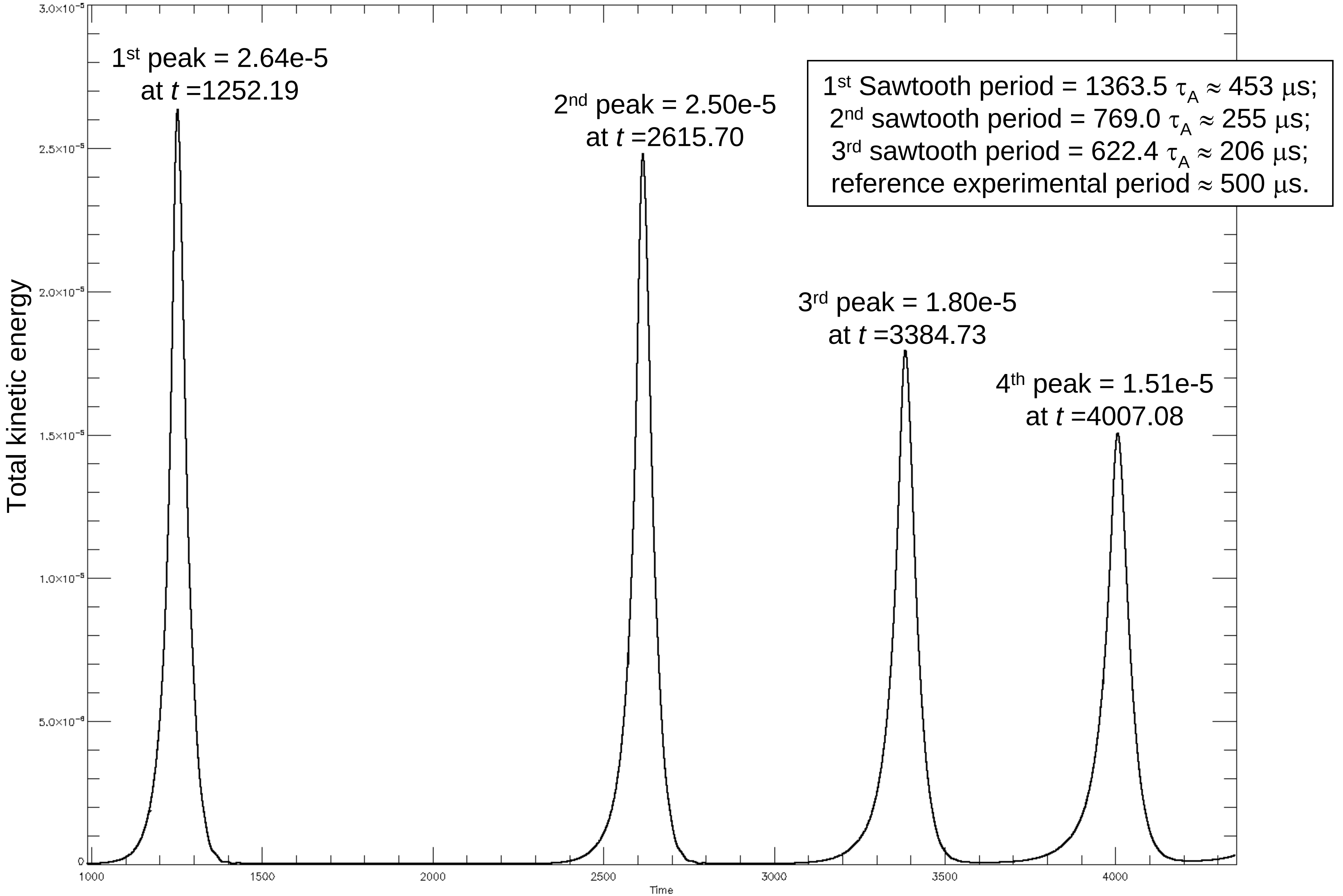


t = 1452.05; q_{min} = 0.9518

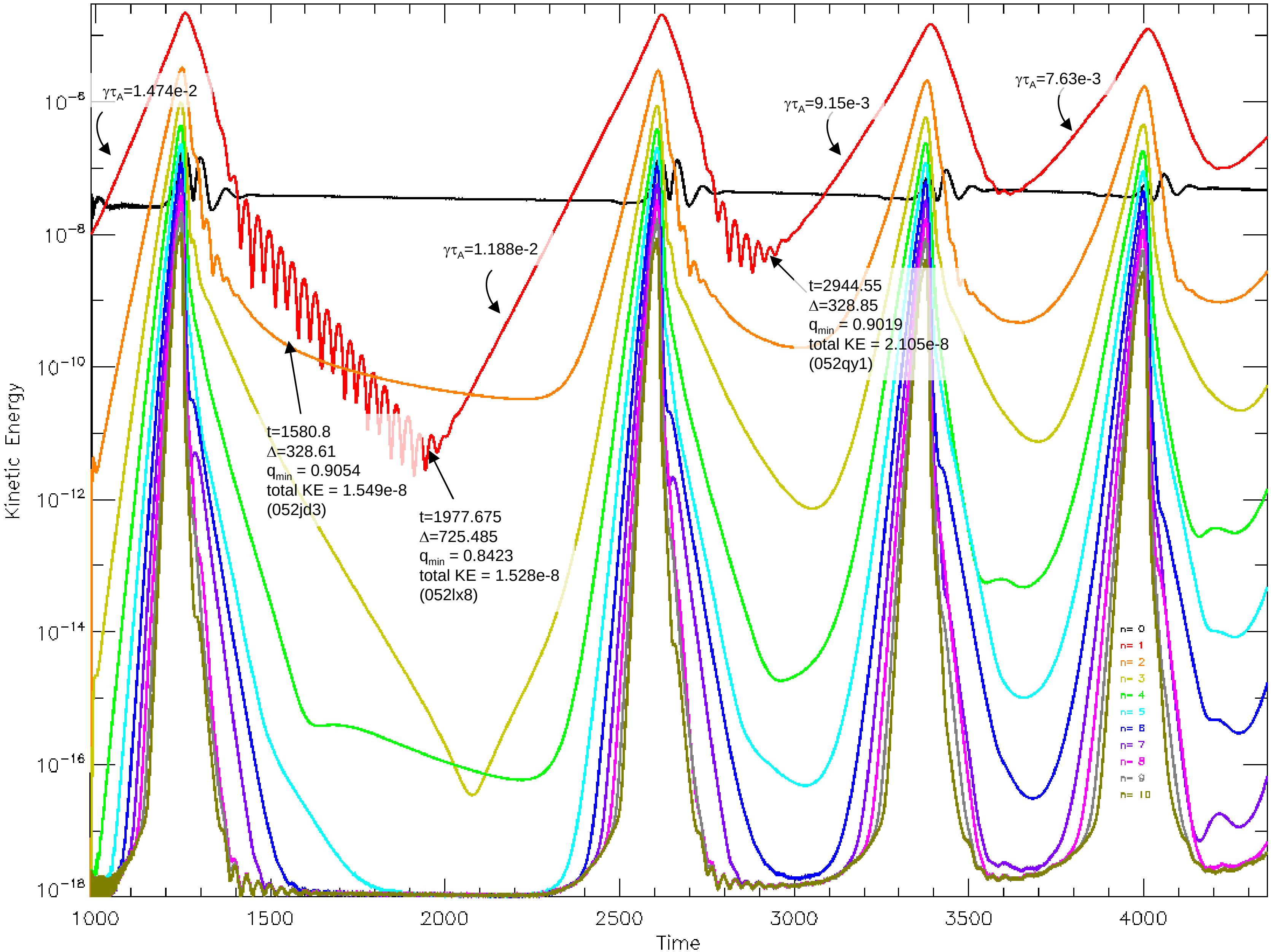


t = 1531.43; q_{min} = 0.9211

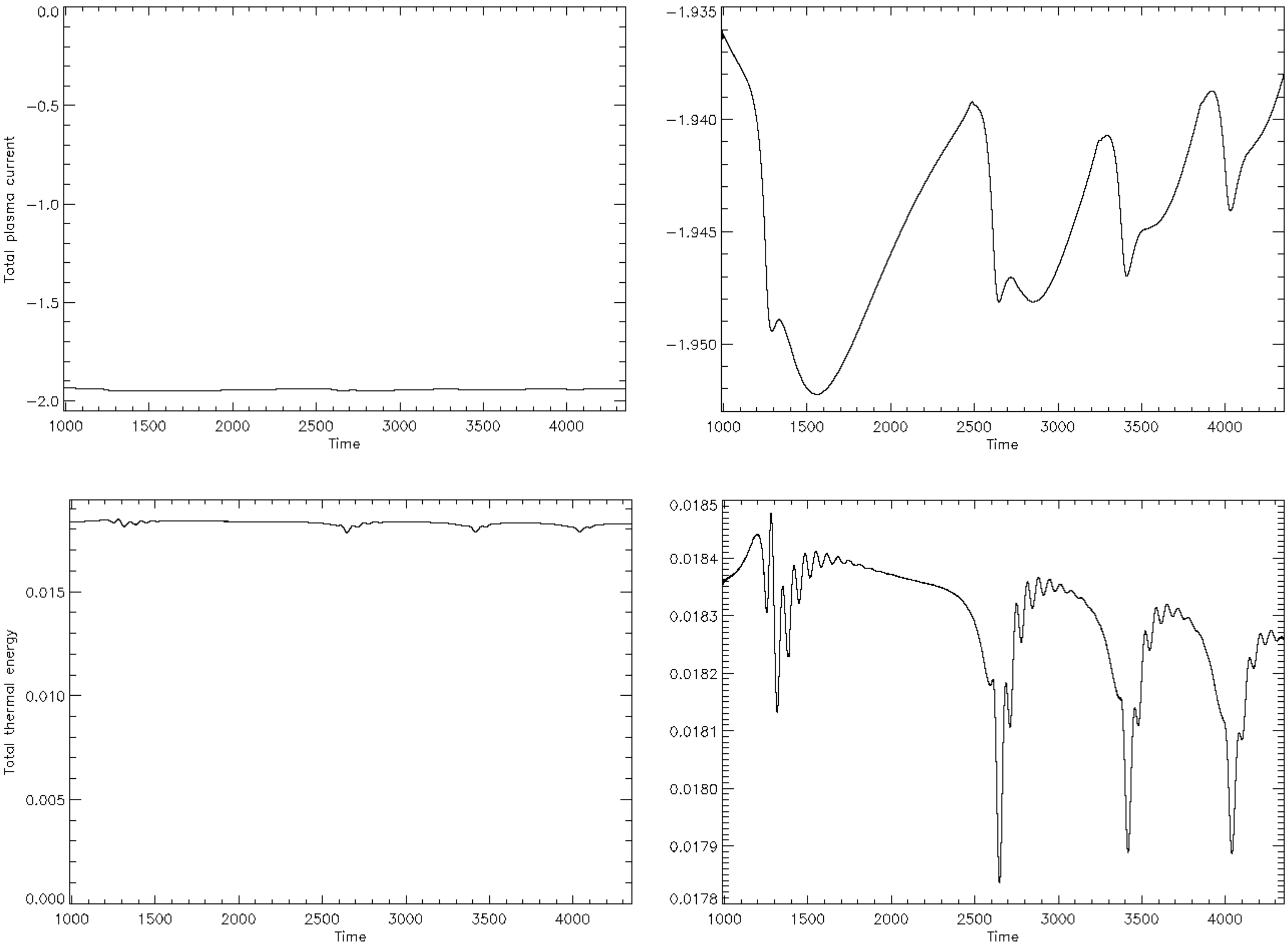
Kinetic Energy History



Kinetic Energy Mode History



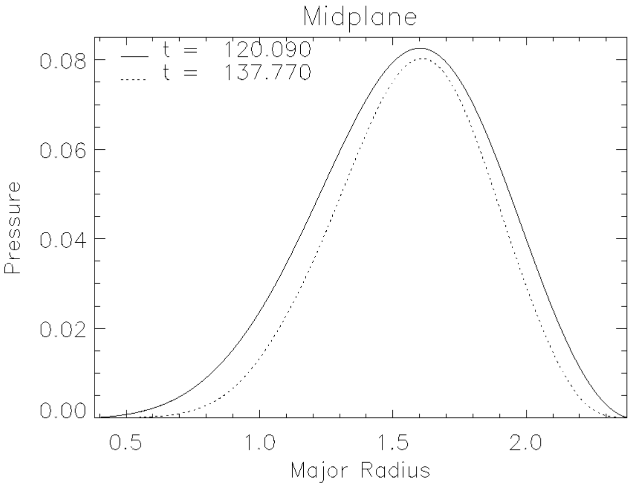
Nonlinear Conservation



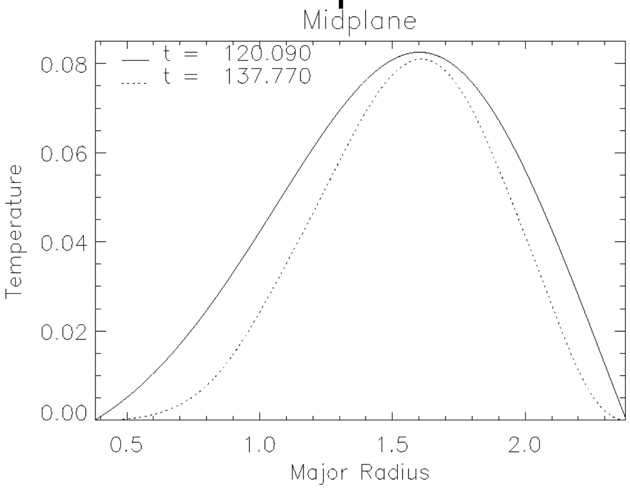
CDX sawtooth with more peaked analytic temperature profile

Original	**New**	
$p = p_0\left(0.1\tilde{\psi} + 0.9\tilde{\psi}^2\right)$	$p = p_0\left(0.1\tilde{\psi}^2 + 0.9\tilde{\psi}^3\right)$	$\tilde{\psi} \equiv \dfrac{\psi - \psi_{\text{lim}}}{\psi_{\text{axis}} - \psi_{\text{lim}}}$
$T = T_0\tilde{\psi}$	$T = T_0\tilde{\psi}^2$	
$n = \dfrac{p}{2k_B T} = \dfrac{p_0}{2k_B T_0}\left(0.1 + 0.9\tilde{\psi}\right)$	$n = \dfrac{p}{2k_B T} = \dfrac{p_0}{2k_B T_0}\left(0.1 + 0.9\tilde{\psi}\right)$	

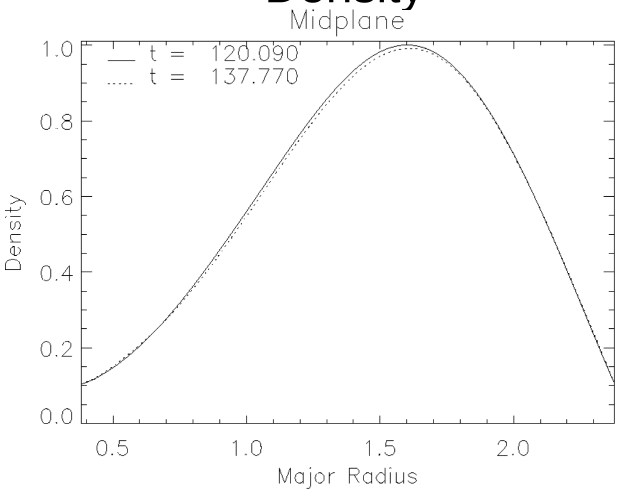
Pressure



Temperature

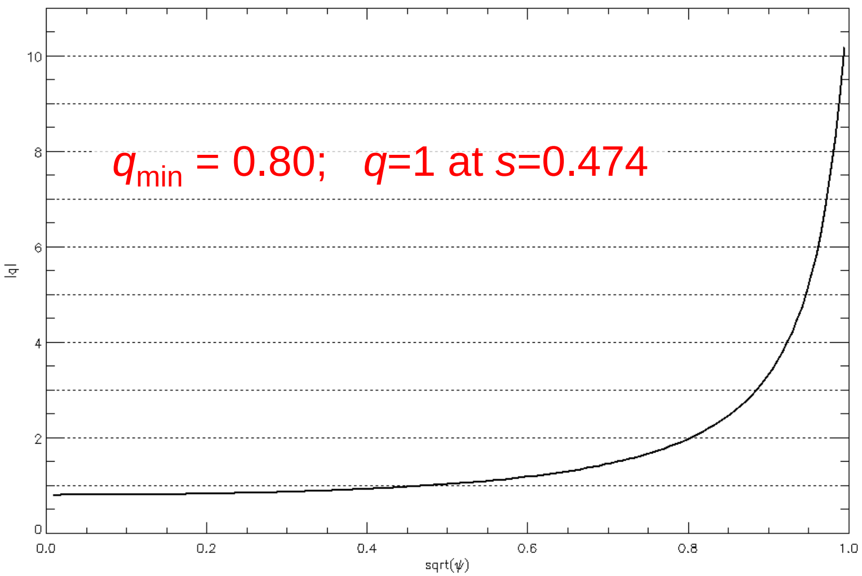


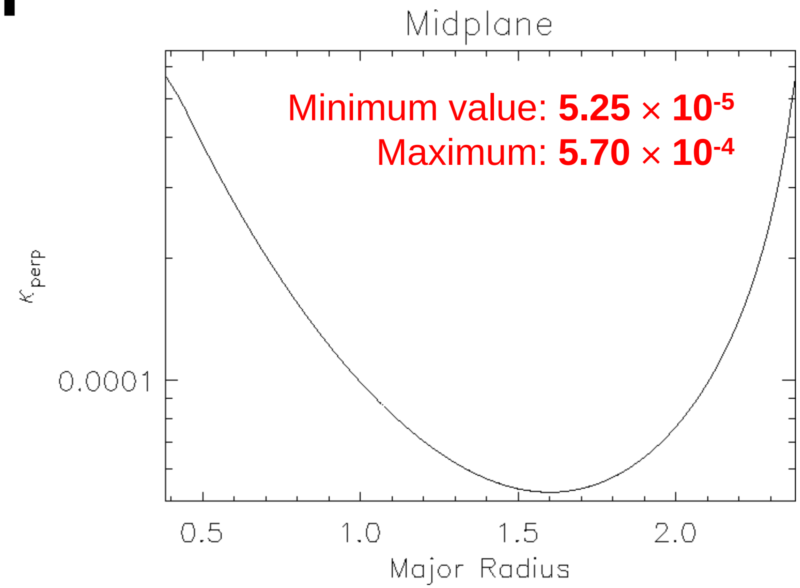
Density



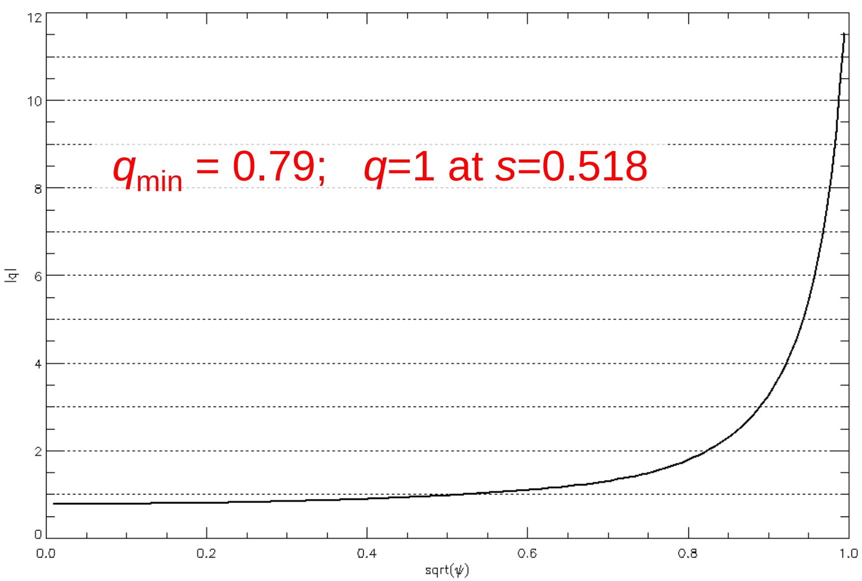
New q, κ profiles

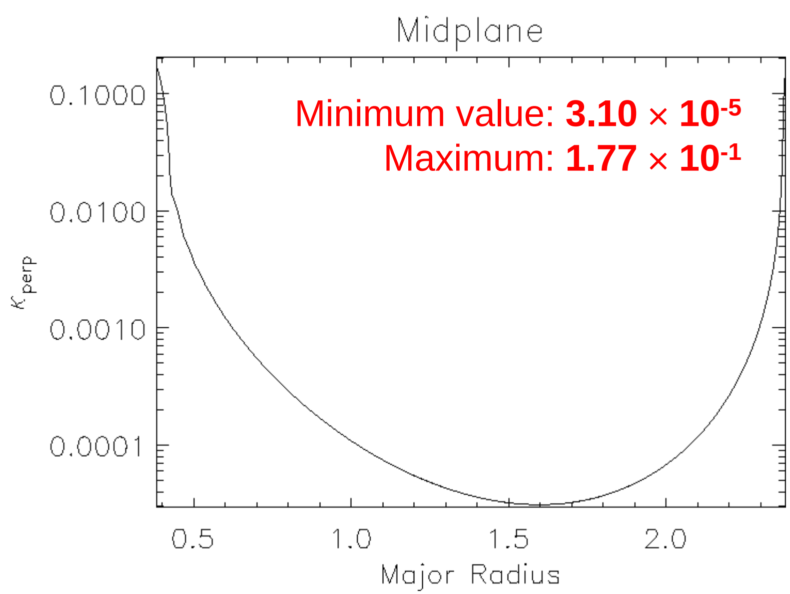
Original





New

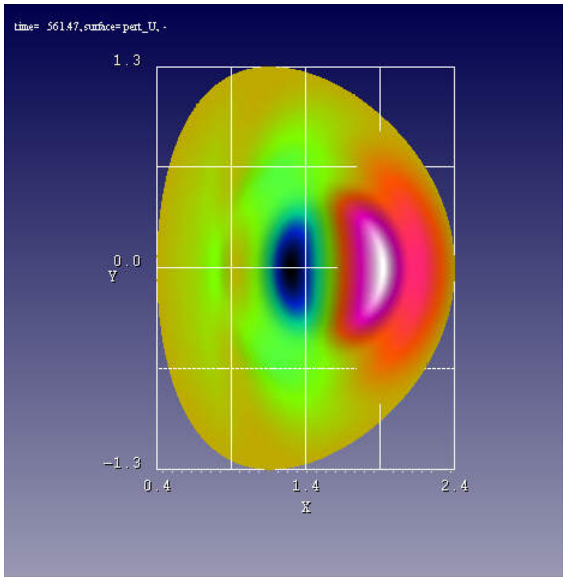




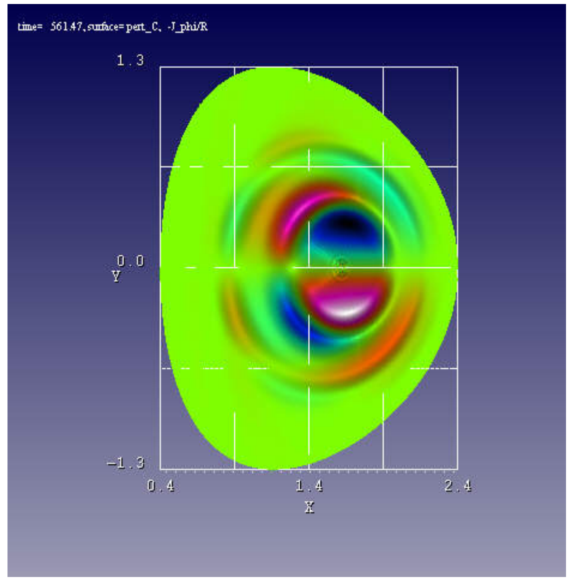
Old case: pkkk $\equiv 9.09 \times 10^{-4}$

New n=1 eigenmode

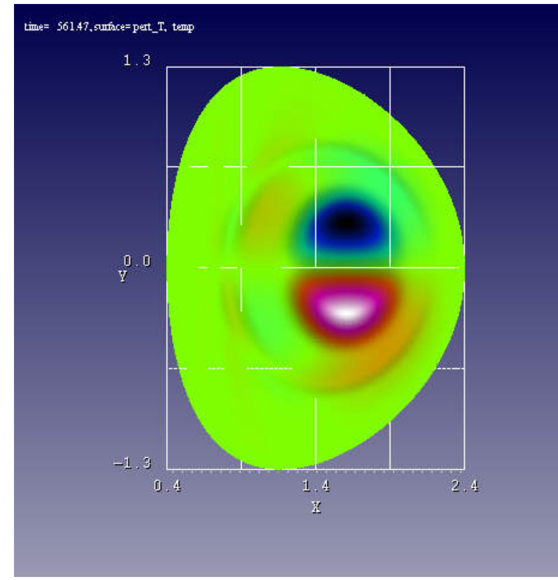
Velocity stream function U



C = -RJ$_\phi$



Temperature



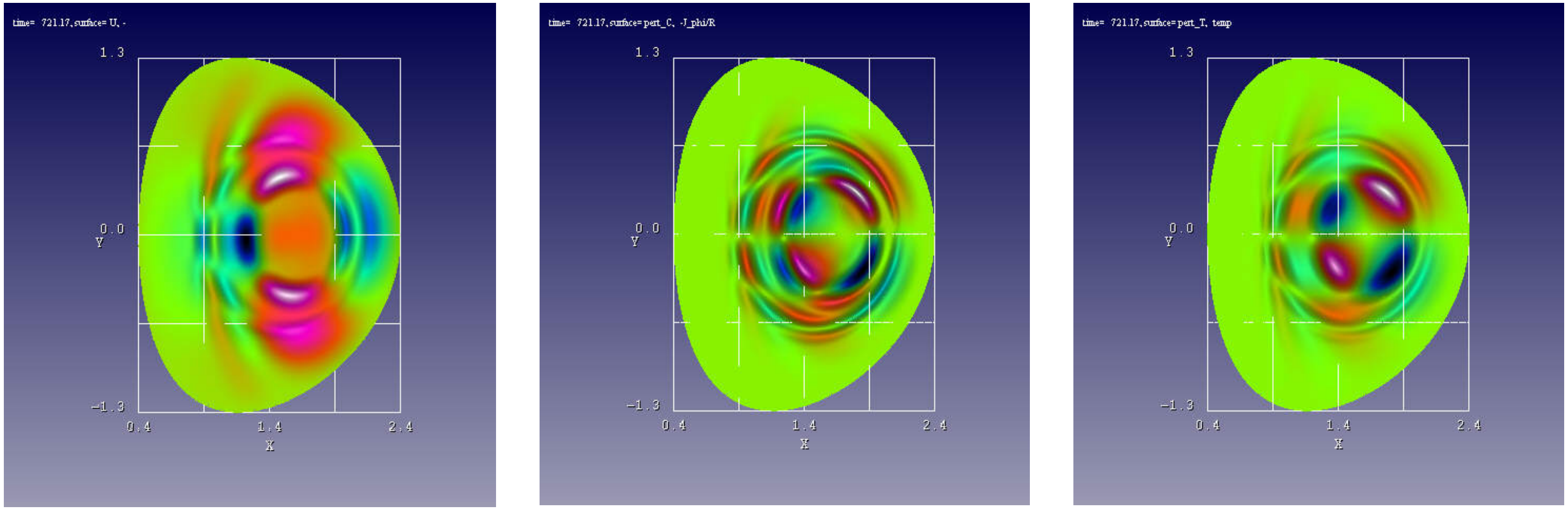
1,1 mode; $\gamma\tau_A \approx (2.9557 \pm 0.0001) \times 10^{-2}$

(Original was 1.415×10^{-2})

Higher *n* eigenmodes

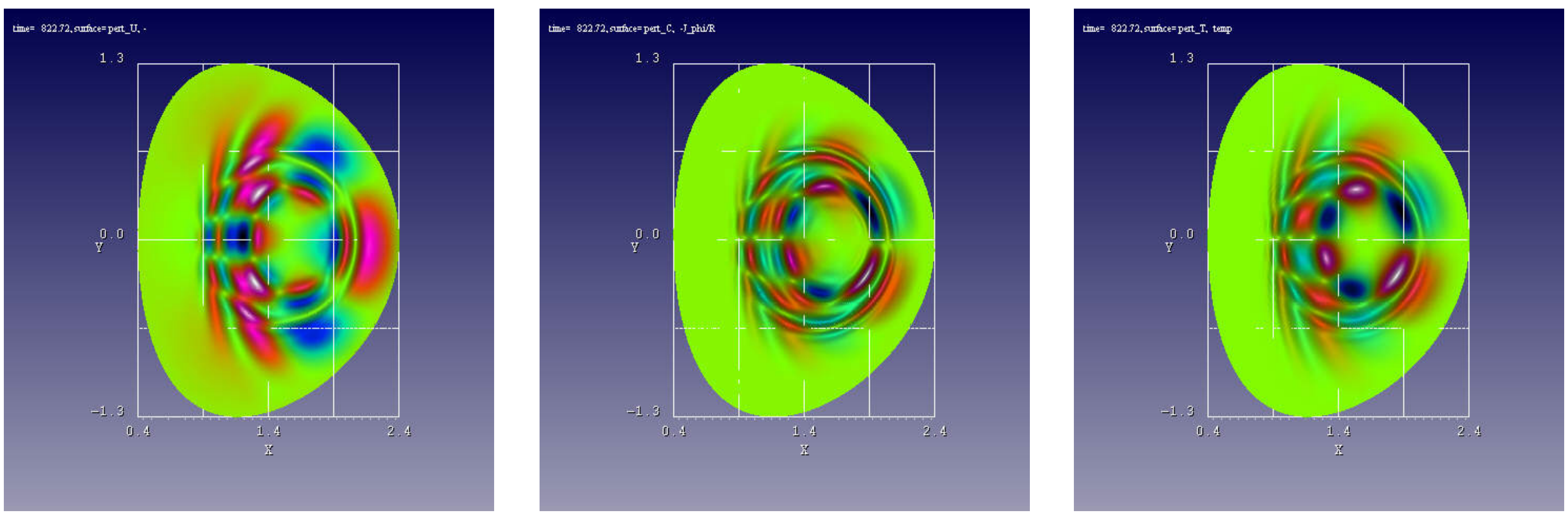
Velocity stream function U | $C = -RJ_\phi$ | Temperature

n = 2



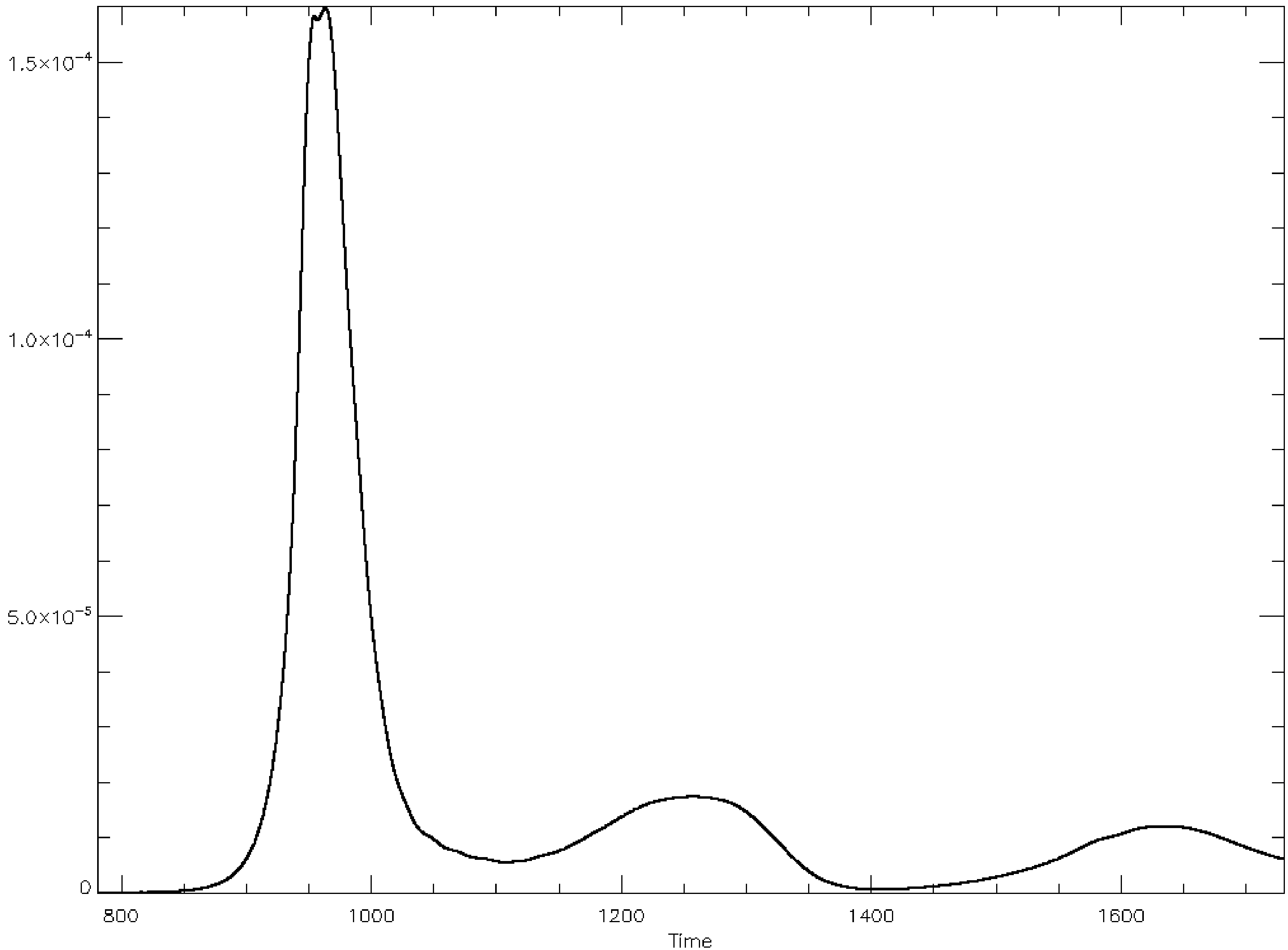
Mostly 2,2; $\gamma\tau_A \approx 1.02\times10^{-2}$

n = 3

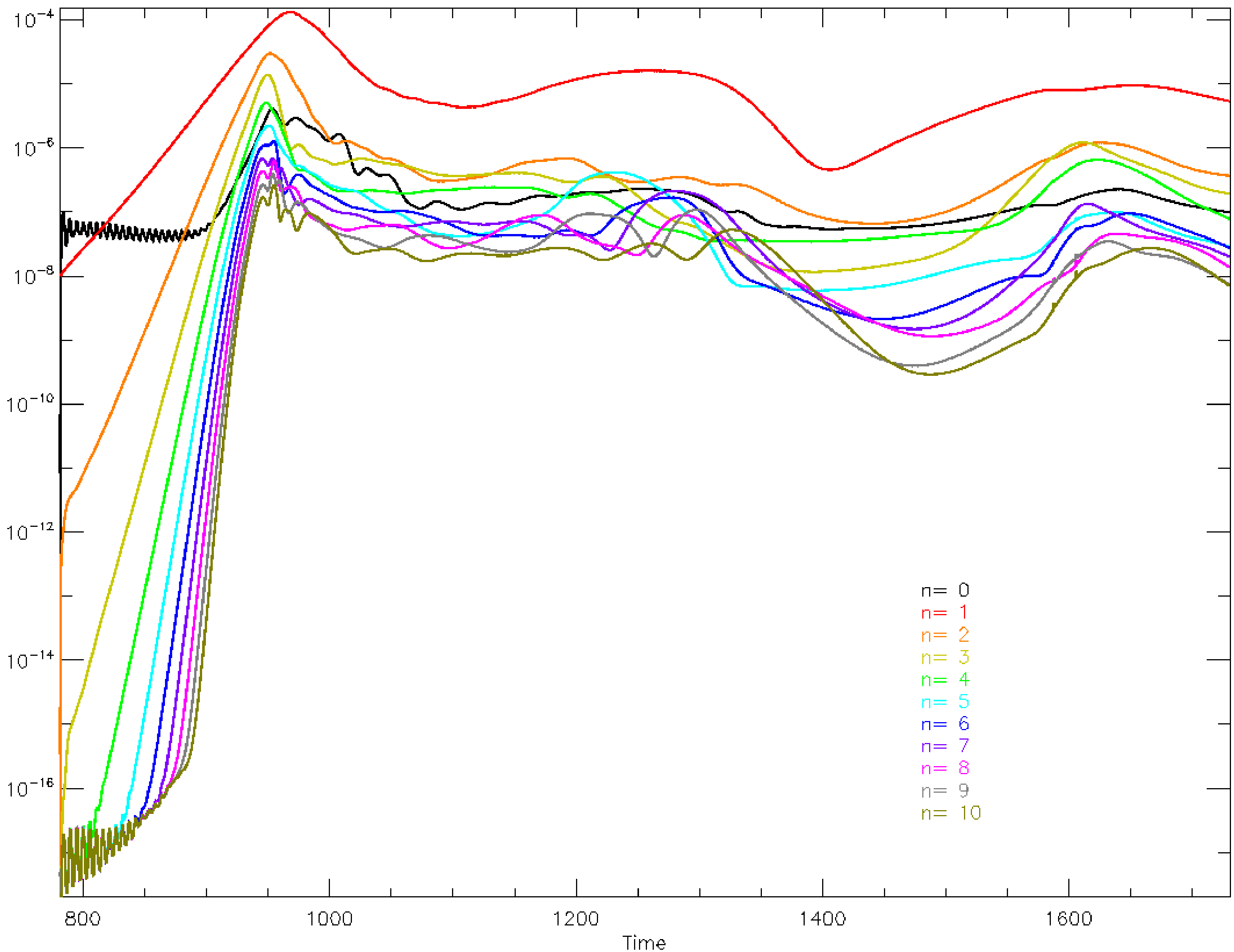


Mostly 3,3; $\gamma\tau_A \approx 1.93\times10^{-3}$

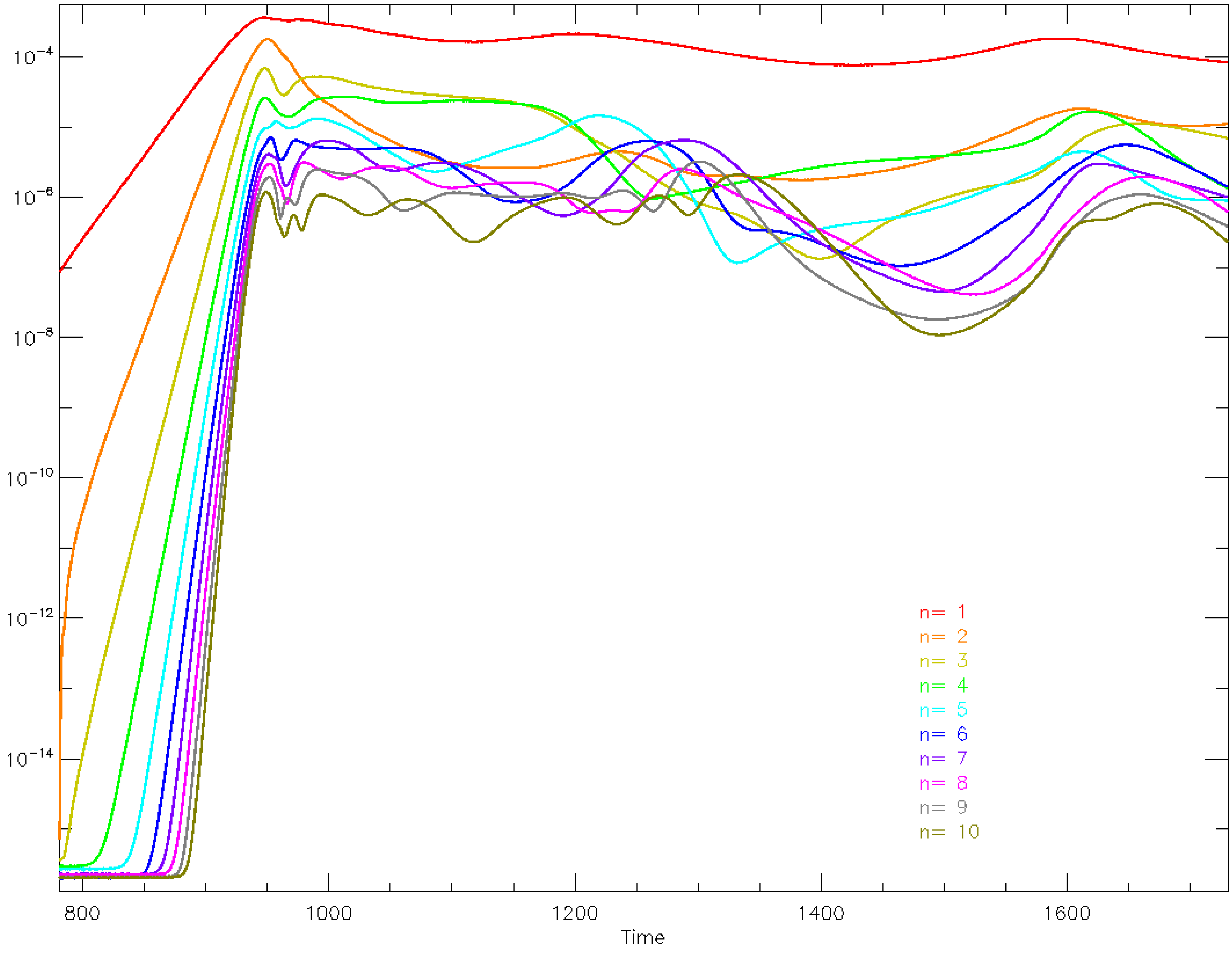
Total Kinetic Energy History



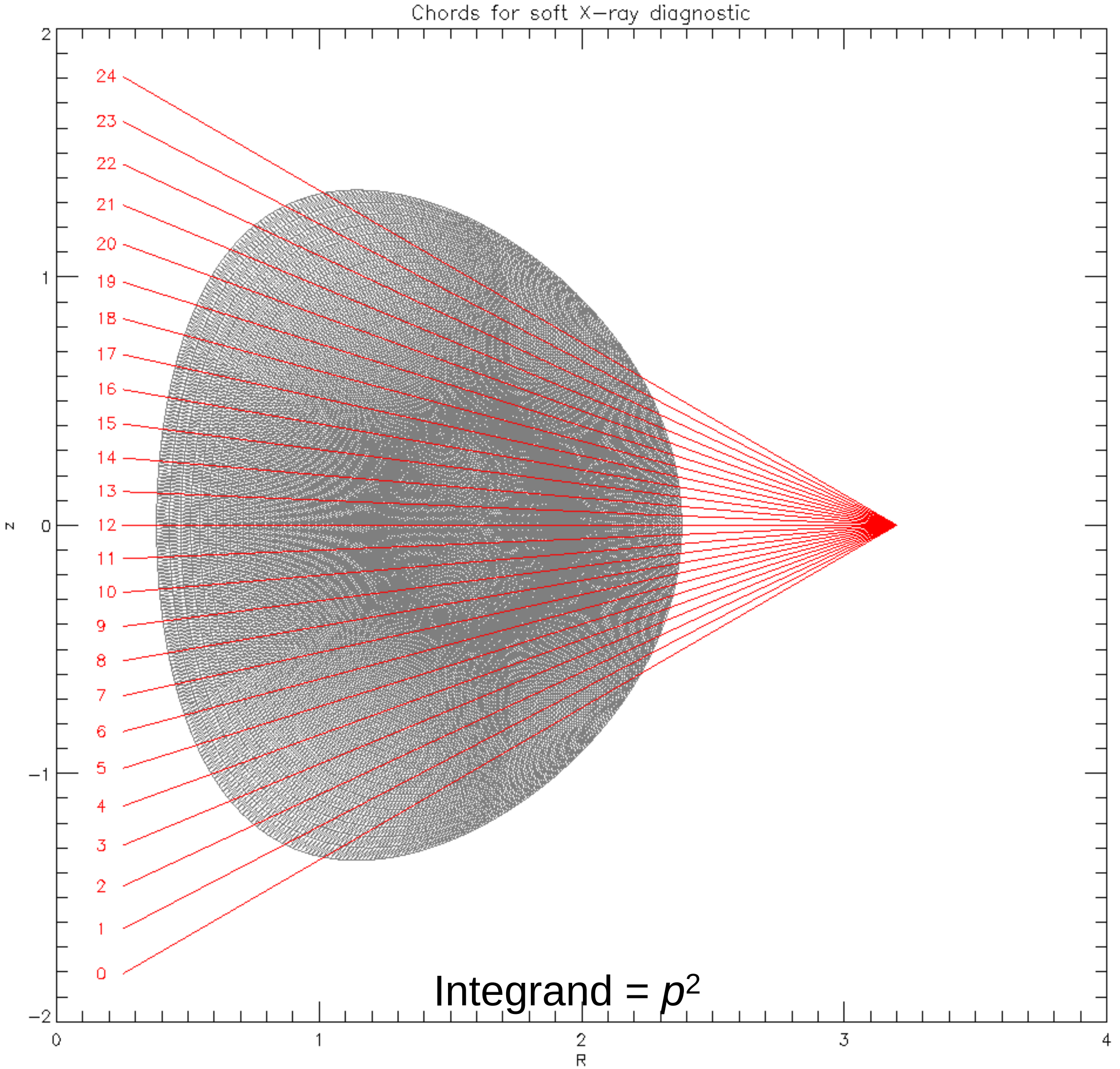
Kinetic Energy History by Mode Number



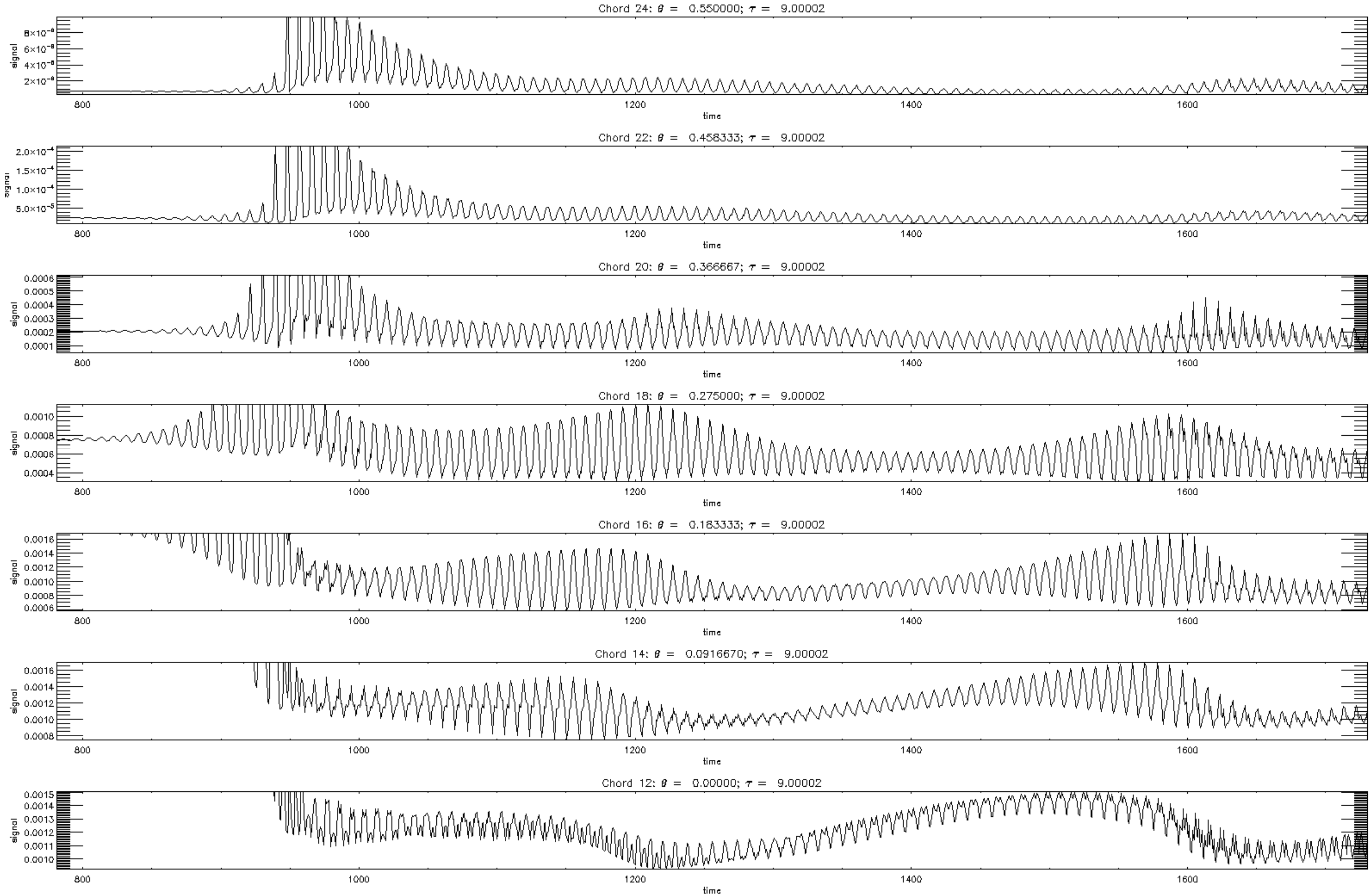
Magnetic Energy History by Mode Number



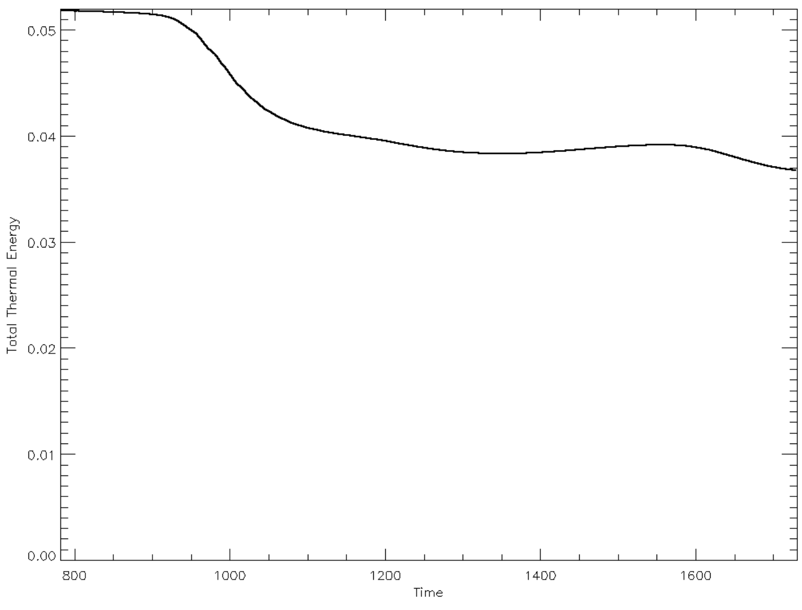
Chords for soft X-ray diagnostic

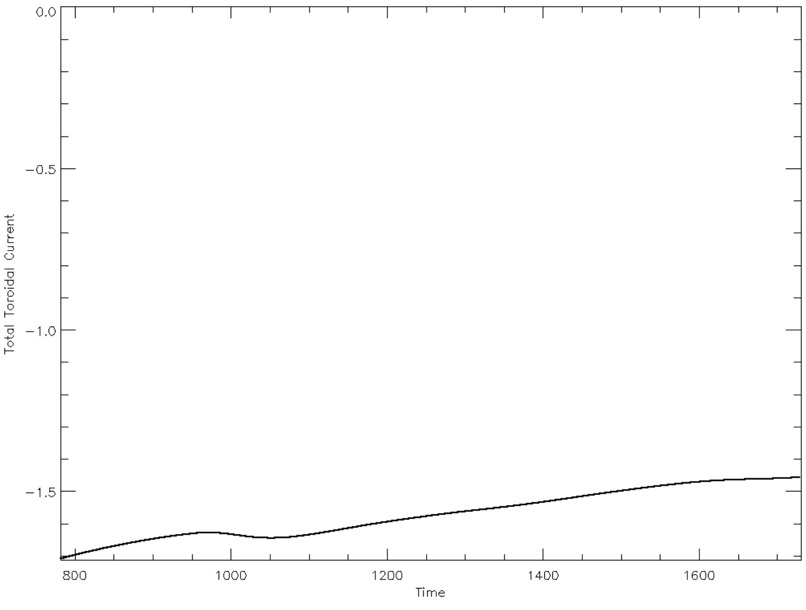


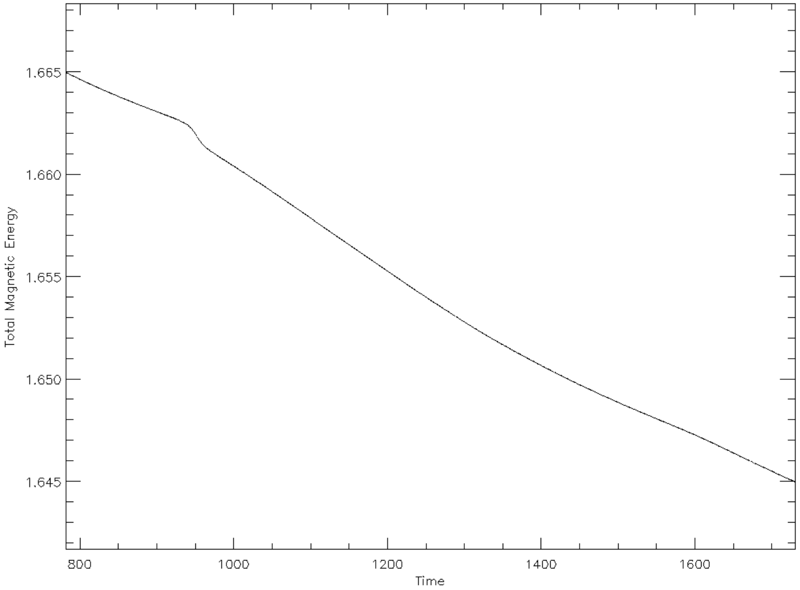
Simulated soft X-ray signal



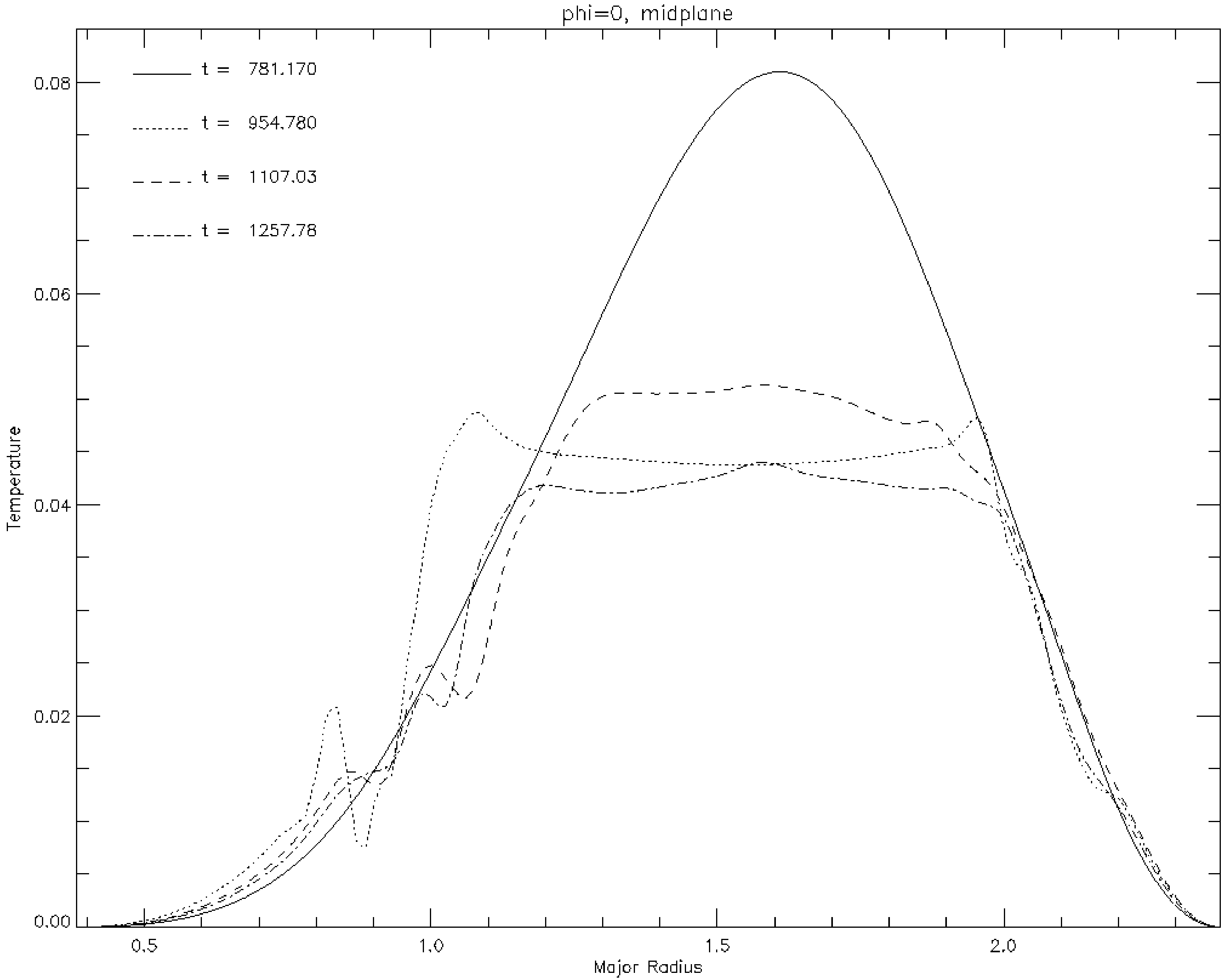
Conservation Properties



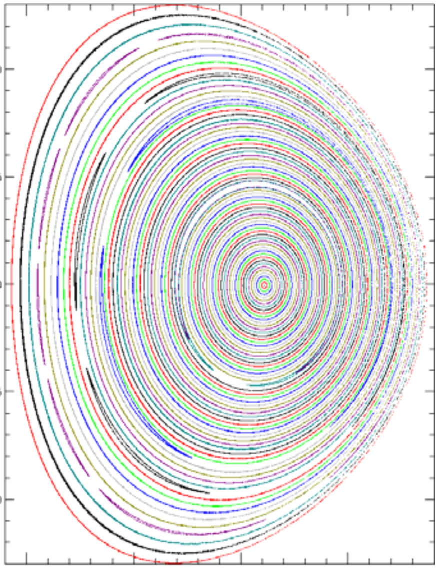




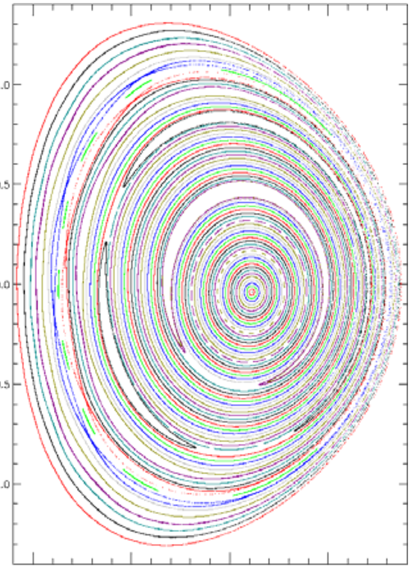
Temperature profiles



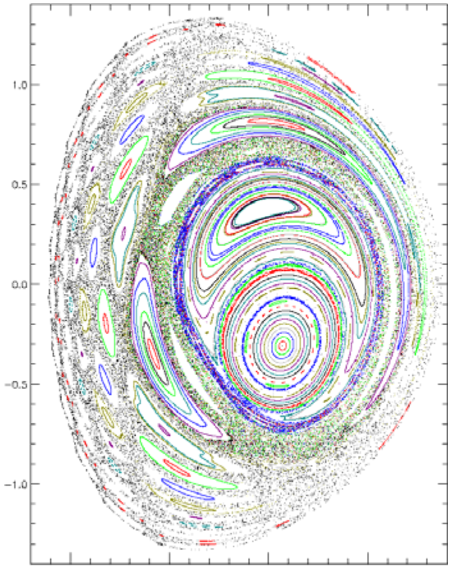
Poincaré plots



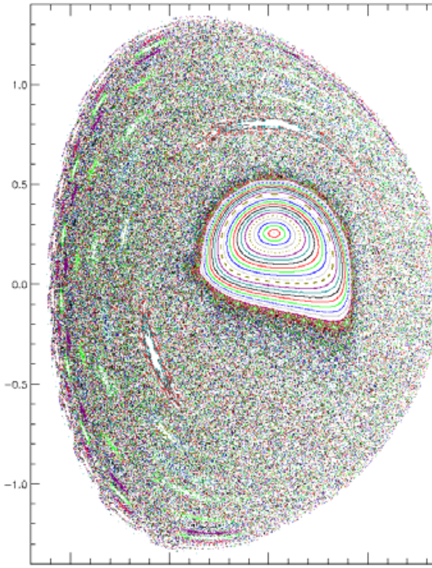
t = 781.17



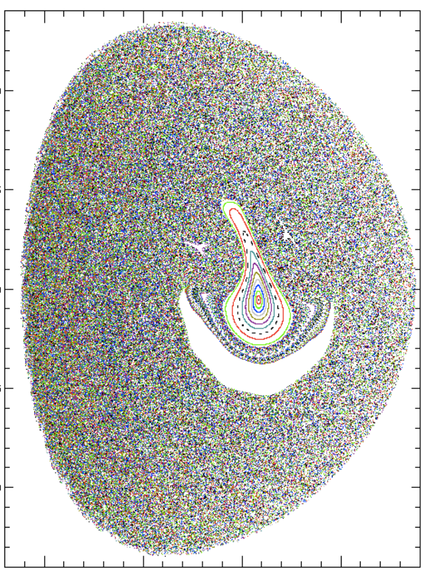
t = 859.53



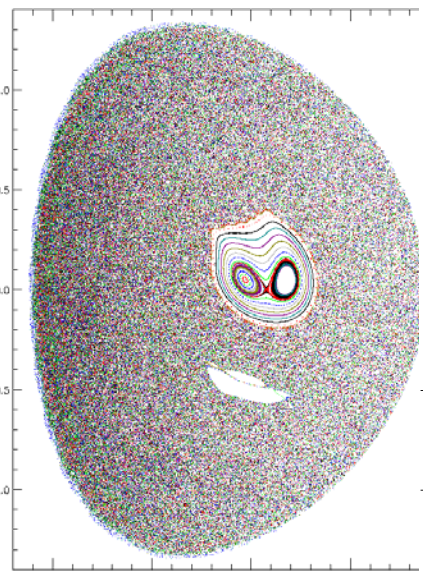
t = 930.03



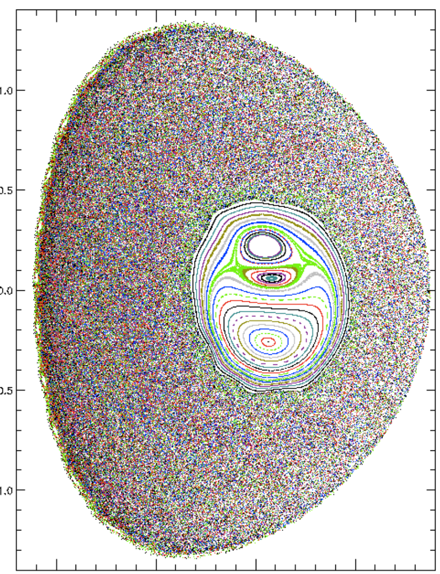
t = 954.78



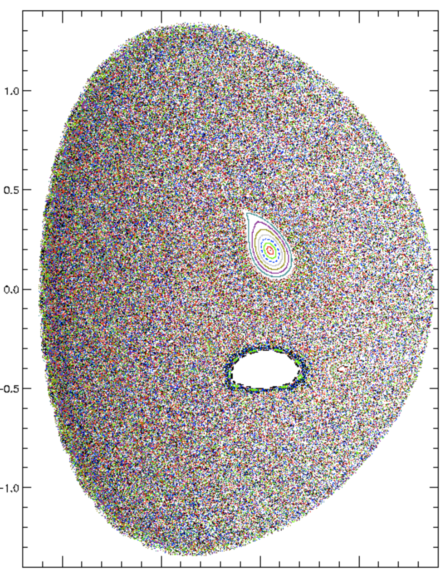
t = 1107.03



t = 1257.78



t = 1407.03



t = 1635.78

Summary

- Previous analytic equilibrium yielded insufficiently robust sawteeth; reconnection became incomplete, with period decreasing over time.

- New proposed equilibrium has more peaked T, p profiles giving a more unstable kink mode.

- Initial nonlinear run with new equilibrium shows large drop-off in energy after first crash, but less after second.
  - Violent crash produces extremely stochastic field.
  - Poor conservation of steady state thermal energy, plasma current suggests non-optimal coefficient for thermal conductivity profile.

- Next trial to use readjusted coefficient. Initial state is non-physical (q inaccessibly small), so conductivity should be adjusted to allow rapid convergence on steady repeating behavior in subsequent cycles.