# Two-Fluid NSTX Steady-States with Flow

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# Overview

- We are using M3D-$C^1$ to calculate axisymmetric toroidal steady-states of a comprehensive two-fluid model.
- These steady-states are steady on all timescales self-consistently include two-fluid effects, gyroviscosity, flow, and anisotropic transport.
- In particular, we would like to understand the effects of two-fluid terms and gyroviscosity on the steady-states.
- These steady-states may be used as accurate equilibria for three-dimensional stability studies.

# Physical Model

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = \Sigma$$

$$n\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = \mathbf{J}\times\mathbf{B} - (\nabla p + \nabla\cdot\Pi) - \mathbf{u}\Sigma$$

$$\frac{1}{\Gamma-1}\left[\frac{\partial p}{\partial t} + \nabla\cdot(p\mathbf{u})\right] = -p\nabla\cdot\mathbf{u} + \frac{d_i}{\Gamma-1}\frac{\mathbf{J}}{n}\cdot\left(\nabla p_e - \Gamma\frac{p_e}{n}\nabla n\right) - \nabla\cdot\mathbf{q} - \Pi : \nabla\mathbf{u} + d_i\Pi_e : \nabla\frac{\mathbf{J}}{n} + \frac{1}{2}u^2\Sigma$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla\times\mathbf{E}$$

$$\mathbf{E} + \mathbf{u}\times\mathbf{B} = \eta\mathbf{J} + \frac{d_i}{n}(\mathbf{J}\times\mathbf{B} - \nabla p_e - \nabla\cdot\Pi_e)$$

$$\mathbf{J} = \nabla\times\mathbf{B} \qquad \Sigma = \sigma + D\nabla^2 n$$

$$\Pi = \Pi_\circ + \Pi_\wedge + \Pi_\parallel \qquad \Pi_e = \lambda n\nabla\mathbf{J}$$

$$\mathbf{q} = -\kappa_\circ\nabla T - \kappa_\parallel\mathbf{B}\mathbf{B}\cdot\nabla T$$

# Digression: Interpretation of Gyroviscosity

- Ramos shows that dominant contribution of $\Pi_\wedge$ (absent parallel gradients) is

$$\nabla \cdot \Pi_\wedge \approx -mn\mathbf{u}_* \cdot \nabla \mathbf{u}$$

where

$$\mathbf{u}_* = -\frac{1}{mn}\nabla \times \left(\frac{p}{B^2}\mathbf{B}\right)$$

- Braginskii shows that difference between fluid drift velocity and average gyro-center drift velocity is

$$\mathbf{u} - \langle \mathbf{v}_c \rangle = -\frac{1}{mn}\nabla \times \left(\frac{p}{B^2}\mathbf{B}\right) = \mathbf{u}_*$$

- Therefore,

$$mn\mathbf{u} \cdot \nabla \mathbf{u} + \nabla \cdot \Pi_\wedge \approx mn(\mathbf{u} - \mathbf{u}_*) \cdot \nabla \mathbf{u} = mn \langle \mathbf{v}_c \rangle \cdot \nabla \mathbf{u}$$

- "Gyroviscous cancellation" cancels fluid drifts from inertia.

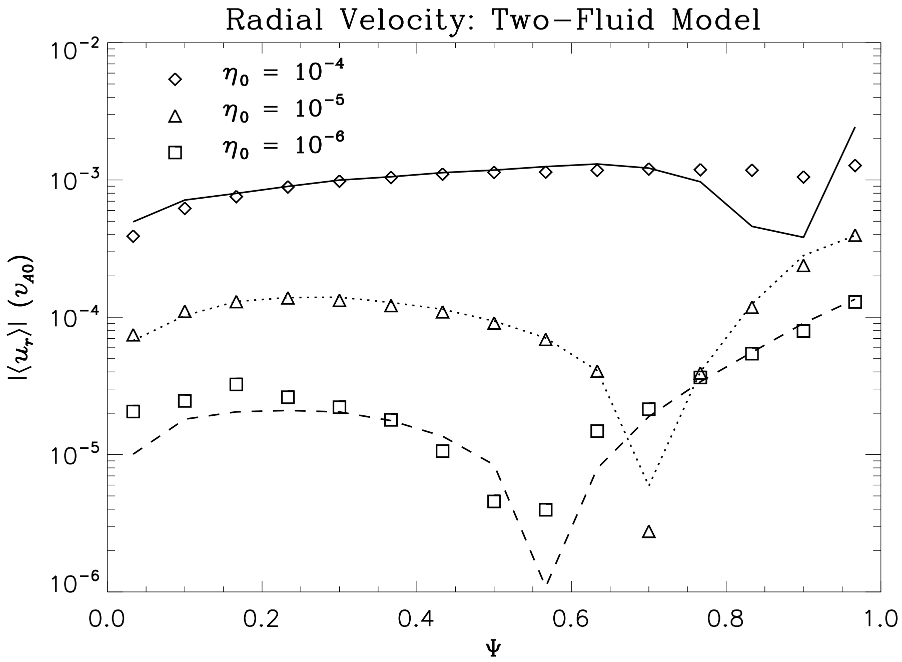
# Method

- The simulation is initialized with a solution to the Grad-Shafranov equation.
- A loop voltage is applied by changing the flux at the boundary of the simulation domain at a constant rate $\dot{\psi} = V_L/2\pi$.
- A localized density source is included to offset diffusive flux out of the simulation domain.
- The simulation is run until a steady state in all hydrodynamic quantities is reached (may not be stationary).
- $\eta = \eta_0 T^{-3/2}$. The vacuum region is simply a low temperature region outside the separatrix.

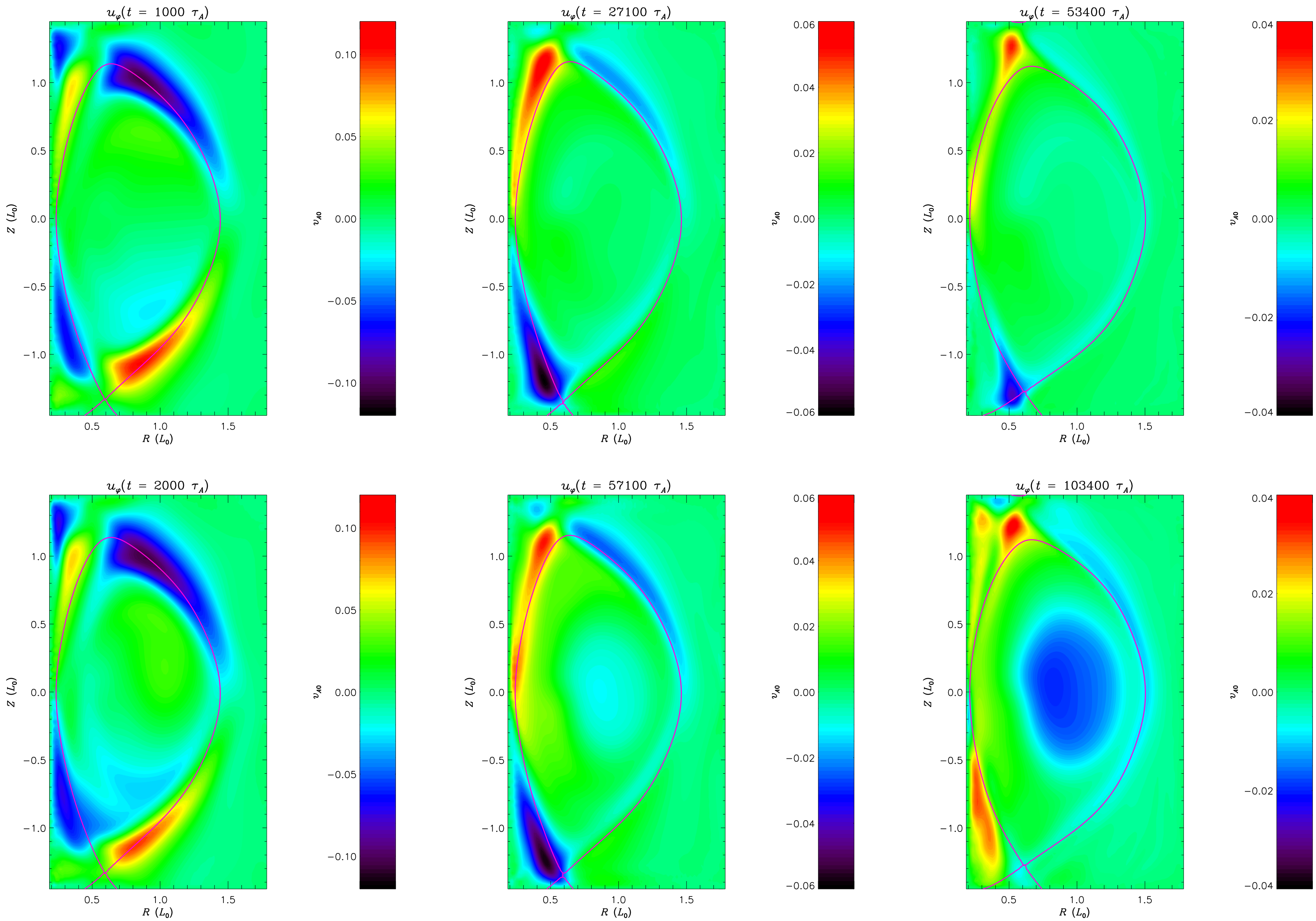
# Radial Flows

- Pfirsch-Schlüter theory of radial flows is well satisfied.

- Radial flows are proportional to $\eta$ and $V_L$.



# Toroidal Flow: Edge Flows

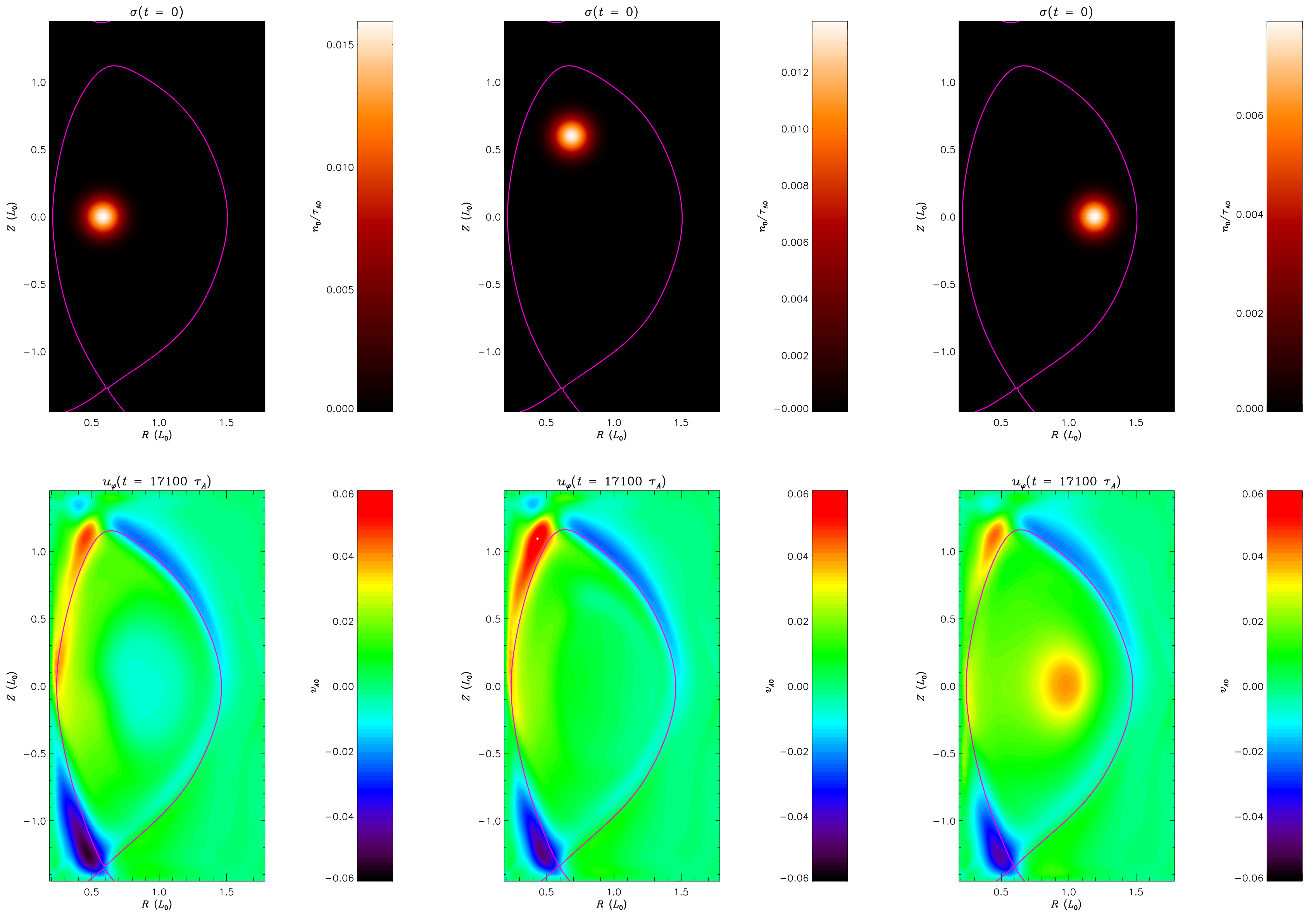



1F

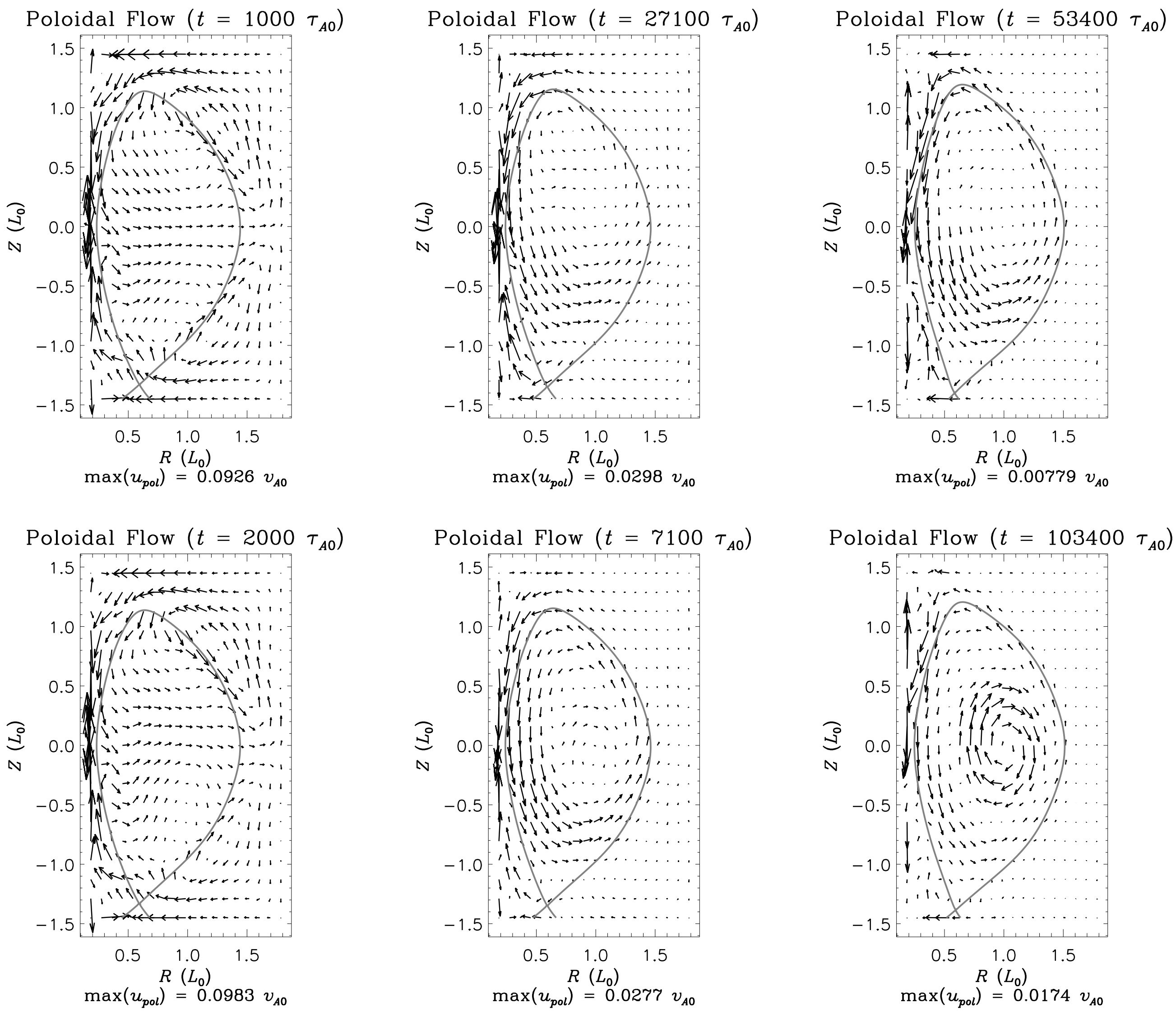
2F + GV

$\eta_0 = 10^{-4}$ $\eta_0 = 10^{-5}$ $\eta_0 = 10^{-6}$

# Toroidal Flow: Core Rotation



# Poloidal Flow



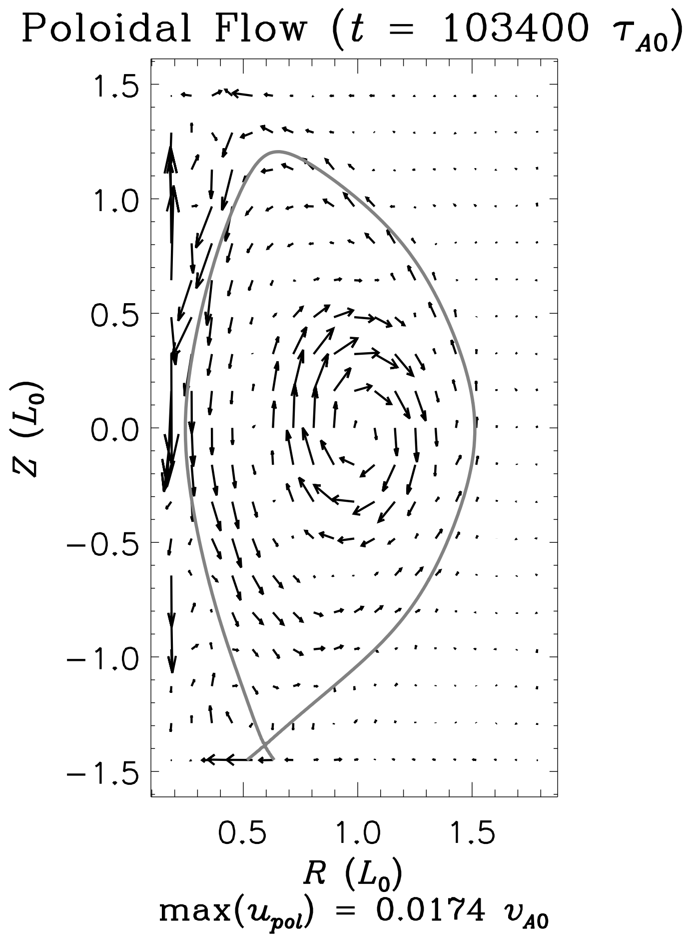

1F

2F + GV

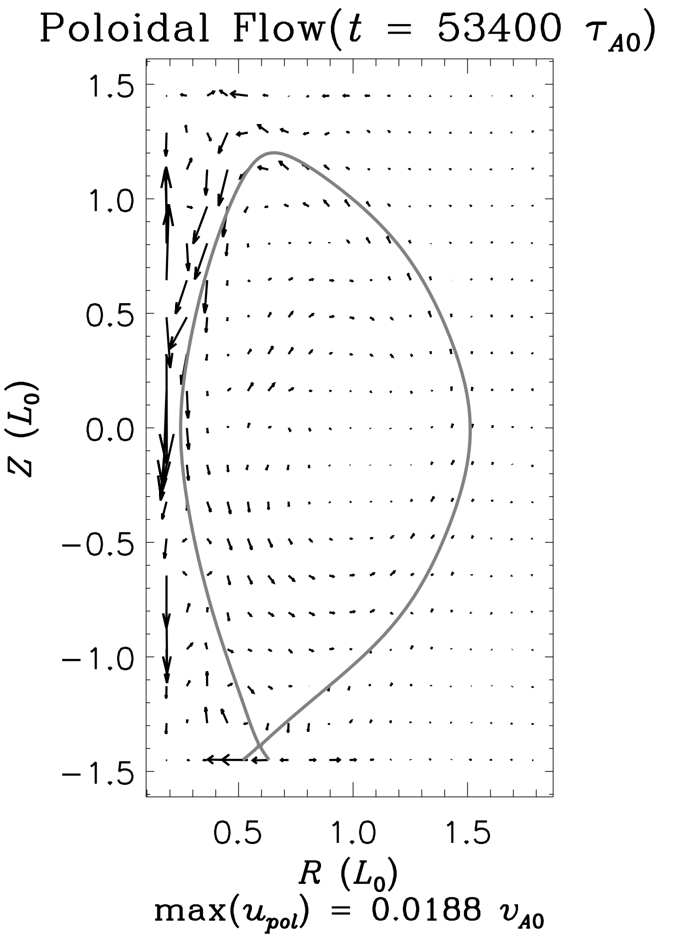
$\eta_0 = 10^{-4}$ $\eta_0 = 10^{-5}$ $\eta_0 = 10^{-6}$

# Parallel Viscosity

- Collisional parallel viscosity damps poloidal flows.
- Does not damp toroidal flows.




$\mu_{\|} = 0$




$\mu_{\|}/\mu_{\perp} = 10^5$

# Conclusions

- We have been able to obtain self-consistent steady-states of the extended-MHD equations for realistic plasma configurations with free boundaries.
- The flows observed in the steady-states are in good agreement with Pfirsch-Schlüter theory.
- Strong up-down antisymmetric toroidal edge flows exist in highly resistive SOLs.
- Hall term and electron pressure gradient have little effect on steady-state.
- In highly resistive, low viscosity case, gyroviscosity may lead to persistent, large-scale oscillations.

# Future Work

- Better modeling of edge/SOL
  - Realistic boundary shapes/conditions
  - Pedestal modeling for H-mode
  - "Rice Scaling" for spontaneous rotation $M_A \sim \beta_N$.
- Need some model for neoclassical parallel viscosity (bootstrap current).
- Coupling to realistic transport models (TGLF?)
- 3D nonlinear simulations