Solver Experience in M3D-C^1

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Overview

- Time-step in M3D-C^1
  - Eliminating steady-state error
  - Improving stability
- Solver issues
  - Moving to iterative methods

Time Step

- Currently use variation of "Caramana" method:

$$n\dot{\mathbf{u}} = (\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla p$$

$$\dot{B} = \nabla \times (\mathbf{u} \times \mathbf{B})$$

$$\dot{p} = -\mathbf{u} \cdot \nabla p - \Gamma p \nabla \cdot \mathbf{u}$$

Taylor expand velocity in each equation..

$$n\dot{\mathbf{u}} = [\nabla \times (\mathbf{B} + \theta\delta t\dot{\mathbf{B}})] \times (\mathbf{B} + \theta\delta t\dot{\mathbf{B}}) - \nabla(p + \theta\delta t\dot{p}) \tag{1}$$

$$\dot{B} = \nabla \times (\mathbf{u} + \theta\delta t\dot{\mathbf{u}}) \times \mathbf{B} \tag{2}$$

$$\dot{p} = -(\mathbf{u} + \theta\delta t\dot{\mathbf{u}}) \cdot \nabla p - \Gamma p \nabla \cdot (\mathbf{u} + \theta\delta t\dot{\mathbf{u}}) \tag{3}$$

Parabolize equations by using (2) and (3) to eliminate $\dot{\mathbf{B}}$ and $\dot{p}$ in (1)...

Time Step

$$(n - \theta^2 \delta t^2 \mathcal{L})\mathbf{u}^{n+1} = (n - \theta(\theta - 1)\delta t^2 \mathcal{L})\mathbf{u}^n + \delta t[(\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla p]$$

where $\mathcal{L}$ is ideal MHD operator

$$\mathcal{L}(\mathbf{u}) = [\nabla \times \nabla \times (\mathbf{u} \times \mathbf{B})] \times \mathbf{B} + (\nabla \times \mathbf{B}) \times [\nabla \times (\mathbf{u} \times \mathbf{B})] + \nabla(\mathbf{u} \cdot \nabla p + \Gamma p \nabla \cdot \mathbf{u})$$

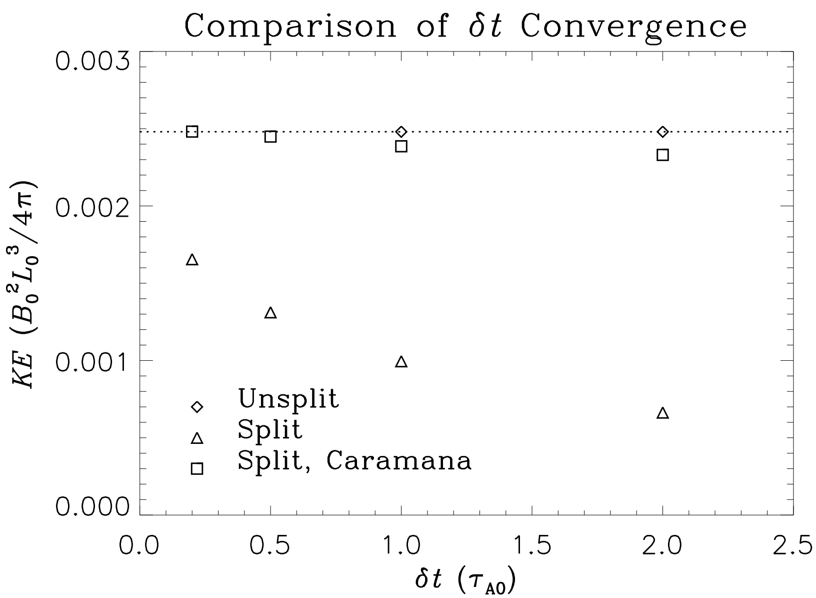
Caramana method:

$$(n - \theta^2 \delta t^2 \mathcal{L})\mathbf{u}^{n+1} = (n - \theta^2 \delta t^2 \mathcal{L})\mathbf{u}^n + \delta t[(\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla p]$$

eliminates steady-state error.

- n, p, $\mathbf{B}$ then implicitly advanced using $\mathbf{u}^{n+1}$.

Time Step: Comparison with Other Methods



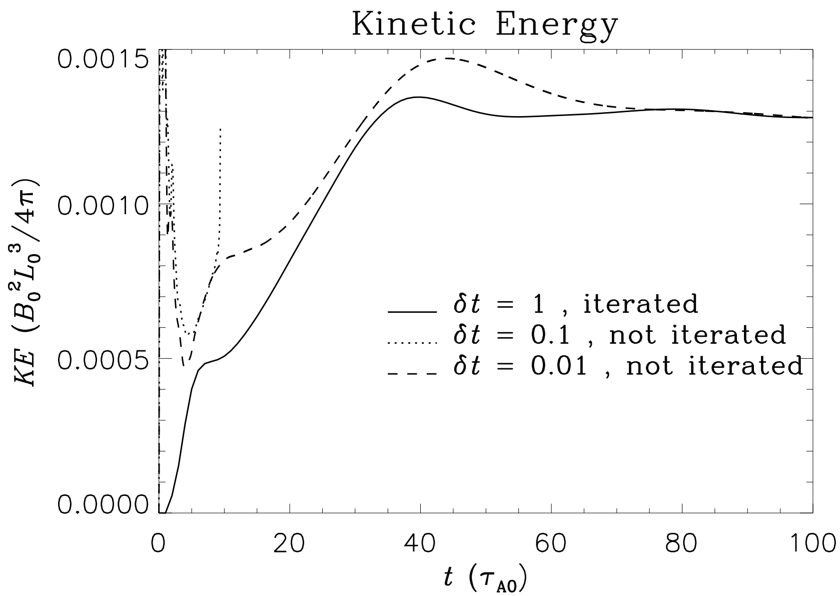
- Also implemented: split θ-implicit method without Caramana modification; unsplit θ-implicit method.

- Caramana method is nearly as accurate as "unsplit" method.

- Caramana method with iteration is found to be more stable than unsplit method.

- For simple case, unsplit time step takes $\sim$ 6–7 times as long as split step

Time Step: Field Iteration

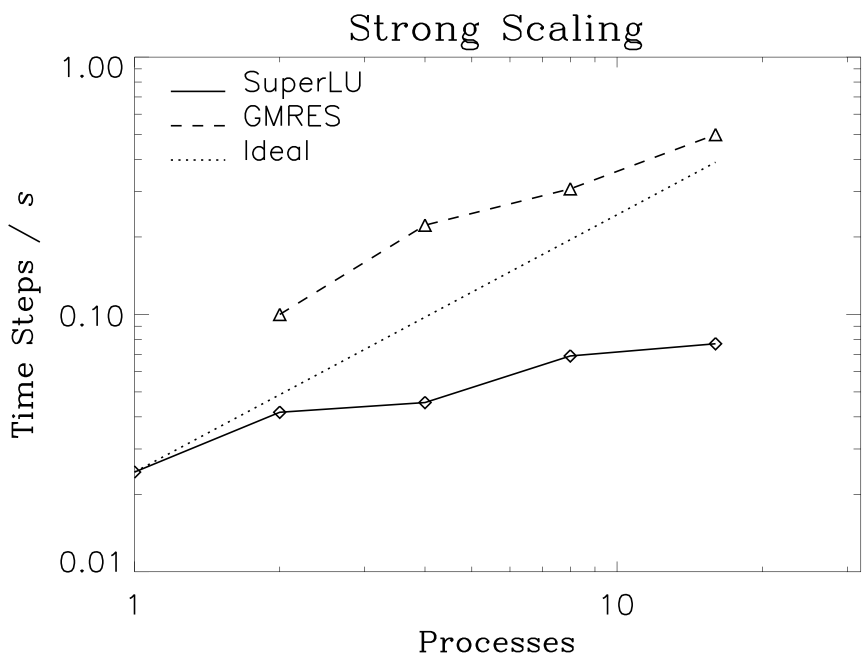


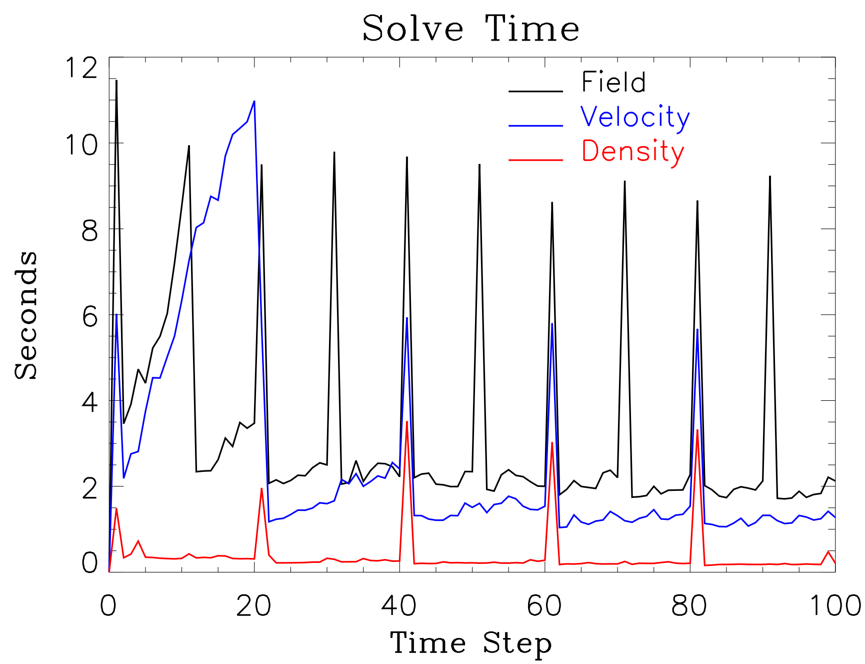
- Stability problems can arise when convection and strongly anisotropic thermal conduction is included in low-resistivity core.
- Improved stability is achieved by calculating transport coefficients after field/pressure advance, then re-calculating field/pressure advance.
- CPU time per time step increased by 50%; maximum stable timestep increased by factor of 10–100.

Linear Solves

- M3D-C^1 uses a linear implicit time step.
- Compactness of matrix allows relatively efficient direct (LU) solves for 2D problems.
- Iterative methods are necessary for 3D problems.
- A. Bauer, X. Luo, J. Chen have implemented PETSc in M3D-C^1
- Early results with iterative solves are encouraging.

Linear Solves: LU vs. GMRES





- LU factorization from earlier timestep is used as preconditioner.
- For some applications, the preconditioner need not be updated frequently.

Conclusions

- Steady-state solution dramatically improved by modifying semi-implicit method in manner of Caramana.
- Stability of this method dramatically improved by iterating field/resistivity calculation (in nonlinear, core plasma scenario).
- Iterative solve results are encouraging.