

Continuum solution to Chapman-Enskog-like drift kinetic equation (CEL-DKE) in NIMROD

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Motivation for continuum solution to closure problem.

- Treat time-dependent problems such as Landau damping or coupling of closures to rapidly evolving instabilities.
- Easily incorporate nonlinearities and particle trapping effects.
- Increase the efficiency of the closure calculation.
- Incorporate accelerations effects.
- Solve lowest-order Chapman-Enskog-like drift kinetic equation:

$$\begin{aligned} & \frac{\partial F}{\partial t} + \vec{v}_{\parallel} \cdot \vec{\nabla} F + \left(\frac{q}{m} \frac{\vec{v}_{\parallel}}{v} \cdot \vec{E} \frac{\partial}{\partial v} - \frac{\mu}{B} \frac{\partial B}{\partial t} \frac{\partial}{\partial \mu} \right) F - \mathcal{C}(F + f_M) = \\ & -\frac{2}{3} L_1^{1/2} \left(\frac{d \ln T}{dt} + \vec{\nabla} \cdot \vec{V}_1 \right) f_M + L_1^{3/2} \vec{v}_{\parallel} \cdot \nabla \ln T f_M + \vec{v}_{\parallel} \cdot (\vec{\nabla} \cdot \Pi - \vec{R}) f_M \\ & + 2s^2 P_2 \left(\frac{v_{\parallel}}{v} \right) (\mathbf{b}\mathbf{b} - \frac{\mathbf{I}}{3}) : \vec{\nabla} \vec{V}_1 f_M \end{aligned}$$

Simplify.

- Consider solving CEL-DKE by expanding $F = \sum F_i(\mathbf{x}, t) \phi_i(v_{\parallel}, v)$:

$$\frac{\partial F}{\partial t} + v_L L(F) + \vec{v}_{\parallel} \cdot \vec{\nabla} F + \left(\frac{q}{m} \frac{\vec{v}_{\parallel}}{v} \cdot \vec{E} \frac{\partial}{\partial v} - \frac{\mu}{B} \frac{\partial B}{\partial t} \frac{\partial}{\partial \mu} \right) F =$$

$$C_{(-L)}(F + f_M) + \text{CEL drives}$$

- **Acceleration** and collision terms, $C_{(-L)}(F + f_M)$, couple velocity expansion coefficients in speed variable, v .
- Preliminary implementation ignores acceleration term and uses a moment approach for $C_{(-L)}$ (J-Y Ji's work and J. James' thesis).

Existing implementation solves for coefficients of F expansion on grid in $s=v/v_T$.

- Expanding $F = \sum F_l(\mathbf{x}, v, t) P_l(v_{||}/v)$ yields:

$$\left(\mathbf{I} \frac{\partial}{\partial t} - v_L \mathbf{L} \right) \vec{F} + \mathbf{A} v \hat{\mathbf{b}} \cdot \vec{\nabla} \vec{F} - \mathbf{B} v (\hat{\mathbf{b}} \cdot \vec{\nabla} \ln B) \vec{F} = \text{drives},$$

$$\text{where } v_L = \sum_b \frac{v_{ab}}{s_a^3} \left(\left(2 - \frac{1}{s_b^2} \right) E(s_b) + \frac{E'(s_b)}{s_b} \right)$$

- Matrices **A**, **B**, and **L** represent free-streaming, |B| and pitch-angle collisional couplings, respectively.
- With Lorentz pitch-angle scattering operator, speed enters as a parameter only.
- Solve equation on grid in s .

Coupling to the fluid equations: time discretization and parallelization issues (I).

- Nimrod uses staggered advance.

$$\vec{V} \quad (n, \vec{B}, T)$$

- For parallel heat flow closure, can couple fully implicit solves for T and F.

$$\vec{V} \quad (n, \vec{B}, (T, F))$$

- Must solve simultaneously for coefficients, F_i , on speed grid, $s_i, i=1, \dots, n$.
- Closure moment $q_{||}$ couples to T equation.
- Leads to large system of equations with “parallelization” performed inside solver.

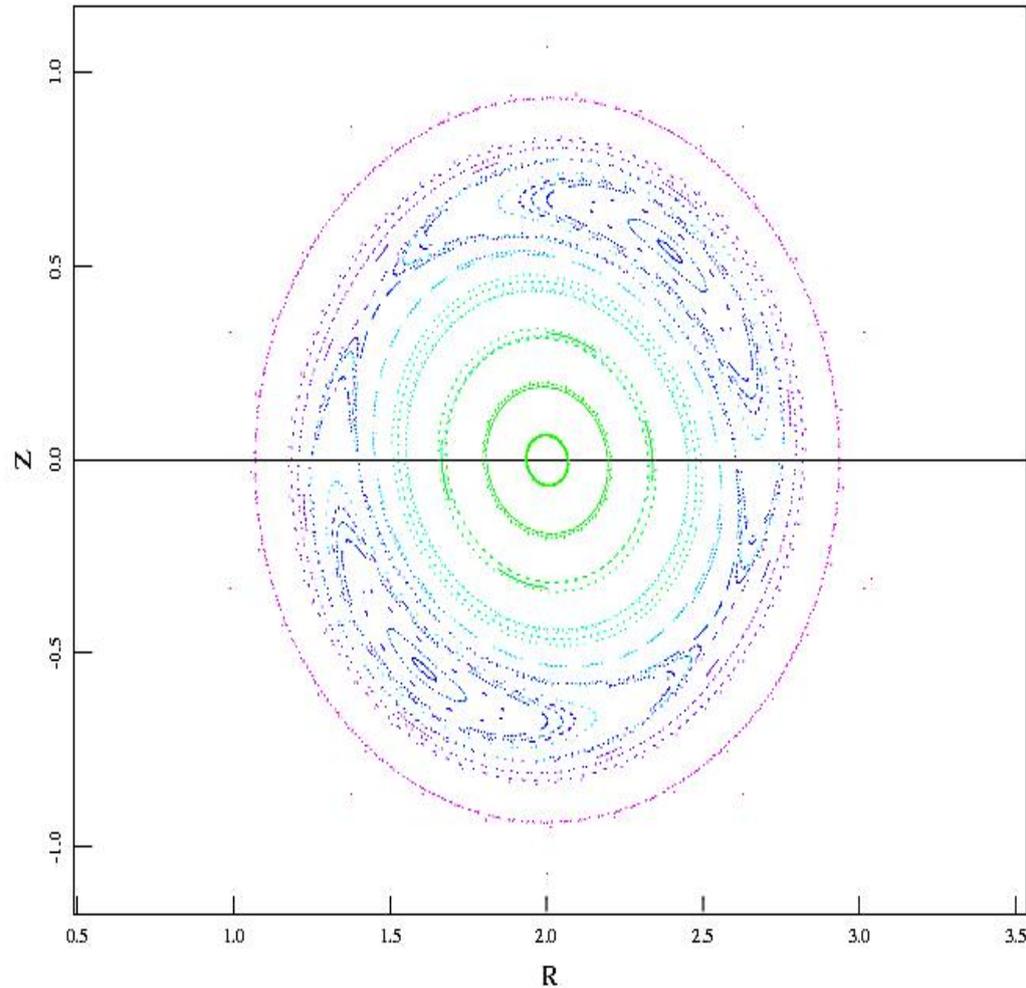
Coupling to the fluid equations: time discretization and parallelization issues (I).

$$\frac{3}{2} n \Delta T + \Delta t \vec{\nabla} \cdot q_{\parallel} \hat{b} = \dots, \quad T \text{ equation}$$

$$\begin{aligned} & ((a_{l,l} / s_i) + \theta \Delta t (v_L(s_i) / s_i) L_{l,l}) \Delta F_l + \\ & \theta \Delta t \hat{b} \cdot \vec{\nabla} (a_{l,l+1} \Delta F_{l+1} + a_{l,l-1} \Delta F_{l-1}) + \\ & \theta \Delta t (\hat{b} \cdot \vec{\nabla} \ln(B)) [b_{l,l+1} \Delta F_{l+1} + b_{l,l-1} \Delta F_{l-1}] + \\ & \delta_{ll} a_{l,l} L_1^{3/2} f_M \hat{b} \cdot \Delta \ln T = \dots, \quad \text{coupled equations for } F_l \end{aligned}$$

where $q_{\parallel} = -T \int d\vec{v} v_{\parallel} L_1^{3/2} P_1(v_{\parallel}/v) F_1$

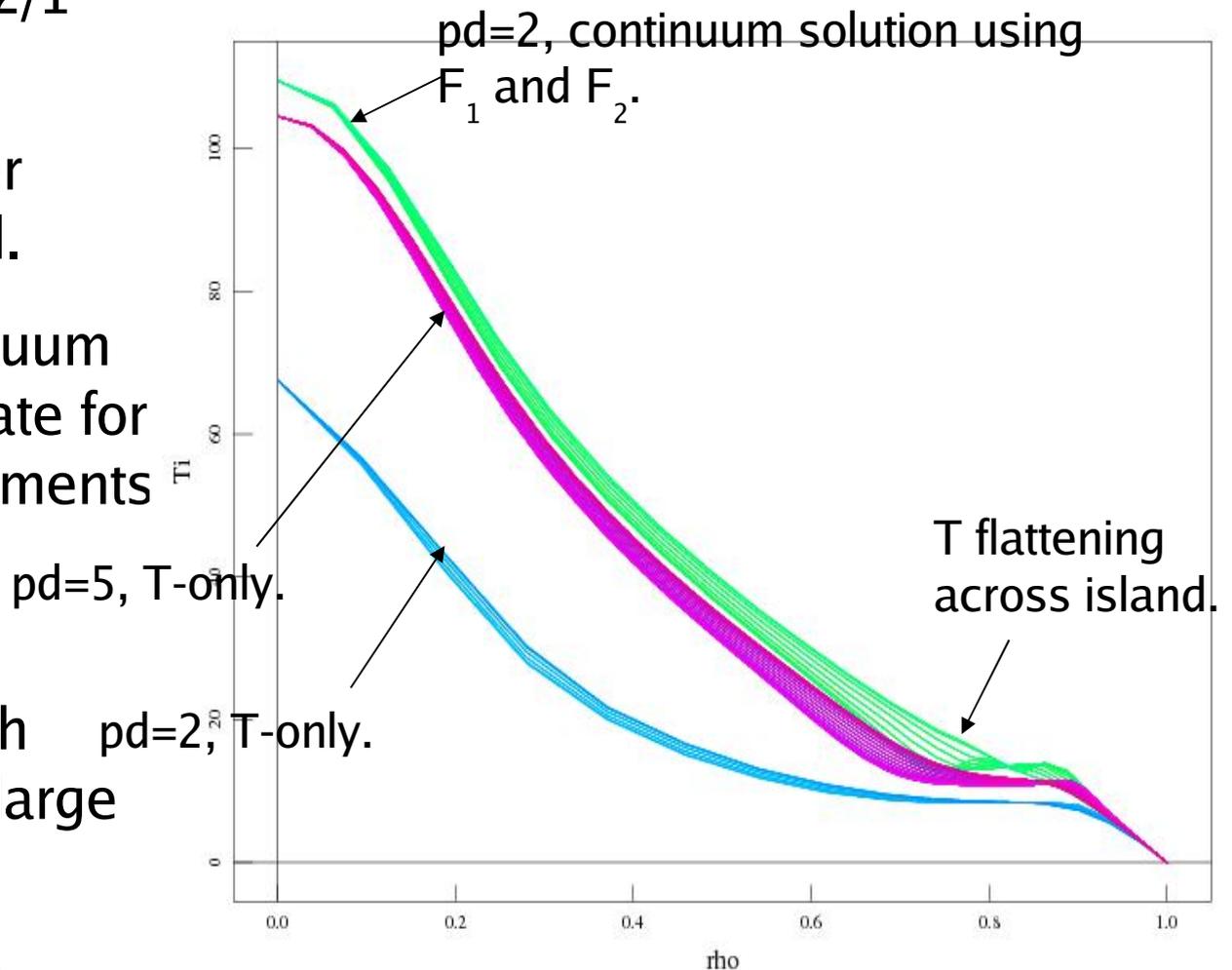
Applied to heat transport problem in cylindrical geometry (Hölzl *et al.*, POP 2008).



2/1 island added into zero pressure, cylindrical eq.
Heat source finite for $r < 0.2$ and zero outside.

Can solve for T coupled to F equations.

- T profiles as function of flux show flattening across 2/1 island.
- All cases used 3 Fourier modes and 10 x 10 grid.
- Predicted T from continuum solution spatially accurate for bicubic (pd=2) finite elements
- Here ν chosen to yield heat flow consistent with Braginskii closure with large parallel conductivity.
- **Solution not resolved in velocity variables!**



Generalize velocity space basis to include 2-D finite elements.

- Expand $F = \sum F_i(\mathbf{x}, t) \phi_i(\xi, \mu)$, where ξ and μ are appropriate velocity variables and insert into CEL-DKE:

$$\frac{\partial F}{\partial t} + \vec{v}_{\parallel} \cdot \vec{\nabla} F + \left(\frac{q}{m} \frac{\vec{v}_{\parallel}}{v} \cdot \vec{E} \frac{\partial}{\partial v} - \frac{\mu}{B} \frac{\partial B}{\partial t} \frac{\partial}{\partial \mu} \right) F - C(F + f_M) = \text{drives}$$

- Integrate equation using $\int d\vec{x} \int d\vec{v} \phi_{i'}(\xi, \mu) \alpha_{j'}(R, Z) e^{-in'\phi}$
- Use NIMROD finite element and Gaussian quadrature machinery to compute velocity integrals.
- Consider C^0 or possibly discontinuous basis functions.
- Requires new data types and solver development.

Future work.

- At present, can couple solution for coefficients of Legendre expansion, $F_l(\mathbf{x}, t)$, on grid in s to T in fully implicit advance.
- Future work includes:
 - testing convergence of fully implicit advance as Legendre polynomials and grid points in speed are added.
 - implementing staggered advance and comparing with fully implicit solutions.
 - generalizing continuum solution to allow for 2-D finite-element basis functions for velocity dependence.