Plasma Heat Transport in Around a Single Helicity Island

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In most simulations, Braginskii closure used for heat flux

- Rigorously only valid in highly collisional plasmas with short mean free-path
- Anisotropic, Ficksian diffusion

$$\mathbf{q}_\alpha = -\kappa_{\|}\mathbf{b}\mathbf{b}\cdot\nabla T - \kappa_{\wedge}\mathbf{b}\times\nabla T - \kappa_{\perp}\nabla_{\perp}T$$

- Coefficients are strongly temperature-dependent:

$$\kappa_{\|} \sim T^{5/2}; \quad \kappa_{\wedge} \sim T; \quad \kappa_{\perp} \sim T^{-1}$$

DIII-D parameters

$$\sim 10^{10}\frac{m^2}{s} \quad \sim 100\frac{m^2}{s} \quad \sim 1\frac{m^2}{s}$$

Measured values Anomalous

- Used widely in plasma fluid modeling

How well does this work for high-temperature plasmas?

Non-local closure more rigorously derived for collisionless regime

- After a long derivation:

$$q_{\|}(L) = \frac{n v_{th}}{\pi^{3/2}} \int K(L',L)\left[T(-L') - T(L')\right]\, dL'$$

Collisional, magnetic field geometry physics

- Equation is non-local: integral must be taken over many mean free-paths (~10 km in DIII-D)
- Changes differential equation into an integro-differential equation
- It is a form of resolving multiscale problem: Allows parallel electron velocity time scale to be captured on MHD time scales

Non-locality is complicated and makes parallelism more complex

But is it needed to model such effects as RMP cases?

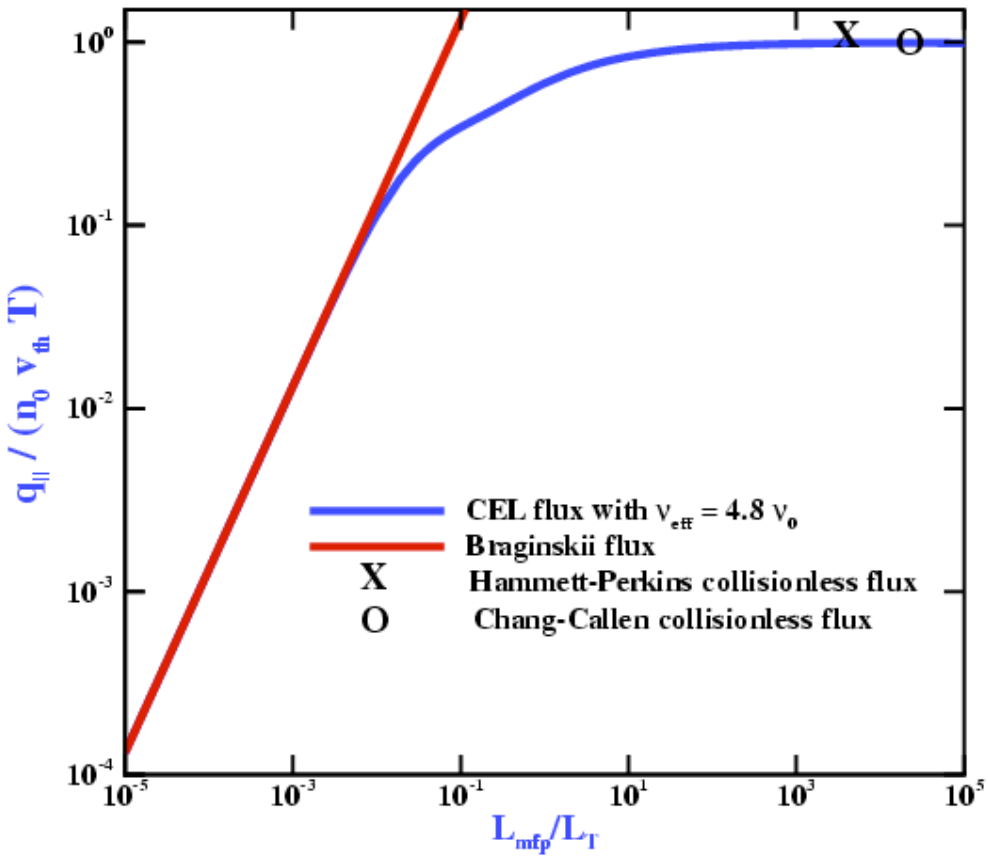
Is it possible to use a local closure to approximate a nonlocal closure?

- **Interest in several fields in addition to fusion extended MHD:**
  - Laser-plasma simulations
    e.g., *Tahraoui and Bendib, Phys. Plasmas* ***9****, 3089 (2002).*
  - Edge transport simulations
    Review article: Fundamenski, PPCF ***47****, R163 (2005).*
  - Space plasmas
    e.g., *Sharma et.al, Astrophysical Journal* ***637****, 952 (2006)*
  - Landau-fluid efforts in the 90's
    Hammett, Dorland, Beer, Snyder, …

What guidance does the non-local closure give for guidance on forms of a local closure?

Closure allows for calculation of heat flux in all collisionality regimes

- In a uniform plasma



Held et.al., PoP 2001

Several local closures have been proposed to model non-local effects

- **Flux-limited Braginskii**

$$\kappa_{||} = Max(CT^{5/2}, \mathrm{v}_T L)$$

  - Fitting factors can be used to match more accurate kinetic results (*e.g., Fundamenski)*
- **Wave equilibration**
  - M3D subsycling of a wave-equation for parallel equilibration
- **Polytopic model**
  - Set ratio of specific heats (Gamma) to near 1
  - Equivalent to a parallel heat flux of the form q=alpha V
  - Takes infinite energy to perfectly equilibrate flux
- **Landau-fluid closure**
  - Use Fourier decomposition in two directions approximate $k_{||}$

Key questions:

What is the proper test for determining which model gives most accurate answer?

Near Rational Surfaces, Parallel Thermal Conduction Ineffective

- Temperature equation:

$$n\frac{dT_s}{dt} \sim \kappa_{\parallel}\nabla_{\parallel}{}^2 T + \kappa_{\perp}\nabla_{\perp}{}^2 T$$

- Recall, parallel gradient goes to zero at rational surface:

$$\mathbf{B}\bullet\nabla T \sim \left(B^{\theta}\frac{\partial}{\partial\theta} + B^{\phi}\frac{\partial}{\partial\phi}\right)T \sim B^{\theta}T_{mn}(m-nq)$$

- Time for || equilibriation near r_s:

$$\tau_{\parallel} \sim \frac{1}{\kappa_{\parallel}\nabla_{\parallel}^2} \sim \frac{1}{\kappa_{\parallel}(m-nq)^2} \sim \frac{a^4}{\kappa_{\parallel}x^2}$$

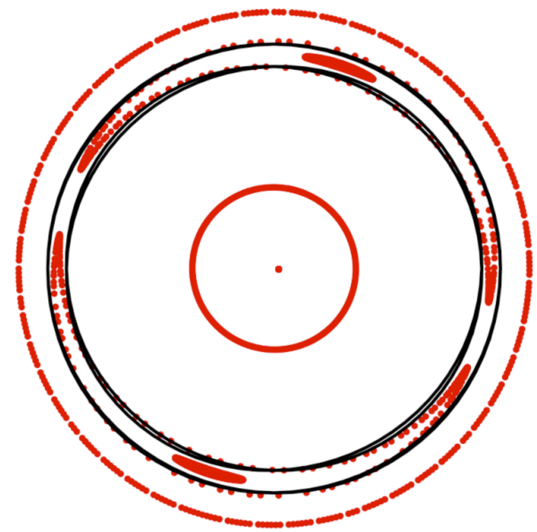
where x = distance from rational surface

- Time for perpendicular diffusion:

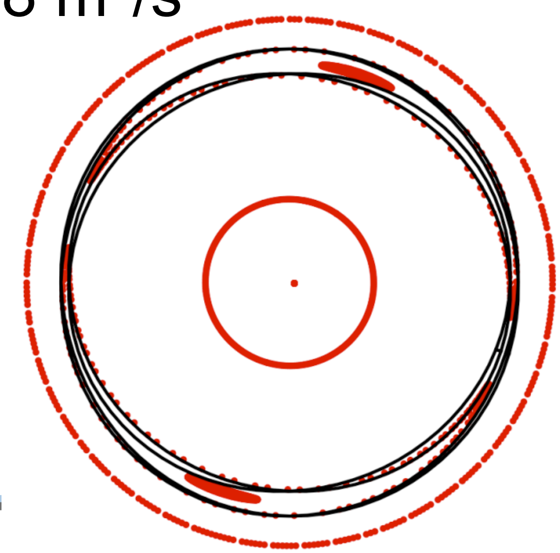
$$\tau_{\perp} \sim \frac{1}{\kappa_{\perp}\nabla_{\perp}^2} \sim \frac{x^2}{\kappa_{\perp}}$$

[Fitzpatrick, PoP 2, 825 (1995)]

$\chi_{\parallel}$=2E5 m^2/s



$\chi_{\parallel}$=2E8 m^2/s



Scale length exists over which parallel equilibriation competes with perpendicular equilibration

- Time for || equilibriation near r_s:

$$\tau_{\parallel} \sim \frac{1}{\kappa_{\parallel}\nabla_{\parallel}^{2}} \sim \frac{1}{\kappa_{\parallel}(m-nq)^{2}} \sim \frac{a^{4}}{\kappa_{\parallel}x^{2}}$$

where x = distance from rational surface

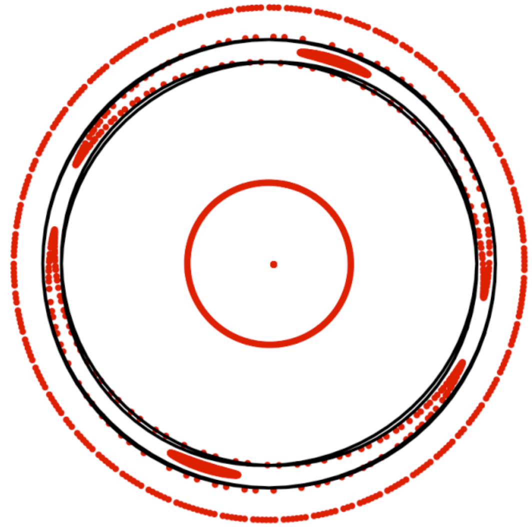
- Time for perpendicular diffusion:

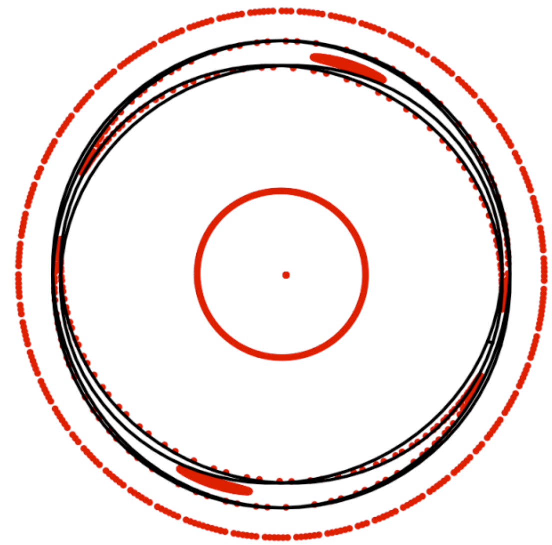
$$\tau_{\perp} \sim \frac{1}{\kappa_{\perp}\nabla_{\perp}^{2}} \sim \frac{x^{2}}{\kappa_{\perp}}$$

- Equating times gives characteristic length scale w_d for competition: $\frac{w_d}{a} \sim \left(\frac{\kappa_{\parallel}}{\kappa}\right)^{-\frac{1}{4}}$

- Temperature scaling: $\kappa_{\parallel} \sim T^{5/2}$ $\quad T \sim w_d^{\frac{8}{5}}$

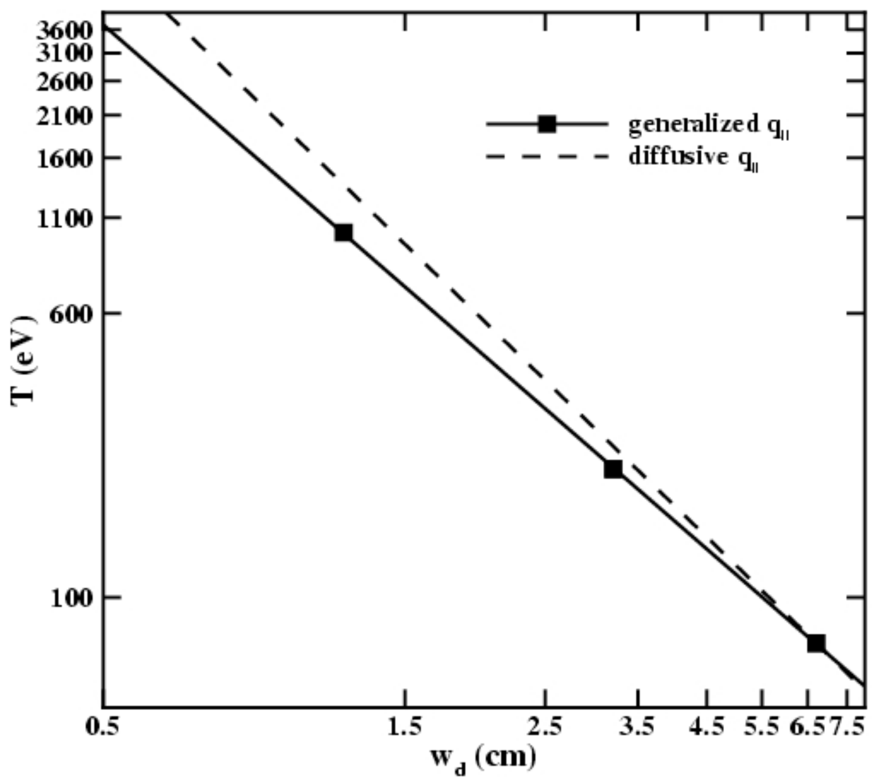
Use test of temperature scaling to determine accuracy of local model





Non-local closure gives different temperature scaling of w_d

- Empirical scaling: $T \sim w_d^{-3/2}$
  - Fit to low-order fraction is empical measurement
- Compare to Braginskii which gives: $T \sim w_d^{-8/5}$



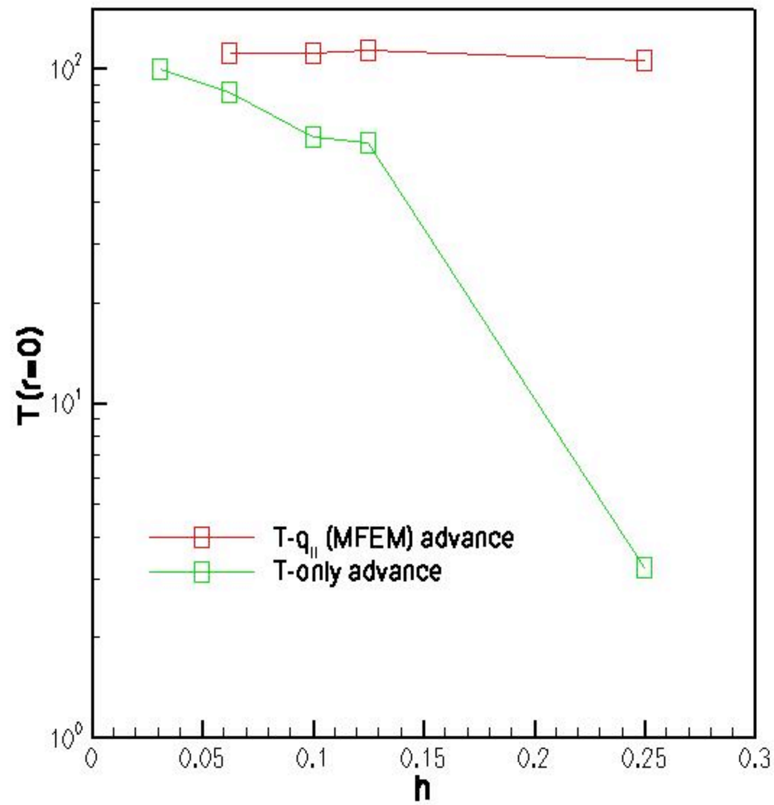
Also worry about total magnitude of w_d

Recent NIMROD developments have enabled advanced closure investigations

- Some of the closures require non-symmetric solves for treating implicitly
  - **New GMRES solver (Sovinec, Barnes)**
- NIMROD's new Mixed Finite Element Method implementation enables implementation of new models easily **(Held, Sovinec)**

$$n\frac{\partial T}{\partial t} = \nabla \bullet \left[\kappa_{\perp} \nabla T - \sqrt{f_0} B \tilde{q}_{\|} \hat{b} \right]$$

$$\frac{f_0 B \tilde{q}_{\|}}{f_1} = \frac{\kappa_{\perp} \hat{b} \bullet \nabla T - q_{\|}}{f_1}$$



- Scaling factors needed for accurate convergence of linear solve

Plans for future work

- **Do systematic scan of heat flux at various temperatures and perturbation sizes**
  - For 6 local models and the non-local model
  - First in single helicity cases in both cylindricial and toroidal geometries (to elucidate mod-B effects)
  - Next in stochastic transport cases, especially the edge plasmas relevant for RMP cases
  - Also plan to extend studies to time-dependent studies once time-dependent DKE-continuum closure becomes available