Recent Developments and Stability Results with M3D-C1

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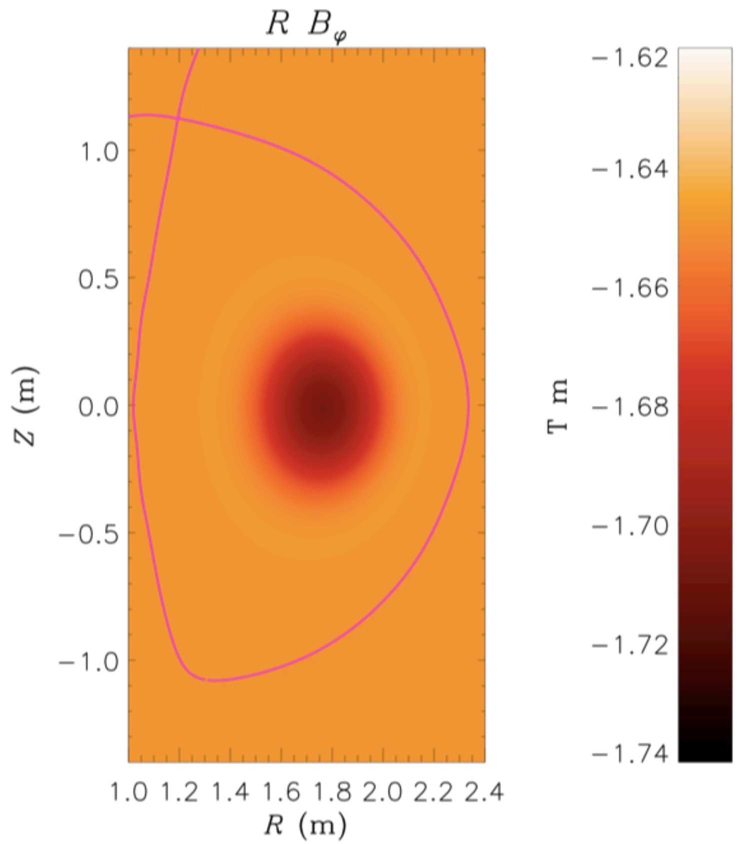
CEMM Meeting
Atlanta, GA
November 1, 2009

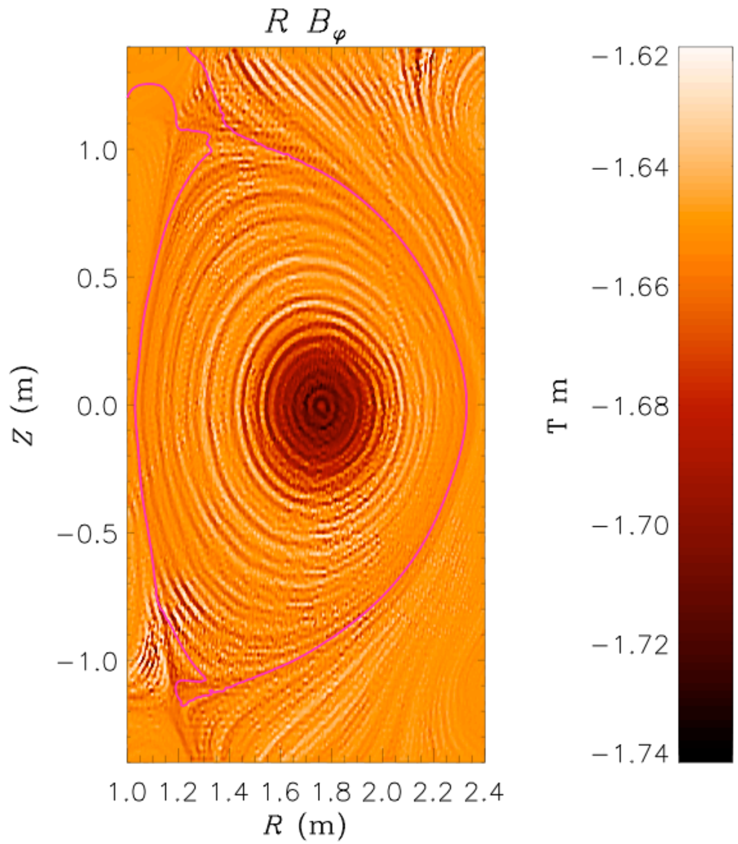
What's New in M3D-C[1]

- **Non-axisymmetric linear equations**
  - Gyrovisc., parallel visc., parallel kappa
- **Ability to read in EFIT, GATO, TOQ, etc.**
- **Semi-implicit Hall operator**
- **New method for specifying boundary conditions**
- **Non-axisymmetric external currents**
- **Capability for discontinuous resistivity**
- **New physics/benchmarking results**
  - See posters TP8.22 and UP8.97 on Thursday

Two Fluid "Noodling" Instability

- **Hall term leads to noodling instability at low η**





Two-Fluid "Noodling" Instability

- One solution is Arakawa differencing (symmetrize differencing operator)[1]
- Another solution is Harned-Mikic dispersion[2]

$$\left(1-\theta^2\delta t^2 L\right)\mathbf{B}^{n+1} = \left(1-\theta^2\delta t^2 L\right)\mathbf{B}^n - \delta t\,\frac{\mathbf{J}\times\mathbf{B}}{ne}$$

$$L \propto -(\mathbf{B}\cdot\nabla)^2\nabla^2$$

- We have implemented "Hall" operator:

$$L(\underline{\mathbf{B}}) = \nabla\times\left\{\left[\nabla\times\nabla\times(\underline{\mathbf{J}}\times\mathbf{B})\right]\times\mathbf{B} + \underbrace{\mathbf{J}\times\left[\nabla\times(\mathbf{J}\times\mathbf{B})\right]}_{\text{not yet implemented}}\right\}$$

[1]Salmon and Talley, *JCP* 83:247 (1988)

[2]Harned and Mikic, *JCP* 83:1 (1989)

Two-Fluid "Noodling" Instability

- **Derivation follows "parabolization" method**
- **Taylor expand** $\dot{\mathbf{B}} = -\nabla \times (\mathbf{J} \times \mathbf{B})$

$$\begin{aligned}
\dot{\mathbf{B}} &= -\nabla \times (\mathbf{J} \times \mathbf{B}) - \theta\,\delta t\,\nabla \times \left[\dot{\mathbf{J}} \times \mathbf{B} + \mathbf{J} \times \dot{\mathbf{B}}\right] \\
&= -\nabla \times (\mathbf{J} \times \mathbf{B}) - \theta\,\delta t\,\nabla \times \left[(\nabla \times \dot{\mathbf{B}}) \times \mathbf{B} + \mathbf{J} \times \dot{\mathbf{B}}\right] \\
&= -\nabla \times (\mathbf{J} \times \mathbf{B}) + \theta\,\delta t\,L(\mathbf{B})
\end{aligned}$$

$$L(\mathbf{B}) = \nabla \times \left\{\left[\nabla \times \nabla \times (\mathbf{J} \times \mathbf{B})\right] \times \mathbf{B} + \mathbf{J} \times \left[\nabla \times (\mathbf{J} \times \mathbf{B})\right]\right\}$$

Boundary Conditions with C^1 Elements

- **Boundary conditions are of the form**

$$\mathbf{L}'x = \mathbf{X}' \qquad \mathbf{L}' = \begin{pmatrix} 1 & \partial_n & \partial_t & \partial_n\partial_n & \partial_t\partial_n & \partial_t\partial_t \end{pmatrix}^T$$

- **Reduced quintic basis obeys:**

$$\mathbf{L}\cdot\nu = \mathbf{I} \qquad \mathbf{L} = \begin{pmatrix} 1 & \partial_R & \partial_Z & \partial_R\partial_R & \partial_R\partial_Z & \partial_Z\partial_Z \end{pmatrix}^T$$

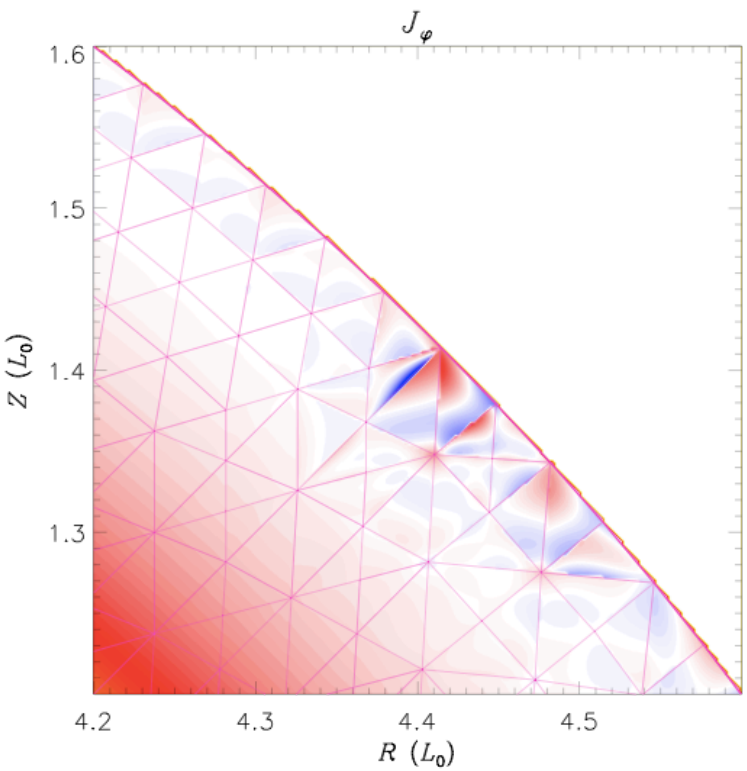
- **Want trial functions obeying** $\mathbf{L}'\cdot\mu = \mathbf{I}$
  - μ is maximally coupled with BC equation
  - This implies $\mu = \left(\mathbf{J}^{-1}\right)^T \cdot \nu$ where $\mathbf{L}' = \mathbf{J}\cdot\mathbf{L}$
- **Therefore, before applying BC's, do**

$$\mathbf{A}\cdot\mathbf{x} = \mathbf{B} \quad \rightarrow \quad \left(\mathbf{J}^{-1}\right)^T \cdot \left[\mathbf{A}\cdot\mathbf{x} = \mathbf{B}\right]$$

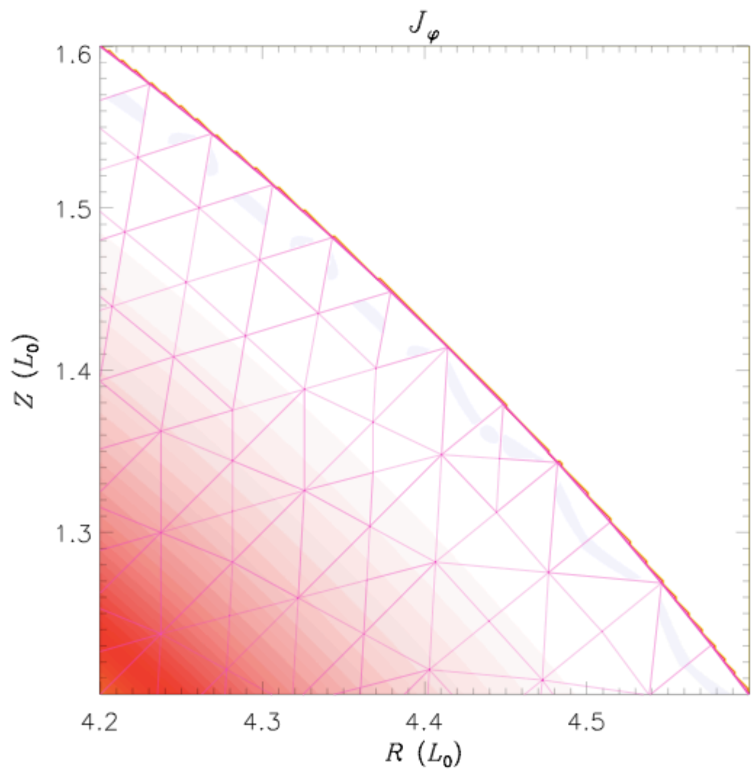
- **For curved boundaries** $\mathbf{J} \neq \left(\mathbf{J}^{-1}\right)^T$

Boundary Conditions with C¹ Elements

- **Correctly treating BCs is important**



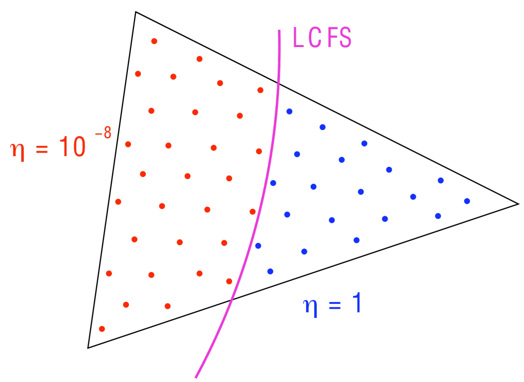
Without transformation

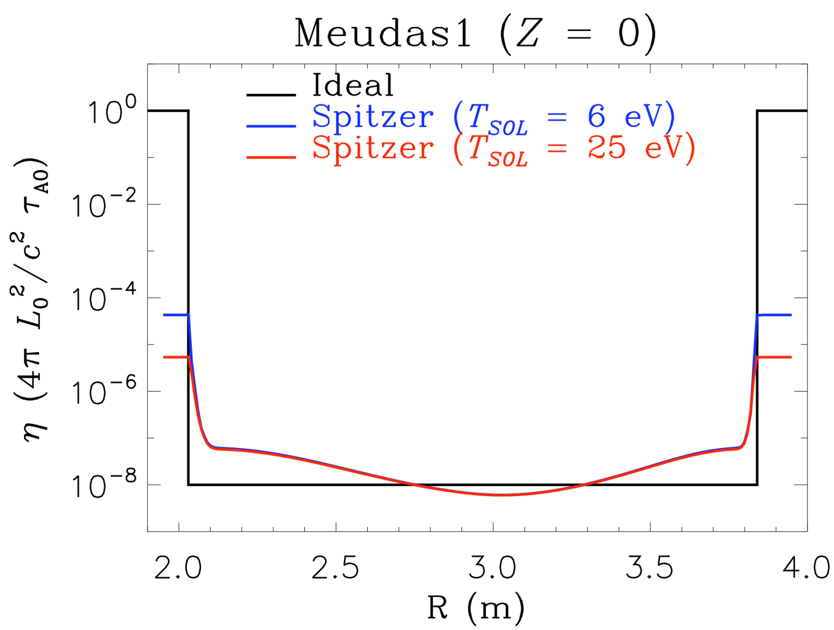


With transformation

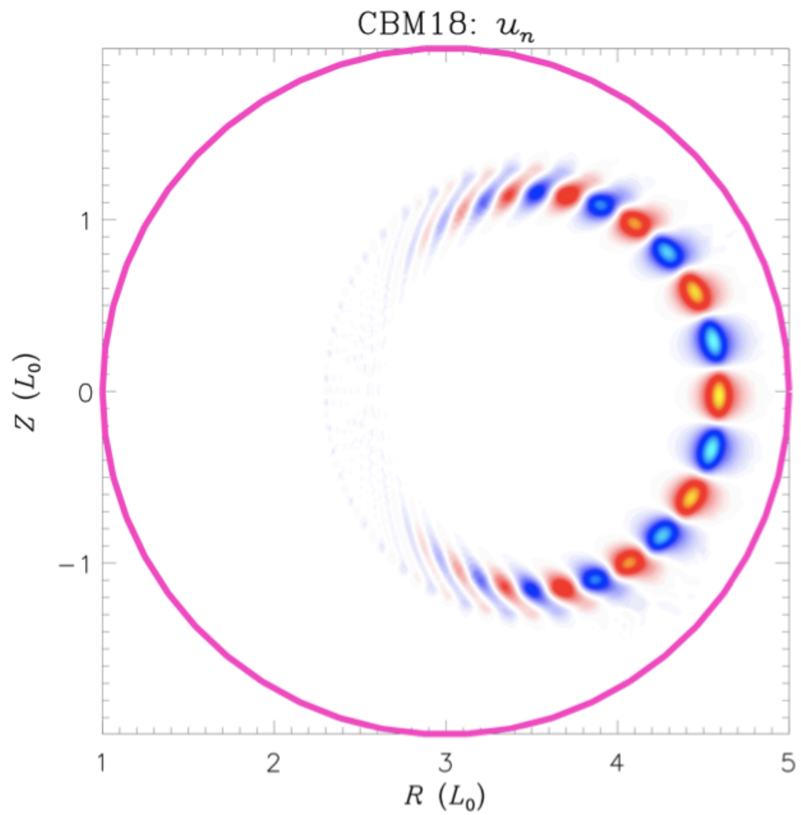
ELM Stability: Ideal Limit

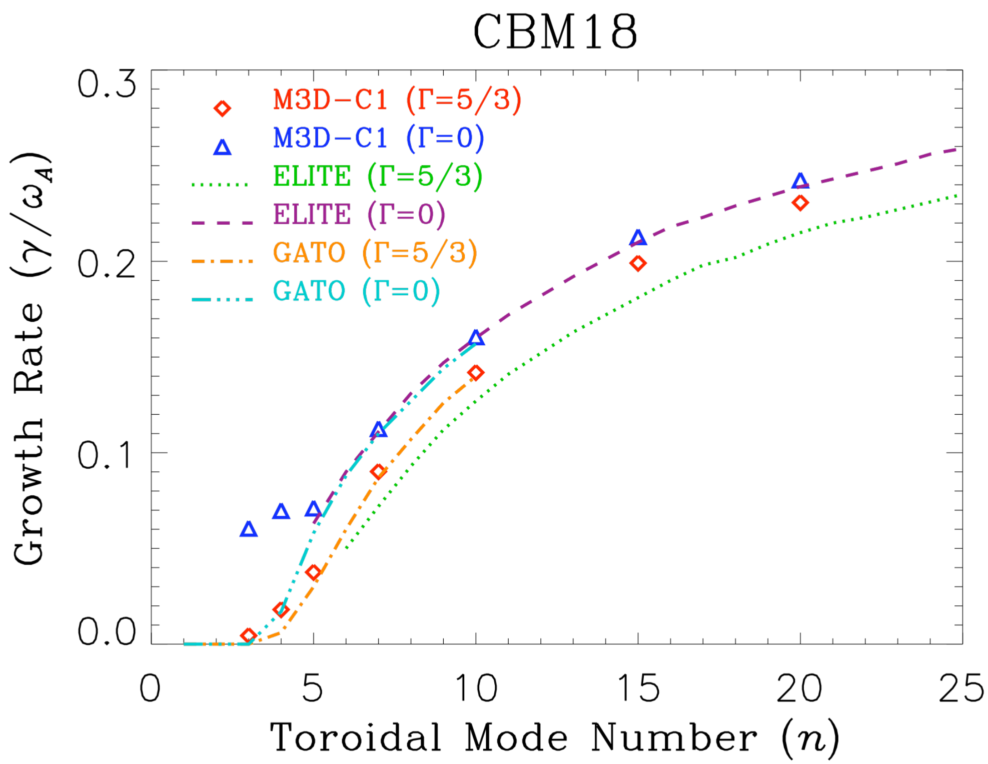
- **To approximate ideal limit, M3D-C^1 uses a discontinuous resistivity profile**
  - In this case, resistivity is not represented on the finite element basis (which is C^1)
  - η is calculated as a function of ψ at each sampling point





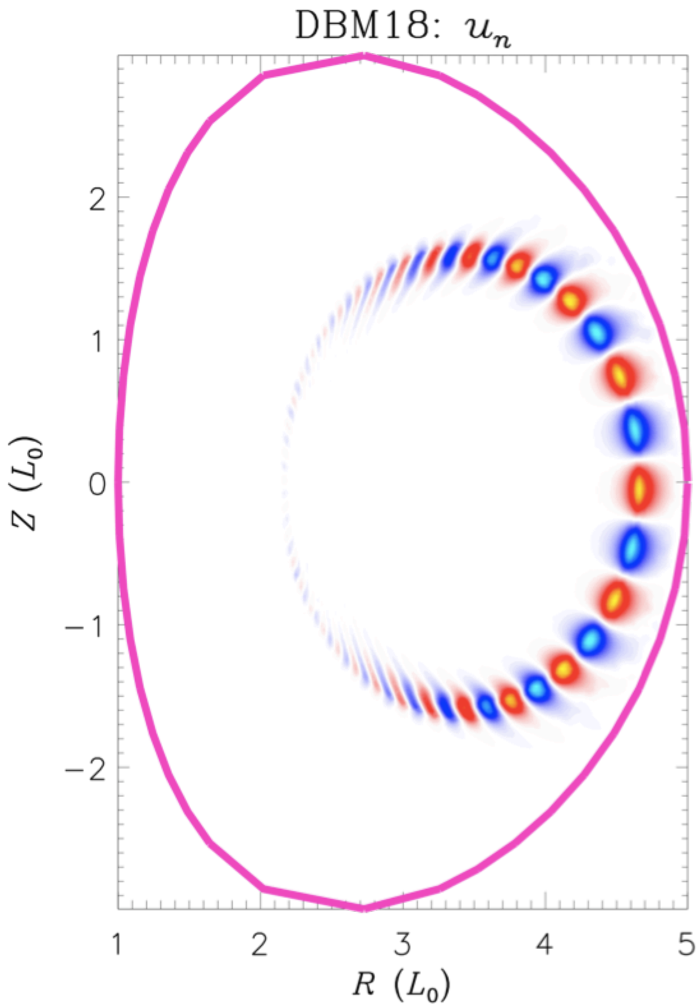
ELM Stability: CBM18

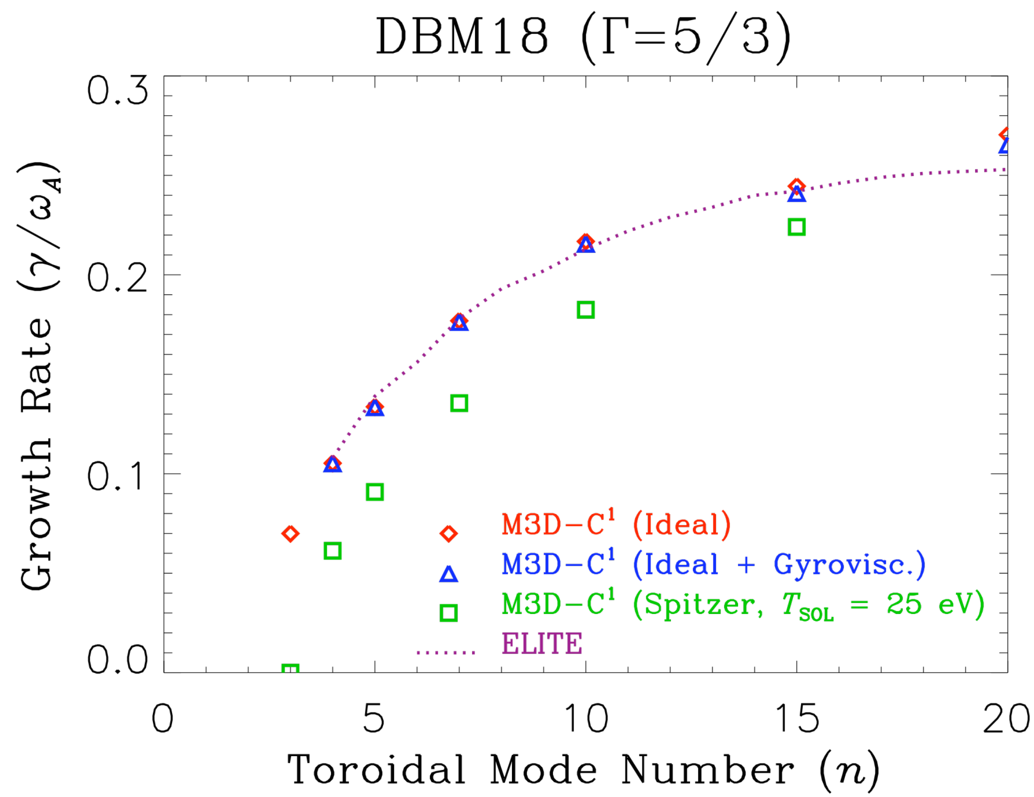




- Relatively low resolution requirement
- Good agreement with ELITE/GATO up to n=20

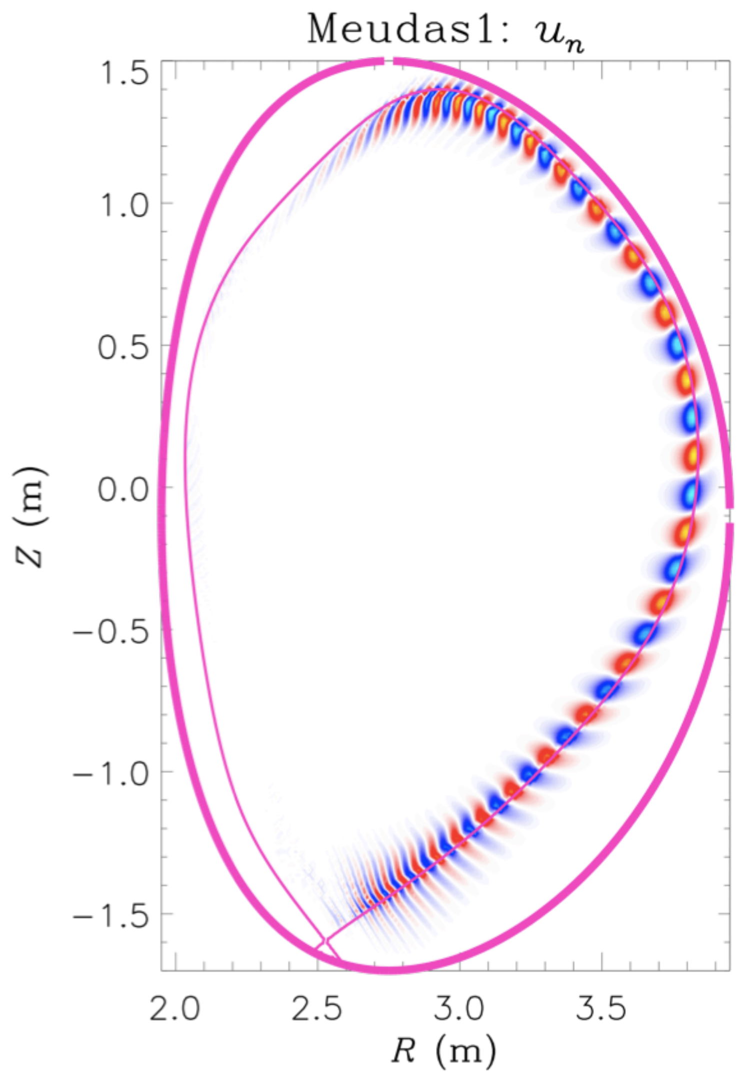
ELM Stability: DBM18

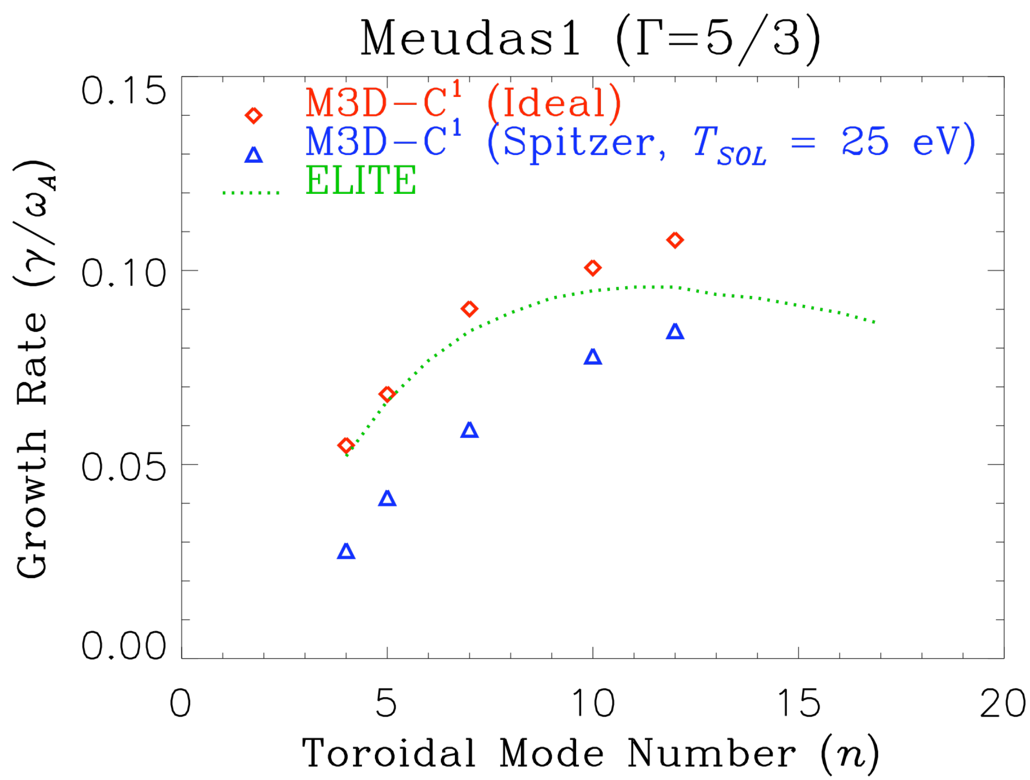




- **Effect of gyroviscosity (FLR) is small**

ELM Stability: Meudas1

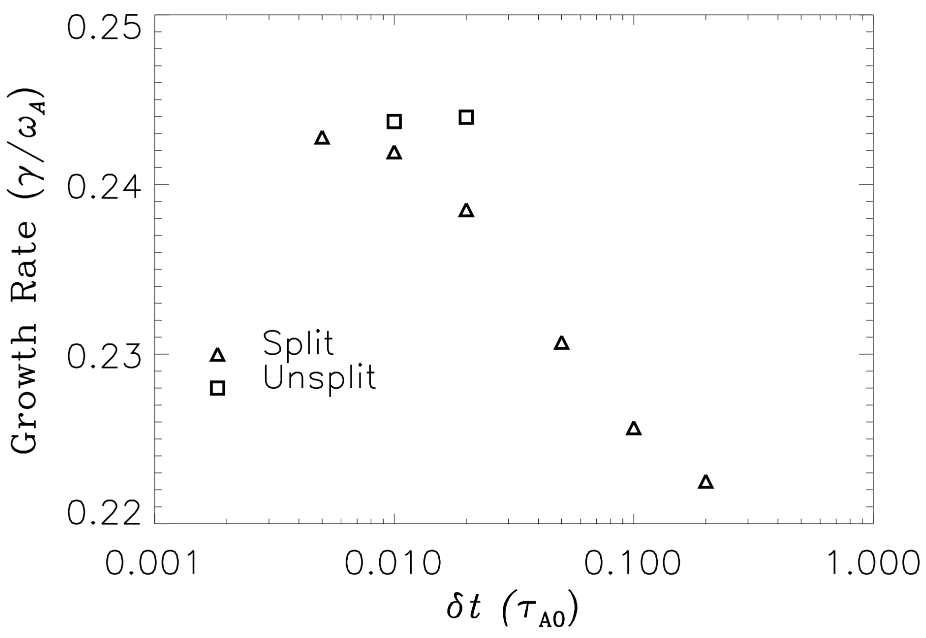




- **Very high resolution requirement near x-point**
- **Growth rate is sensitive**
- **ELITE assumes infinite vacuum**

ELM Stability: Time Step

- The (unsplit) Crank-Nicholson time discretization is more accurate
- The (split) semi-implicit step is faster, allows larger problem size
- Split convergence might not always be this bad



Non-Axisymmetric Fields: Vacuum

- **The vector components of B are known from Biot-Savart**
- **Need to translate to M3D-C^1 variables**
  - Solve:

$$B_R = -\frac{1}{R}\frac{\partial\psi}{\partial Z} - \frac{\partial^2 f}{\partial R\,\partial\varphi}$$

$$B_Z = \frac{1}{R}\frac{\partial\psi}{\partial R} - \frac{\partial^2 f}{\partial Z\,\partial\varphi}$$

$$B_\varphi = R\nabla_\perp^2 f$$

Over-determined!

$$\mathbf{L}\cdot\begin{pmatrix}\psi\\ f\end{pmatrix} = \mathbf{B}$$

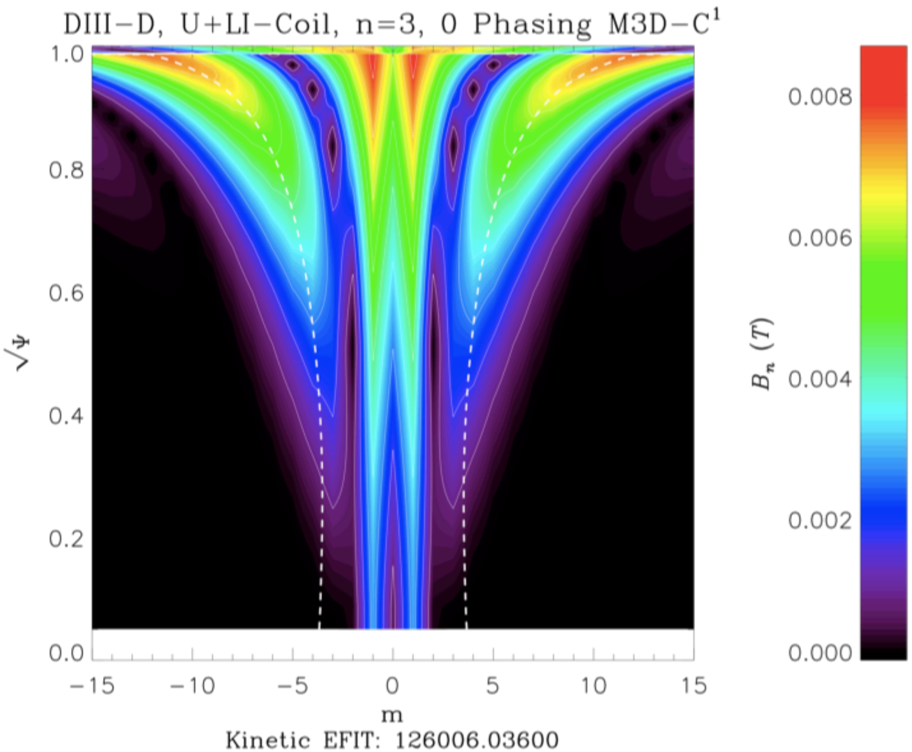
Least-squares solution:

$$\mathbf{L}^T\cdot\mathbf{L}\cdot\begin{pmatrix}\psi\\ f\end{pmatrix} = \mathbf{L}^T\cdot\mathbf{B}$$

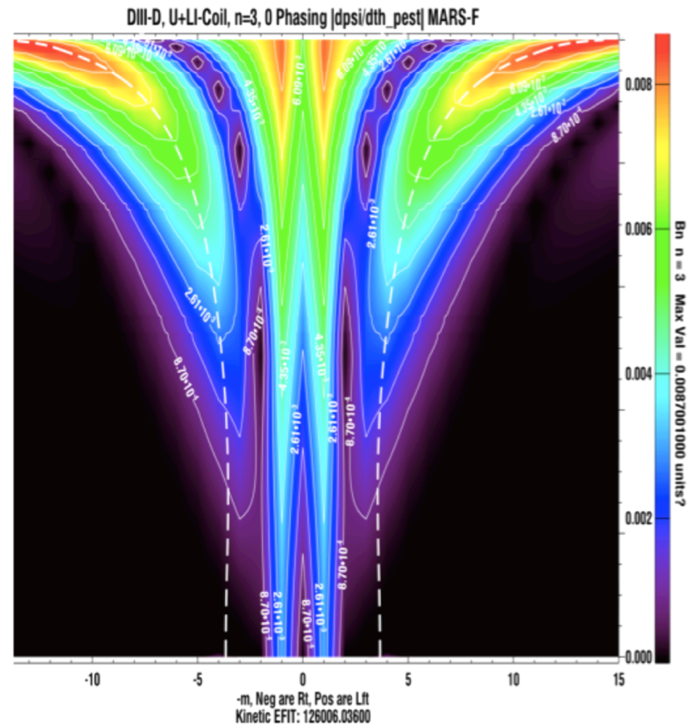
Non-Axisymmetric Fields: Vacuum

Even Parity

M3D-C[1]



SURFMN

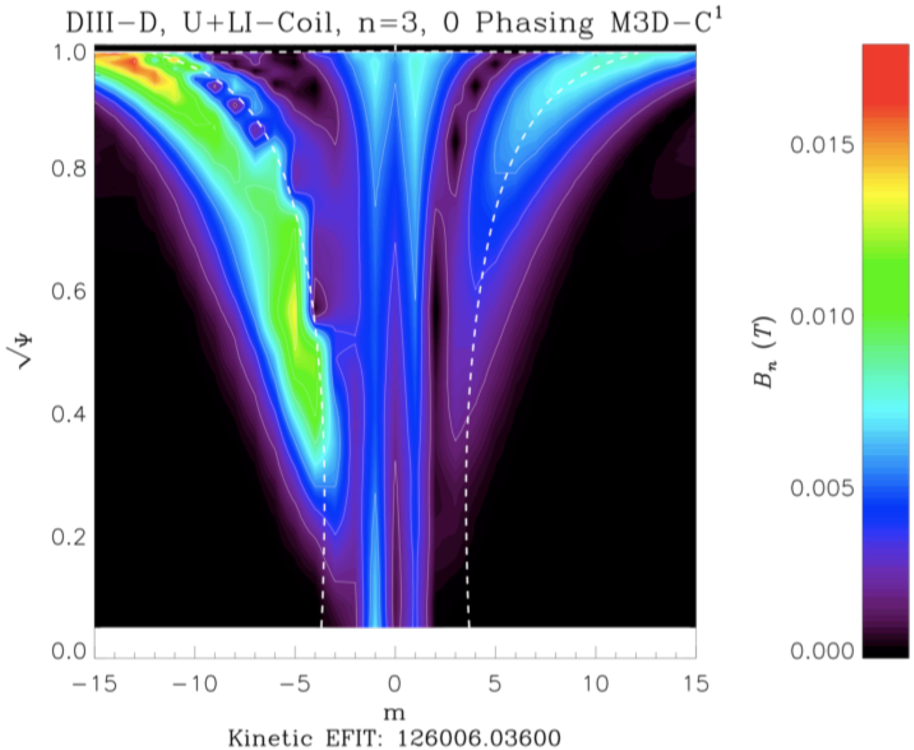


Dashed white line is *m = nq*

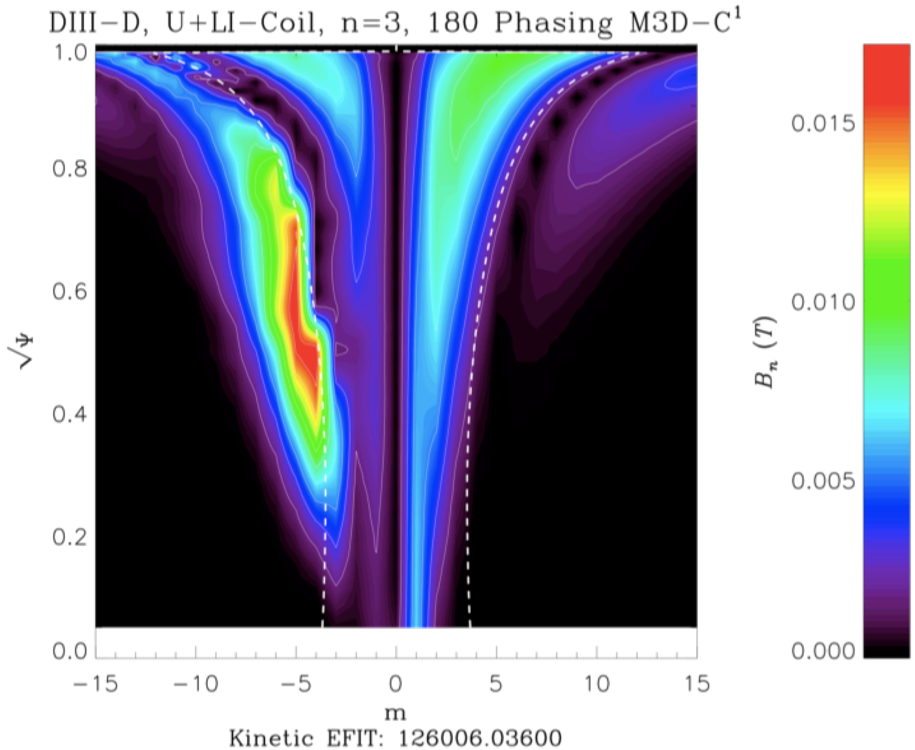
Non-Axisymmetric Fields: Quasi-Ideal Response

- **Perturbation on resonant surface is suppressed**

Even Parity



Odd Parity



Dashed white line is *m* = *nq*

$$\eta = 10^{-8}, \quad \mu = 10^{-5}, \quad \kappa = 10^{-4}$$

GENERAL ATOMICS

Conclusions

- **Semi-implicit Hall operator works well in axisymmetric simulations**
- **New BC method greatly improves stability**
- **M3D-C^1 finds excellent agreement with ideal codes on ELM stability for low/intermediate *n***
  - Resolution requirements are onerous at high *n*
  - Effect of gyroviscosity is negligible at low *n*
- **Vacuum RMP calculation agrees with Surfmn**
- **Linear RMP response shows both expected and unexpected features**

Future Work

- **Comprehensive Study of Non-Ideal Effects in ELM Stability**
  - Resistive (Type III)
  - Two-Fluid
  - Non-stationary equilibrium
- **Comparison of linear RMP response with experimental results**
  - Dynamical response explored in DIII-D experiment
- **Nonlinear extension**