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LONG-PARALLEL-MEAN-FREE-PATH KINETIC CLOSURES FOR SLOW EXTENDED-MHD*

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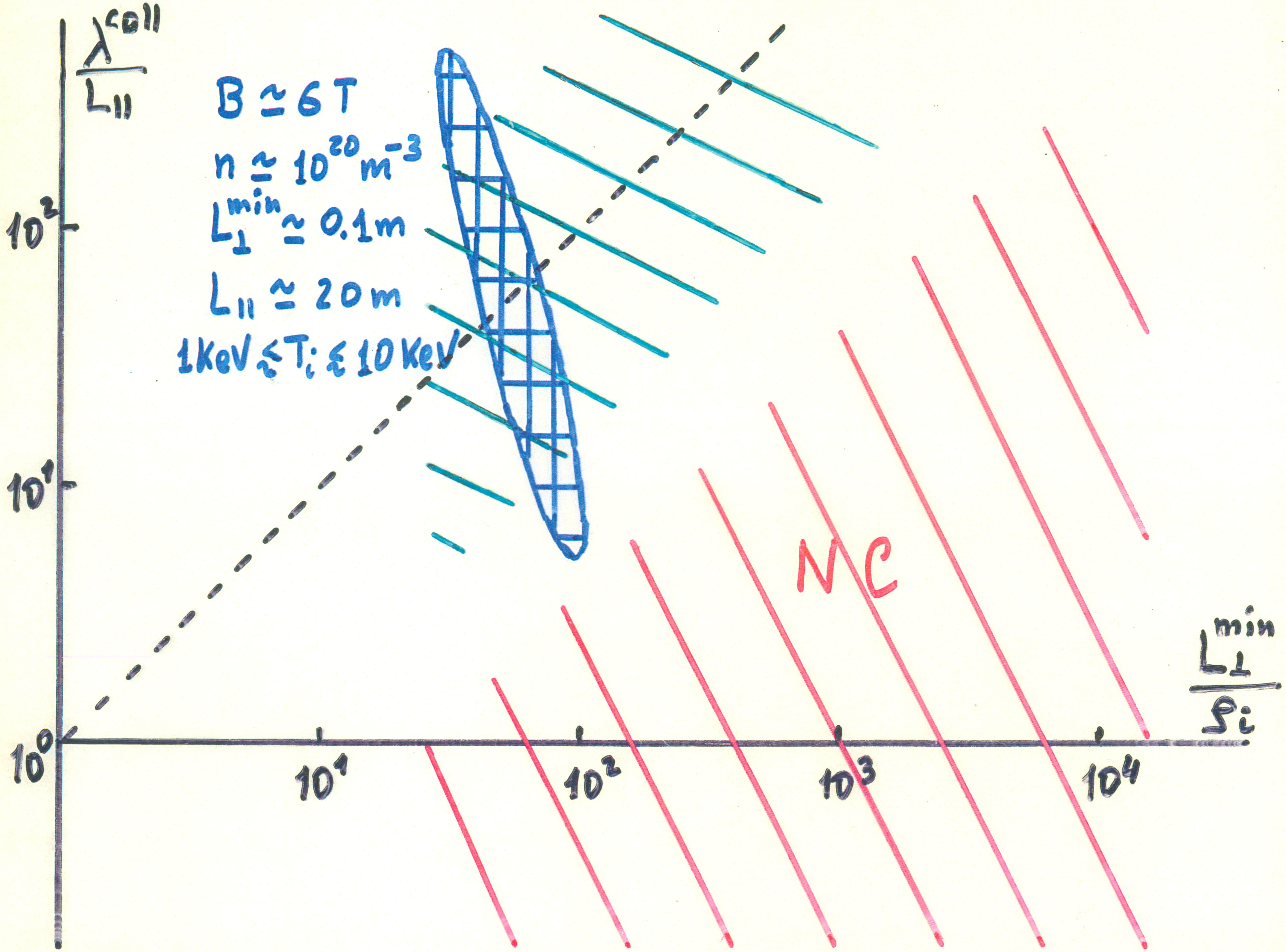
INTRODUCTION

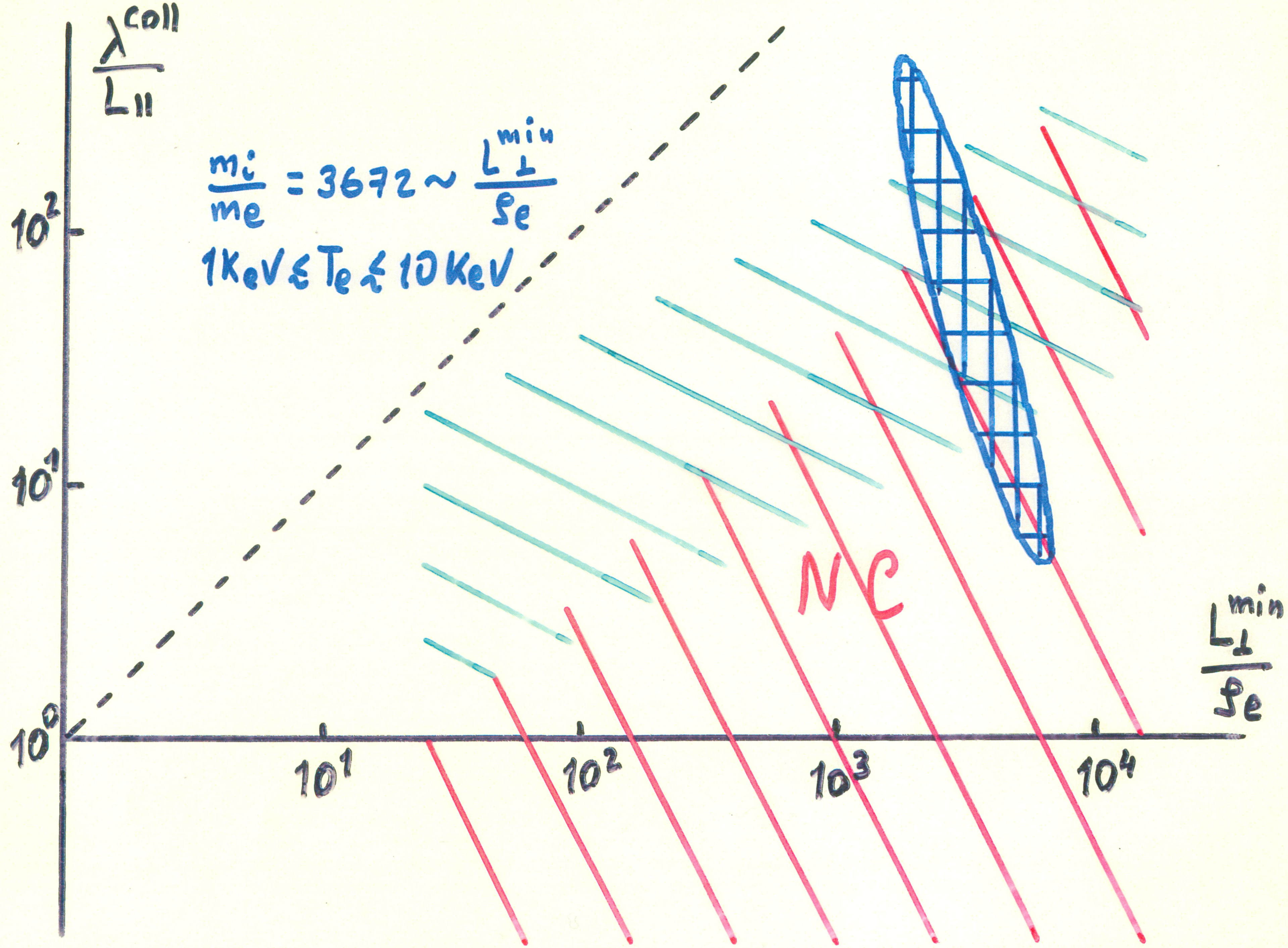
A theoretical model of the electron dynamics for slow, macroscopic plasma processes (such as the "neoclassical" tearing instabilities) in a long-parallel-mean-free-path collisionality regime will be presented.

The model is a hybrid one, with fluid conservation equations for particle number, momentum and energy, and drift-kinetic closures.

Key to this work is a careful choice of the orderings relating fundamental parameters, aimed at describing as realistically as possible the low-collisionality, fusion-relevant plasmas of interest. The conventional ordering of the collisionality in neoclassical theory is deemed too high for the ions, even in the banana regime. Instead, the orderings $\rho_\iota/L \sim L/\lambda^{coll} \sim (m_e/m_\iota)^{1/2} \ll 1$ are adopted, which still yield a theory equivalent to the one based on the neoclassical banana orderings for the electrons.



$\frac{\lambda^{coll}}{L_{\parallel}}$
$B \simeq 6T$
$n \simeq 10^{20} m^{-3}$
$L_{\perp}^{min} \simeq 0.1 m$
$L_{\parallel} \simeq 20 m$
$1 KeV \lesssim T_i \lesssim 10 KeV$
10^2
10^1
10^0
10^1
10^2
10^3
10^4
NC
$\frac{L_{\perp}^{min}}{\rho_i}$



λ^coll / L∥
mi/me = 3672 ~ L⊥^min/ρe
1 KeV ≤ Te ≤ 10 KeV
10^2
10^1
10^0
10^1
10^2
10^3
10^4
L⊥^min/ρe
NC

.

BASIC FRAMEWORK AND ORDERING ASSUMPTIONS

Quasineutral plasma with one ion species of unit charge:

$$n_\iota = n_e = n \; , \qquad \frac{\partial n}{\partial t} \; + \; \nabla \cdot (n\mathbf{u}_\iota) \; = \; \frac{\partial n}{\partial t} \; + \; \nabla \cdot (n\mathbf{u}_e) \; = \; 0 \; ,$$

$$\frac{\partial \mathbf{B}}{\partial t} \; = \; - \; \nabla \times \mathbf{E} \; , \qquad \mathbf{j} \; = \; en(\mathbf{u}_\iota - \mathbf{u}_e) \; = \; \nabla \times \mathbf{B} \; .$$

Small ion Larmor radius fundamental expansion parameter:

$$\delta \sim \rho_\iota / L \sim k\rho_\iota \ll 1 \; .$$

Small mass ratio and low collisionality orderings, linked to δ:

$$(m_e/m_\iota)^{1/2} \sim \delta \; , \qquad \textbf{hence} \qquad \rho_e/L \sim k\rho_e \sim \delta^2$$

and

$$\nu_\iota \sim \delta\nu_e \sim \delta^2 \Omega_{c\iota} \; , \qquad \textbf{hence} \qquad \lambda^{coll} \sim v_{th\iota}/\nu_\iota \sim v_{the}/\nu_e \sim \delta^{-1} L \; .$$

Macroscopic flows of the order of the diamagnetic drifts:

$$u_\iota \sim u_e \sim u_{*\iota,e} \sim \delta v_{th\iota} \sim \delta^2 v_{the} \ .$$

Close to Maxwellian distribution functions with comparable ion and electron temperatures and small parallel temperature gradients:

$$f_s = f_{Ms} + f_{NMs} = \frac{n}{(2\pi)^{3/2}\ v_{ths}^3} \exp\left(-\frac{|\mathbf{v} - \mathbf{u}_s|^2}{2\ v_{ths}^2} \right) + f_{NMs} \quad \textbf{with} \quad v_{ths}^2 \equiv T_s/m_s \ ,$$

$$\mathbf{b} \cdot \nabla T_s \sim \delta^2\ T_s/L, \qquad T_e \sim T_\iota, \qquad f_{NM\iota} \sim \delta\ f_{M\iota}, \qquad f_{NMe} \sim \delta^2\ f_{Me} \ .$$

Using $\Omega_{c\iota}$ as reference, we have the following hierarchy of time scales:

$$O(\delta^{-2}): \ \Omega_{ce} = v_{the}/\rho_e$$

$$O(1): \quad \Omega_{c\iota} = v_{th\iota}/\rho_\iota \sim v_{the}/L$$

$$O(\delta): \quad \nu_e \sim \omega_A = kc_A \sim \omega_S = kc_S \sim v_{th\iota}/L$$

$$O(\delta^2): \quad \nu_\iota \sim ku_{\iota,e} \sim \omega_{*\iota,e} = ku_{*\iota,e}$$

$$O(\delta^3): \quad collisional \quad dynamics$$

FLUID AND DRIFT-KINETIC APPROACH

Non-Maxwellian parts of the distribution functions, f_{NMs}, evaluated in the moving reference frames of their macroscopic flows, like the Maxwellian parts.

1, $\mathbf{v} - \mathbf{u}_s$ and $|\mathbf{v} - \mathbf{u}_s|^2$ velocity moments of f_{NMs} equal to zero.

Density, flow velocities and temperatures determined by fluid moment equations.

Solution of drift-kinetic equations for f_{NMs} to provide the fluid closure terms. Since f_{NMs} are obtained in the reference frames of their macroscopic flows, the evaluation of the stress and heat flux tensors is direct without the need of substracting the mean flows.

FLUID MOMENTUM AND TEMPERATURE EQUATIONS

$$m_\iota n \frac{\partial \mathbf{u}_\iota}{\partial t} = en(\mathbf{E} + \mathbf{u}_\iota \times \mathbf{B}) - \nabla(nT_\iota) - m_\iota n(\mathbf{u}_\iota \cdot \nabla)\mathbf{u}_\iota - \nabla \cdot \left[(p_{\iota\|} - p_{\iota\perp})(\mathbf{bb} - \mathbf{I}/3) + \mathbf{P}_\iota^{GV}\right] - \nabla \cdot \mathbf{P}_{\iota\perp}^{coll} + \mathbf{F}_\iota^{coll}$$

$$m_e n \frac{\partial \mathbf{u}_e}{\partial t} = -\, en(\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) - \nabla(nT_e) - \nabla \cdot \left[(p_{e\|} - p_{e\perp})(\mathbf{bb} - \mathbf{I}/3)\right] + \mathbf{F}_e^{coll} \; .$$

$O(nm_e v_{the}^2/L)$ $\qquad O(\delta^2 nm_e v_{the}^2/L)$ $\qquad O(\delta^3 nm_e v_{the}^2/L)$

$$\frac{3n}{2}\frac{\partial T_\iota}{\partial t} = -\, \frac{3n}{2}\mathbf{u}_\iota \cdot \nabla T_\iota - nT_\iota \nabla \cdot \mathbf{u}_\iota - \nabla \cdot \left(q_{\iota\|}\mathbf{b} + \frac{5nT_\iota}{2eB}\mathbf{b} \times \nabla T_\iota\right) + G_\iota^{coll} \; .$$

$$\frac{3n}{2}\frac{\partial T_e}{\partial t} = -\, \frac{3n}{2}\mathbf{u}_e \cdot \nabla T_e - nT_e \nabla \cdot \mathbf{u}_e - \nabla \cdot \left(q_{e\|}\mathbf{b} - \frac{5nT_e}{2eB}\mathbf{b} \times \nabla T_e\right) + G_e^{coll} \; .$$

$O(\delta^2 nm_e v_{the}^3/L)$ $\qquad O(\delta^3 nm_e v_{the}^3/L)$

KINETIC EQUATION FOR THE NON-MAXWELLIAN PART OF THE ELECTRON DISTRIBUTION FUNCTION

In terms of velocity-space coordinates (v', χ, α) in the reference frame of the electron macroscopic flow:

$$\mathbf{v} \; = \; \mathbf{u}_e(\mathbf{x},t) \; + \; v' \cos\chi \; \mathbf{b}(\mathbf{x},t) \; + \; v' \sin\chi \; [\cos\alpha \; \mathbf{e}_1(\mathbf{x},t) + \sin\alpha \; \mathbf{e}_2(\mathbf{x},t)] \; ,$$

the non-Maxwellian part of the electron distribution function can be represented as

$$f_{NMe}(v', \chi, \alpha, \mathbf{x}, t) \; = \; \bar{f}_{NMe}(v', \chi, \mathbf{x}, t) \; + \; \tilde{f}_{NMe}(v', \chi, \alpha, \mathbf{x}, t)$$

with

$$\langle \tilde{f}_{NMe} \rangle_\alpha \equiv (2\pi)^{-1} \oint d\alpha \; \tilde{f}_{NMe} = 0 \; .$$

Then, keeping the accuracy of $O(\delta^2 f_{Me}) + O(\delta^3 f_{Me})$:

$$\tilde{f}_{NMe} \; = \; f_{Me} \, \frac{m_e v'}{2eBT_e} \left(\frac{m_e v'^2}{T_e} - 5 \right) \; \sin\chi \; (\cos\alpha \; \mathbf{e}_2 - \sin\alpha \; \mathbf{e}_1) \cdot \nabla T_e$$

and $\bar{f}_{NMe}$ obeys the following drift-kinetic equation:

$$\frac{\partial \bar{f}_{NMe}}{\partial t} + \cos\chi\left(v'\mathbf{b}\cdot\frac{\partial \bar{f}_{NMe}}{\partial \mathbf{x}} + \frac{T_e}{m_e}\mathbf{b}\cdot\nabla\ln n\,\frac{\partial \bar{f}_{NMe}}{\partial v'}\right) - \frac{\sin\chi}{v'}\left(\frac{T_e}{m_e}\mathbf{b}\cdot\nabla\ln n - \frac{v'^2}{2}\mathbf{b}\cdot\nabla\ln B\right)\frac{\partial \bar{f}_{NMe}}{\partial \chi} =$$

$$= \Big\{\cos\chi\,\frac{v'}{2T_e}\left(5-\frac{m_e v'^2}{T_e}\right)\mathbf{b}\cdot\nabla T_e + \cos\chi\,\frac{v'}{nT_e}\,\mathbf{b}\cdot\left[\frac{2}{3}\nabla(p_{e\parallel}-p_{e\perp}) - \left(p_{e\parallel}-p_{e\perp}\right)\nabla\ln B - \mathbf{F}_e^{coll}\right] +$$

$$+ P_2(\cos\chi)\,\frac{m_e v'^2}{3T_e}\left(\nabla\cdot\mathbf{u}_e - 3\mathbf{b}\cdot[(\mathbf{b}\cdot\nabla)\mathbf{u}_e]\right) + \frac{1}{3nT_e}\left(\frac{m_e v'^2}{T_e}-3\right)\left[\nabla\cdot(q_{e\parallel}\mathbf{b}) - G_e^{coll}\right] +$$

$$+ \frac{1}{6eB}\left[2P_2(\cos\chi)\frac{m_e v'^2}{T_e}\left(\frac{m_e v'^2}{T_e}-5\right) + \frac{m_e^2 v'^4}{T_e^2} - 10\frac{m_e v'^2}{T_e} + 15\right](\mathbf{b}\times\boldsymbol{\kappa})\cdot\nabla T_e +$$

$$+ \frac{1}{6eB}\left[-P_2(\cos\chi)\frac{m_e v'^2}{T_e}\left(\frac{m_e v'^2}{T_e}-5\right) + \frac{m_e^2 v'^4}{T_e^2} - 10\frac{m_e v'^2}{T_e} + 15\right](\mathbf{b}\times\nabla\ln B)\cdot\nabla T_e +$$

$$+ P_2(\cos\chi)\,\frac{m_e v'^2}{3eBT_e}(\mathbf{b}\times\nabla\ln n)\cdot\nabla T_e\,\Big\}f_{Me} +$$

$$+ \langle C_{ee}^{(3)}(f_{Me}, f_{NMe}) + C_{ee}^{(3)}(f_{NMe}, f_{Me})\rangle_\alpha + \langle C_{e\iota}^{(3)}(f_{Me}, f_\iota) + C_{e\iota}^{(3)}(f_{NMe}, f_{M\iota})\rangle_\alpha\ .$$

With the 1, $v'\cos\chi$ and v'^2 moments of $\bar{f}_{NMe}$ equal to zero, the 1, $v'\cos\chi$ and v'^2 moments of this drift-kinetic equation are satisfied identically.

COLLISION OPERATORS

Based on the complete form of the linearized Fokker-Planck operators and using the electron collision frequency definition

$$\nu_e \;\equiv\; \frac{c^4 e^4 n \ln \Lambda_e}{4\pi m_e^2 v_{the}^3} \;,$$

the gyrophase averaged collision operators that enter in the drift-kinetic equation are as follows:

$$\langle C_{e\iota}^{(3)}(f_{Me}, f_\iota)\rangle_\alpha \;=\; \frac{\nu_e m_e}{m_\iota}\left(\frac{T_i}{T_e}-1\right) f_{Me}(v')\left[\frac{v_{the}}{v'}\;\phi\left(\frac{v'}{v_{th\iota}}\right) - \frac{4\pi v_{the}^3}{n}\; f_{M\iota}(v')\right] \;+$$

$$+\;\frac{\nu_e j_\parallel v_{the}}{e n v_{th\iota}^3}\; f_{Me}(v')\;\left[\frac{v_{th\iota}}{v'}\;\xi\left(\frac{v'}{v_{th\iota}}\right) - \frac{4\pi v_{th\iota}^3}{n}\; f_{M\iota}(v')\right]\; v'\cos\chi$$

where

$$\phi(x) \;=\; \frac{2}{(2\pi)^{1/2}}\int_0^x dt\;\; \exp(-t^2/2) \qquad \textbf{and} \qquad \xi(x) \;=\; \frac{1}{x^2}\left[\phi(x) - x\frac{d\phi(x)}{dx}\right] \;.$$

$\langle C^{(3)}_{ee}(f_{Me}, f_{NMe}) + C^{(3)}_{ee}(f_{NMe}, f_{Me})\rangle_\alpha + \langle C^{(3)}_{e\iota}(f_{NMe}, f_{M\iota})\rangle_\alpha \;\equiv\; \mathcal{C}[\bar{f}_{NMe}]$ **is Legendre diagonal:**

$$\mathcal{C}\left[\sum_{l=0}^{\infty} f_l(v')P_l(\cos\chi)\right] \;=\; \sum_{l=0}^{\infty} P_l(\cos\chi)\;\mathcal{C}_l[f_l(v')]$$

with

$$\begin{aligned}
\mathcal{C}_l[f_l(v')] \;&=\; \frac{\nu_e}{n} f_{Me}(v')\Big\{4\pi v^3_{the} f_l(v') \;-\; v_{the}\Phi^R_l[f_l(v')] \;+\; v^{-1}_{the}\Xi^R_l[f_l(v')]\Big\} \;+ \\
&+\; \frac{\nu_e v^3_{the}}{v'^2}\frac{\partial}{\partial v'}\left\{\xi\left(\frac{v'}{v_{the}}\right)\left[v'\frac{\partial f_l(v')}{\partial v'} \;+\; \frac{v'^2}{v^2_{the}}\; f_l(v')\right]\right\} \;- \\
&-\; \frac{\nu_e l(l+1)v^3_{the}}{2v'^3}\left[\phi\left(\frac{v'}{v_{the}}\right) - \xi\left(\frac{v'}{v_{the}}\right) + \phi\left(\frac{v'}{v_{th\iota}}\right) - \xi\left(\frac{v'}{v_{th\iota}}\right)\right] f_l(v')
\end{aligned}$$

and

$$\frac{1}{v'^2}\frac{\partial}{\partial v'}\left\{v'^2\frac{\partial\Phi^R_l[f_l(v')]}{\partial v'}\right\} \;-\; \frac{l(l+1)}{v'^2}\Phi^R_l[f_l(v')] \;=\; -4\pi f_l(v')\;,$$

$$\Xi^R_l[f_l(v')] \;=\; v'^2\frac{\partial^2\Psi^R_l[f_l(v')]}{\partial v'^2}\;,$$

$$\frac{1}{v'^2}\frac{\partial}{\partial v'}\left\{v'^2\frac{\partial\Psi^R_l[f_l(v')]}{\partial v'}\right\} \;-\; \frac{l(l+1)}{v'^2}\Psi^R_l[f_l(v')] \;=\; \Phi^R_l[f_l(v')]\;.$$

ELECTRON CLOSURE VARIABLES

The kinetically defined closure terms in the electron fluid equations are:

$$(p_{e\|} - p_{e\perp}) \;=\; 2\pi m_e \int_0^\infty dv'\; v'^4 \int_0^\pi d\chi\;\sin\chi\; P_2(\cos\chi)\;\; \bar{f}_{NMe} \;=\; O(\delta^2 n m_e v_{the}^2) \;+\; O(\delta^3 n m_e v_{the}^2)\;,$$

$$q_{e\|} =\; \pi m_e \int_0^\infty dv'\; v'^5 \int_0^\pi d\chi\;\sin\chi\;\cos\chi\;\; \bar{f}_{NMe} \;=\; O(\delta^2 n m_e v_{the}^3) \;+\; O(\delta^3 n m_e v_{the}^3)\;,$$

$$\mathbf{F}_e^{coll} \;=\; \frac{2 m_e \nu_e}{3(2\pi)^{1/2} e}\,\mathbf{j} \;-\; \frac{m_e \nu_e n}{(2\pi)^{1/2} e B}\,(\mathbf{b}\times\nabla T_e) \;-$$

$$-\; 2\pi m_e \nu_e v_{the}^3 \int_0^\infty dv' \;\int_0^\pi d\chi\;\sin\chi\;\cos\chi\;\bar{f}_{NMe}\;\mathbf{b} \;=\; O\left(\frac{\delta^3 n m_e v_{the}^2}{L}\right)\;,$$

$$G_e^{coll} \;=\; \frac{2 m_e \nu_e n}{(2\pi)^{1/2} m_\iota}\,(T_\iota - T_e) \;=\; O\left(\frac{\delta^3 n m_e v_{the}^3}{L}\right)\;.$$

APPLICATION: STATIONARY AXISYMMETRIC SYSTEM

$$\nabla \cdot \mathbf{B} = 0 \ , \qquad \mathbf{j} = \nabla \times \mathbf{B} \ , \qquad \mathbf{E} = -\nabla\Phi - V_0 \nabla\varphi$$

$$\nabla \cdot (n\mathbf{u}_\imath) = \nabla \cdot (n\mathbf{u}_e) = 0 \ , \qquad \mathbf{u}_e = \mathbf{u}_\imath - \frac{1}{en}\mathbf{j}$$

$$\mathbf{b} \cdot \nabla T_e \ = \ O(\delta^2 \ T_e/L) \ , \qquad \mathbf{b} \cdot \nabla T_\imath \ = \ O(\delta^2 \ T_\imath/L) \ , \qquad T_e - T_\imath \ll T_e$$

$$-en(\mathbf{E} \ + \ \mathbf{u}_\imath \times \mathbf{B}) \ + \ \nabla(nT_\imath) \ = \ O(\delta^2 \ T_\imath/L)$$

$$en(\mathbf{E} \ + \ \mathbf{u}_e \times \mathbf{B}) \ + \ \nabla(nT_e) + \nabla \cdot \left[(p_{e\|} - p_{e\perp})(\mathbf{bb} - \mathsf{I}/3)\right] - \ \mathbf{F}_e^{coll} \ = \ 0$$

$$\frac{3n}{2}\mathbf{u}_e \cdot \nabla T_e \ + \ nT_e \nabla \cdot \mathbf{u}_e \ + \ \nabla \cdot \left(q_{e\|}\mathbf{b} \ - \ \frac{5nT_e}{2eB}\mathbf{b} \times \nabla T_e\right) = \ 0$$

LOWEST-ORDER STATIONARY FLUID RELATIONS

The axisymmetric magnetic field is

$$\mathbf{B} \;=\; \nabla\psi \times \nabla\varphi \;+\; RB_\varphi \nabla\varphi$$

and the lowest-order stationary fluid system (valid on the MHD time scale $t \lesssim \delta^{-1}\Omega_{ci}^{-1}$) yields the well known relations:

$$n = N^{(0)}(\psi), \quad T_s = T_s^{(0)}(\psi), \quad \Phi = \Phi^{(1)}(\psi) = O(T_s/e), \quad RB_\varphi = (RB_\varphi)^{(0)}(\psi) \;\equiv I(\psi)$$

$$\mathbf{u}_s \;=\; \mathbf{u}_s^{(1)} \;=\; U_s(\psi)\mathbf{B} + R^2 \left[\frac{d\Phi^{(1)}}{d\psi} + \frac{1}{e_s N^{(0)}} \frac{d(N^{(0)}T_s^{(0)})}{d\psi}\right] \nabla\varphi \;=\; O(\delta v_{thi})$$

From these, it follows that:

$$\nabla\psi \cdot (\mathbf{b} \times \boldsymbol{\kappa}) \;=\; \nabla\psi \cdot (\mathbf{b} \times \nabla \ln B) \;=\; I(\psi)\; \mathbf{b} \cdot \nabla \ln B$$

$$\nabla \cdot \mathbf{u}_s \;=\; 0$$

$$\mathbf{b} \cdot [(\mathbf{b} \cdot \nabla)\mathbf{u}_s] \;=\; U_s(\psi)\; \mathbf{b} \cdot \nabla \ln B$$

HIGHER-ORDER STATIONARY ELECTRON FLUID RELATIONS

Keeping the highest accuracy of $O(\delta^3 n m_e v_{the}^2/L)$, the parallel component of the stationary electron momentum equation yields

$$\mathbf{b}\cdot\left[\frac{2}{3}\nabla(p_{e\|}-p_{e\perp})-\left(p_{e\|}-p_{e\perp}\right)\nabla\ln B-\mathbf{F}_e^{coll}\right] \;=\; N^{(0)}T_e^{(0)}\;\mathbf{b}\cdot\nabla\left(\frac{e\Phi}{T_e^{(0)}}-\frac{n}{N^{(0)}}-\frac{T_e}{T_e^{(0)}}\right)\;+\;\frac{eV_0N^{(0)}I}{BR^2}$$

and keeping $O(\delta^3 n m_e v_{the}^3/L)$, the stationary electron temperature equation yields

$$\nabla\cdot\left(q_{e\|}\mathbf{b}\right)\;=\;\nabla\cdot\left(\frac{5N^{(0)}T_e^{(0)}}{2eB}\mathbf{b}\times\nabla T_e^{(0)}\right)\;=\;\frac{5N^{(0)}T_e^{(0)}I}{eB}\;\frac{dT_e^{(0)}}{d\psi}\;\mathbf{b}\cdot\nabla\ln B\;.$$

Notice that, within this highest available accuracy, the stationary electron temperature equation does not provide any information on the higher-order correction $T_e(\mathbf{x})-T_e^{(0)}(\psi)$!

STATIONARY ELECTRON DRIFT-KINETIC EQUATION

Using the previous stationary fluid results and calling $g \equiv \bar{f}_{NMe}/f_{Me}^{(0)} = \Sigma_{l=0}^{\infty}\ g_l\ P_l(\cos\chi)$, the stationary electron drift-kinetic equation can be written as:

$$v' \left(\cos\chi\ \mathbf{b}\cdot\frac{\partial g}{\partial \mathbf{x}} + \frac{1}{2}\mathbf{b}\cdot\nabla \ln B\ \sin\chi\ \frac{\partial g}{\partial \chi} \right) =$$

$$= v'\cos\chi \left[\mathbf{b}\cdot\nabla\left(\frac{e\Phi}{T_e^{(0)}} - \frac{n}{N^{(0)}}\right) + \left(\frac{3}{2} - \frac{m_e v'^2}{2T_e^{(0)}}\right)\mathbf{b}\cdot\nabla\left(\frac{T_e}{T_e^{(0)}}\right) + \frac{eV_0 I}{T_e^{(0)} B R^2}\right] -$$

$$- \left\{ P_2(\cos\chi)\ \frac{m_e v'^2}{T_e^{(0)}}\ U_e B + \left[2 + P_2(\cos\chi)\right] \frac{m_e v'^2}{3T_e^{(0)}}\left(\frac{5}{2} - \frac{m_e v'^2}{2T_e^{(0)}}\right)\frac{I}{eB}\ \frac{dT_e^{(0)}}{d\psi}\right\} \mathbf{b}\cdot\nabla\ln B +$$

$$+ v'\cos\chi\ \mathcal{D}_1 + \hat{\mathcal{C}}[g]$$

where

$$\mathcal{D}_1 \equiv \frac{\nu_e j_\parallel v_{the}}{e n v_{th\iota}^3} \left[\frac{v_{th\iota}}{v'}\ \xi\left(\frac{v'}{v_{th\iota}}\right) - \frac{4\pi v_{th\iota}^3}{n}\ f_{M\iota}(v')\right] \qquad \textbf{and} \qquad \hat{\mathcal{C}}[g] \equiv \frac{1}{f_{Me}^{(0)}}\ \mathcal{C}\left[g f_{Me}^{(0)}\right] = \sum_{l=0}^{\infty} P_l(\cos\chi)\ \hat{\mathcal{C}}_l[g_l]$$

The stationary electron drift-kinetic equation can be solved with the methods of neoclassical theory in the banana regime. Thus, using the variable

$$\lambda(\mathbf{x},\chi) = \sin^2\chi\ B_{max}(\psi)/B(\mathbf{x}), \qquad 0 \le \lambda \le B_{max}/B, \qquad v'_\parallel(\mathbf{x},v',\lambda) \ = \ \pm v'(1-\lambda B/B_{max})^{1/2}\ ,$$

one gets

$$g \ = \ g_{p0} + (v'_\parallel/v')\ g_{p1} + h^{(2)} + h^{(3)} \ = \ \frac{e\Big(\Phi-\Phi^{(1)}\Big)}{T_e^{(0)}} - \frac{n-N^{(0)}}{N^{(0)}} + \left(\frac{3}{2}-\frac{m_e v'^2}{2T_e^{(0)}}\right)\frac{T_e-T_e^{(0)}}{T_e^{(0)}} +$$

$$+\ v'_\parallel\ \left[-\frac{m_e U_e B}{T_e^{(0)}} + \frac{m_e I}{eBT_e^{(0)}}\left(\frac{5}{2}-\frac{m_e v'^2}{2T_e^{(0)}}\right)\frac{dT_e^{(0)}}{d\psi}\right] + \sigma(v'_\parallel)\ H(1-\lambda)\ \hat{K}(\psi,v',\lambda) + h^{(3)}(\mathbf{x},v',\lambda)\ ,$$

where $g_{p1}\sim h^{(2)}\sim\delta^2$ **and** $g_{p0}\sim h^{(3)}\sim\delta^3$**.**

The function $h^{(3)}(\mathbf{x},v',\lambda)=O(\delta^3)$ **satisfies**

$$v'_\parallel\ \mathbf{b}\cdot\frac{\partial h^{(3)}(\mathbf{x},v',\lambda)}{\partial\mathbf{x}} \ = \ v'_\parallel\left[\frac{eV_0 I}{T_e^{(0)}BR^2} + \mathcal{D}_1(\mathbf{x},v')\right] + \hat{\mathcal{C}}\Big[\sigma(v'_\parallel)\ H(1-\lambda)\ \hat{K}(\psi,v',\lambda) \ + \ (v'_\parallel/v')\ g_{p1}\Big]$$

which has the integrability constraint

$$\oint\frac{dl}{v'_\parallel}\ \hat{\mathcal{C}}\Big[\sigma(v'_\parallel)\ H(1-\lambda)\ \hat{K}(\psi,v',\lambda) \ + \ (v'_\parallel/v')\ g_{p1}\Big] \ = \ -\ \oint dl\ \left[\frac{eV_0 I}{T_e^{(0)}BR^2} + \mathcal{D}_1(\mathbf{x},v')\right]$$

and the solution of this Spitzer problem determines the function $\hat{K}(\psi,v',\lambda)$**.**

ODD PARALLEL CLOSURES

Once the solution of the Spitzer problem $\hat{K}(\psi, v', \lambda)$ is known, it specifies the poloidal flow stream function $U_e(\psi)$ and the odd parallel closures $q_{e\|}(\mathbf{x})$ and $F_{e\|}^{coll}(\mathbf{x})$:

$$U_e(\psi) \;=\; \frac{2\pi}{N^{(0)}(\psi)\;B_{max}(\psi)}\;\int_0^\infty dv'\;v'^3 f_{Me}^{(0)}(\psi, v')\;\int_0^1 d\lambda\;\hat{K}(\psi, v', \lambda)\;,$$

$$q_{e\|} \;=\; -\;\frac{5N^{(0)}T_e^{(0)}}{2}\left(\frac{I}{eB}\frac{dT_e^{(0)}}{d\psi} + U_e B\right) \;+\; \frac{\pi m_e B}{B_{max}}\;\int_0^\infty dv'\;v'^5 f_{Me}^{(0)}(\psi, v')\;\int_0^1 d\lambda\;\hat{K}(\psi, v', \lambda)\;,$$

$$F_{e\|}^{coll} \;=\; \frac{2m_e\nu_e}{3(2\pi)^{1/2}}\left(\frac{j_\|}{e} + N^{(0)}U_e B - \frac{3N^{(0)}I}{2eB}\frac{dT_e^{(0)}}{d\psi}\right) \;-\; \frac{2\pi m_e \nu_e v_{the}^3 B}{B_{max}}\;\int_0^\infty dv'\;f_{Me}^{(0)}(\psi, v')\;\int_0^1 d\lambda\;\hat{K}(\psi, v', \lambda)\;.$$

CLOSURE PROBLEM FOR THE PRESSURE ANISOTROPY $(p_{e\|} - p_{e\perp})$

With a Legendre series expansion, $\left(h^{(2)} + h^{(3)}\right)(\mathbf{x}, v', \chi) \; = \; \Sigma_{l=0}^{\infty} \; h_l(\mathbf{x}, v') \; P_l(\cos\chi)$**, the** $l = 2$ **projection of the drift-kinetic equation can be expressed after algebraic elimination of the electric potential using the fluid momentum equation as:**

$$B^{3/2} \; \mathbf{b} \cdot \frac{\partial}{\partial \mathbf{x}} \left\{ B^{-3/2} \left[\frac{2}{5} h_2(\mathbf{x}, v') \; - \; \frac{2(p_{e\|} - p_{e\perp})}{3N^{(0)} T_e^{(0)}} \right] \right\} \; + \; \mathbf{b} \cdot \frac{\partial g_0(\mathbf{x}, v')}{\partial \mathbf{x}} \; =$$

$$= \; \left(\frac{5}{2} - \frac{m_e v'^2}{2T_e^{(0)}} \right) \mathbf{b} \cdot \nabla \left(\frac{T_e}{2T_e^{(0)}} \right) \; - \; \frac{F_{e\|}^{coll}}{N^{(0)} T_e^{(0)}} \; + \; \mathcal{D}_1(\mathbf{x}, v') \; + \; \frac{1}{v'} \; \hat{\mathcal{C}}_1[h_1 + g_{p1}] \; ,$$

where $g_0 = h_0 + g_{p0}$ **is an unknown function that must satisfy**

$$\int_0^\infty dv' \; v'^2 \; g_0(\mathbf{x}, v') \; f_{Me}^{(0)}(\psi, v') \; = \; 0 \qquad \textbf{and} \qquad \int_0^\infty dv' \; v'^4 \; g_0(\mathbf{x}, v') \; f_{Me}^{(0)}(\psi, v') \; = \; 0 \; ,$$

and

$$h_1(\mathbf{x}, v') \; = \; \frac{3B(\mathbf{x})}{2B_{max}(\psi)} \; \int_0^1 d\lambda \; \hat{K}(\psi, v', \lambda) \; .$$

The pressure anisotropy is given by $(p_{e\|} - p_{e\perp}) = (4\pi m_e/5) \int_0^\infty dv' \; v'^4 \; h_2 \; f_{Me}^{(0)}$**, but the** $\int_0^\infty dv' \; v'^4 \; f_{Me}^{(0)}$ **moment of the above equation results in an identity!**

The conclusion is reached that, within the available accuracy of $O(\delta^3)$, the stationary and axisymmetric drift-kinetic equation does not contain information on the pressure anisotropy moment $(p_{e\|} - p_{e\perp})(\mathbf{x}) = O(\delta^3 n T_e)$.

This is consistent with the fluid moment equation for $(p_{e\|} - p_{e\perp})$ which, in the stationary and axisymmetric case and within $O(\delta^3)$ accuracy, reduces to

$$\nabla \cdot \left[(2q_{eB\|} - q_{eT\|})\mathbf{b}\right] \; + \; 3q_{eT\|}\mathbf{b} \cdot \nabla \ln B \; + \; \nabla \cdot \left(\frac{N^{(0)}T_e^{(0)}}{eB} \mathbf{b} \times \nabla T_e^{(0)} \right) \; + \; \frac{3N^{(0)}T_e^{(0)}}{eB} \left(\mathbf{b} \times \nabla T_e^{(0)} \right) \cdot \boldsymbol{\kappa} = 0.$$

So, the determination of $(p_{e\|} - p_{e\perp})(\mathbf{x})$ as well as $T_e(\mathbf{x}) - T_e^{(0)}(\psi)$ in a stationary and axixymmetric system requires carrying the analysis at least to $O(\delta^4)$. This would necessitate a drift-kinetic equation accurate to the second order of the electron gyroradius and including the quadratic parts of the collision operators.

SUMMARY

A closed fluid and drift-kinetic electron system for slow dynamics has been put forward. It is accurate to the third order in a small ion gyroradius and large parallel collisional mean free path expansion, and is compatible with the neoclassical theory for electrons in their banana regime.

The stationary and axisymmetric limit of such electron system has been studied. Here, the lowest significant order expressions for the poloidal flow and the odd parallel closures, $q_{e\|}$ and $F_{e\|}^{coll}$, have been derived. On the other hand, it has been shown that the lowest significant order of the stationary pressure anisotropy, $(p_\| - p_\perp)/(nT_e) = O(\delta^3)$, is not determined by the available third order equilibrium system. Therefore, even though the parallel collisional friction force is known with third order accuracy, $F_{e\|}^{coll}/(nT_e) = O(\delta^3/L)$, the equilibrium parallel electric field can only be determined in its collisionless second order: $\mathrm{b}\cdot\nabla(e\Phi/T_e) = \mathrm{b}\cdot\nabla \ln n = O(\delta^2/L)$.