Wall Force produced during a Disruption

H. Strauss, *HRS Fusion*

R. Paccagnella, *Instituto Gas Ionizzati del C.N.R.*

J. Breslau, *PPPL*

L. Sugiyama, *MIT*

- Nonaxisymmetric wall forces in ITER
  - are produced on conducting structures during a disruption
  - calculate with a thin resistive wall model
  - obtain scaling of wall force with current and wall constant
- Simulations with M3D
  - upwind method to dissipate fine scale turbulence
  - thin resistive wall
    - double ITER wall model

Resistive Wall

The normal component of magnetic field is continuous at the wall,

$$B_n^v = B_n^p,$$

where B_n^v, B_n^p are the normal component of magnetic field in the vacuum, just outside the wall, and the plasma (or blanket), just inside the wall. Integrating the resistive diffusion equation across the wall gives

$$\frac{\partial B_n}{\partial t} = -\frac{\eta_w}{\delta}\nabla \cdot \left[\hat{\mathbf{n}} \times (\mathbf{B}^v - \mathbf{B}^p) \times \hat{\mathbf{n}}\right] \tag{1}$$

where η_w is the wall resistivity, δ is wall thickness, and $\hat{\mathbf{n}}$ is the normal unit vector. The vacuum field is solved using Green's functions, with the GRIN code.

The vacuum field is represented as

$$\mathbf{B}_v = \nabla\psi^v \times \nabla\phi + \nabla\lambda + I_0 \nabla\phi$$

where I_0 is a constant. From Green's identity one has an integral equation relating $\partial\psi^v/\partial n$ to given ψ^v, and λ_n to given $\partial\lambda_n/\partial n$ on the boundary. When discretized, these integral equations become matrix equations which are set up and solved by GRIN. Given a set of boundary points, R_i, Z_i

$$\left(\frac{\partial\psi^v}{\partial n}\right)_i = \sum_j K^0_{ij}\psi^p_j + S_{xi}, \quad (2)$$

$$\lambda^n_i = \sum_j K^n_{ij}(B^p_n)_j \quad (3)$$

where K^0_{ij}, K^n_{ij} are matrices that can be precomputed given the set of boundary points. The source term S_x is chosen so that at the initial time, $\partial\psi^v/\partial n = \partial\psi^p/\partial n$.

Wall Pressure

The current in the wall is given by

$$\mathbf{J}_w = \nabla \times \mathbf{B} \approx \frac{\hat{\mathbf{n}}}{\delta} \times (\mathbf{B}_v - \mathbf{B}_p).$$

The normal component of the force density is

$$F_{wn} = \hat{\mathbf{n}} \cdot \mathbf{J}_w \times \mathbf{B}_w = -\frac{1}{\delta}(\mathbf{B}_v - \mathbf{B}_p) \cdot \mathbf{B}_w.$$

Inside the wall assume that

$$\mathbf{B}_w = \frac{1}{2}(\mathbf{B}_v + \mathbf{B}_p).$$

The normal wall force density can be expressed

$$F_{wn} = \frac{1}{2\delta}(|\mathbf{B}_p|^2 - |\mathbf{B}_v|^2). \tag{4}$$

It has a simple physical meaning. It is the difference in magnetic pressure across the wall, divided by the wall thickness.

Integrating over the wall thickness δ gives the magnetic pressure on the wall. The normalized wall pressure P_w is

$$P_w = \frac{(|\mathbf{B}_p|^2 - |\mathbf{B}_v|^2)}{2B_0^2}$$

where B_0 is the vacuum toroidal magnetic field on axis.

The tangential component of the wall force is

$$F_{w\ell} = \frac{1}{\delta}(\hat{\mathbf{n}} \cdot \mathbf{B})(\mathbf{B}_\ell^v - \mathbf{B}_\ell^p).$$

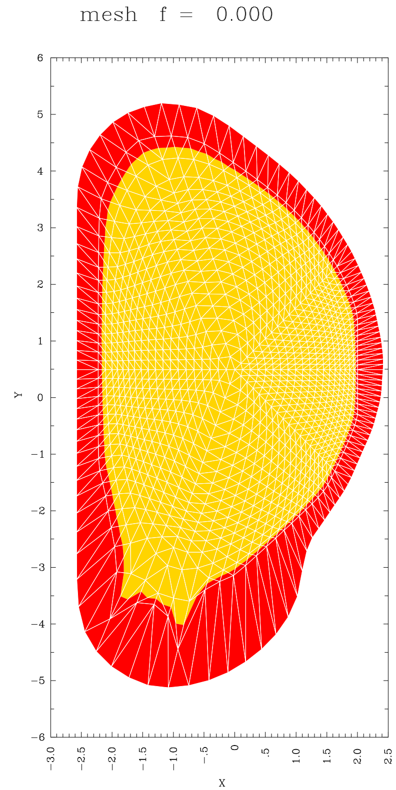
where $\hat{\ell} = -\hat{\mathbf{n}} \times \hat{\phi}$. The physical interpretation is

$$F_{w\ell} = \mathbf{J}_\phi \hat{\mathbf{n}} \cdot \mathbf{B}$$

In general,

$$F_{w\ell} \sim |B_1|^2 << F_{wn} \sim |B_1|.$$

Mesh ITER Double wall

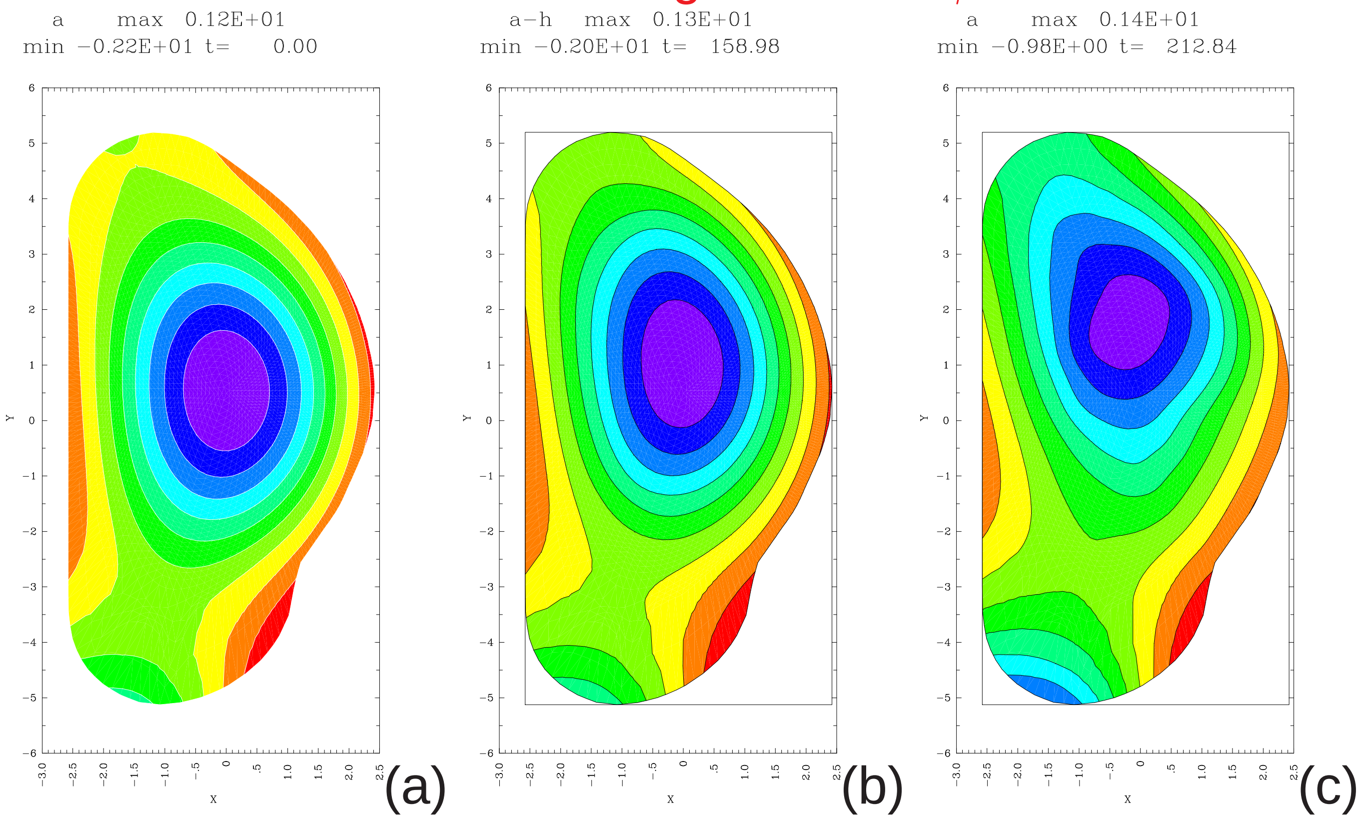


Low resolution ITER mesh. Simulation regions: inner plasma region (gold), first wall, blanket (red), outer wall, outer vacuum (white). The velocity vanishes outside the first wall, but the magnetic field is continuous up to the outer wall. Resistive wall boundary conditions are applied at the outer wall. The assumption is that the first wall is much more resistive than the outer wall.

Disruption Simulation

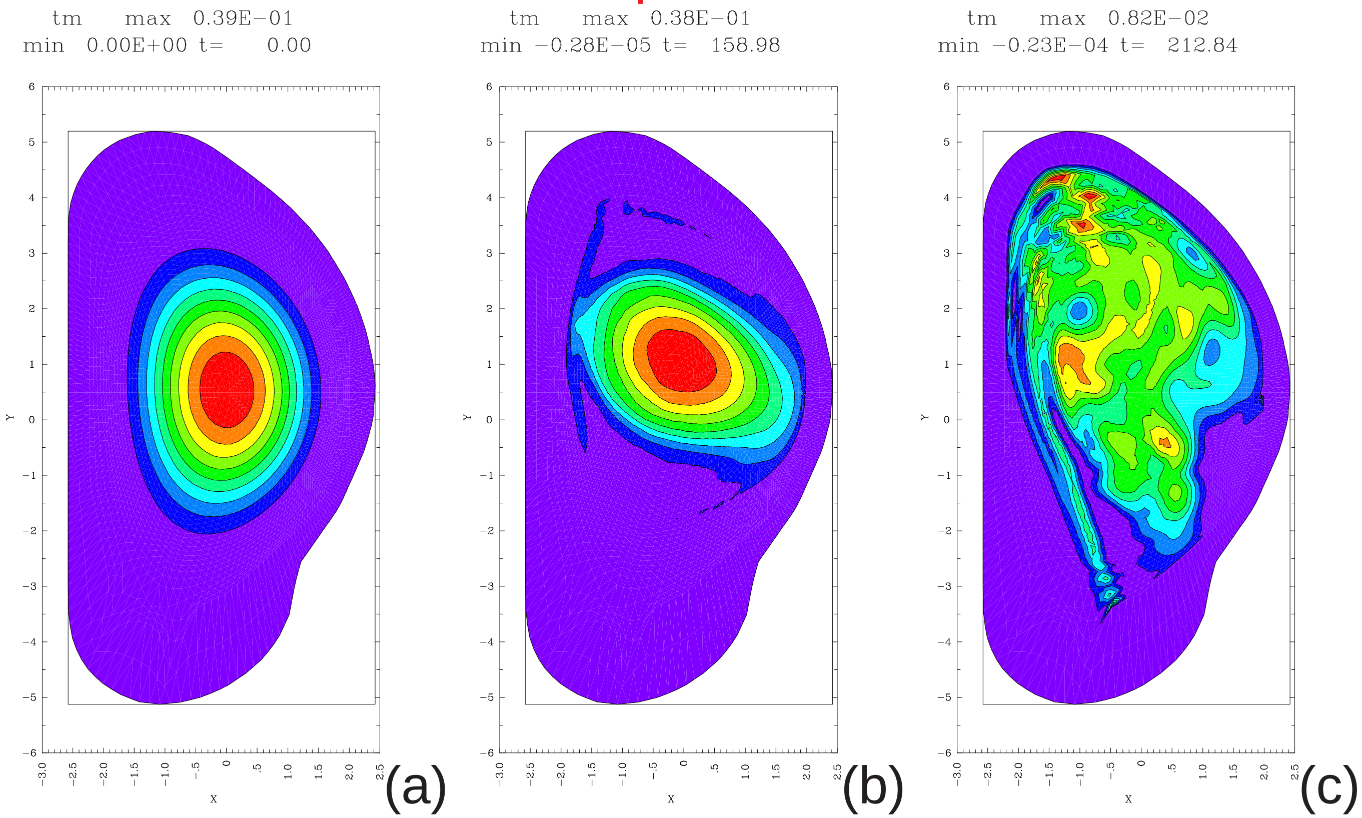
The initial state is an ITER reference case equilibrium (FEAT15MA). calculated by CHEASE, and written to a file in EQDSK format. This was read into M3D and used to generate a mesh and initialize a nonlinear simulation. The initial equilibrium had $q = 1.1$ on axis. The magnetic flux ψ, and the other equilibrium quantities were rescaled to generate a sequence of equilibria with $0.6 < q < 1.1$ on axis, and $3 < I/(aB) < 9$. This models what might have occurred if outer layers of plasma were scraped off during a VDE. The resulting state is unstable to a resistive wall mode or external kink, depending on q. A small perturbation was added to the plasma and it was allowed to evolve nonlinearly.

Poloidal Magnetic Flux ψ



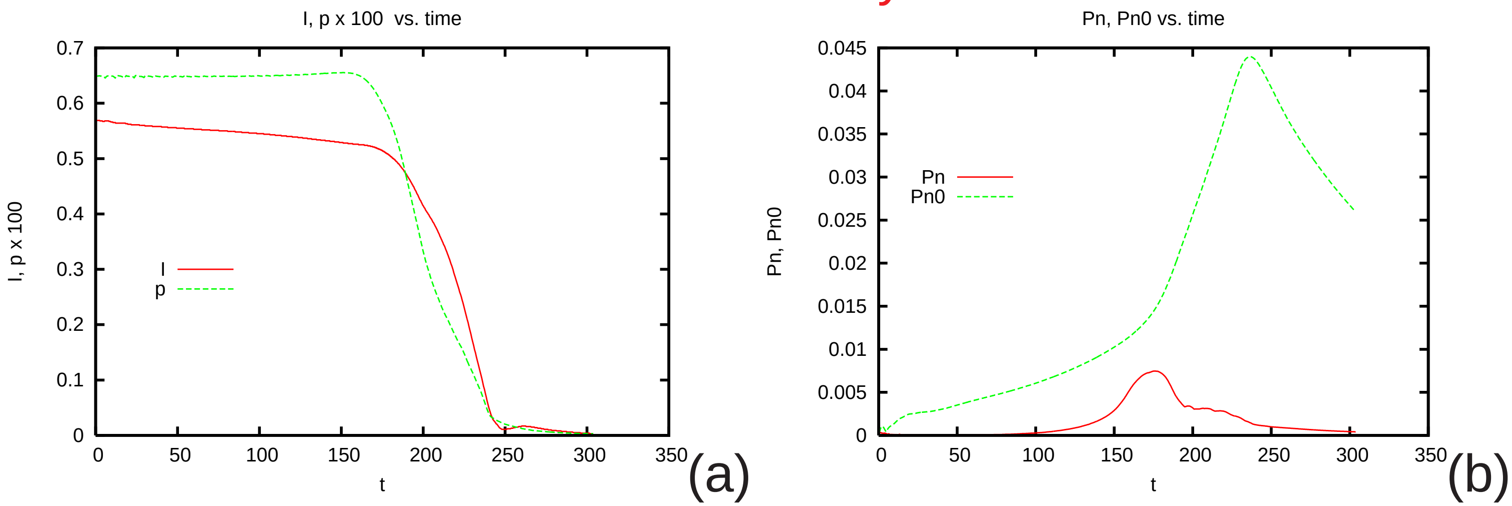
(a) initial magnetic flux contours of ITER equilibrium reconstruction. This case was not rescaled. (b) magnetic flux contours in the poloidal plane with toroidal angle $\phi = 0$, at time $t = 159\tau_A$. (c) at time $t = 213\tau_A$ with nonlinear VDE and RWM.

Temperature



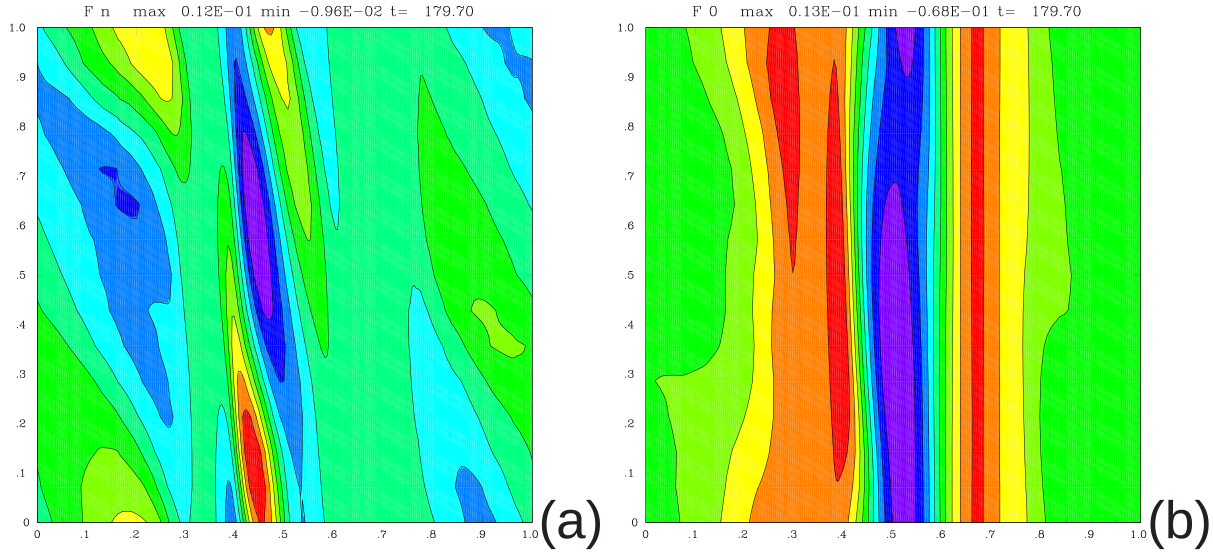
(a) initial temperature contours in the poloidal plane with toroidal angle $\phi = 0$. (b) temperature contours at $t = 159\tau_A$, showing predominantly $(m, n) = (2, 1)$ structure of nonlinear RWM. (c) at time $t = 213\tau_A$. Temperature is unconfined due to parallel transport along stochastic magnetic lines.

Time history



(a) time history of the total plasma pressure (red) and total toroidal current (green) in the RWM case, showing temperature and current quench. (b) time history of the peak normalized symmetric (green) and asymmetric (red) normalized wall pressure. The peak wall pressure coincides with temperature and current quench. The asymmetric part is caused by the nonlinear n=1 mode; the symmetric part by equilibrium loss.

Spatial structure of wall magnetic pressure

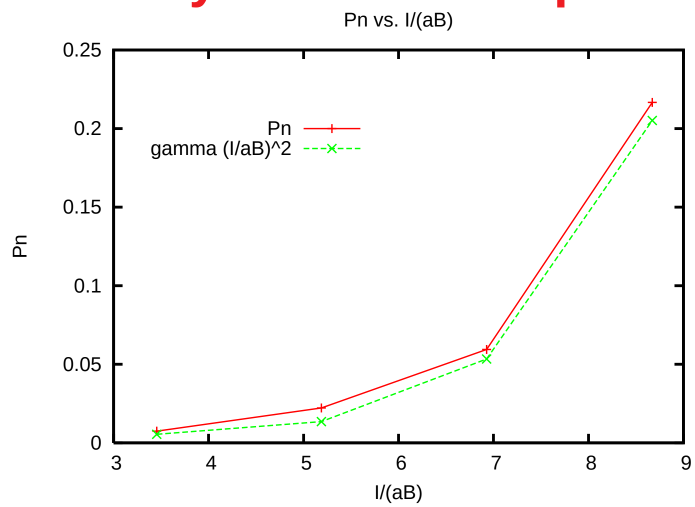


Asymmetric magnetic pressure on the wall as a function of poloidal angle (horizontal) and toroidal angle (vertical) (a) $n \geq 1$ part of RWM at time $t = 180\tau_A$. The normal wall pressure has mostly $(m, n) = (2, 1)$ structure. (b) $n \geq 0$ wall pressure at $t = 180\tau_A$.

Scaling of Wall Pressure with Current

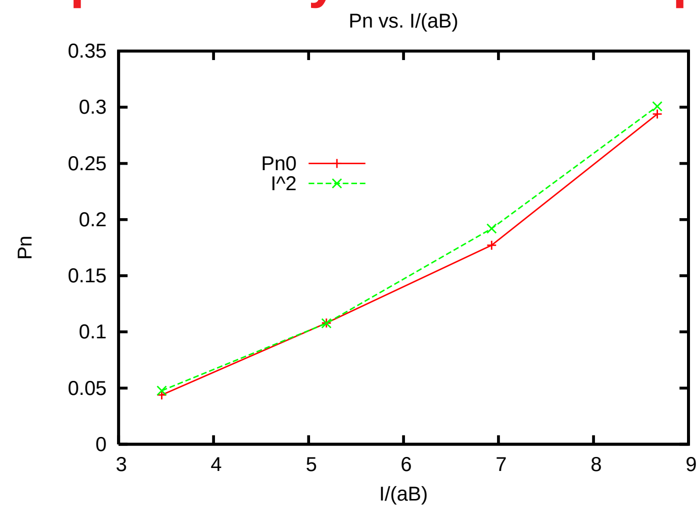
In JET, the net vertical symmetric wall force and the net horizontal asymmetric wall scale as the square of the current (Riccardo et al., NF 2000;Riccardo et al., NF 2004).

Peak non symmetric wall pressure



Scaling of peak non axisymmetric normal force density with total current I/B_0 and $0.01\gamma(I/B_0)^2$, where γ is the growth rate.

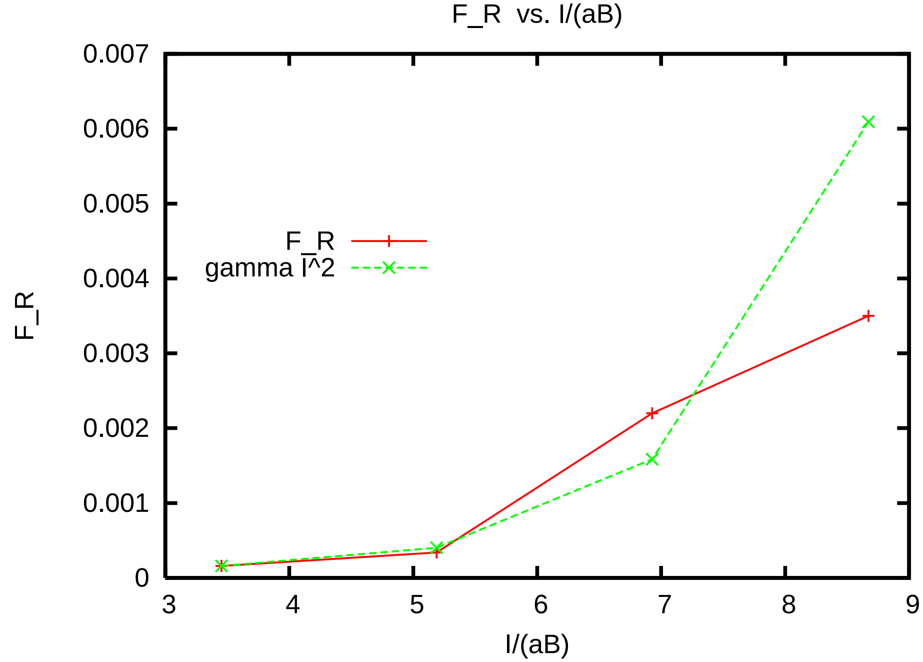
Scaling of peak axisymmetric wall pressure



Scaling of peak axisymmetric normal force density $< P_{wn} >$ with total current I/B_0. Also shown is $0.004(I/B_0)^2$. The scalings are consistent with JET data.

The peak pressure is rather high; but the wall averaged net horizontal force and the net vertical force are about an order of magnitude smaller. The ITER disruption database only includes $I/(aB) \leq 7$.

Scaling of wall averaged horizontal force



Scaling of wall averaged non axisymmetric horizontal force density with total current I/B_0. Also shown is $\gamma(I/B_0)^2$.

The horizontal normalized pressure force is

$$P_h \approx \frac{1}{2\pi L} \int d\phi dl R P_n n_R$$

It is about 5% of the peak normal $n = 1$ pressure. In terms of ITER parameters, the toroidal field $B_\phi = 5.3T$, the plasma surface area is $2\pi \int dlR = 917\mathrm{m}^2$. In the RWM case $I/(aB) = 3.4$, the total horizontal force is $F_R = 1.25$ MN.

Scaling of Force with Current and Wall Resistivity

The scaling of wall force with current and wall resistivity can be estimated using a simple model. The magnetic field is approximately,

$$\mathbf{B} = \nabla\psi \times \widehat{\phi} + B\widehat{\phi},$$

and assuming a simple circular flux surface geometry, $\psi = \psi_0(r) + \psi_{mn}\,\exp(im\theta + in\phi)$, with constant toroidal current $\nabla^2\psi_0 = 2B/(q_0 R_0)$ inside the plasma boundary at $r = a$. A perturbed equilibrium satisfies

$$\mathbf{B}_0 \cdot \nabla\nabla^2\psi_1 + \mathbf{B}_1 \cdot \nabla\nabla^2\psi_0 = 0.$$

Integrating this across the plasma - blanket interface at $r = a$ gives

$$\frac{\partial\psi^b_{mn}}{\partial r} - \frac{\partial\psi^p_{mn}}{\partial r} - \frac{2k_\perp}{k_{||} q_0 R_0}\psi^p_{mn} = 0. \tag{5}$$

where $k_{||} q_0 R_0 = (m - nq_0)$, and $k_\perp = m/a$. At the vacuum - blanket interface, $r = b$, from (1),

$$\gamma\psi_{mn} = \frac{\eta_w}{\delta}\left(\frac{\partial\psi^v_{mn}}{\partial r} - \frac{\partial\psi^b_{mn}}{\partial r}\right) \tag{6}$$

The force density (4) is

$$F\delta = B_\theta \left(\frac{\partial \psi^v_{mn}}{\partial r} - \frac{\partial \psi^b_{mn}}{\partial r} \right)$$

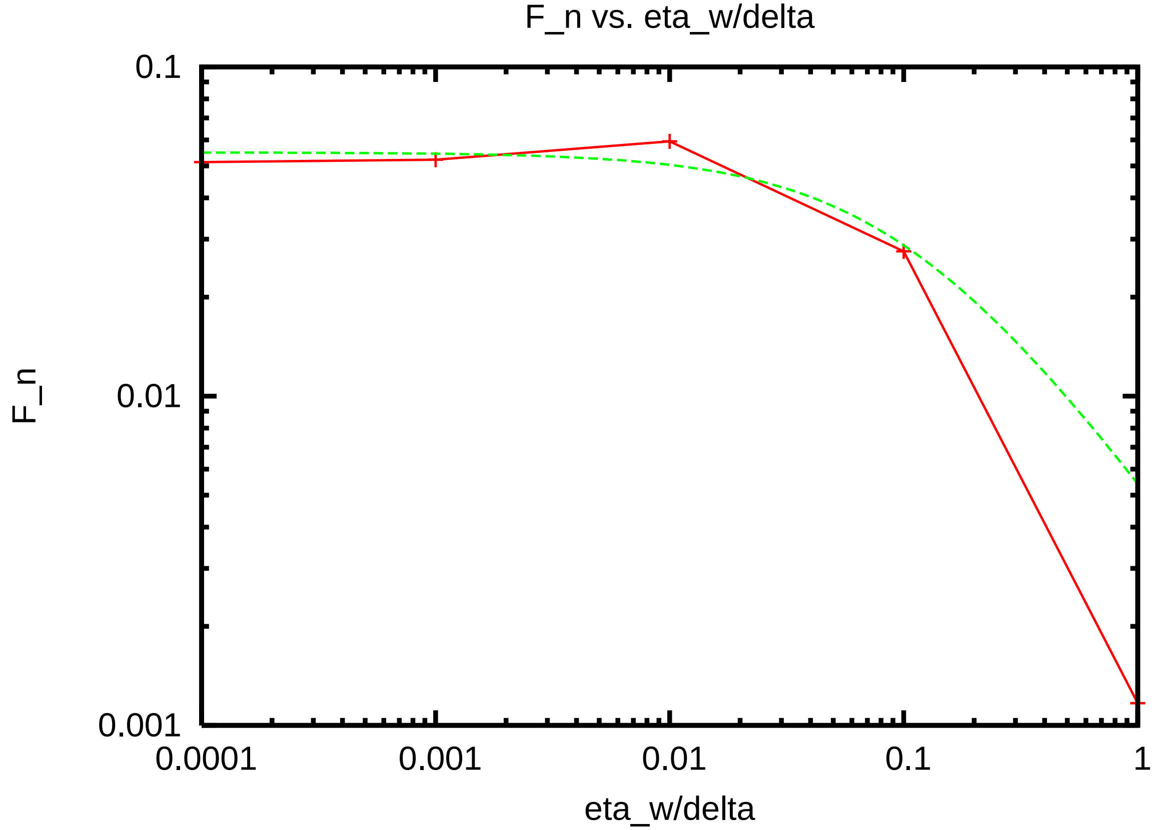
In the plasma, $\psi_{mn} = \psi_1 r^m$, in the blanket $a < r < b$, $\psi_{mn} = \psi_2 r^m + \psi_3 r^{-m}$, and in the vacuum, $r > b$, $\psi_{mn} = \psi_4 r^{-m}$. Using continuity of ψ_{mn} at $r = a$ and at $r = b$, and using (5) and (6), it is possible to eliminate ψ_2, ψ_3 and ψ_4 to obtain

$$F\delta = \frac{B_\theta}{1 + \frac{\eta_w m}{\gamma \delta b}} \left[1 + \frac{1 + (\frac{a}{b})^{2m}}{m - nq} \right] \psi_1 b^m. \qquad (7)$$

If ψ_1 is assumed to be proportional to ψ_0, which in turn is proportional to B_θ, then $F\delta$ is proportional to $I^2/(aB)^2$. In the limit $\gamma \ll (\eta_w m)/(\delta b)$, $F\delta \propto \gamma I^2/(aB)^2$. The net horizontal force, averaged over the wall, is

$$< F >= \frac{1}{4\pi^2 R_0} \int \int d\phi d\theta R F \sim \frac{1}{4} (\psi_{11} + \frac{b}{R_0} \psi_{21})$$

Wall pressure as a function of η_w/δ.



Variation of peak wall pressure $P_w = F_{wn}\delta/B_0^2$ as a function of wall resistivity divided by wall thickness, η_w/δ. The data is fit by the formula, $P_w \propto 1/(1+\alpha\eta_w/\delta)$ where $\alpha = 9$. This justifies the two wall model; the wall force goes to zero with the wall time constant.

Numerical Difficulties

- Current generated in disruptions cause magnetic island overlap, stochastic magnetic field. Arbitrarily short spatial scales are generated.

- Pressure driven (ballooning modes) are unstable for all wavelength in ideal MHD.

- Dissipation is required to limit the spatial scales. A large resistivity η varied from 10^{-5} on axis to 10^{-2} at the wall. A spatially constant perpendicular viscosity was used, $\mu = 10^{-4}$.

Dissipative Numerical Methods

- Upwinding to maintain positivity of density and temperature
  - gives an effective dissipation $D \sim v\Delta$ where Δ is the grid size
  - flux limiter subtracts off most of the dissipation if there are no mesh point to mesh point oscillations - not so important when $v \ll v_A$.
  - only diffusive part is used as an "artificial diffusion."
- dealiasing: highest 1/3 of n modes set to zero

Future Work

- higher resolution will be used to improve results.
- try initial states corresponding to different disruption scenarios.
- carry out JET simulations and compare with data.
- contact with Hiro current theory (Zakharov 2008).
- two wall model improvements:
  - add structure to blanket
  - give finite resistivity to first wall; blanket region solved with Green's function or elliptic solver