Update on kinetic effects of energetic particles on nonlinear resistive MHD instability

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Results on Energetic Particle on resistive MHD linear stability

Energetic particles have significant damping and stabilizing effects at experimentally relevant β_N, β_{frac}, and S, and weaker damping and stabilizing effects in the ideal unstable regime, and excite a real frequency of the 2/1 mode.

- PRL 2009
- Nucl. Fusion 2009

We are on a path to research:

"Nonlinear + 2fl effects + Energetic particles."

Recent Results Show Energetic Particle/MHD Coupling Important and Computationally Viable

Historical focus has been on the simplified effects on the 1/1 mode.

Recent Computational Efforts Successful

- Choi, Turnbull, Chan (GA) Show highly accurate prediction of the sawtooth crash in DIII-D (PoP 2007). --> D.D. Schnack et al.(Sherwood 09)
- *Kim (U. Wash.) Shows Linear Evolution of 1/1 with Benchmark against M3D, NOVA-K/GATO. (Comp.Phys.Comm.164.2004, PoP 2008)*

Our resistive MHD analyses suggest possible energetic particle stabilization of resistive 2/1 modes at high energetic particle beta fractions.

PIC noise complicates study of Energetic Particles on resistive MHD stability

- PIC code injects noise into earlier linear stage ($\beta_{frac}/ \beta_{frac\_c} \sim (V_{\phi h} /V_{rh})^2$), (these errors can be decreased by increasing particle numbers), however later driven & saturation stages can be recovered.
- With 2fl, MHD linear stability will be even more complicated. (Localization, long timescale.)

The δf PIC model

- PIC is a Lagrangian simulation of phase space $f(\mathbf{x},\mathbf{v})$
- PIC evolves the $f(\mathbf{x}(t),\mathbf{v}(t))$
- δf PIC reduces the discrete particle noise associate with conventional PIC
- Vlasov equation $\dfrac{\partial f(\mathbf{z})}{\partial t} + \dot{\mathbf{z}} \cdot \dfrac{\partial f(\dot{\mathbf{z}})}{\partial t} = 0$
- Evolution equation for δf, $\quad \delta\dot{f} = -\dot{\delta}\mathbf{z} \cdot \dfrac{\partial f_0}{\partial t}$.
- the drift kinetic equations of motion are used as the particle characteristics

$$\dot{\mathbf{x}} = v_{\|}\hat{\mathbf{b}} + \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{m^2}{eB^4}(v_{\|}^2 + \frac{v_{\perp}^2}{2})(\mathbf{B}\times\nabla\frac{B^2}{2}) - \frac{\mu_0 m v_{\|}^2}{eB^2}\mathbf{J}_{\perp},$$

$$m\dot{v}_{\|} = -\hat{\mathbf{b}}\cdot(\mu\nabla B - e\mathbf{E}).$$

The slowing down distribution function for energetic particles

The slowing down distribution function

The total poloidal magnetic flux

$$f = \frac{P_0 \exp\left(P_\zeta / \psi_n\right)}{\varepsilon^{3/2} + \varepsilon_c^{3/2}}, \quad P_\zeta \propto \psi, \quad \psi_n = C\psi_0$$

A constant matching the equilibrium pressure profile

$P_\zeta = g\rho_\parallel - \psi_p$, is the canonical toroidal momentum.

The initial equilibrium state, $\exp\left(g\rho_\parallel / \psi_n\right)$ is ignored: an energetic isotropic pressure.

The linearized evolution equation for δf becomes

$$\delta\dot{f} = f_0\{\frac{mg}{e\psi_n B^3}[(v_\parallel^2 + \frac{v_\perp^2}{2})\delta\mathbf{B}\cdot\nabla B - \mu_0 v_\parallel \mathbf{J}_\perp \cdot \delta\mathbf{E}$$

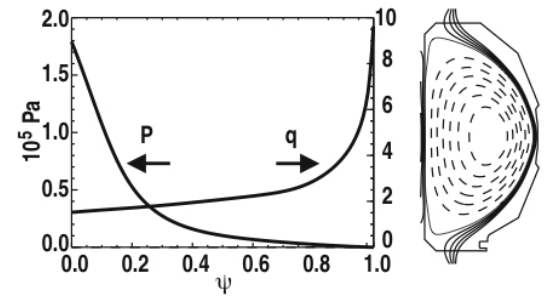
$$+ \frac{\delta\mathbf{v}\cdot(\nabla\psi_p - \rho_\parallel\nabla g)}{\psi_n} + \frac{3}{2}\frac{e\varepsilon^{1/2}}{\varepsilon^{3/2} + \varepsilon_c^{3/2}}\mathbf{v}_D \cdot \delta\mathbf{E}\},$$

$$\mathbf{v}_D = \frac{mg}{eB^3}(v_\parallel^2 + \frac{v_\perp^2}{2})(\mathbf{B}\times\nabla B) + \frac{\mu_0 m v_\parallel^2}{eB^2}\mathbf{J}_\perp,$$

$$\delta\mathbf{v} = \frac{\delta\mathbf{E}\times\mathbf{B}}{B^2} + \mathbf{v}_\parallel \frac{\delta\mathbf{B}}{B}\cdot\delta\mathbf{E}.$$

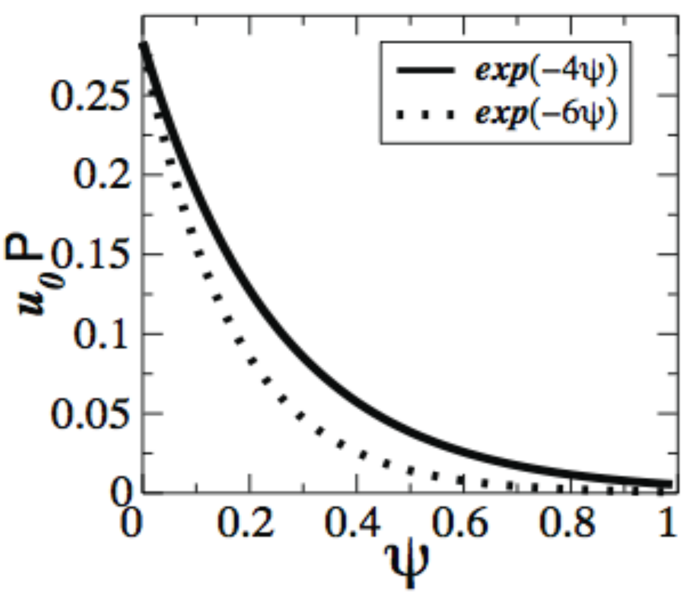
Equilibrium pressure and safety factor profiles as a function of ψ in the D shape

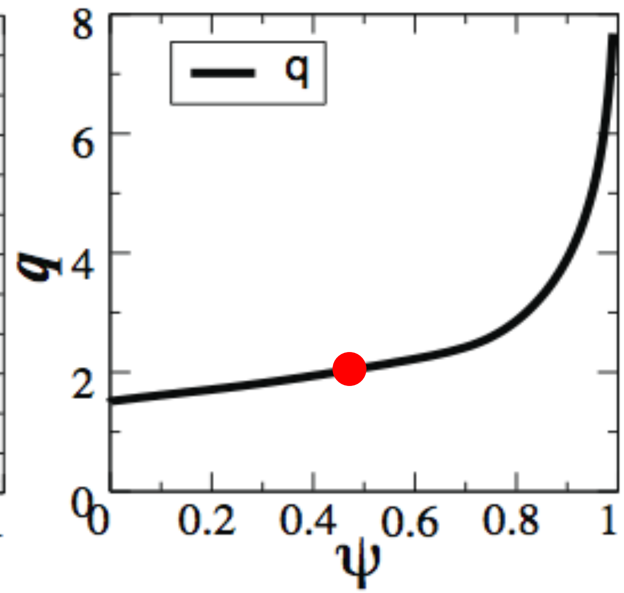
Equilibrium :the D shape tokamaks from Brennan, et al. Nucl. Fusion 45, 1178, 2005



Pr (the ratio of the viscosity to electric diffusivity)=100

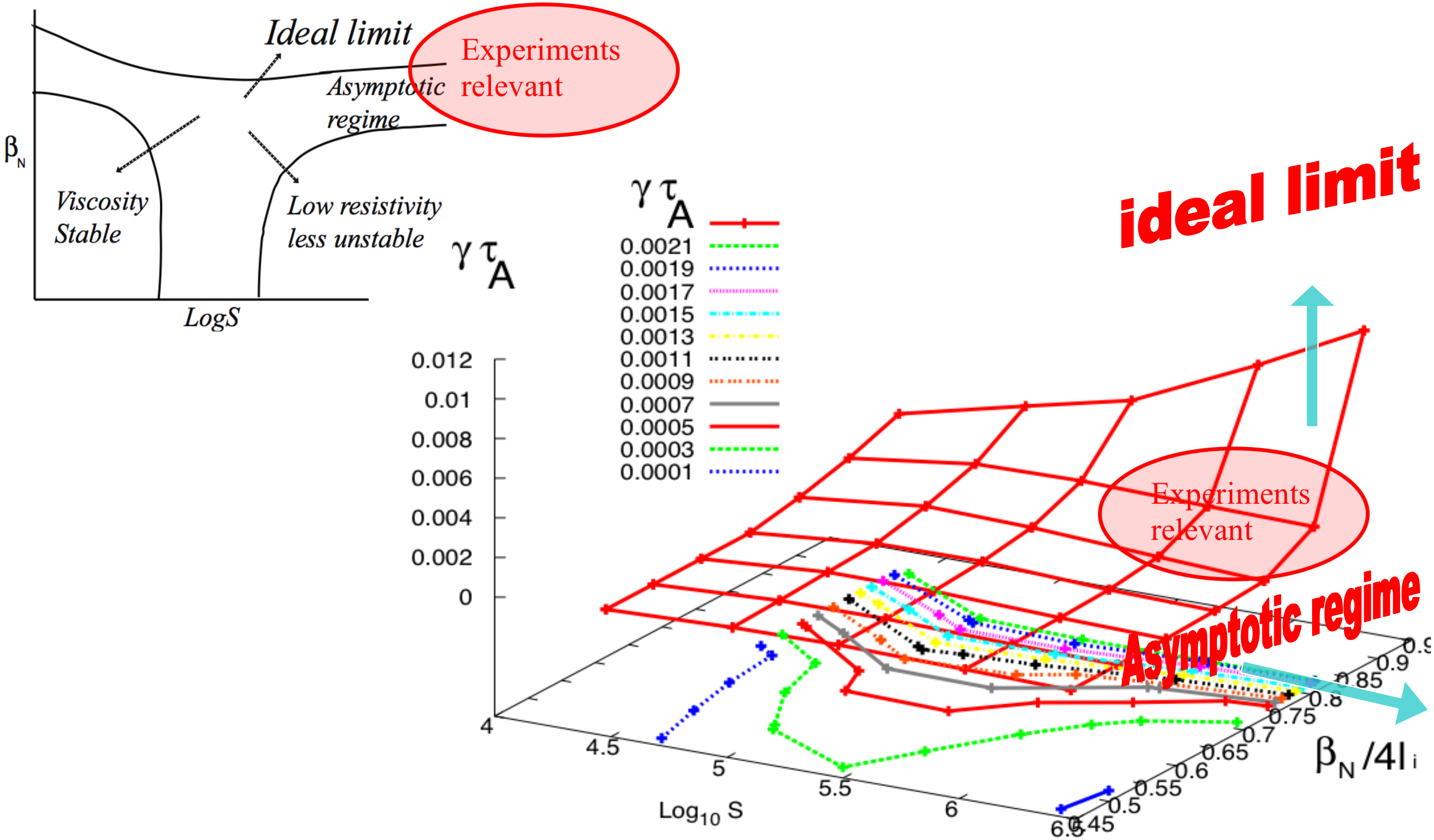
$$f \sim \exp(\psi / C)$$





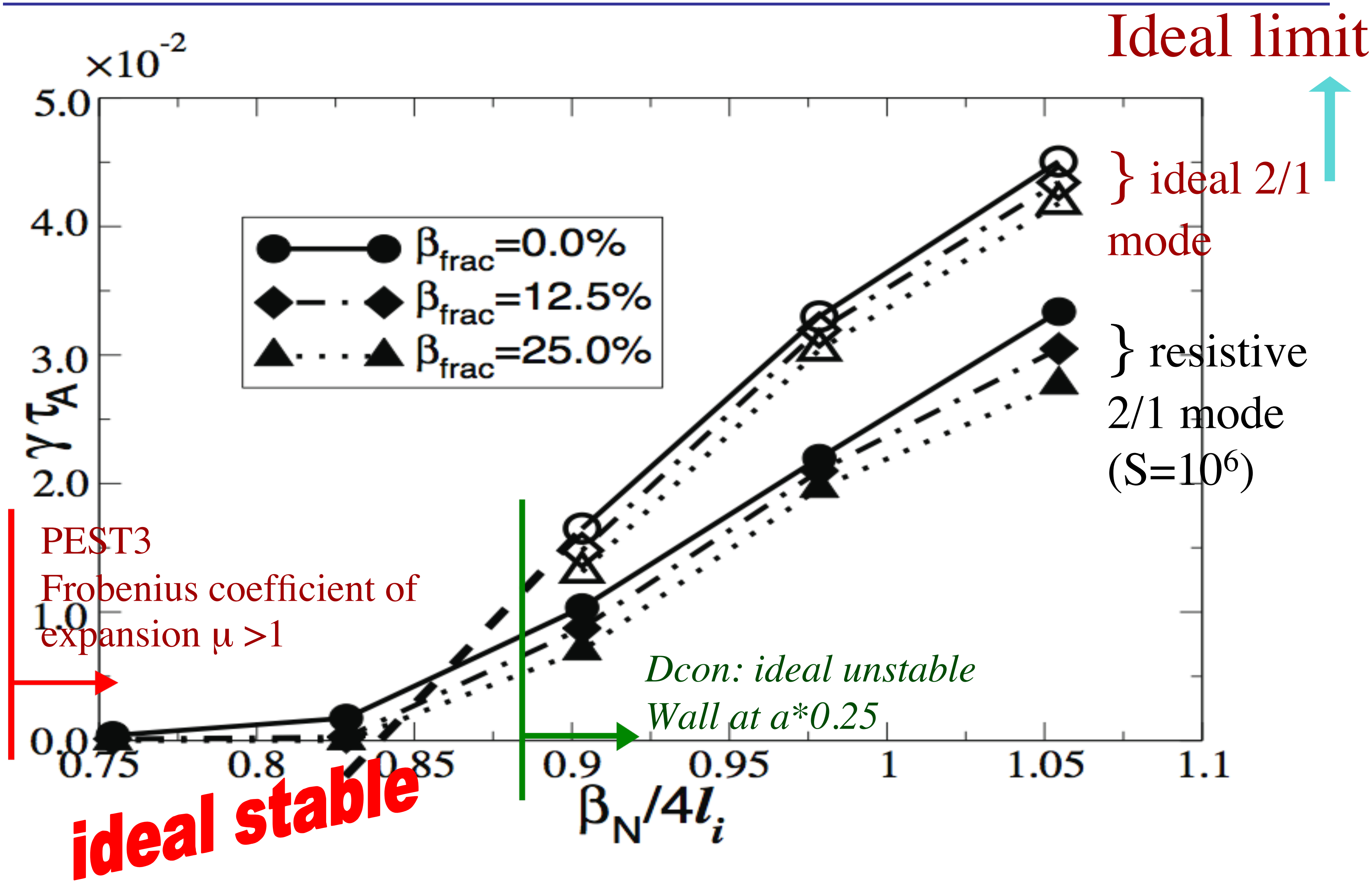
$$q_{\min} \approx 1.5, \quad q_{95} \approx 4.4$$

Linear Growth rates (of the resistive 2/1 mode) as a function of S for MHD only cases, Exp(-4ψ), (single fluid)



Growth rates for series of equilibria ($\beta_N / 4l_i$)

(stability diagram sketch)

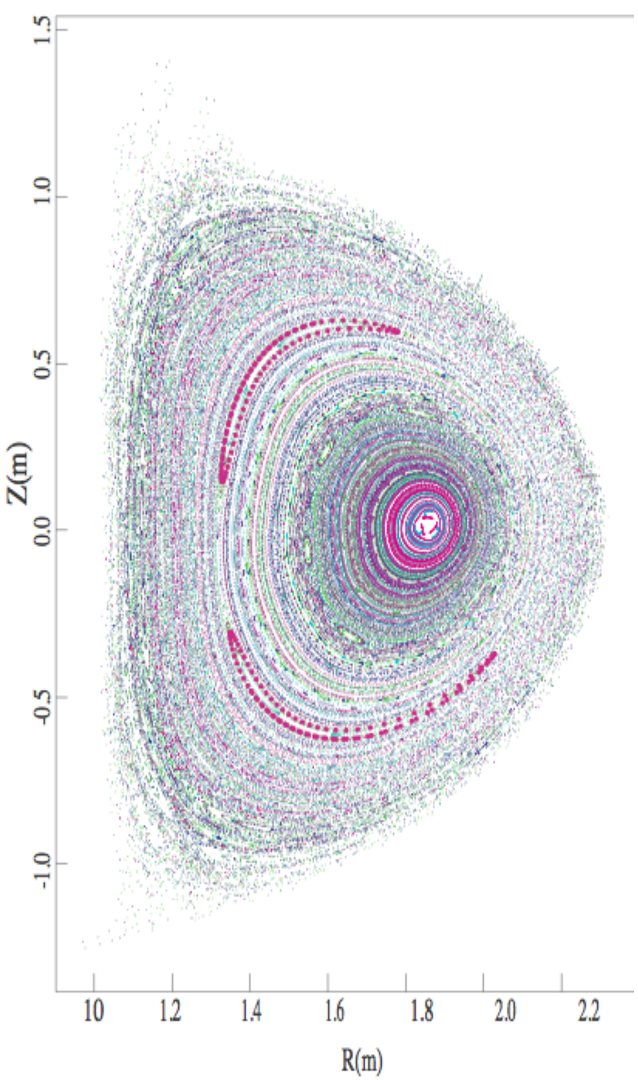


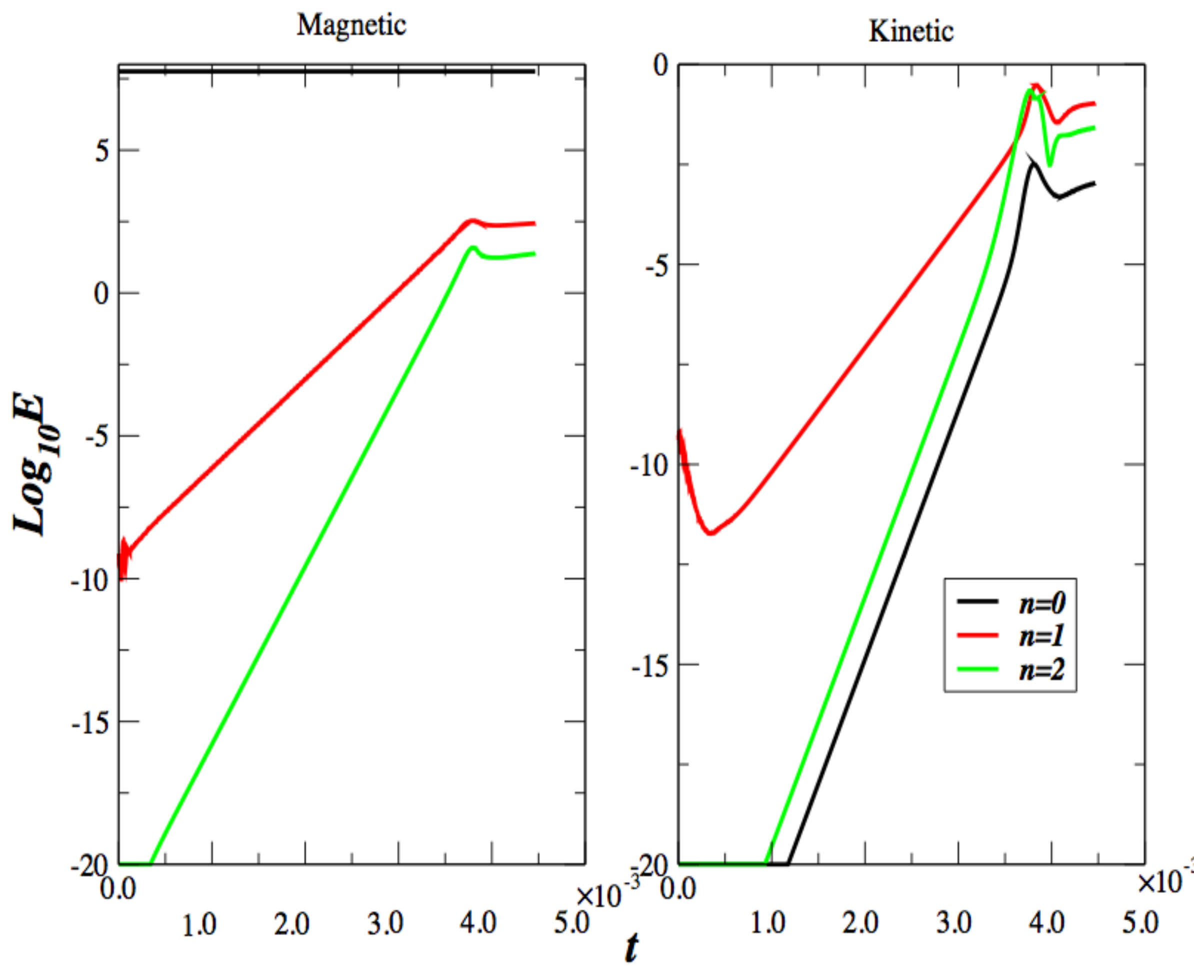
MHD only nonlinear (resistive) results

(β_N / 4l_i =0.83, S=10^6)

t = 4.8(ms)

Saturation stage can be resolved at higher modes
--> too expensive for 2-fl & energetic particles.

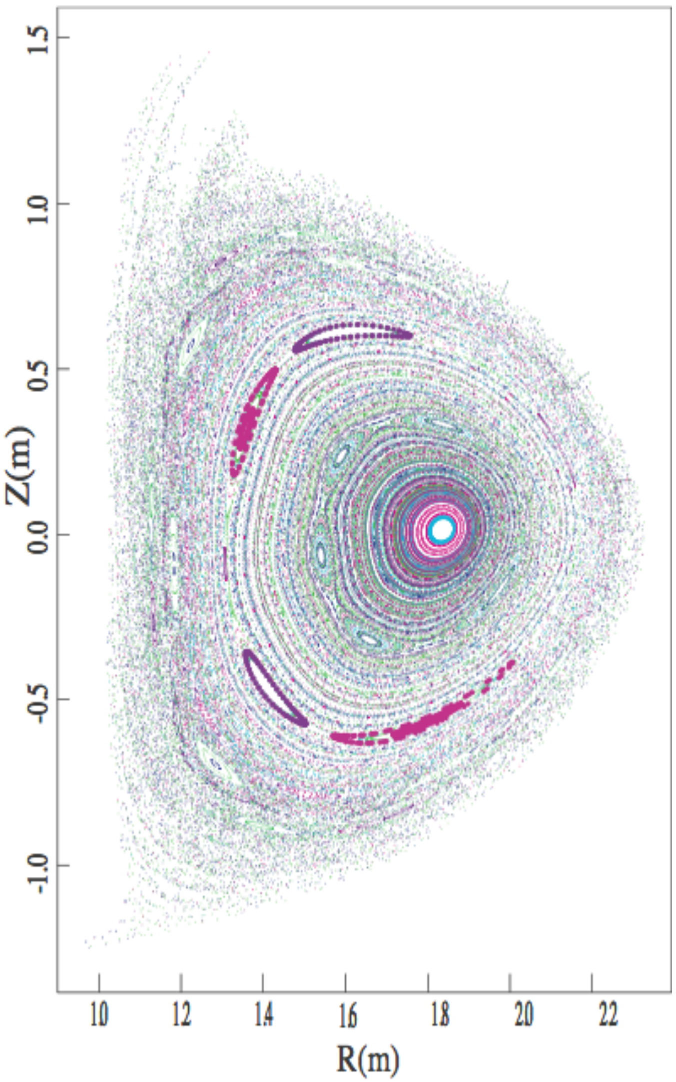




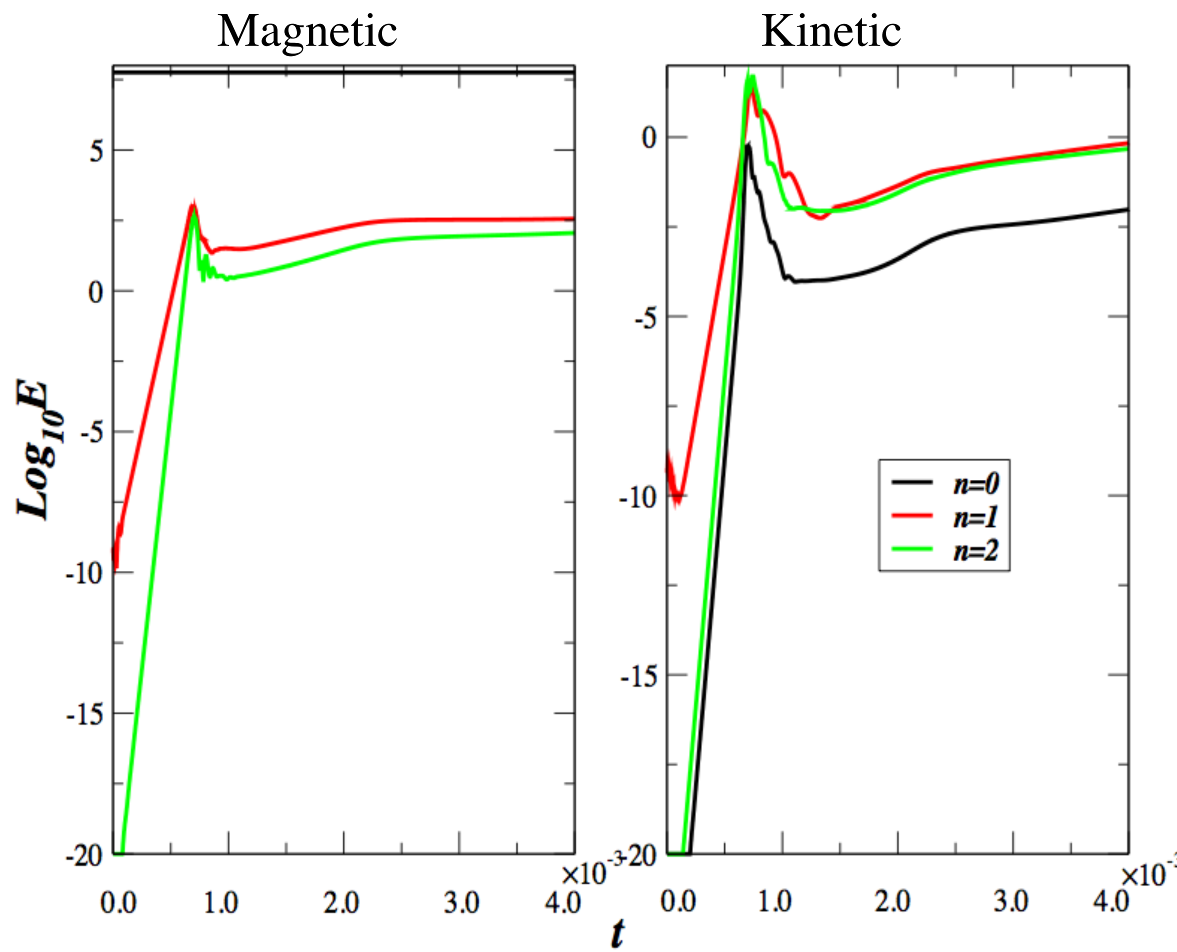
MHD only nonlinear (ideal) results

(β_N / 4l_i =0.90, S=10^6)

t = 4.5(ms)

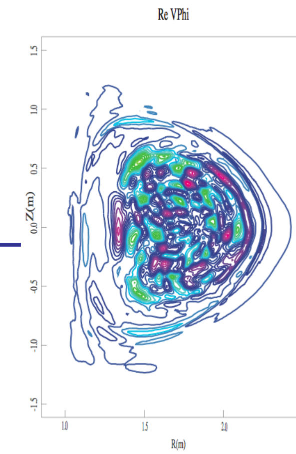


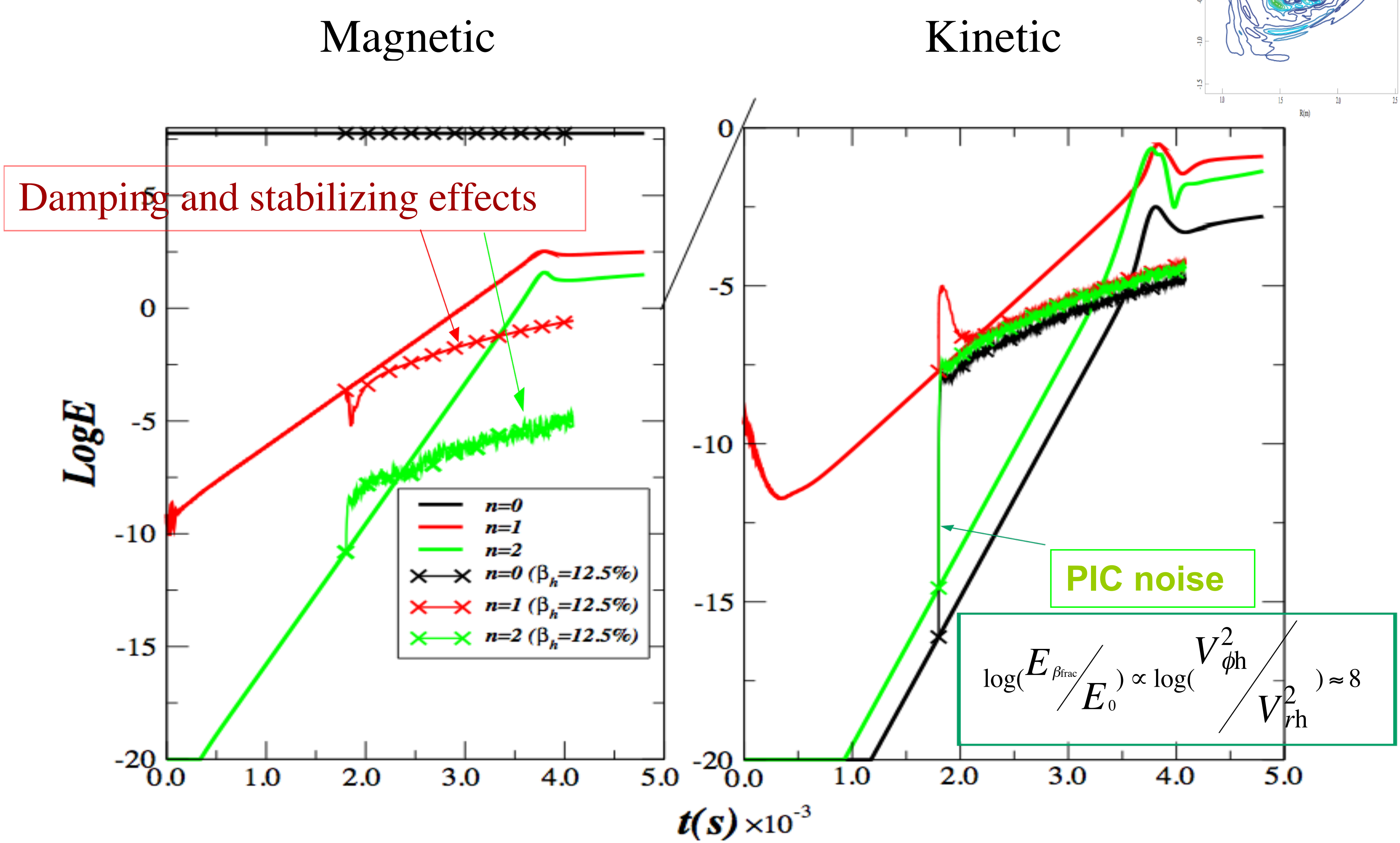
Higher toroidal modes need to resolved …
Also, need to evolved nonlinear stage longer …
m/n=4/2 islands --> n1~n2 (magnetic)



Nonlinear results ($\beta_N / 4l_i = 0.83$, $S=10^6$)

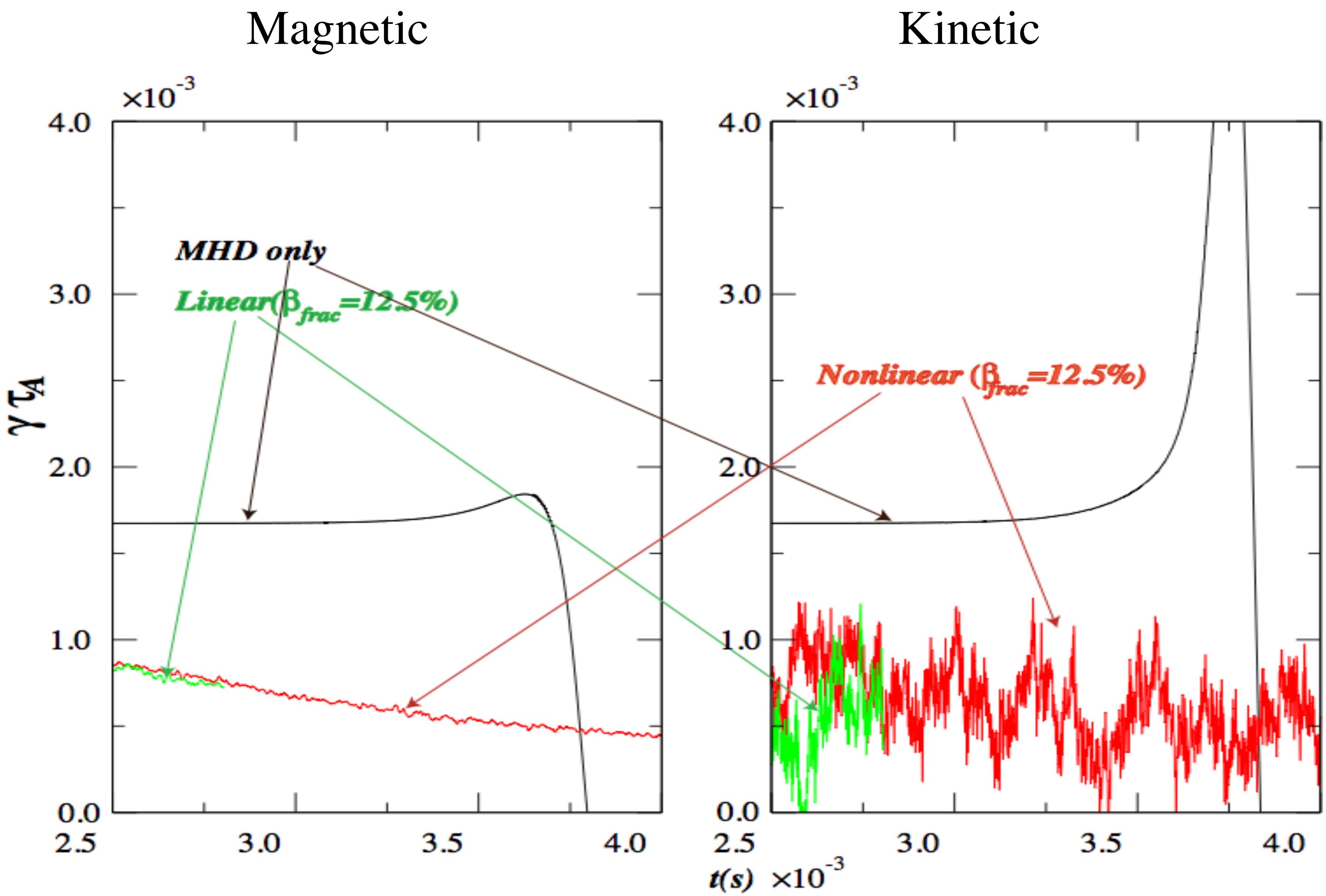
With energetic particles





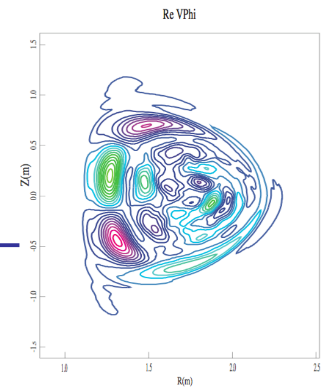
Nonlinear results (β_N / 4l_i =0.83, S=10^6)

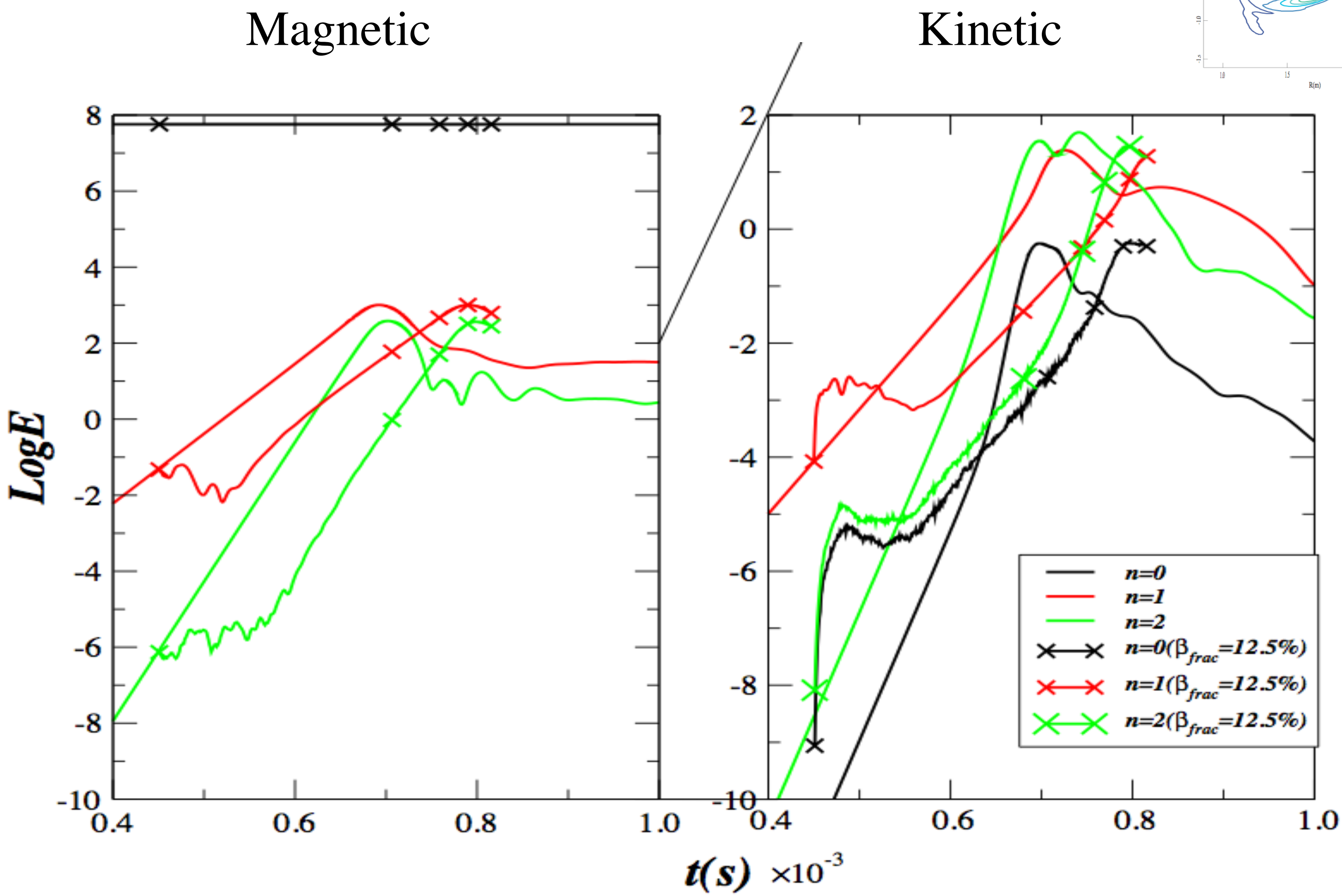
n=1 Growth rates



Nonlinear results ($\beta_N / 4l_i = 0.90$, $S=10^6$)

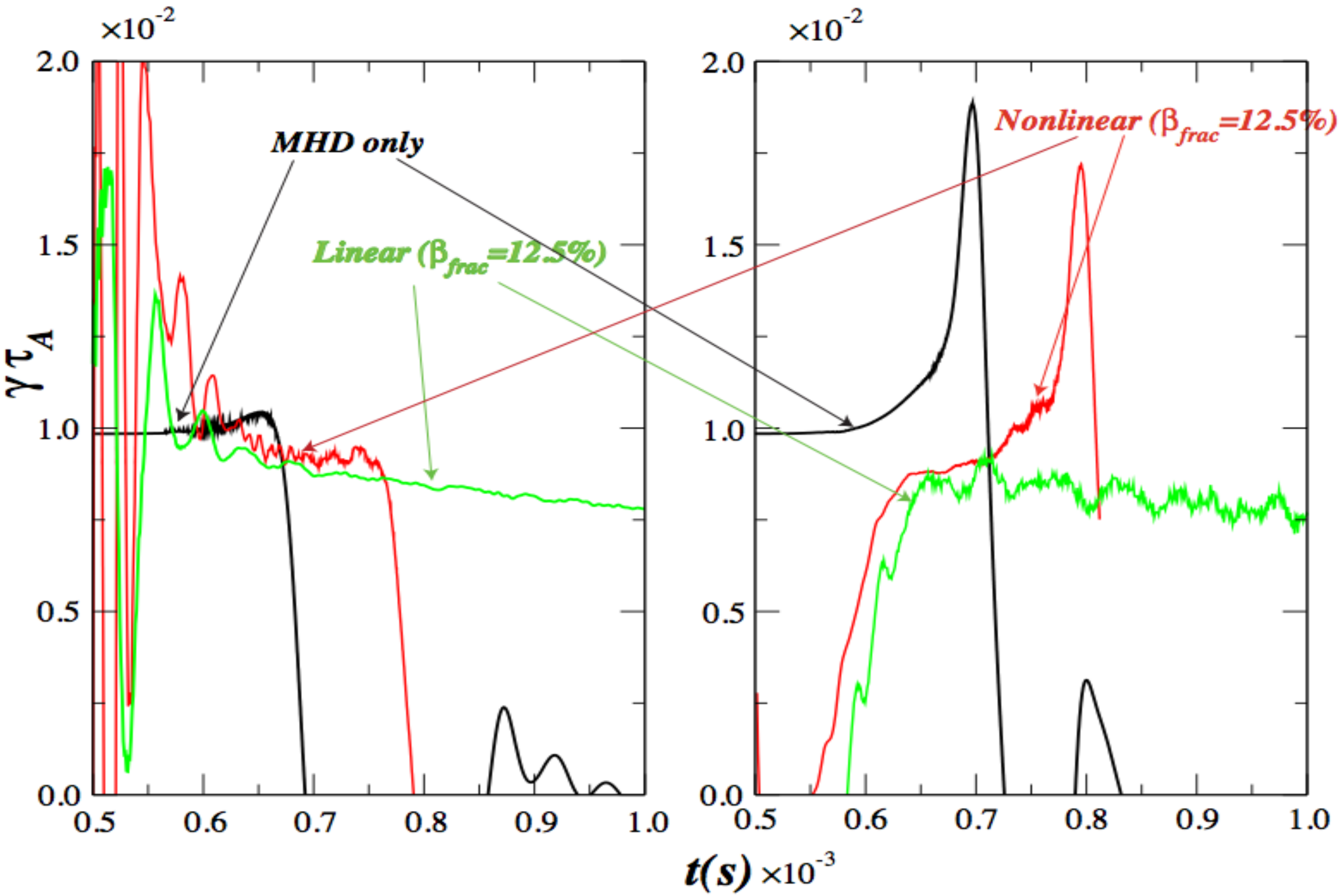
With energetic particles





Nonlinear results (β_N / 4l_i =0.90, S=10^6)

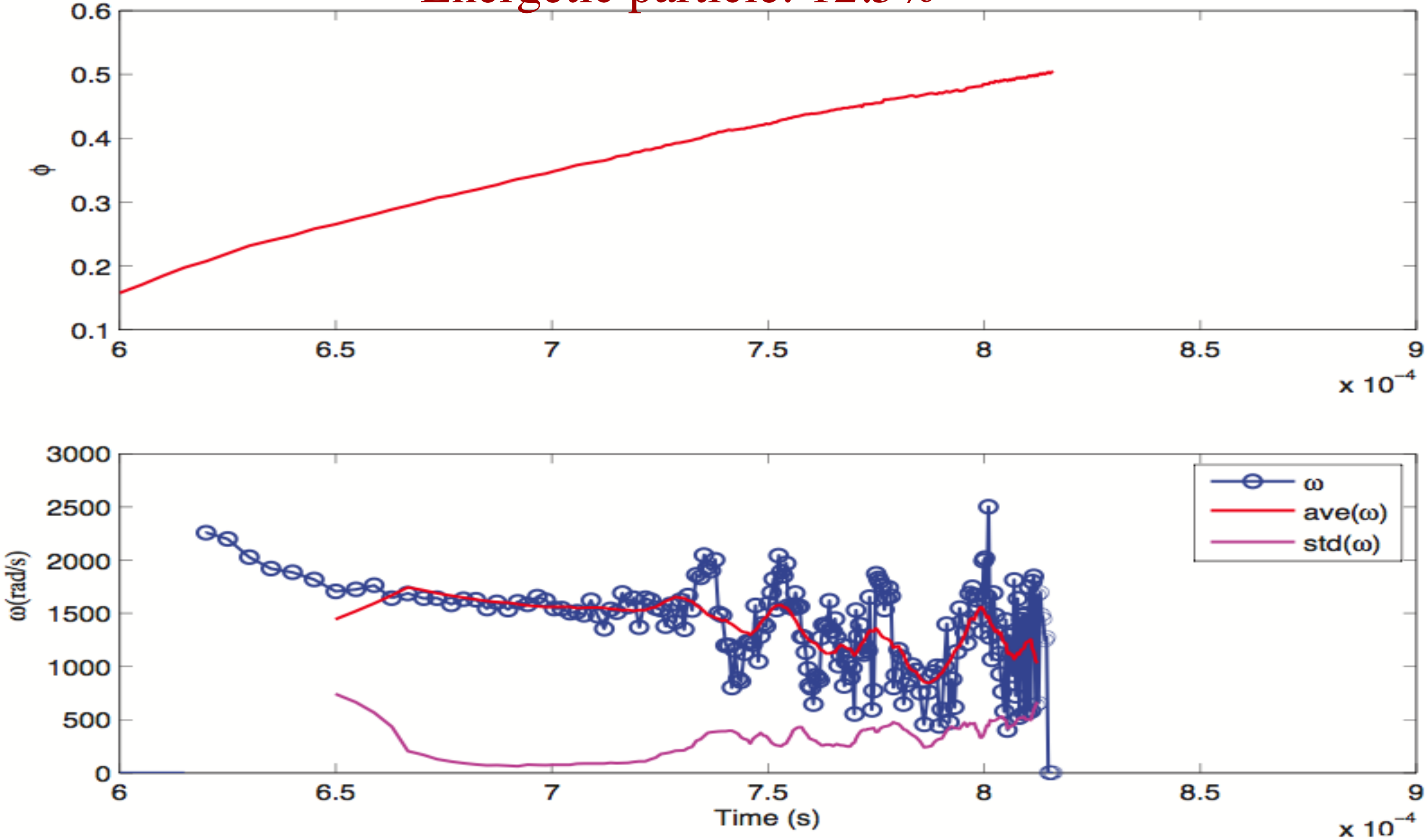
n=1 Growth rate



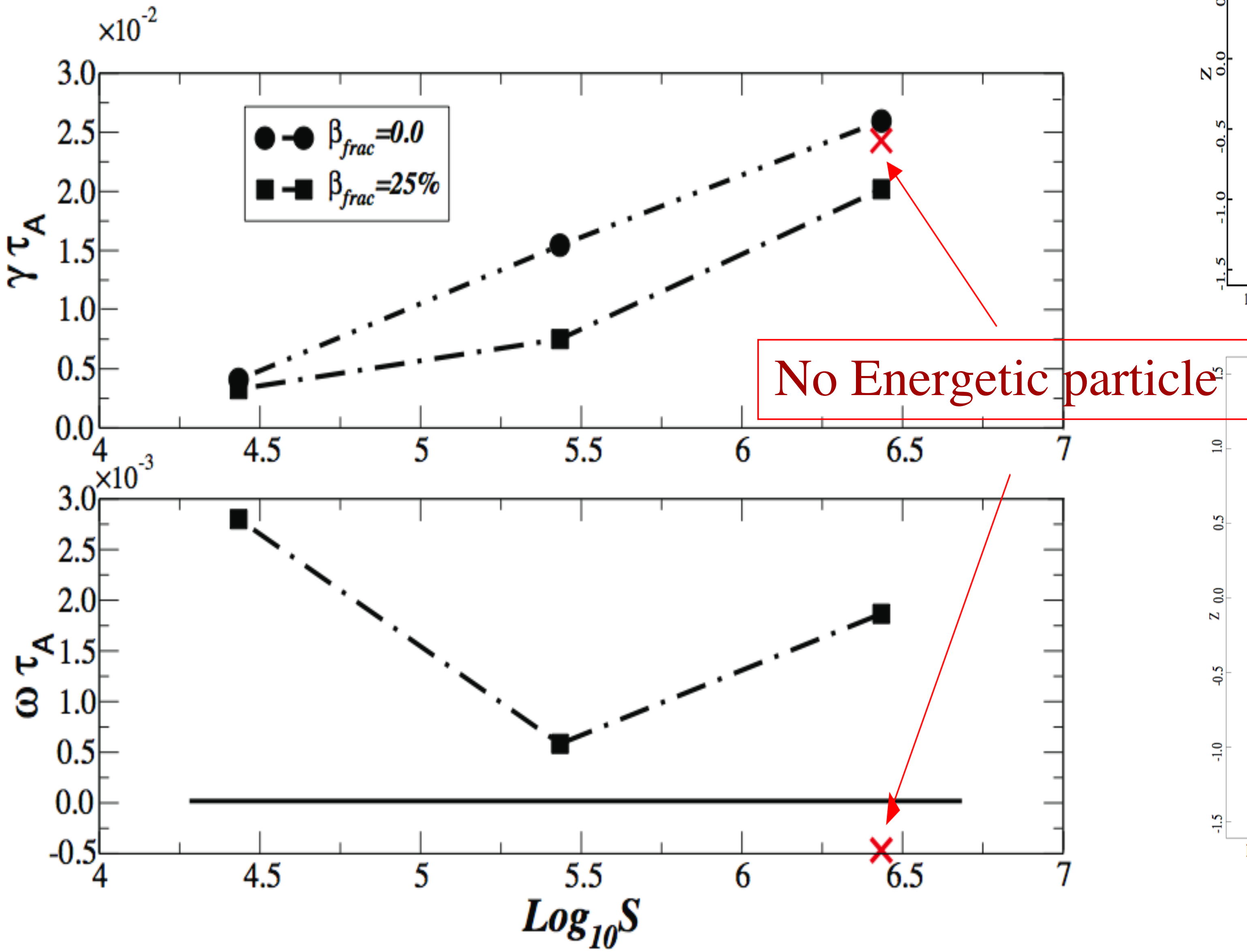
A real frequency of the 2/1 mode (β_N / 4l_i =0.90)

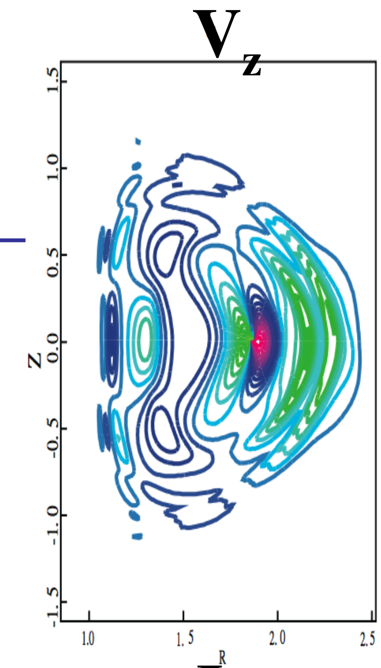
nonlinear (ideal) results

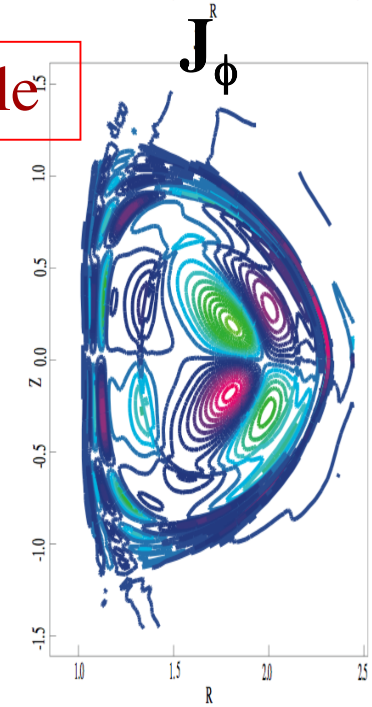
Energetic particle: 12.5%



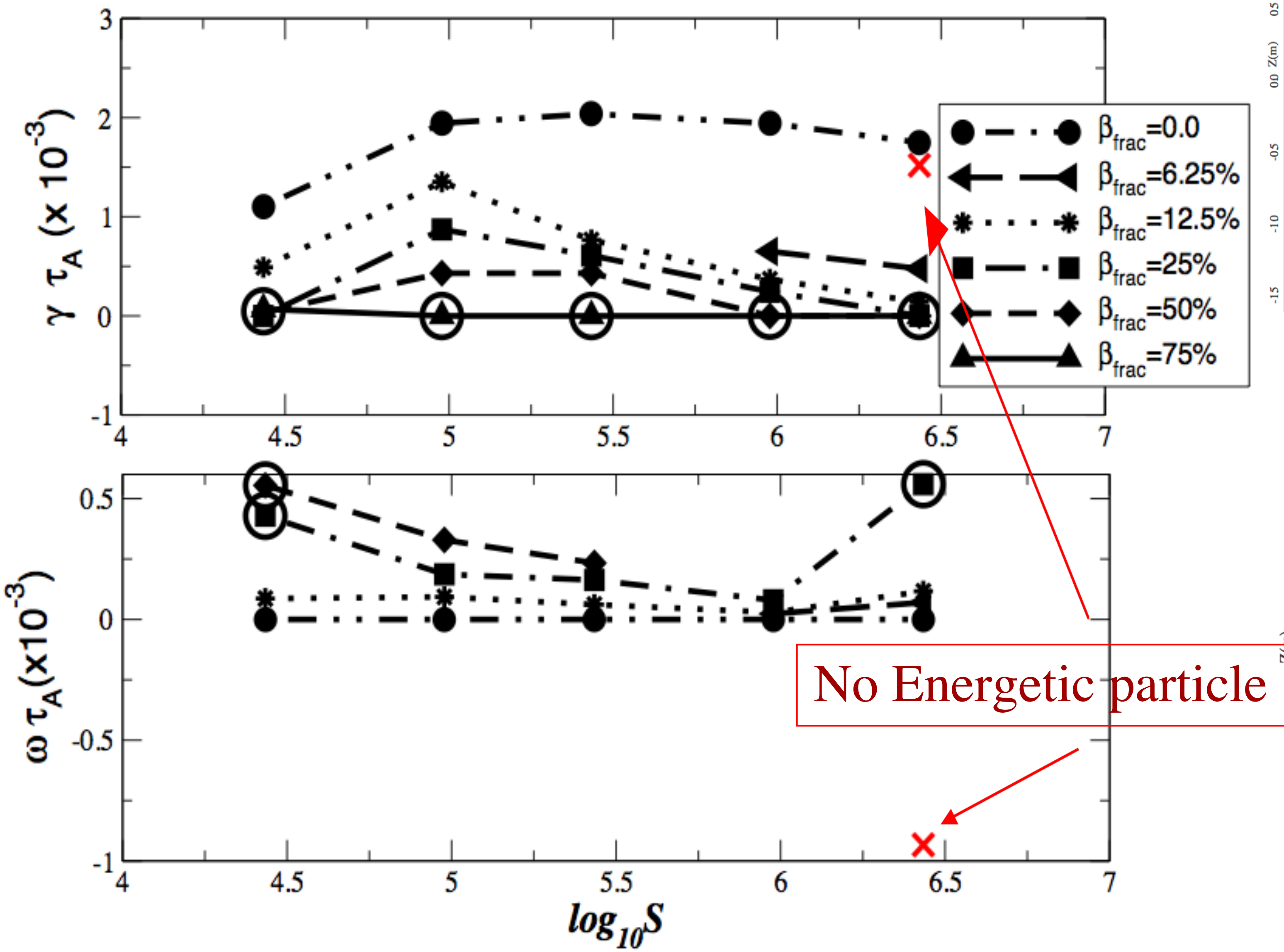
Two-fluid effect (β_N / $4l_i$ =0.98), close to ideal limits,linear results (MHD &Hall, dtm=5x10^{-9})

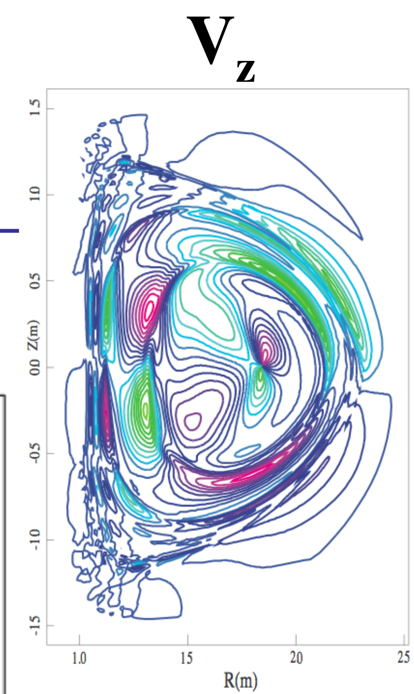


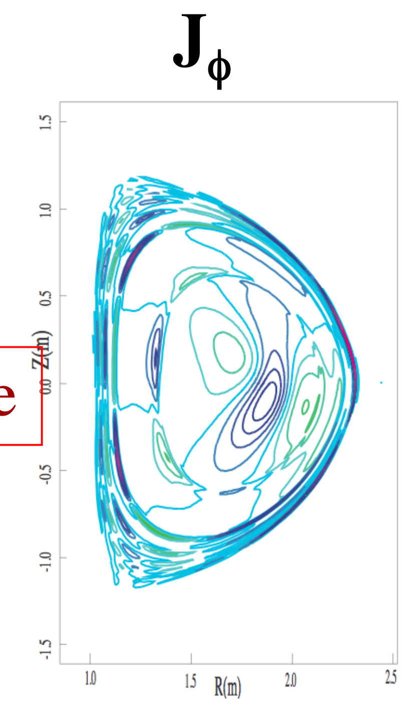




Two-fluid effect ($\beta_N / 4l_i = 0.83$), resistive linear results (MHD & Hall, dtm=5x10^{-9})







Precession rates (analytic calculations)

Ballpark estimation

The ion banana orbits drift toroidally with a frequency ω_B

$$\omega_B \approx \frac{q v_{th}^2}{\Omega_c R r} \le 8.0 \times 10^2, \tau_A \, \omega_B \sim 3.0 \times 10^{-4}$$

(Hu *et al,* PoP 2005)

Diamagetic Rotation

$$\omega_{*e,-i} = \frac{c}{ne\, Br} \frac{dp_{e,i}}{dr} = \frac{1}{m_{e,i} n r \omega_{ce,i}} \frac{dp_{e,i}}{dr}$$

$$\omega_{*e} \sim 2.2 \times 10^3, \omega_{*i} = 1.1 \times 10^3$$

$$(\omega(\omega - \omega_{*i})(\omega - \omega_{*e})^3 = i \gamma_{MHD}^5)$$

(Coppi, PFs 1965)

$$|\omega| \le 1.5 \times 10^3, \tau_A \omega \sim 7.0 \times 10^{-4}$$

Conclusion and Discussion

•**Nonlinear 2fl with energetic particles will be important!**

•**Nonlinear (single fluid with energetic particles)**

- **Real frequencies will increase or decrease at nonlinear stage?**
- **nlayers needs for nonlinear 11 modes, NIM(RE)SET.**

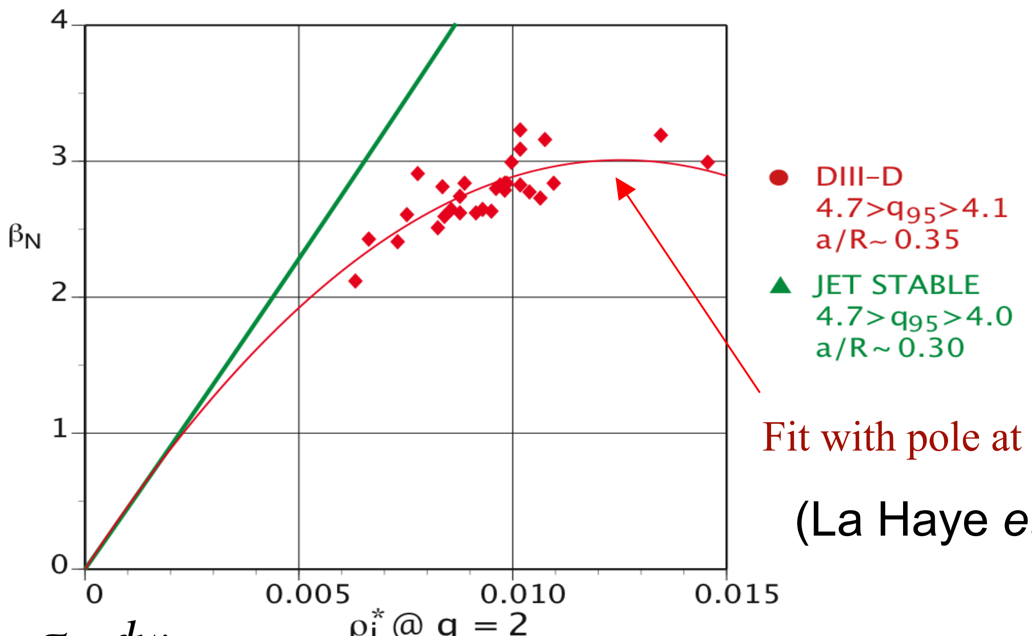
• **2fl linear results**

- **Close to the Ideal limit, small damping effects γ, and small ω**
- **Resistive cases, small damping effects, however ω is larger.**
- **Need to resolve separatrix region, add n_hypd, … etc.**

Can a Kinetic - MHD model Explain the Stabilization of the 2/1 in JET

Experimental data from the DIII-D, Asdex, JT-60U and JET experiments show only JET breaks the model of onset of the 2/1 near ideal MHD limit.

- Model: parametric Δ' near ideal limit (Brennan 2002/3) in modified Rutherford equation for a ρ^*_i dependence of onset (La Haye 2008).



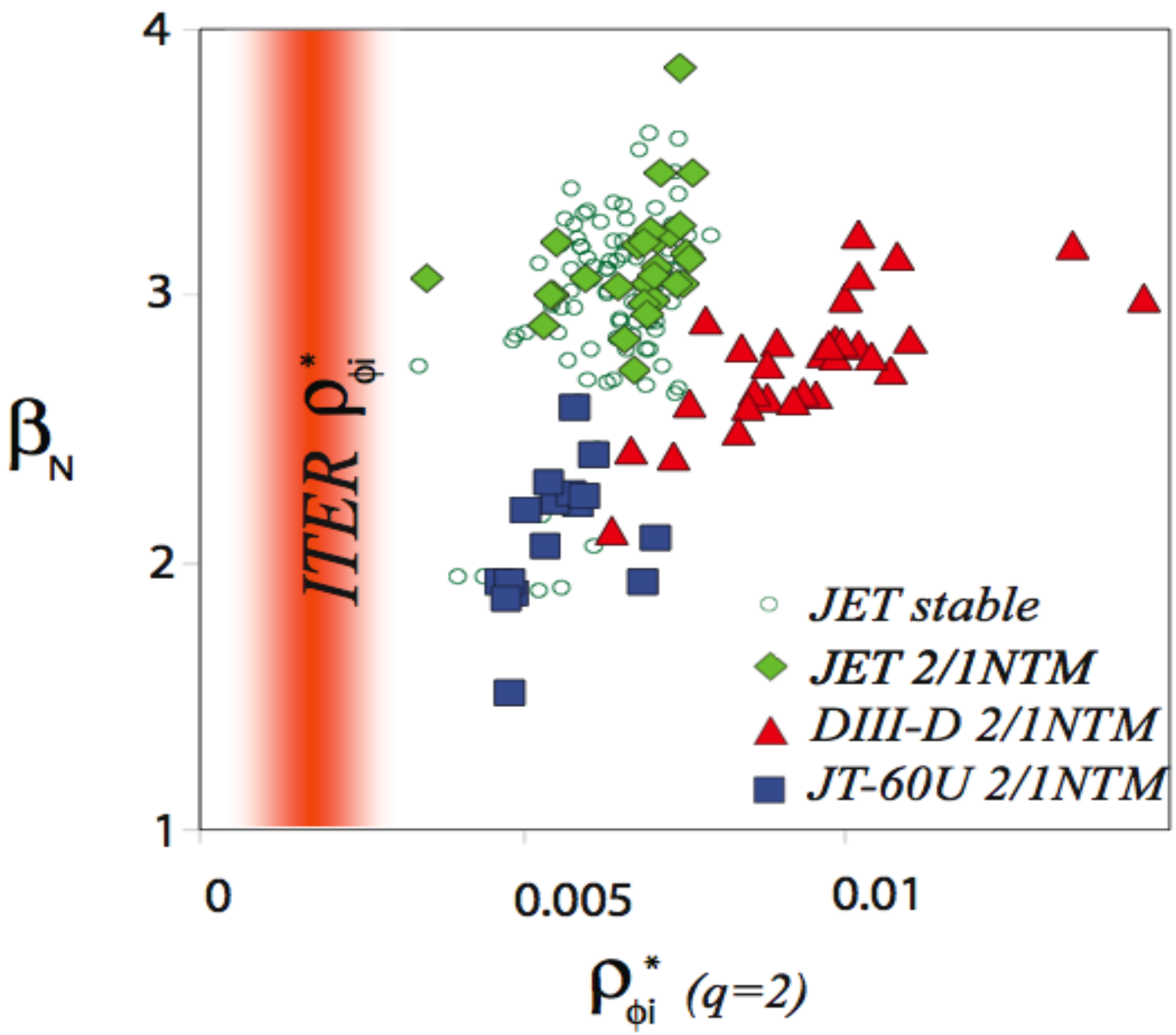
(La Haye *et al.* N. Fusion 2008)

$$\frac{\tau_R}{r}\frac{dw}{dt} = \Delta' r + a_2 \varepsilon^{1/2}(L_q/L_p)\beta_\theta(r/w)(1 - w_{m\arg}^2/3w^2)$$

Classic theory:
The linear tearing stability index

$$\boxed{\Delta' r = -(m-k) - k\alpha x[\cot(\alpha x)],} \quad x \equiv \frac{\beta_N}{4 l_i}$$

Can a Kinetic - MHD model Explain the Stabilization of the 2/1 in JET, the 2/1 is stable in JET



Buttery (2007, APS)

Buttery et al (IAEA, 2008)

Fusion Energy 2008 (Proc. 22nd Int. Conf. Geneva, 2008) (Vienna: IAEA) CD-ROM file IT/P6-8 and http://www-naweb.iaea.org/napc/physics/FEC/FEC2008/html/index.htm

Can a Kinetic - MHD model Explain the Stabilization of the 2/1 in JET

Puzzle: Why does the JET experiment not show instability like the others?

Likely reason: energetic particles stabilize the 2/1 mode.

- JET (β_{frac}) > 30%,
- DIII-D, JT-60U (β_{frac}) < 20%

T. Hender et~al., Nucl. Fusion **44**, 788 (2004)

OTHER Possible Causes?

•Accurate Δ' calculation (Brennan 2002/3/6).
•Accurate equilibrium.
•Other physics, two-fluid effects … ?