New Developments with M3D-C1

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Outline

- Resistive Wall Boundary Conditions
- Linear Non-Axisymmetric Response
- Nonlinear 3D

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- **Resistive Wall Boundary Conditions**
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Thin-Shell Resistive Wall Boundary Conditions

$$\mathbf{B} = \nabla\psi \times \nabla\varphi - \nabla_\perp f' + F\nabla\varphi$$

$$F = F_0 + R^2\nabla_\perp^2 f$$

- M3D-C^1 advances ψ and F
- Essential (Dirichlet) condition on ψ:

$$\frac{\partial\psi}{\partial t} = -\frac{\eta_W}{\delta} R\left(\hat{\mathbf{t}}\cdot\mathbf{B} - \hat{\mathbf{t}}\cdot\mathbf{B}^v\right) - \frac{V_L}{2\pi}$$

- Natural (Neumann) boundary condition on F:

$$\int dV \frac{\mu}{R^2}\frac{\partial F}{\partial t} = -\oint dS \frac{\mu}{R}\frac{\eta_W}{\delta}\left(\frac{F}{R} - \hat{\varphi}\cdot\mathbf{B}^v\right) + \int dV\, \nabla\varphi\cdot\nabla\mu\times\mathbf{E}$$

Vacuum Response Depends on Plasma Response Non-Locally

$$\left(\hat{\mathbf{n}}\cdot\mathbf{B}^{v}\right)_{i}=\left(\hat{\mathbf{n}}\cdot\mathbf{B}\right)_{i}$$

$$\left(\hat{\mathbf{t}}\cdot\mathbf{B}^{v}\right)_{i}=M_{ij}^{t}\left(\hat{\mathbf{n}}\cdot\mathbf{B}\right)_{j}$$

$$\left(\hat{\varphi}\cdot\mathbf{B}^{v}\right)_{i}=M_{ij}^{\varphi}\left(\hat{\mathbf{n}}\cdot\mathbf{B}\right)_{j}$$

i, *j* range over all boundary nodes

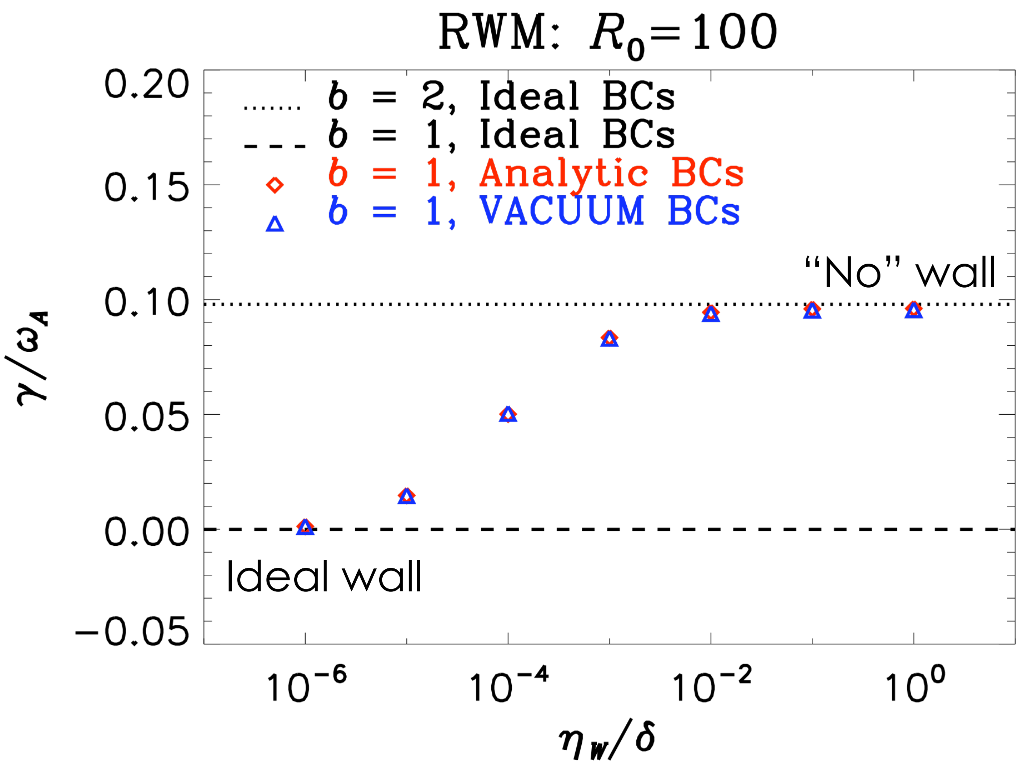
- **VACUUM* calculates response matrices *M* in arbitrary geometry**

- ***M* is dense; all boundary nodes coupled to each other**
  - Adds communication; hurts scalability
  - Not yet supported by SCOREC libraries in parallel

* M.S. Chance, *Phys. Plasmas* **4**, 2161 (1997)

Resistive Wall Mode Test

- **Equilibrium is no-wall unstable**
- **Stable with conducting wall at *b*=1**



- **Growth rate should transition from ideal-wall limit to no-wall limit as η_W/δ is increased.**
- **In the large-aspect limit, we know response matrix analytically**

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Linear Non-axisymmetric Field Response with M3D-C1

- **$B(t) = B_0 + B_1(t)$**
  - **B_0** is the axisymmetric equilibrium field
  - **B_1**(0) is the "vacuum field" from non-axisymmetric coils (I-coils).

- **Conducting-wall boundary condition**
  - **B** held constant in time on simulation domain boundary (approximately vacuum vessel)

- **Simulation is time-advanced until the steady-state is reached.**

- **Final B_1 is applied field + plasma response.**

Response Calculations Require Stable Equilibria

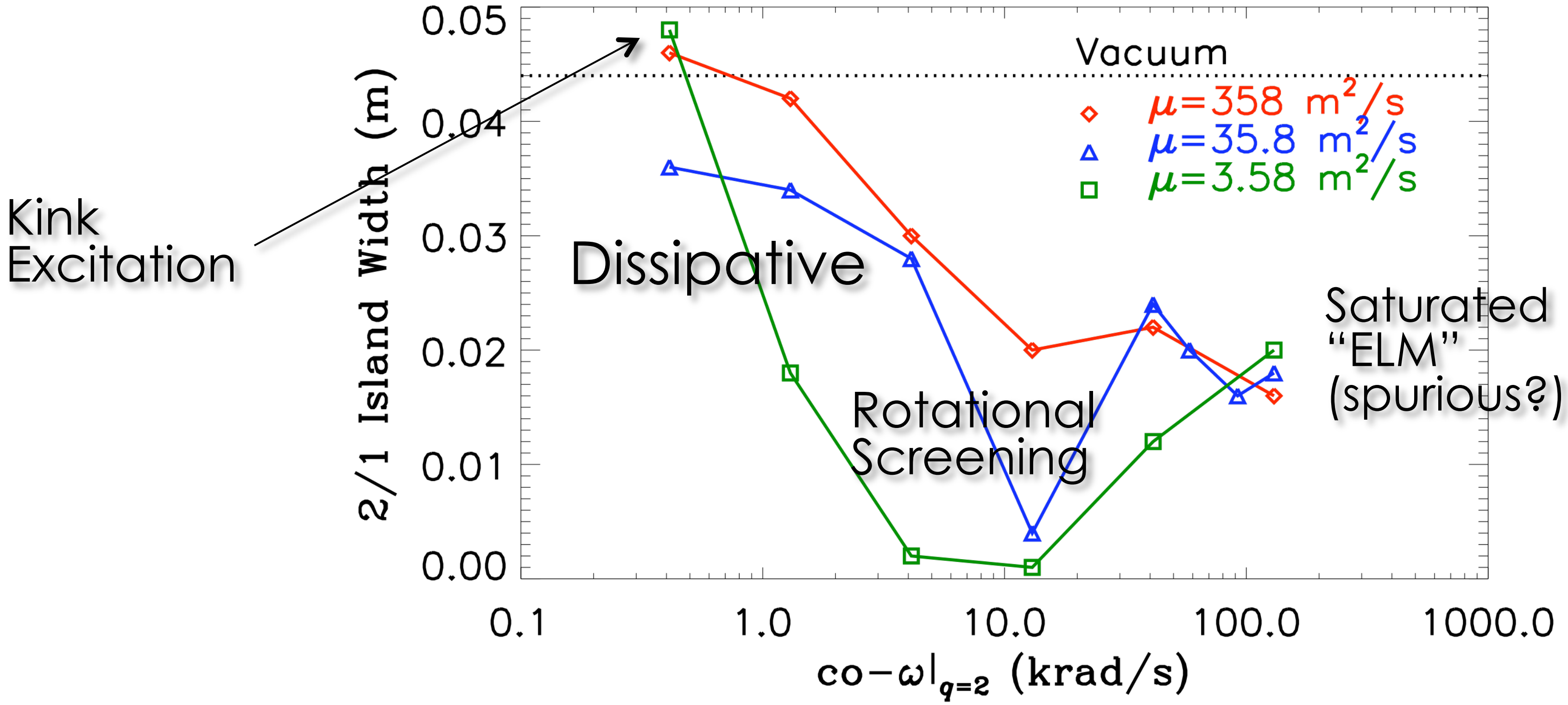
- **In practice, equilibria are almost always weakly unstable to "numerical tearing" modes**
  - Due mainly to lack of resolution around rational surfaces
  - No steady-state ➔ This invalidates response

- **With MARS, this is usually not a problem since the response frequency is chosen *a priori***

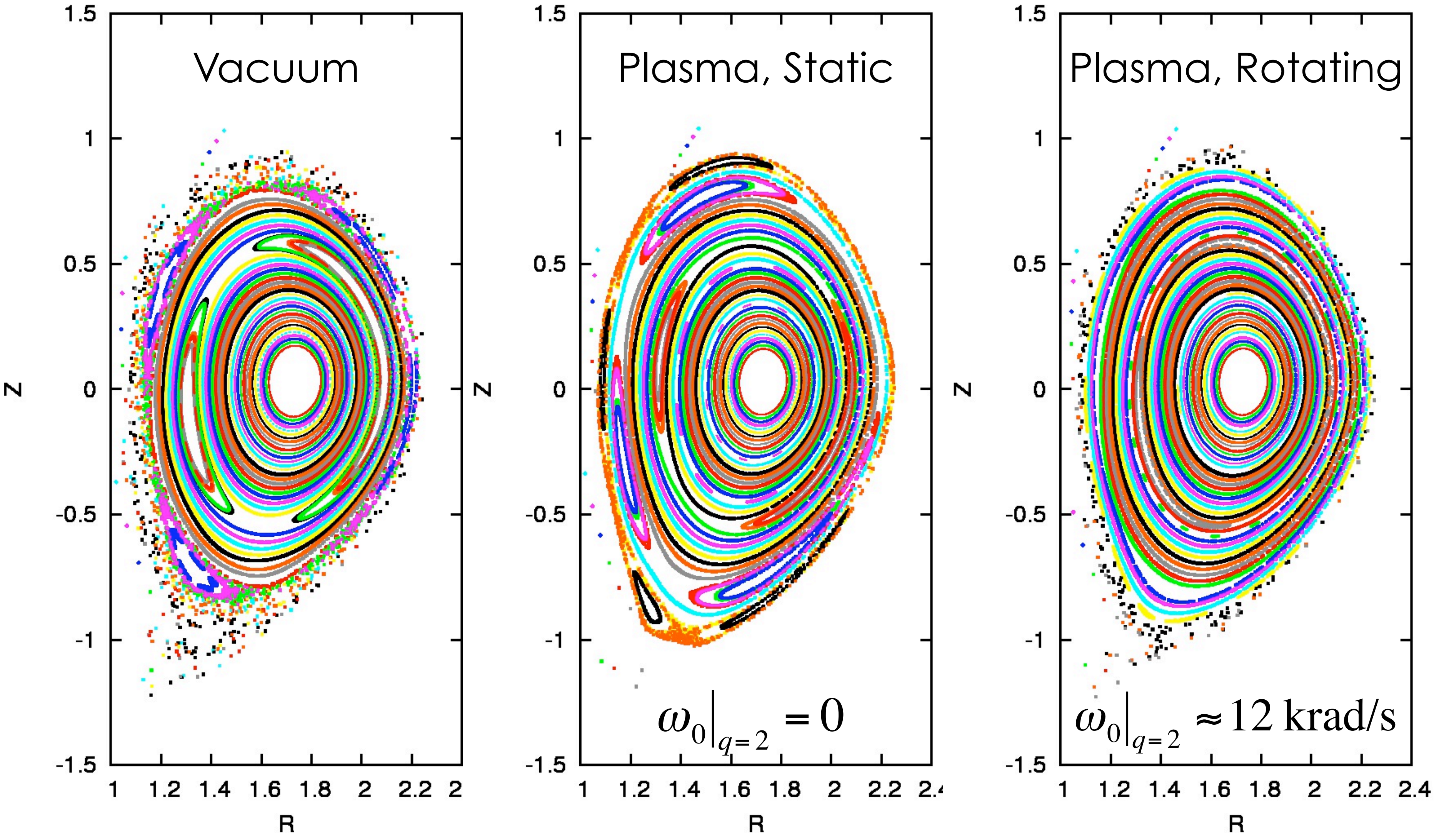
- **With initial value code, the equilibrium must be made to be stable to these spurious modes**
  - Change equations: thermal diffusion, viscosity
  - Change equilibrium: rotation

Rotation & Dissipation Affect Stability & Screening

- **Dissipative terms inhibit screening response**
  - Magnetic islands form
- **Equilibrium rotation enhances screening**



Rotation Improves Core Screening; But Stochasticizes Edge

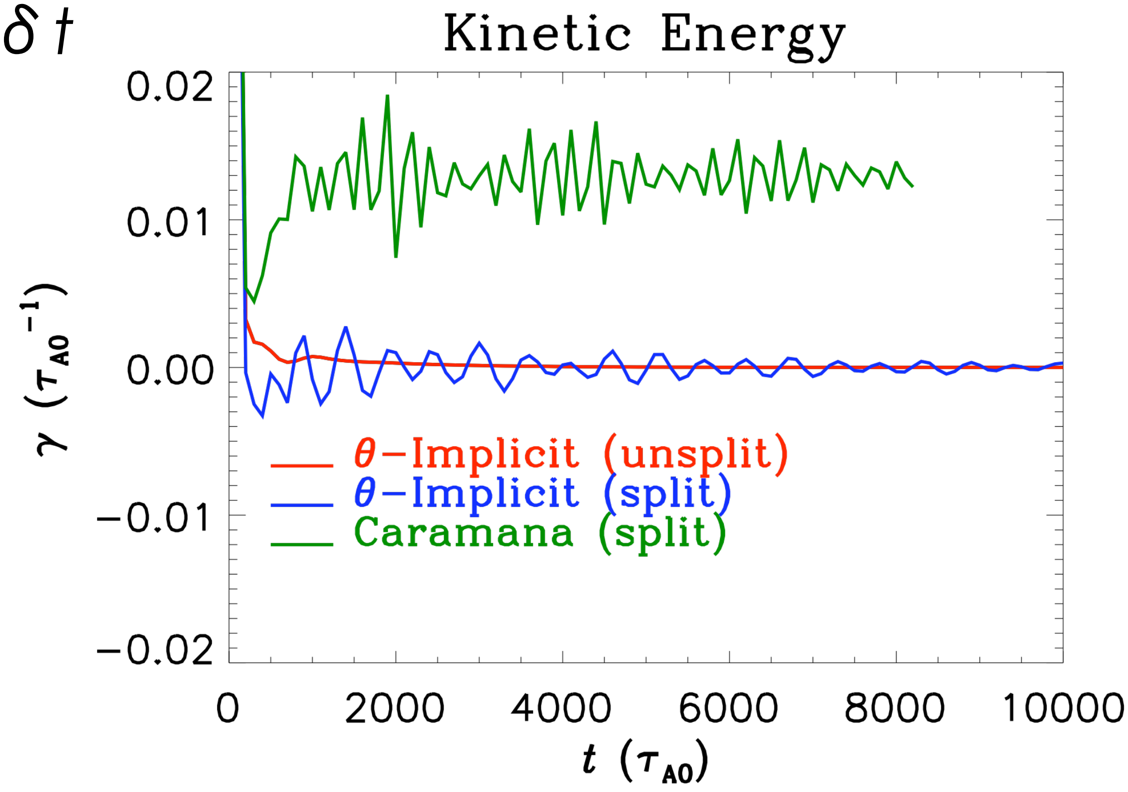


Summary of Non-Axisymmetric Results

- **We are able to calculate response with Spitzer resistivity, rotation, and two-fluid terms**
- **Initial-value calculations require dissipation to ensure stability**
  - Dissipation inhibits screening
- **Rotation enhances screening**
  - Direction of rotation is important *even in single fluid MHD*
- **Poster Tuesday morning**

Unsplit Method Superior to Split Methods for Finding Linear Perturbed Equilibrium

- **Split method has difficulty obtaining perturbed equilibrium**
  - Persistent oscillations at low dissipation
  - More sensitive to δt
  - Caramana method more susceptible to numerical instability in under-resolved cases



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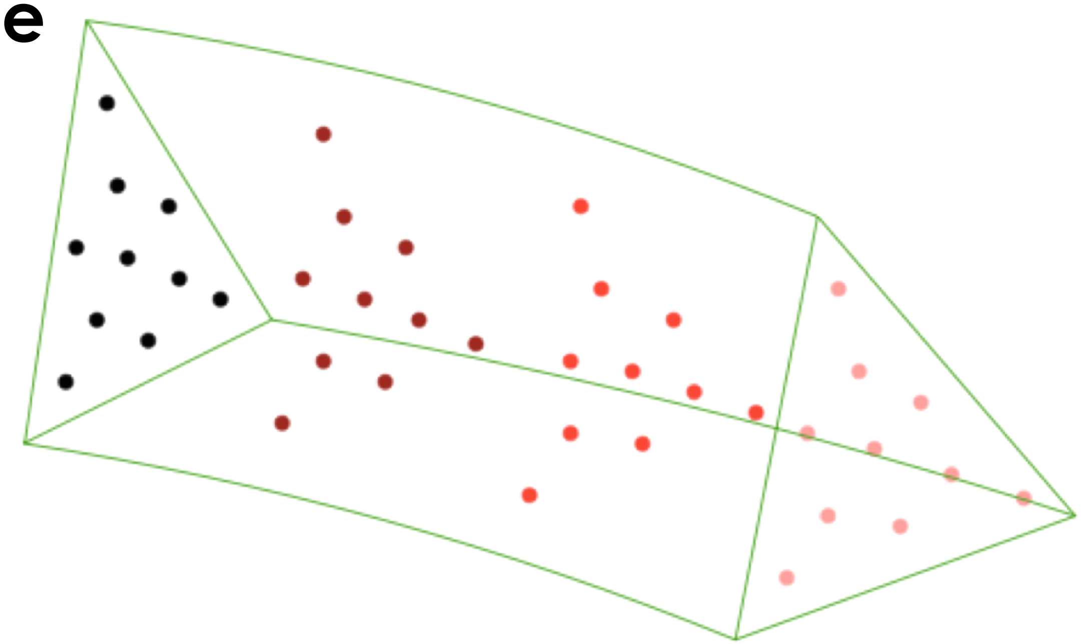
M3D-C1 Uses Wedge-Shaped Elements in 3D

- **Tensor product**
  - Poloidal: reduced quintic (C^1)
  - Toroidal: reduced cubic polynomials (C^1)
  - Integration quadrature is also tensor product
- **6×2=12 DOFs/node**
- **3D mesh is series of 2D planes**
- **Allows packing in toroidal direction**

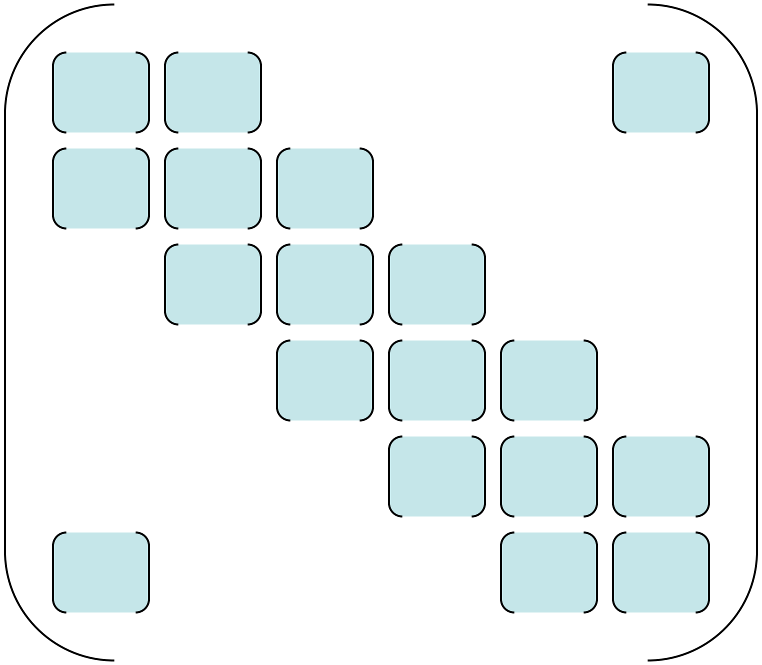


3D Matrix Has Cyclic Block-Tridiagonal Form

- Each plane corresponds to "block"
- Only nearest neighbor planes are coupled

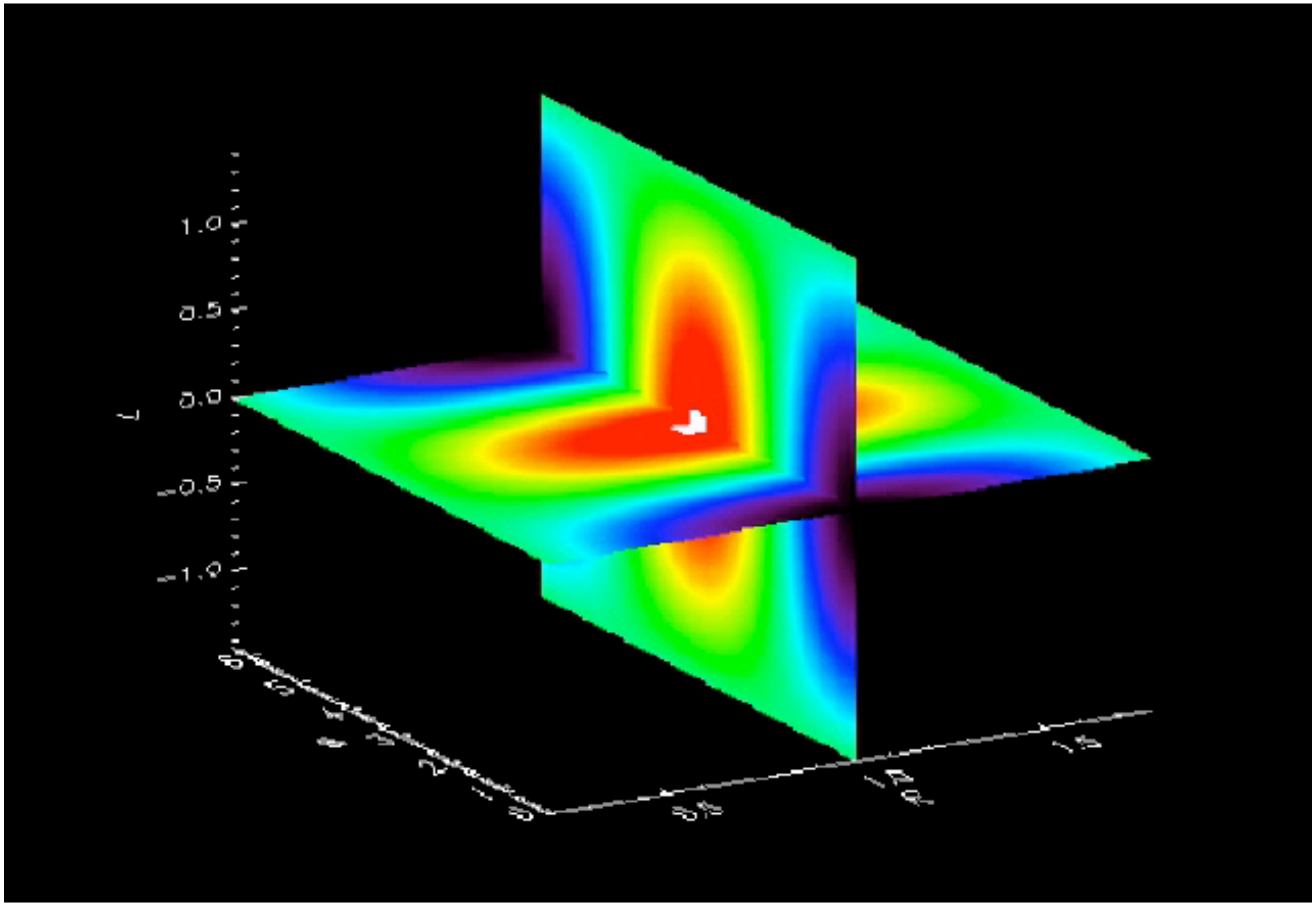
- Presently this is solved without (further) preconditioning

- Typical problem: 10^5 DOFs/block

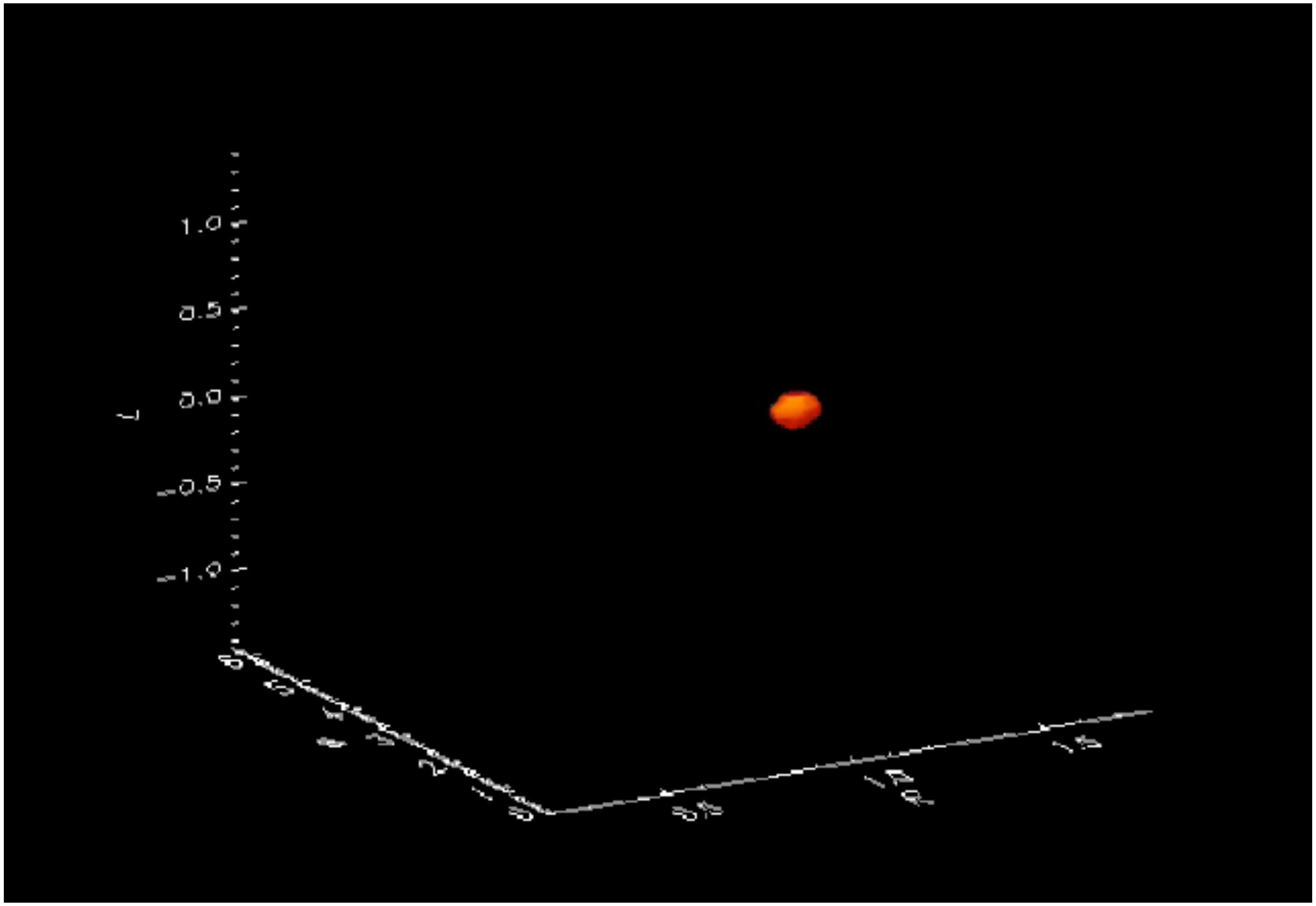


3D Results: Alfven Wave

- With just 2 toroidal planes, ω is correct to 1 part in 10^5



3D Results: Anisotropic Diffusion in Helical Field



Summary

- **Resistive-wall boundary conditions are implemented**
  - Not yet functional in parallel

- **Linear 3D response successfully calculated with Spitzer resistivity, rotation, and two-fluid physics**

- **First fully-3D simulations have been run successfully**
  - Much future work will involve solver strategies