Effects of Energetic Particles on the n=1 β Limit in Hybrid Discharges

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Outline

Experimental onset of 2/1 tearing mode in DIII-D Hybrid

- Typical onset scenario, increasing β_N decreasing q_{min}
- Equilibrium reconstruction for discharge

MHD stability analysis shows stability

- Generate series of equilibria spanning q_{min}, β_N space
- Cases with increased β_N show instabilty -> puzzle

Include Energetic Particle Effects on the n=1 Mode

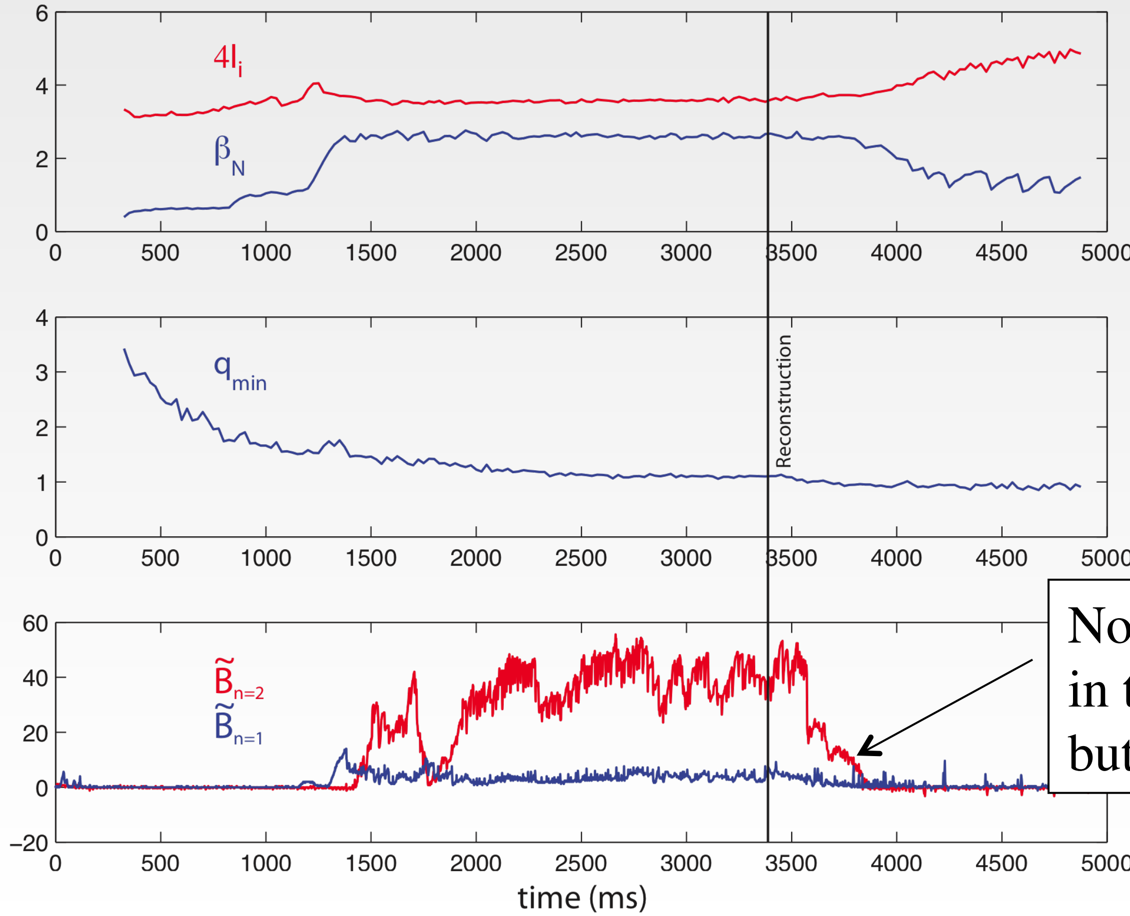
- Simulations show significant change in stability
- Gradient in growth rates now in increasing β_N

Summary

Typical Hybrid discharge: 3/2 tearing mode and hovering q_{min}>~1

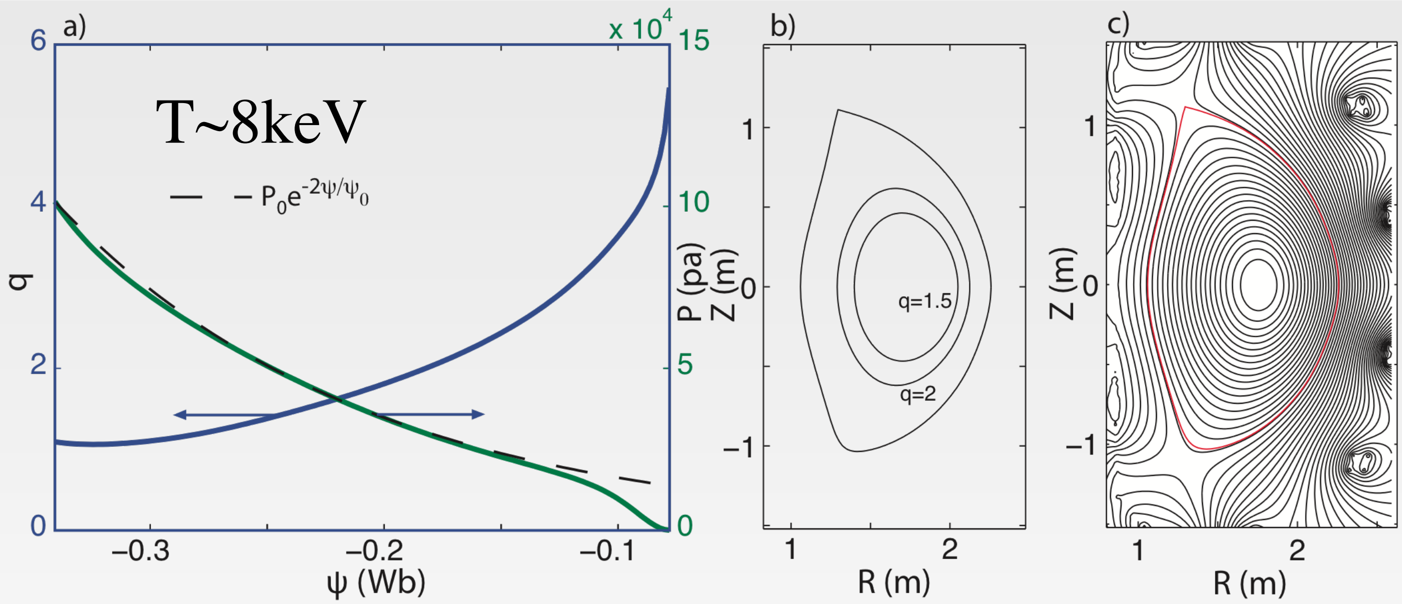
Tokamak hybrid experiments commonly show an m/n=3/2 neoclassical tearing mode (NTM) onset during the β ramp up and flattop before the onset of an m/n=2/1 NTM.

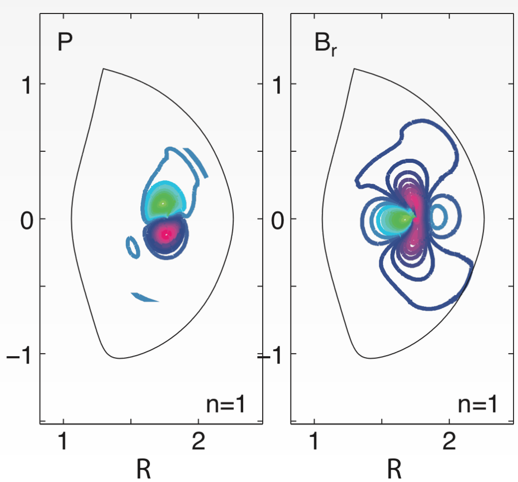
3/2 mode onsets when q=1.5 comes into existence, and continues in nonlinear state.



No 2/1 onset in this case, but in others

Equilibrium reconstruction has $q_{min} \geq 1$: Stability differs from high q_{min} by near axis response





Nonresonant 1/1 component dominates P perturbation

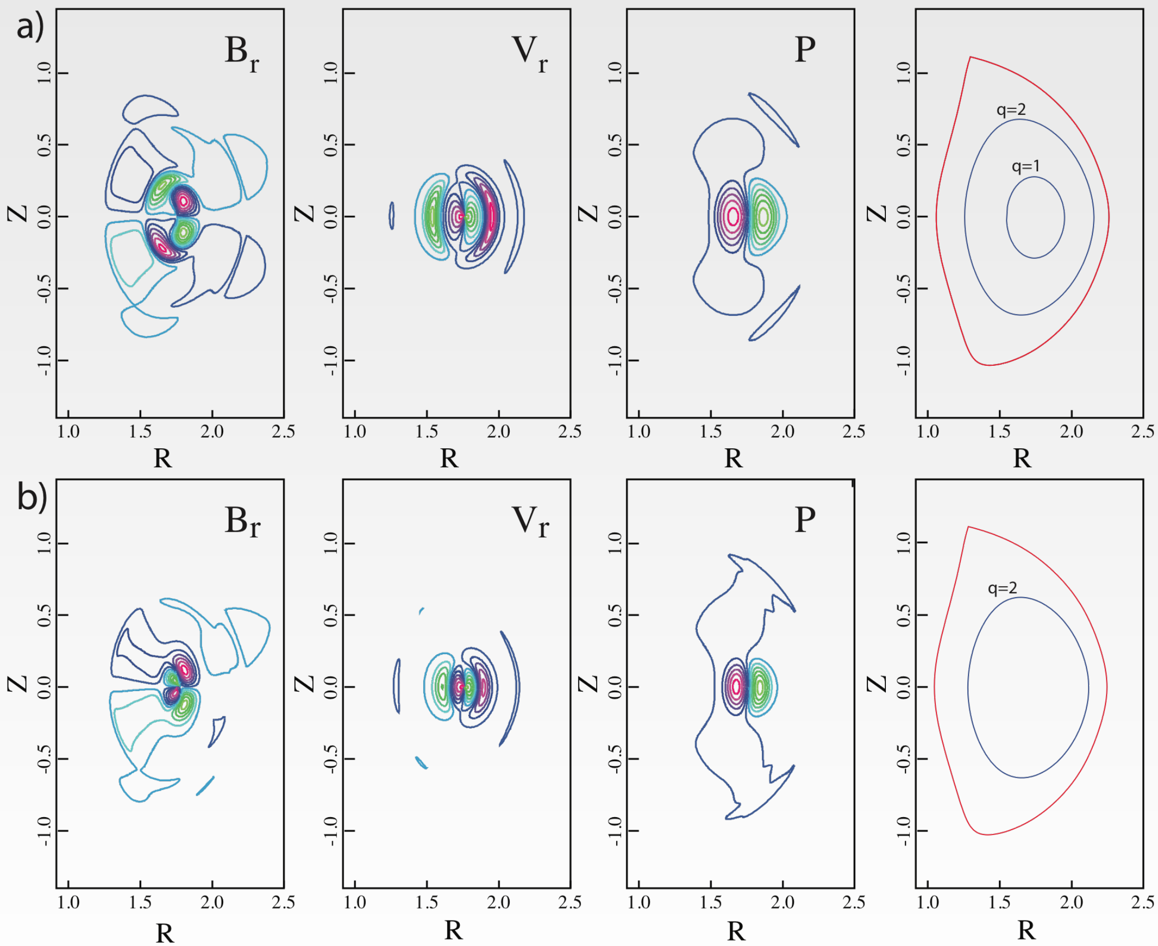
Low shear important, weak continuum damping

Br remains similar to conventional 2/1

MHD eigenmodes show near axis nonresonant mode smoothly changes to resonant as q_{min}<1

q_{min}=0.95
1/1 mode
zero frequency

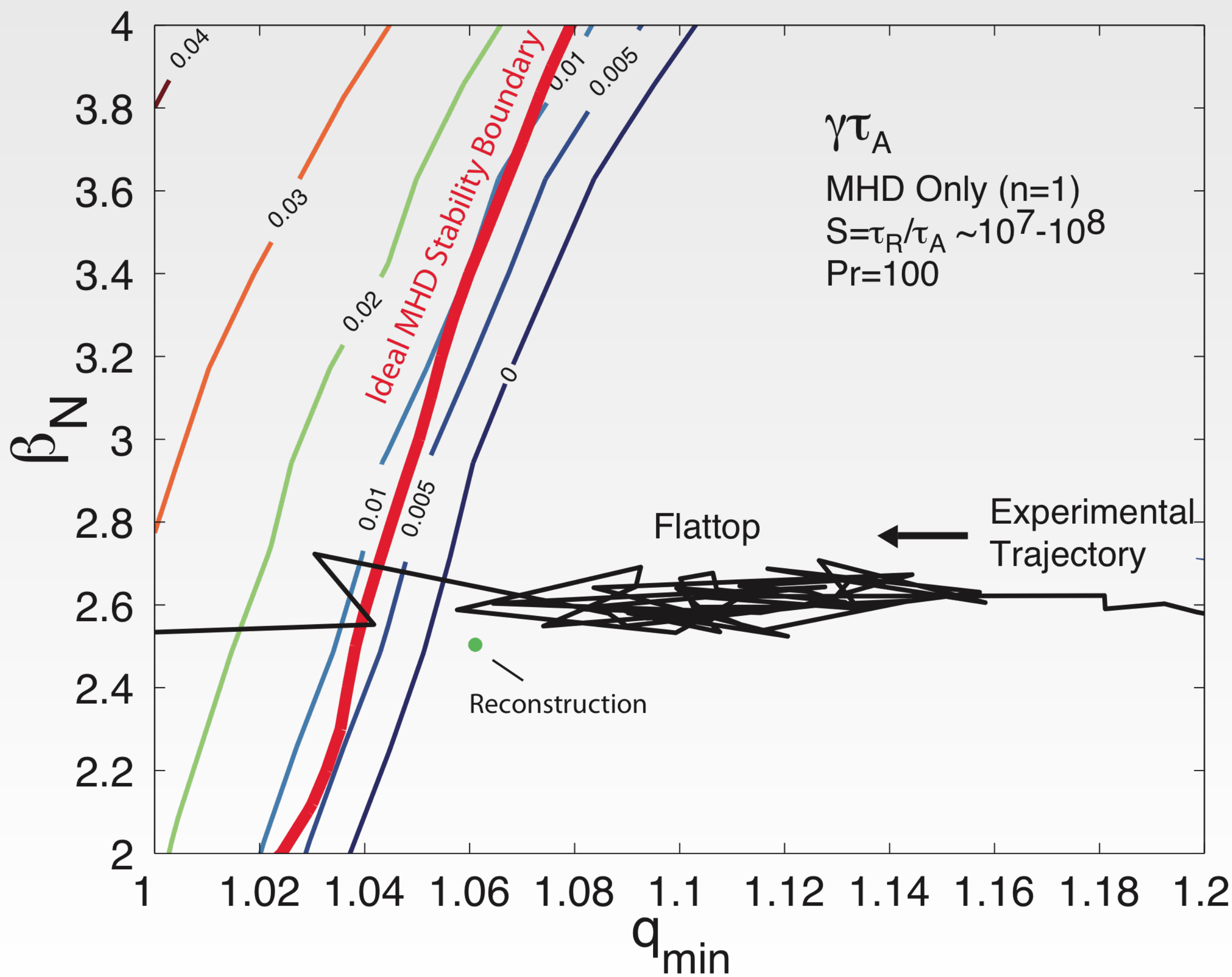
q_{min}=1.05
1/1 core localized
nonresonant



Stable at higher q_{min}

Stability maps of MHD only cases agree with experimental trajectory, but not increase in β_N

Experiment hovers just outside the n=1 unstable zone in this discharge.



For more on experiment: La Haye et al. Nucl. Fusion 2010

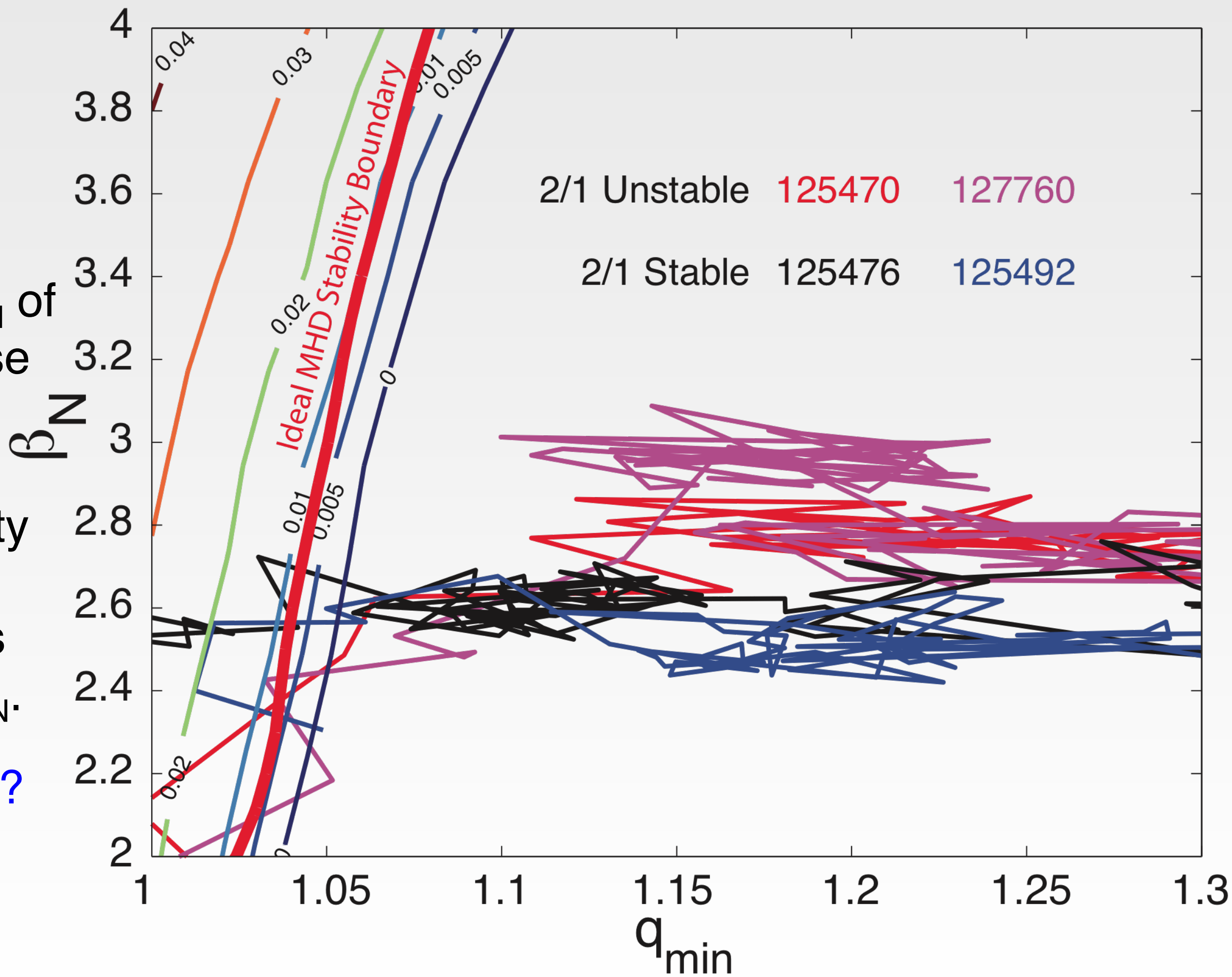
Stability maps of MHD only cases agree with experimental trajectory, but not increase in β_N

Experiment hovers just outside the n=1 unstable zone in this discharge.

However, increases in β_N of small amounts ~0.2 cause onset of the 2/1 mode.

Puzzle: why is the stability boundary not even in the right direction? Indicates stable region at higher β_N.

Are particles responsible?



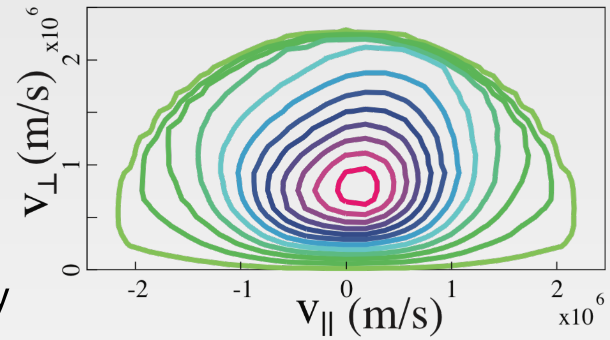
For more on experiment: La Haye et al. Nucl. Fusion 2010

Slowing Down Distribution for Energetic Particles Effectively Represents Neutral Beam Injection

The slowing down distribution function is used

Constant C matches the equilibrium pressure profile

$$f = \frac{P_0 \exp\left(P_\zeta / \psi_n\right)}{\varepsilon^{3/2} + \varepsilon_c^{3/2}}, \quad P_\zeta \propto \psi, \quad \psi_n = C\psi_0$$



ε_c models the peak in f while a max initial v models the birth energy

The linearized evolution equation for δf becomes

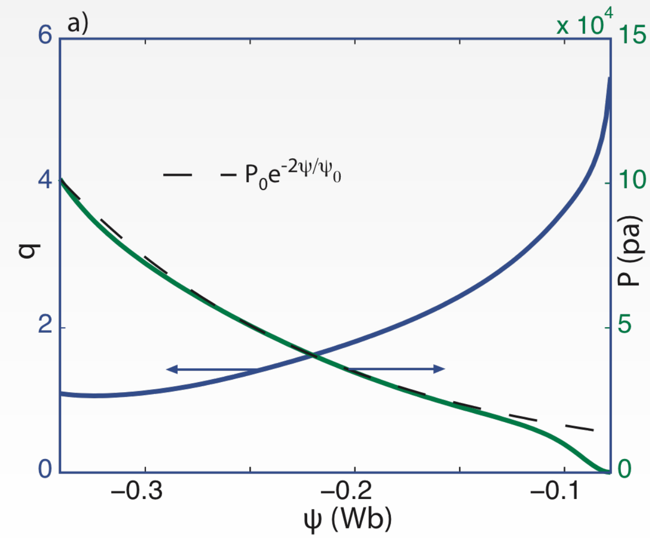
$$\delta\dot{f} = f_0\{\frac{mg}{e\psi_n B^3}[(v_\parallel^2 + \frac{v_\perp^2}{2})\delta\mathbf{B}\cdot\nabla B - \mu_0 v_\parallel \mathbf{J}_\perp\cdot\delta\mathbf{E}]$$

$$+\frac{\delta\mathbf{v}\cdot(\nabla\psi_p - \rho_\parallel\nabla g)}{\psi_n} + \frac{3}{2}\frac{e\varepsilon^{1/2}}{\varepsilon^{3/2}+\varepsilon_c^{3/2}}\mathbf{v}_D\cdot\delta\mathbf{E}\},$$

$$\mathbf{v}_D = \frac{mg}{eB^3}(v_\parallel^2 + \frac{v_\perp^2}{2})(\mathbf{B}\times\nabla B) + \frac{\mu_0 m v_\parallel^2}{eB^2}\mathbf{J}_\perp,$$

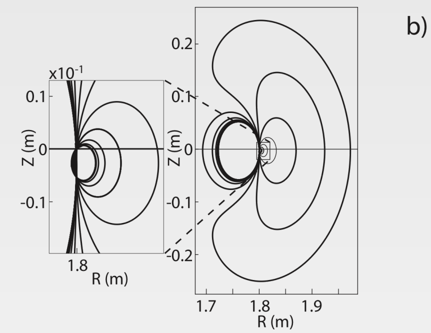
where

$$\delta\mathbf{v} = \frac{\delta\mathbf{E}\times\mathbf{B}}{B^2} + \mathbf{v}_\parallel\frac{\delta\mathbf{B}}{B}$$



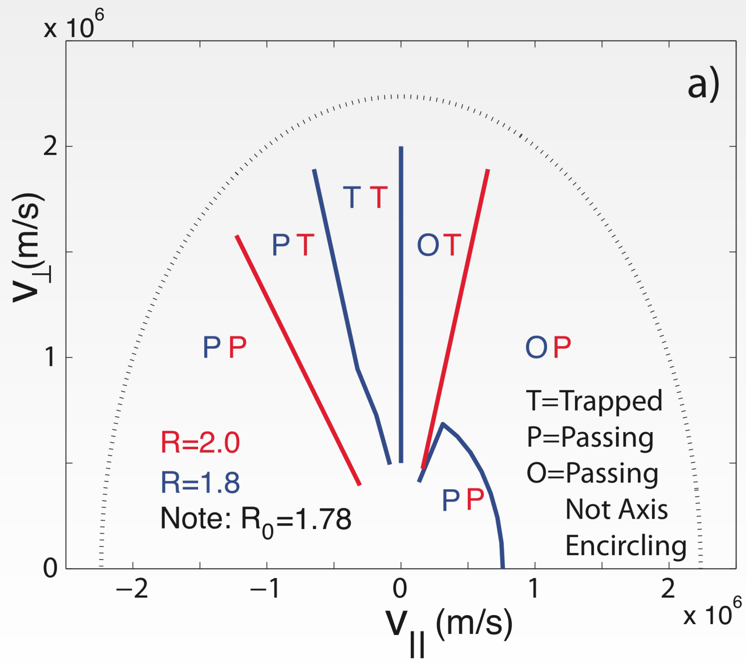
Trajectories of energetic particles can be highly irregular

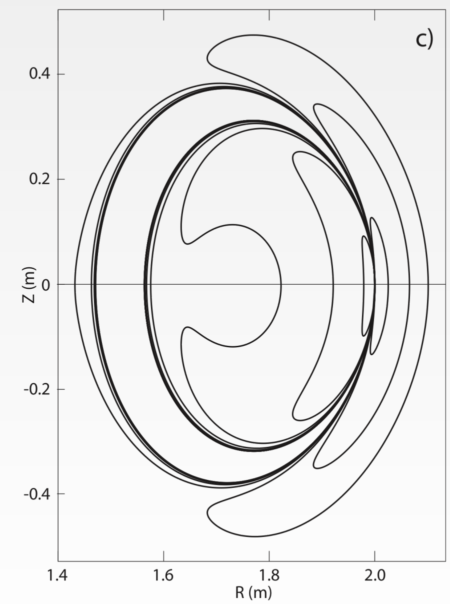
Flux conservation in particle orbit causes deviation from poloidal surface
Deviation increases with energy, high energy-> irregular
Near axis orbits are highly localized.
Trapped orbits have largest contribution to δf, but passing are more numerous



Outer orbits fairly regular T/P with “fat bananas”

Near axis trapped cone closes “Passing not axis encircling” exist





For sim movie http://www.youtube.com/watch?v=_kaSu76_xeo

Frequency discontinuous between three regions of q_{min} as most unstable mode switches

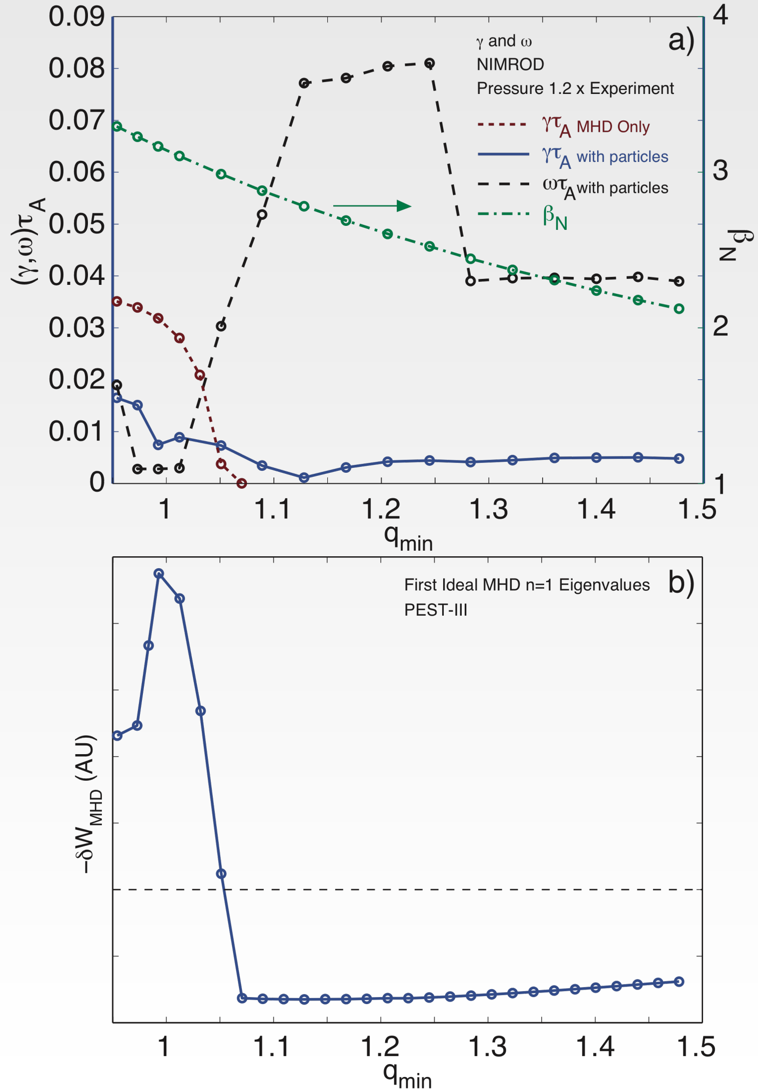
Growth rate curves transition continuously as most unstable mode changes, frequencies discontinuous.

Particle inclusive growth rates damped at low q_{min}, driven unstable at high q_{min}

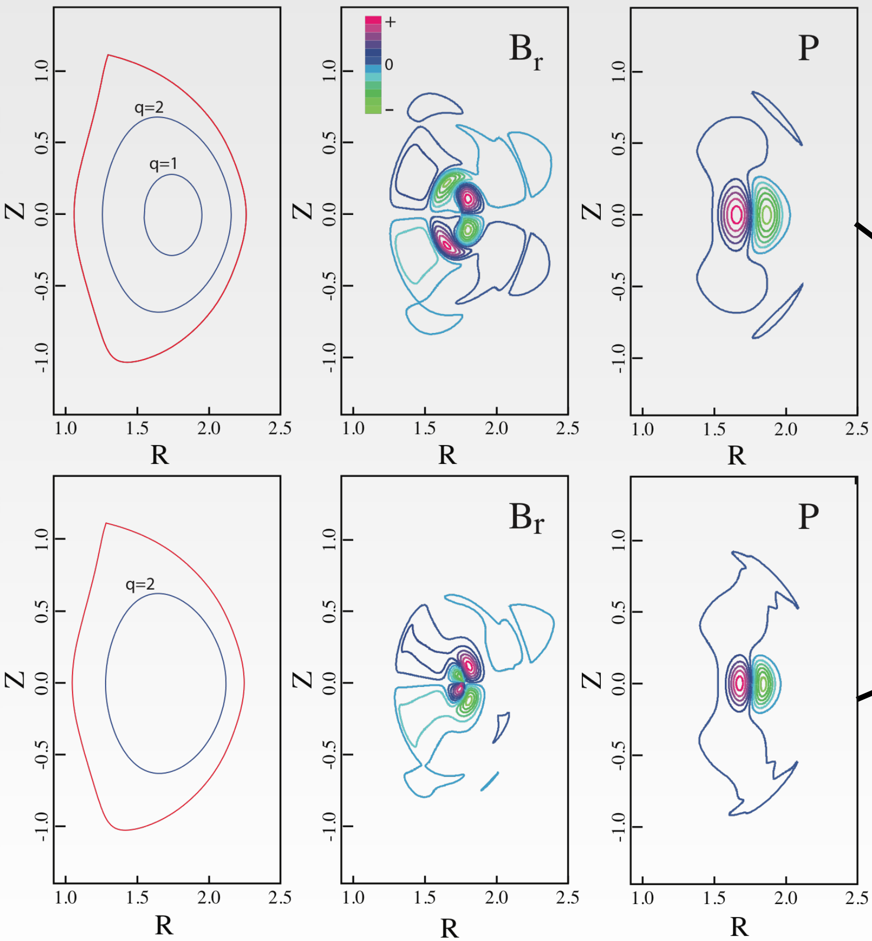
Ideal MHD δW from PEST strongly unstable when 1/1 surfaces come into existence

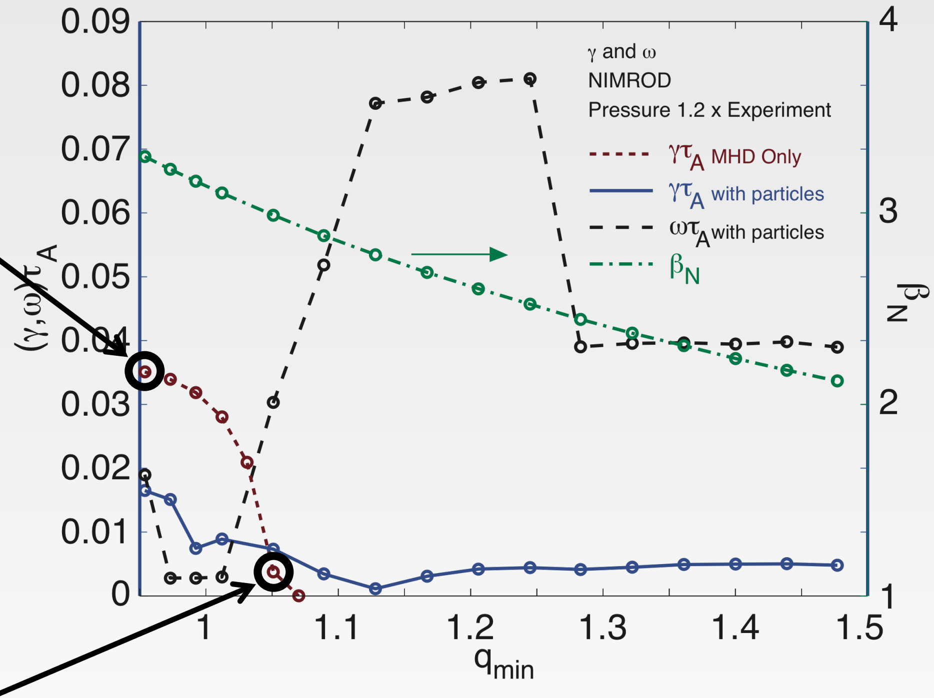
At high q_{min}, broad 2/1 is stable, nearly constant δW with q_{min}

Mode frequency lowest where δW strongest



Series of q_{min} at fixed pressure: MHD-only mode has core localized mode, stabilizes at q_{min}~1.07

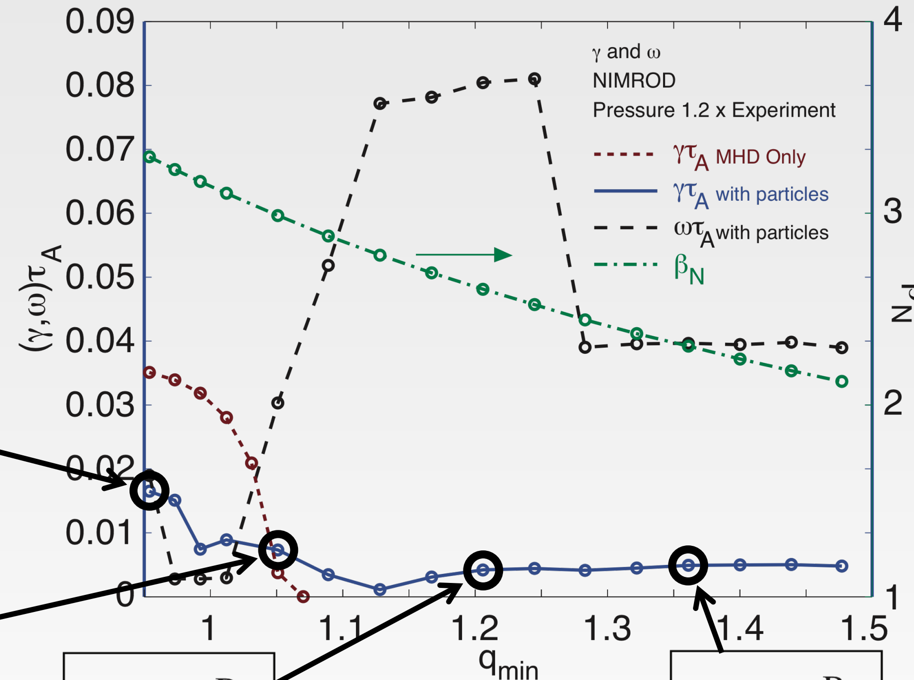




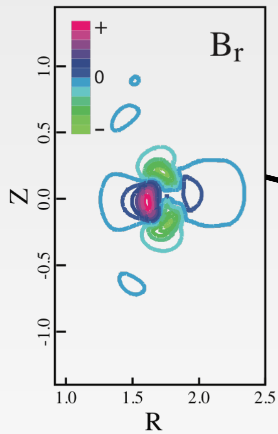
- q varied with adding constant to F^2 preserving equilibrium, β_N varies
- q_{min}=0.95 has 1/1 component in B_r and P on axis
- q_{min}=1.05 has 2/1 component in B_r on axis, highly localized, 1/1 in P remains

Series of q_{min} at fixed pressure: Energetic particle driven modes up to high q_{min}

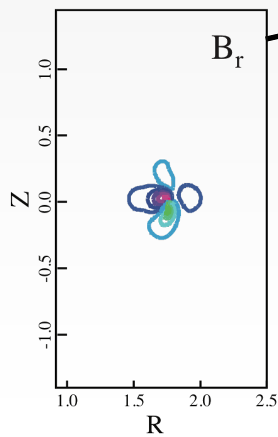
MHD unstable region: damped, low ω
MHD stable region: EP driven, $\omega >> \gamma$
ω lower than ω_{TAE}



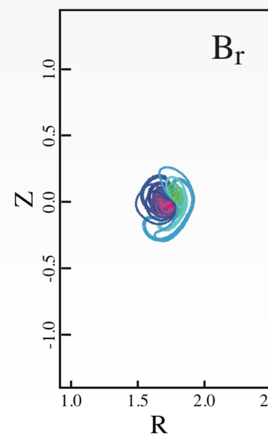
Low q_{min} has broad 1/1 structure moderate ω, damped γ



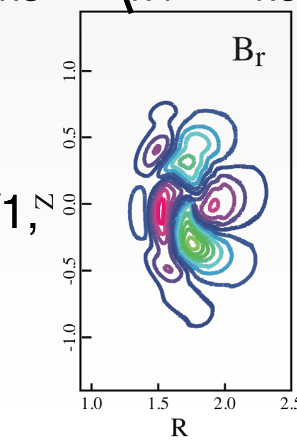
q_{min} >~1 has localized 1/1, increasing ω, decreasing γ, with q



q_0>~1.2 has broader 1/1, high ω, driven γ



q_{min}=1.36 dominantly 2/1, moderate ω, driven γ



Significant differences in particle inclusive eigenmodes in each mode region

Low q_{min}
Structure modified

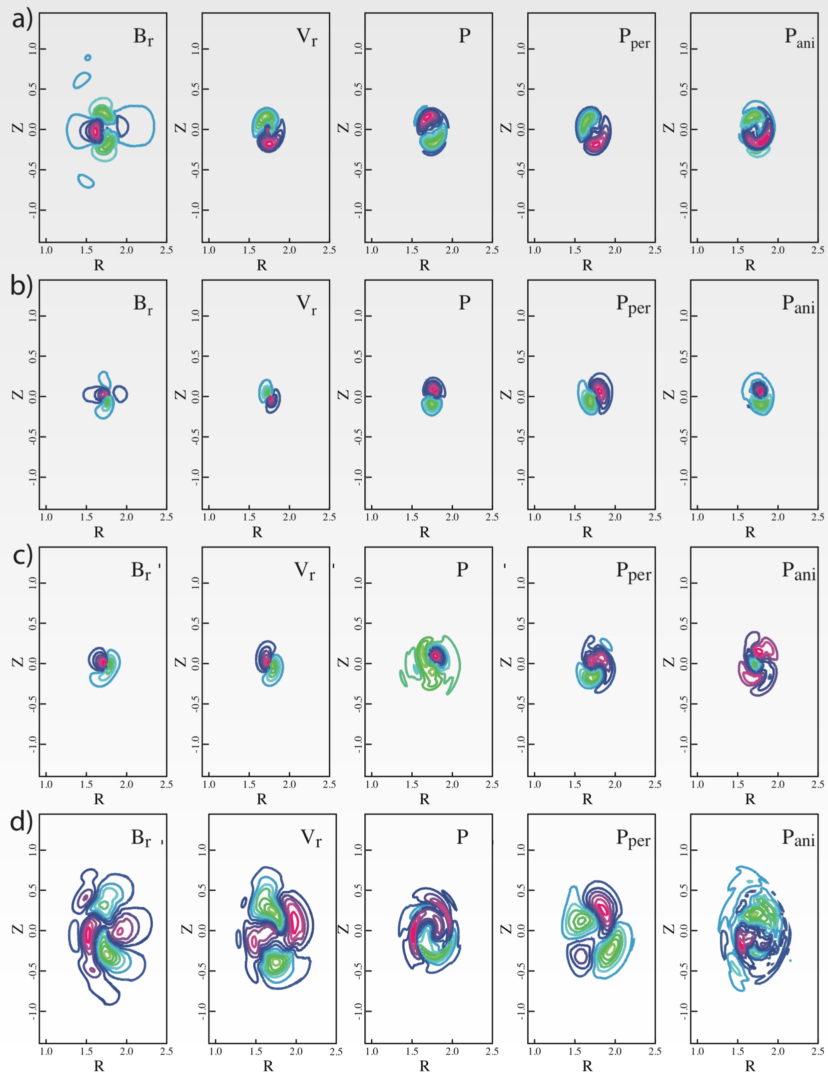
Unstable at higher q_{min}

q_{min}=0.95
1/1 mode
low frequency

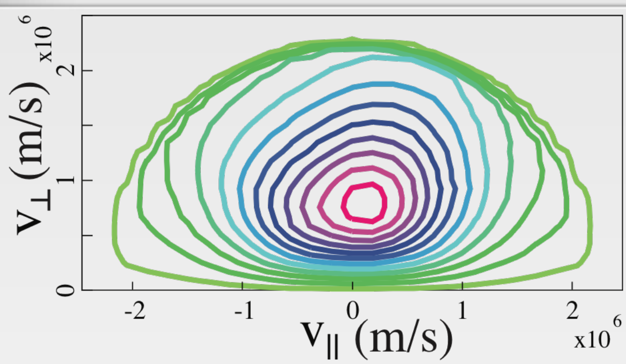
q_{min}=1.05
1/1 core localized

q_{min}=1.21
high frequency
energetic particle mode

q_{min}=1.36
broad 2/1 mode
frequency lower



Trapped particle interaction differs in core and outside of core, and which dominates changes



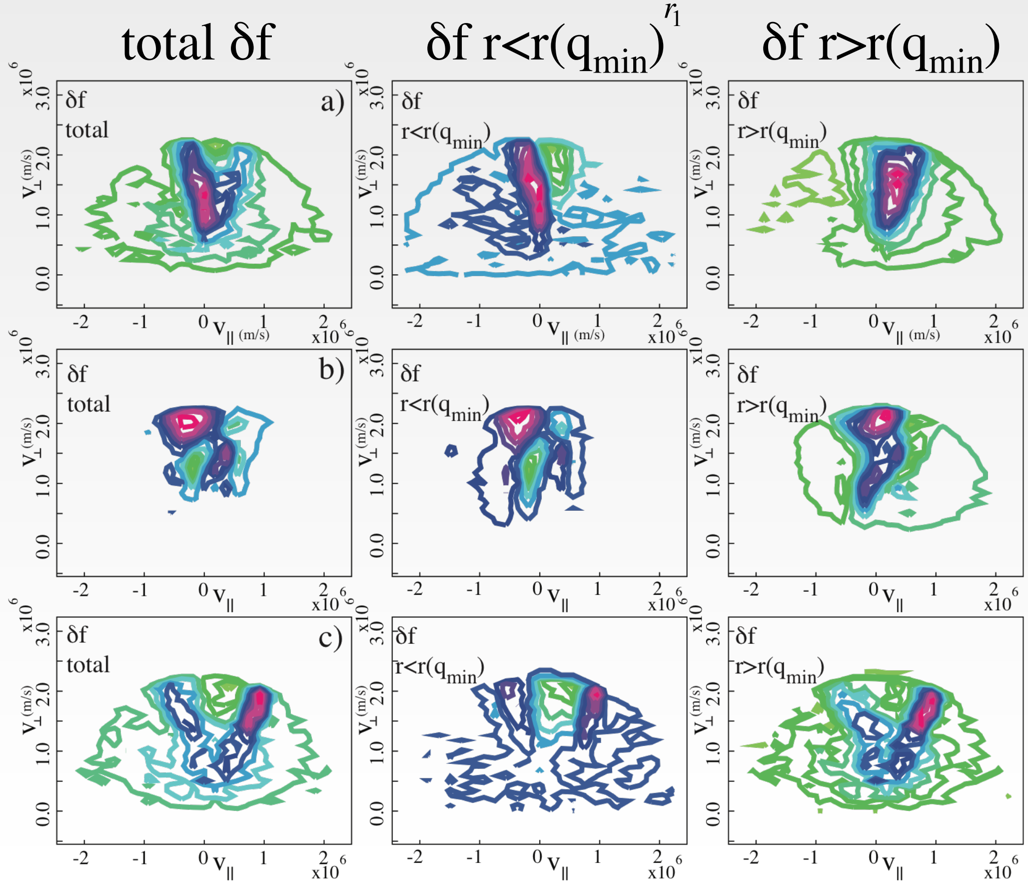
The total distribution at each point as a function of parallel and perpendicular velocity

$$\delta f(v_{\parallel},v_{\perp})_{n=1} = \int_{r_1}^{r_2} \delta f(\mathbf{z})\, d^3x\Big|_{n=1}$$

q_{min}=1.05
1/1 core localized

q_{min}=1.21
high frequency
energetic particle mode

q_{min}=1.36
broad 2/1 mode
frequency lower



Mode frequency comparable to toroidal precession frequency of resonant particles

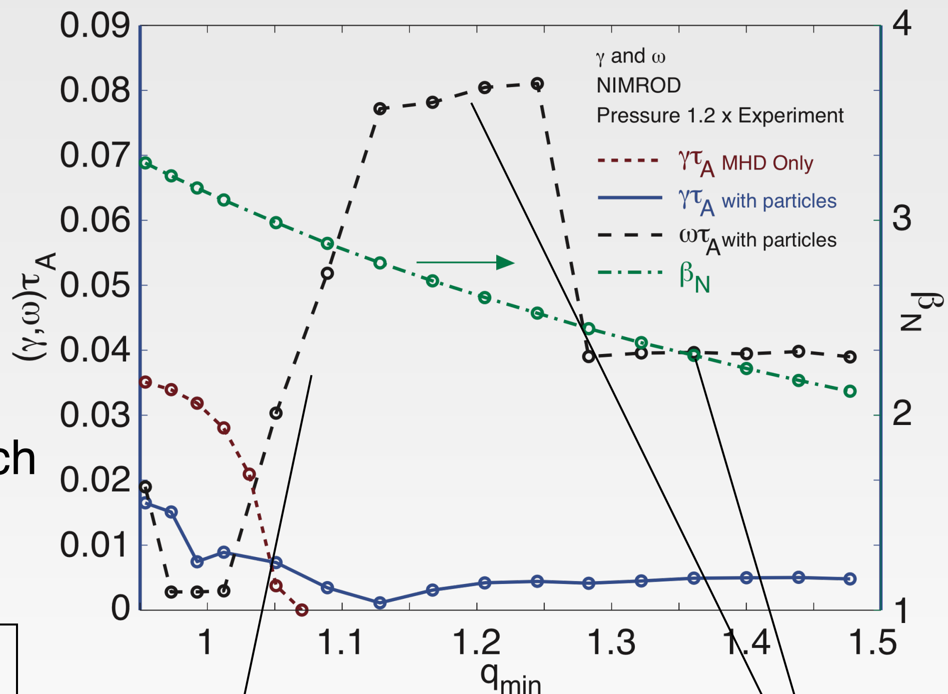
Precession frequency estimate

$$\omega_p \approx \frac{nqE_{eV}}{r_m R_0 B_0}$$

Power flow particles to mode

$$\frac{dU}{dt} = e\omega v_d \cdot (B \times \xi) e^{-i(\omega t - \omega_p t)}$$

is only steady state with frequency match -> mode structure changes cause frequency to change.



For higher q_{min} modes:

resonant q changes=>

ω changes=>

$q_{\min} \approx 1.2$	$q_{\min} \approx 1.4$
$E_{eV} \approx 30keV$	$E_{eV} \approx 30keV$
$B_0 \approx 2T$	$B_0 \approx 2.3T$
$r_m \approx 0.05m$	$r_m \approx 0.25m$
$q \approx 1$	$q = 2$
$\omega_p \tau_A \approx 0.09$	$\omega_p \tau_A \approx 0.04$

For low $q_{\min} \approx 1$ resonant energy increases with q. Fishbone mode.

$\omega \sim \sqrt{\beta q}$ fixed with q. BAE modes.

More details in submitted Nucl. Fusion.

Frequency discontinuous between three regions of q_{min} as most unstable mode switches

For a series of increasing pressures we vary q_{min} and map the growth rates at fixed β_N.

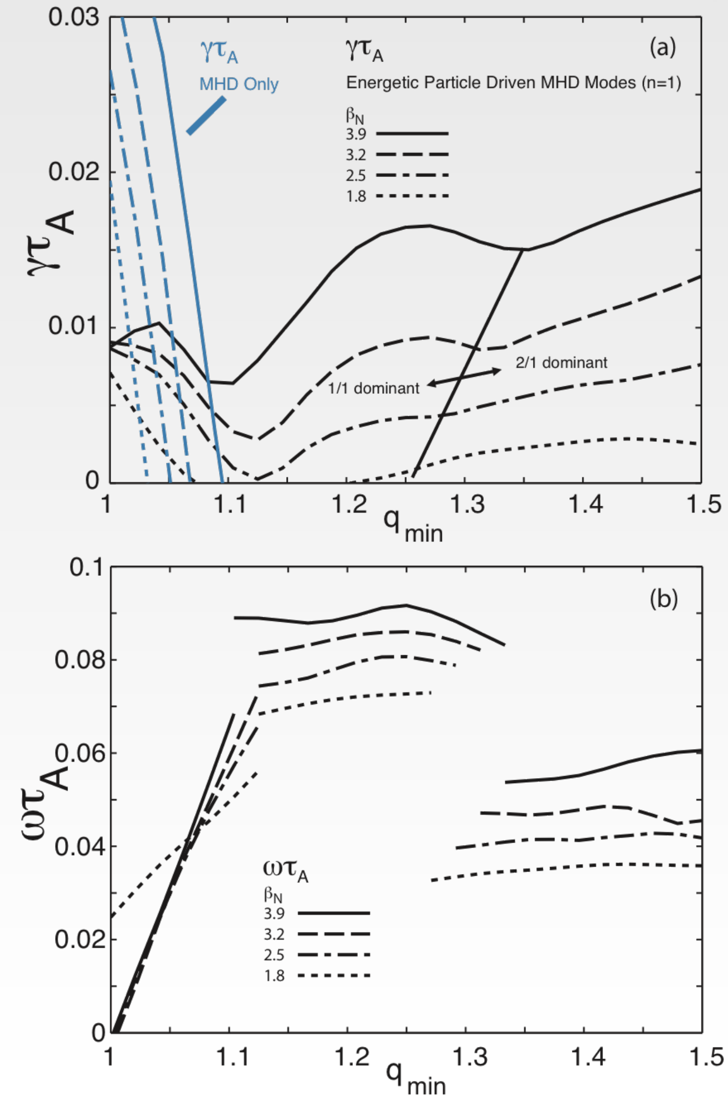
Similar result at each β_N: multiple regions.

Stability boundary at lower β_N.

Mode structures weakly dependent on P in this regime.

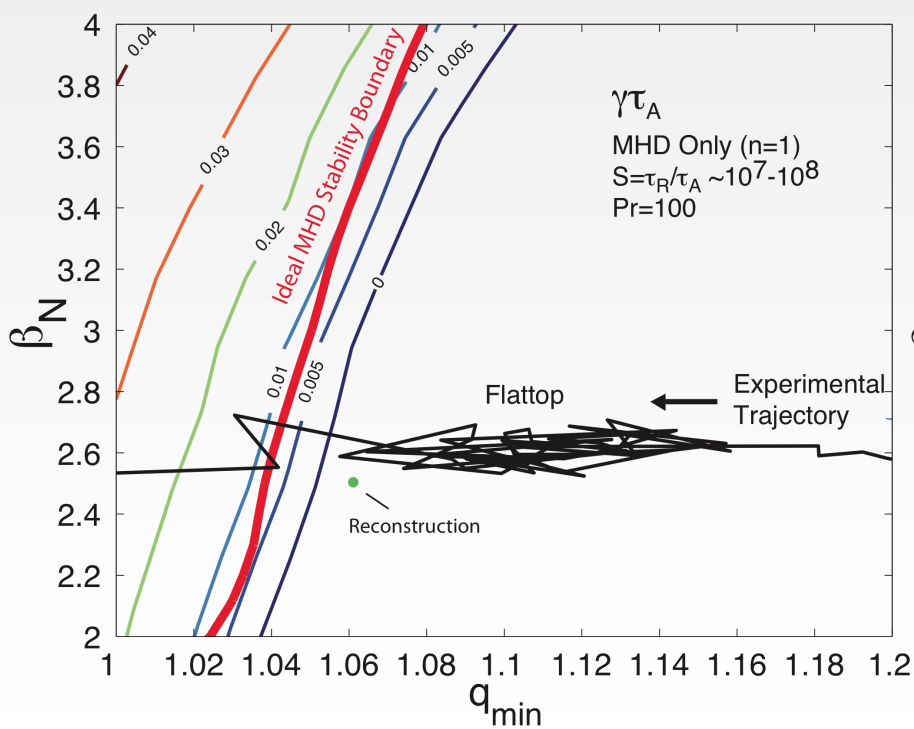
Boundaries between modes predominantly in q_{min}.

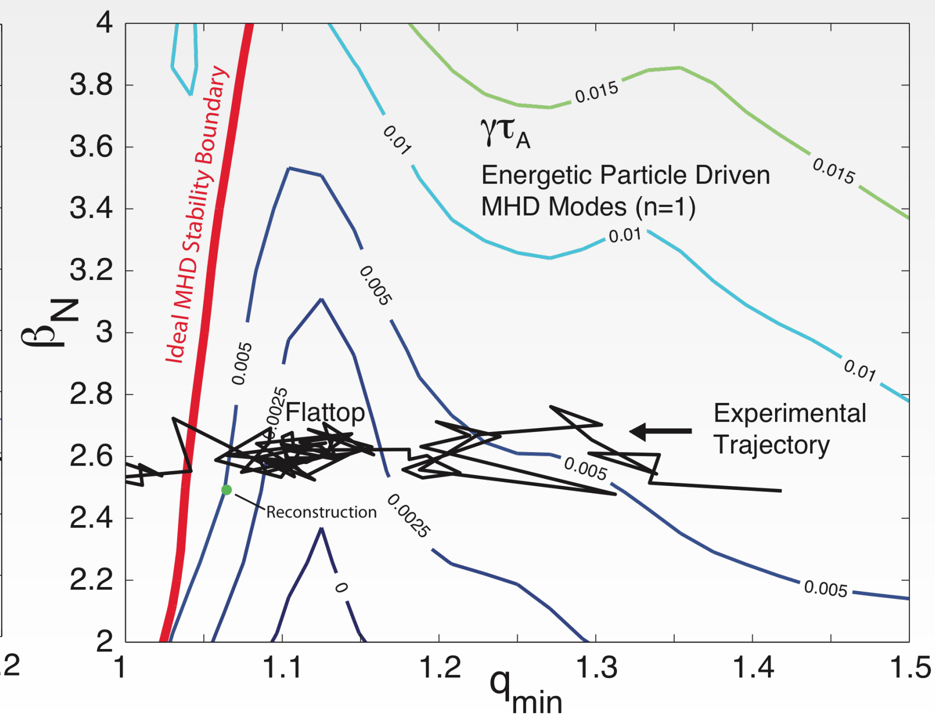
Experimental q_{min},β_N near stable region.



Key Result: Growth rate contours now indicate gradient in β_N

Destabilization well into high q_{min} regime
Experimental trajectory in a low growth rate region
Gradient in increasing β_N direction, mode destabilized in that direction
Resistive instability significant at $\gamma\tau_A$~0.005



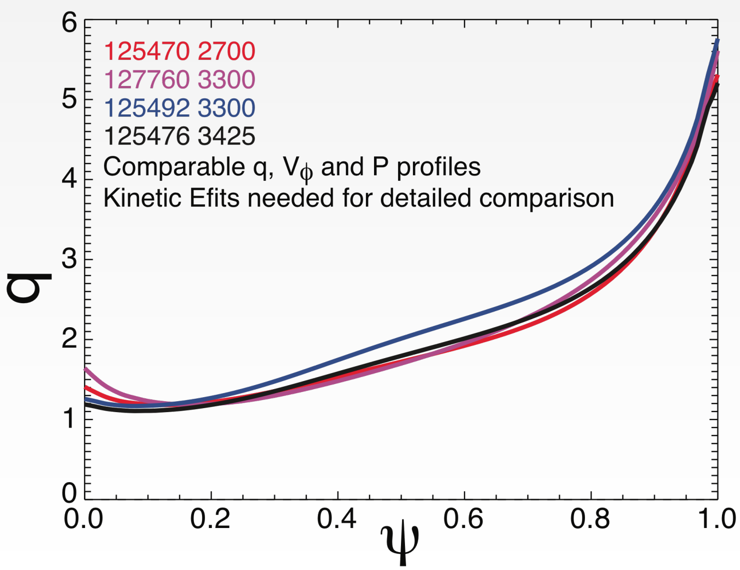


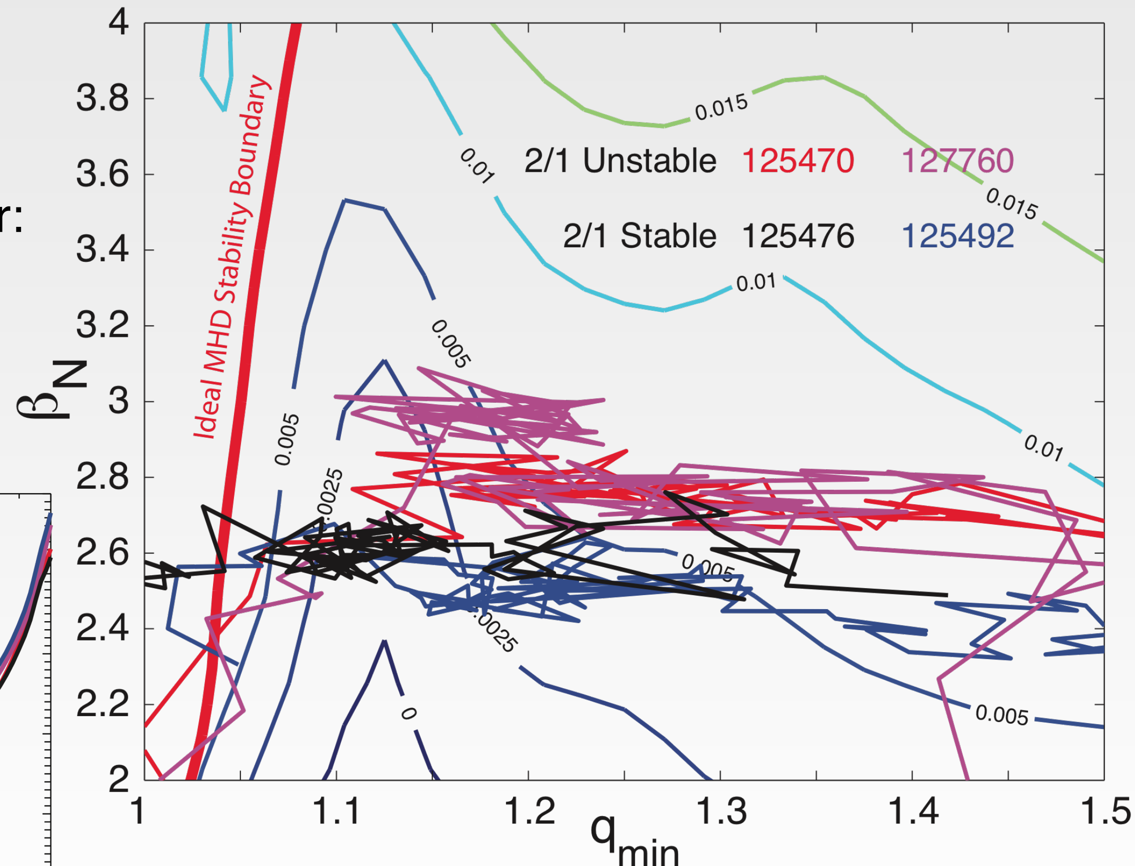
Gradients of growth rate in β_N strongly suggest why 2/1 mode onsets : destabilized by energetic particles

Slight differences between discharges, though similar.

Many other effects to consider:

- •Rotation
- •Two fluid
- •Nonlinear
- •etc…





General trend captured.
More to be done.

Experimental trajectory near stable boundary in β_N q, observes a small saturated 1/1 kink: encouraging!

MHD only results produce a boundary in q_{min}, but far from boundary in β_N

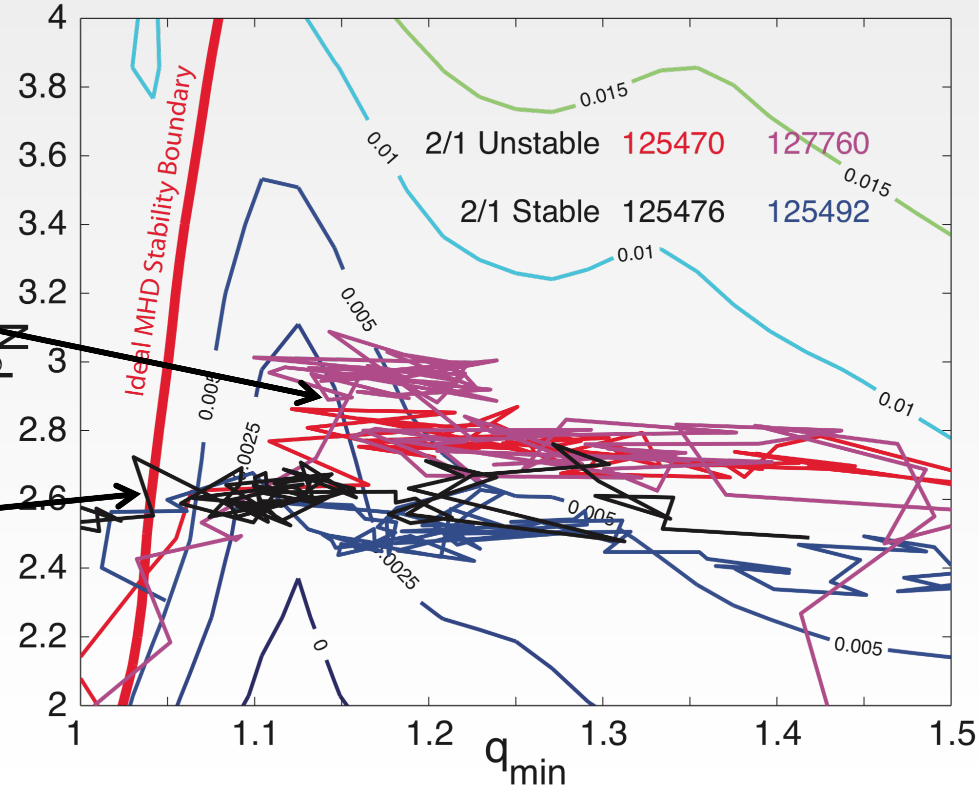
Experimental operation indicates a significant stability boundary in β_N to the n=1 mode, for q_{min}->1.5 at pressures just above experimental

The most important aspect: the boundary is in increasing β_N direction, near exp.

2/1 mode subdominant in this region.

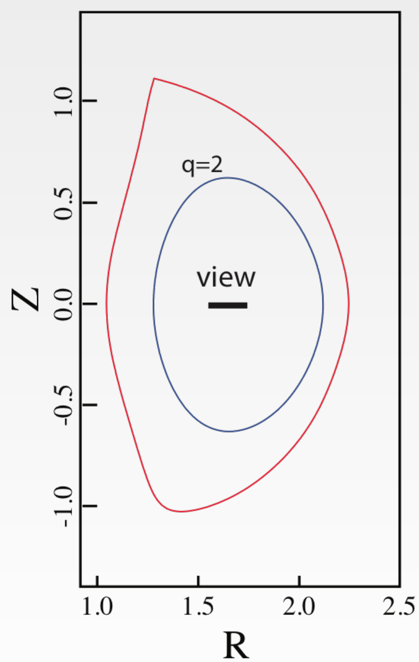
In region of trajectory, axis localized mode is weakly unstable.

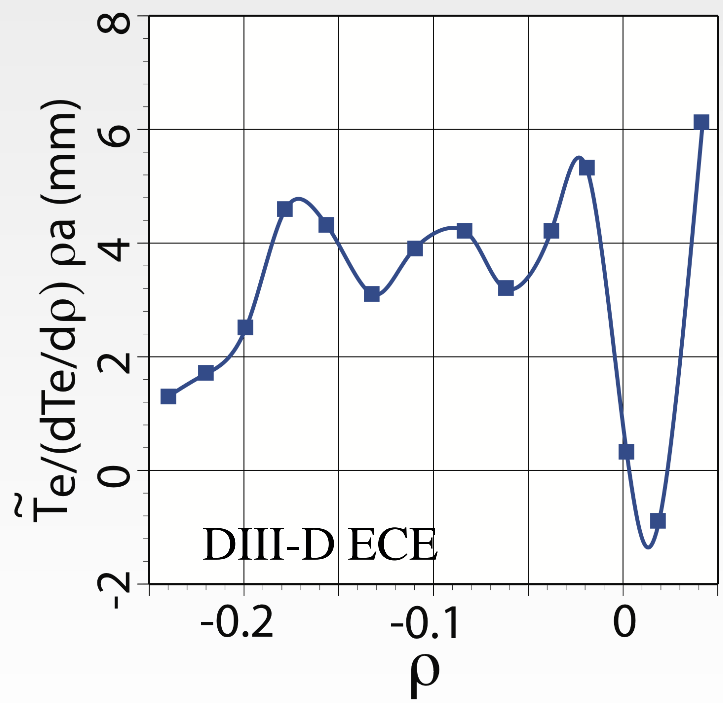
DIII-D also observes a saturated 1/1 kink like mode peaked just off axis near q_{min}.

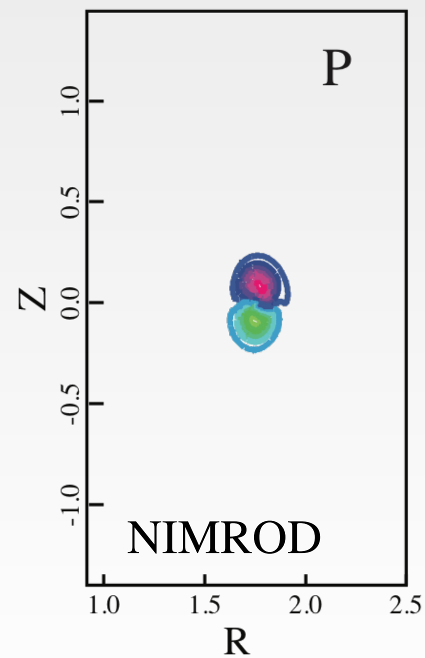


Experiment and computations show n=1 structure inside of q_{min} surrounding axis: further agreement

Minor radial extent of $|\rho|=0.2$ is location of q_{min} at r=0.11m
Frequency in right range $\omega\tau_A \sim 0.05$
Structure nonlinear, can be different but related to linear structure







ECE cutoff outboard of rightmost channel
Further inboard the 3/2 dominates signal

Computed mode
near experimental q_{min},β_N
localized within q_{min}

Summary

Energetic particles in hybrid DIII-D discharge destabilize the n=1 mode in the MHD stable regime and damp the mode in the MHD unstable regime. Three regimes evident.

Particle diagnostics present detailed picture of particle interaction:

- Mainly trapped but passing also influential
- Significant asymmetry in phase space
- Very different interactions between core and surrounding plasma
- Resonant location changes to surrounding plasma at high q_{min}

Results suggest particles could be destabilizing the experimentally observed 2/1 mode in these DIII-D cases.
MHD alone does not capture this boundary.

Experimental observations also indicate a saturated 1/1 kink localized near axis, in rough association with the linear eigenfunctions: encouraging!

Supporting Slides

In Hybrid-Kinetic Approach, Initial value MHD computations coupled to δf model

In the limit $n_h << n_0, \quad \beta_h \sim \beta_0$ and quasi-neutrality, the only modification of the MHD equations is addition of an energetic particle tensor in momentum equation (C.Z.Cheng JGR 91)

$$\rho \frac{d\mathbf{V}}{dt} = \rho\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{V}\right) = \mathbf{J}\times\mathbf{B} - \nabla\cdot\mathbf{p}_b - \boxed{\nabla\cdot\mathbf{p}_h}$$

where $$\mathbf{p}_h = \mathbf{p}_{h0} + \delta\mathbf{p}_h = \begin{pmatrix} p_\perp & 0 & 0 \\ 0 & p_\perp & 0 \\ 0 & 0 & p_\parallel \end{pmatrix} = \int m(\mathbf{v}-\mathbf{V}_h)^2 \delta f(\mathbf{x},\mathbf{v})d\mathbf{v}$$

is computed from a code advancing the change in the distribution function δf

Assumption: Steady state fields satisfy a scalar pressure force balance isotropic => tensorial $\mathbf{p}_{h0}$ reduces to scalar p_{h0}

$$J_0 \times B_0 = \nabla p_0 + \boxed{\nabla p_{h0}}$$

The δf Particle In Cell (PIC) model

- PIC is a Lagrangian simulation of phase space $f(\mathbf{x},\mathbf{v})$
- PIC evolves the $f(\mathbf{x}(t),\mathbf{v}(t))$
- δf PIC reduces the discrete particle noise associate with conventional PIC (Parker PFB 93)

- Vlasov equation $$\frac{\partial f(\mathbf{z})}{\partial t} + \dot{\mathbf{z}}\cdot \frac{\partial f(\mathbf{z})}{\partial \mathbf{z}} = 0$$

- Evolution equation $$\delta\dot{f} = -\delta\dot{\mathbf{z}}\cdot \frac{\partial f_0}{\partial \mathbf{z}}$$

- Drift kinetic equations of motion are used as the particle characteristics

$$\dot{\mathbf{x}} = v_{\|}\hat{\mathbf{b}} + \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{m^2}{eB^4}\left(v_{\|}^2 + \frac{v_{\perp}^2}{2}\right)\left(\mathbf{B}\times\nabla\frac{B^2}{2}\right) - \frac{\mu_0 m v_{\|}^2}{eB^2}\mathbf{J}_{\perp},$$

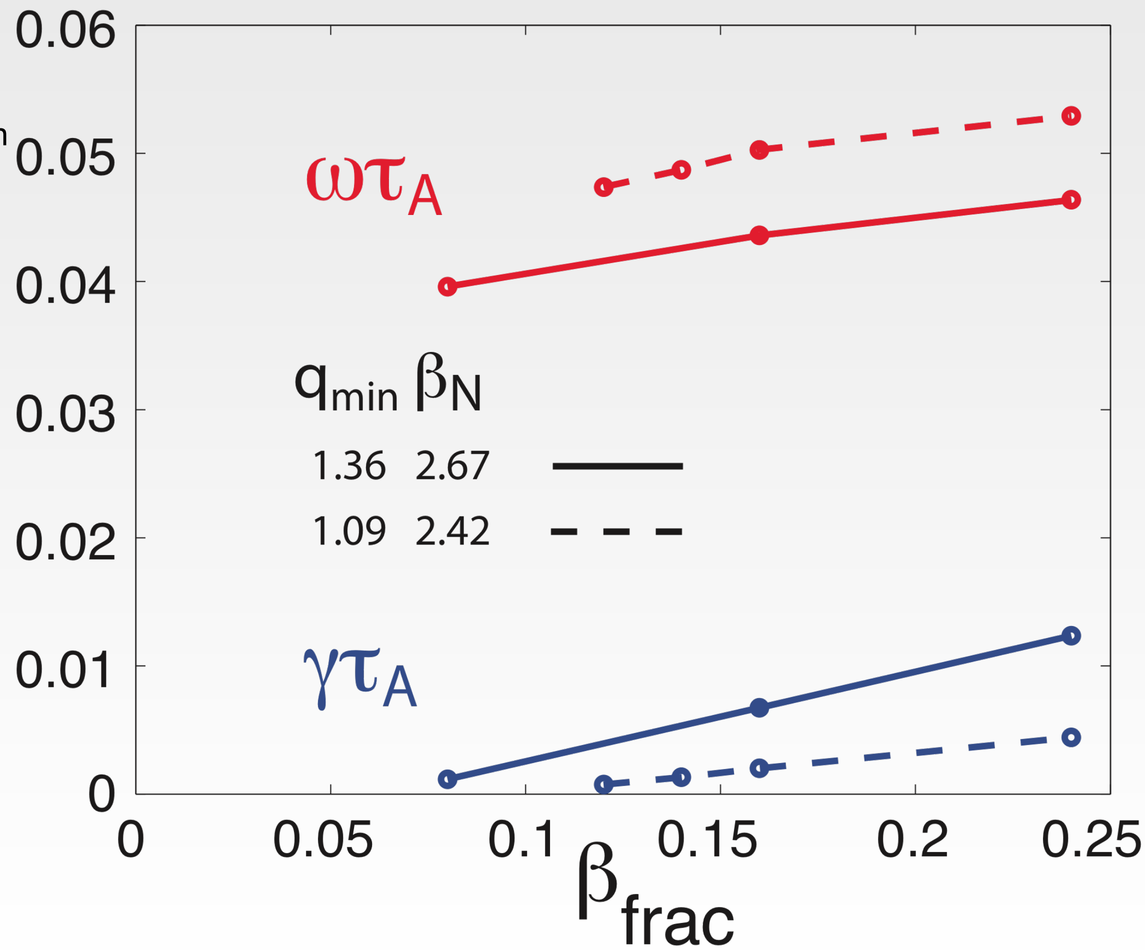
$$m\dot{v}_{\|} = -\hat{\mathbf{b}}\cdot(\mu\nabla B - e\mathbf{E}).$$

Modes stabilize at lower β_{frac}

As an example of our tests, vary β_{frac} for high q_{min} and near experimental cases

Modes stabilize at β_{frac} significantly below experimental regime ~0.16-0.2

Real frequencies finite and significant at stability boundary, proportional to β_{frac}



For high q_{min}>1.3, $\omega \propto \sqrt{\beta}$, while for 1.1<$q_{min}$<1.3, fit less strong

Related to the Beta induced Alfven Eigenmode (BAE)

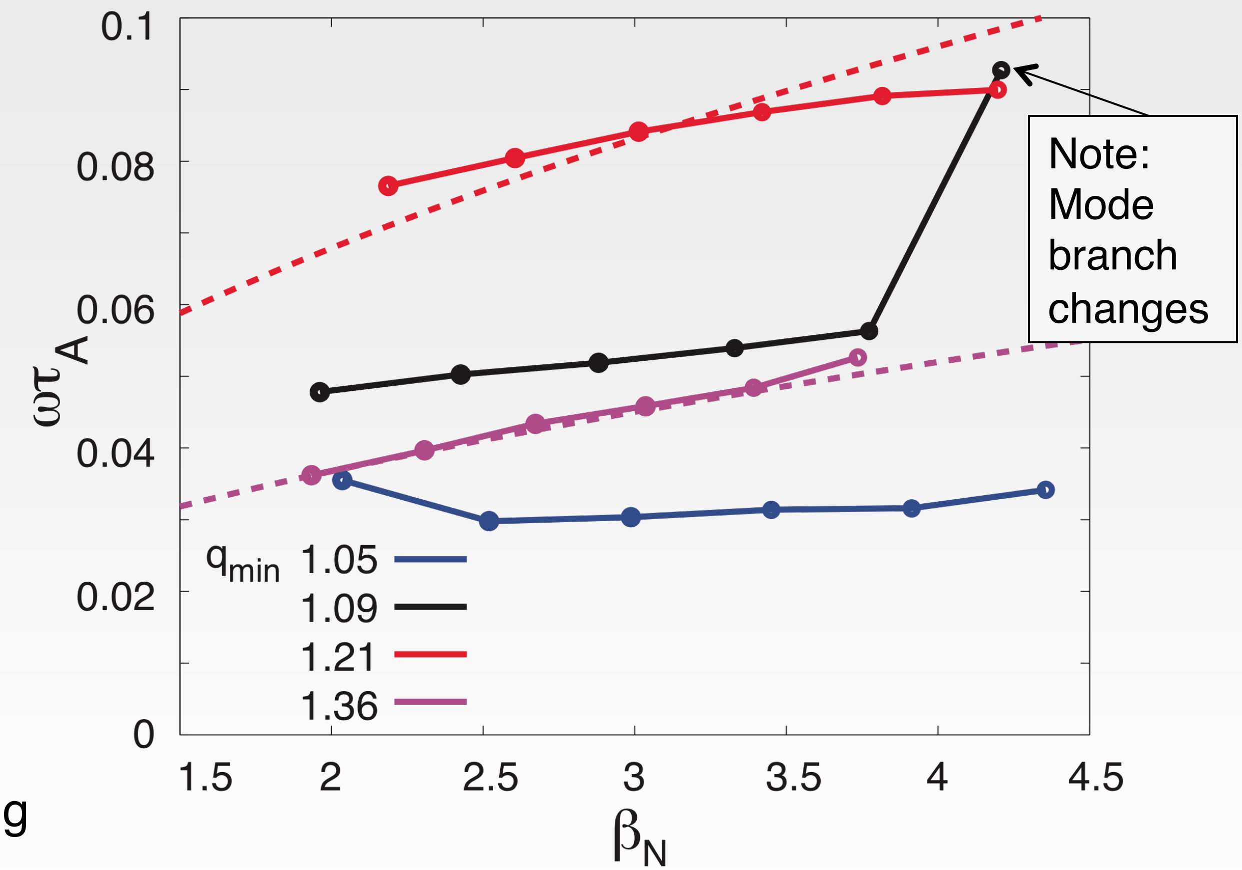
$$0 \leq \omega\tau_A \leq \sqrt{\beta q}$$

ω constant with fixed $\sqrt{\beta q}$ shown earlier.

How does ω scale with β Within each region at fixed q_{min}?

High q_{min}>1.3 fit is strong
1.1<q_{min}<1.3 fit less strong

TAE freq far too high $\omega\tau_A$~0.25+



Also $\xi_{\parallel} > \xi_{\perp}$
Indicative of BAE

More on BAE: A.D. Turnbull *et al.*, Phys. Fluids B 5, 2546 (1993).