PPPL PRINCETON PLASMA PHYSICS LABORATORY

U.S. DEPARTMENT OF ENERGY | Office of Science

TORWARD HYBRID NEOCLASSICAL-MAGNETOHYDRONAMIC SIMULATIONS OF CORE PLASMA INSTABILITIES

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Introduction

How can one model neoclassical tearing modes?

Neoclassical tearing mode modeling

- NTM stability place a severe limit on maximum β
- Most common cause of disruptions on JET[1]
- High-fidelity simulations required for prediction, control, avoidance, and understanding of NTMs
  - Especially important for ITER operation, in which very few disruptions can be tolerated[2]
- NTMs incorporate a lot of physics
  - Cause: Neoclassical kinetic theory (bootstrap current)
  - Effect: MHD destabilization (island growth)
  - Requires a hybrid model

[1] P.C. de Vries, et al., Nucl. Fusion **51**, 053018 (2011)

[2] T.C. Hender, et al., Nucl. Fusion **47**, S128-S202 (2007)

Framework for hybrid solver

- Use existing MHD time-evolution code (e.g., M3D-*C*[1], NIMROD)
- Desirable traits for neoclassical drift–kinetic equation (DKE) solver
  - Three-dimensional toroidal geometry
    - Study nonaxisymmetric geometries with magnetic islands
  - Full Fokker-Planck-Landau collision operator
    - Use of model collision operators can lead to errors of 5%-10%[3]
  - Continuum model
    - Good convergence properties, especially for long times
    - Straight-forward coupling to MHD solvers
    - Potentially more computationally efficient than PIC
- <u>No code currently exists that has all of these traits</u>

[3] E.A. Belli and J. Candy, Plasma Phys. Control. Fusion **54**, 015015 (2012)

Ramos Form of DKE

- J.J. Ramos (Phys. Plasmas 2010 & 2011) provides analytic framework for a neoclassical solver appropriate for core plasma instability simulations
- Velocity taken in frame of each species' macroscopic flow
- Two expansion parameters for high-temperature fusion plasmas

$$\delta \sim \rho_i / L \ll 1 \qquad \nu_* \sim L/\lambda_{\mathrm{mfp}} \sim \delta$$

  - Maintain equations to lowest order exhibiting collisional dynamics
    - Electron DKE is maintained to order $\delta_e \nu_*$, or δ^3
    - Ion DKE should be maintained to order δ^2
  - For now, ion DKE is maintained to order $\delta\nu_*$, with $\delta \ll \nu_* \ll 1$
    - This is conventional neoclassical banana dynamics for both species
- DKE evolves difference between the full distribution function and a shifting Maxwellian (like delta-f)

Neoclassical Ion-Electron Solver

NIES solves for the non-adiabatic distribution function to zeroth-order in collisionality in a stationary, axisymmetric toroidal plasma

NIES overview

- Work in 4D phase space w/ flux coordinates: $(\psi, \theta, v, \lambda)$
- Distribution function: $\bar{f}_s = \left(1 + g_{s,0} + g_{s,1}\ v_{\|}\right) f_{Ms} + h_s$
- Non-adiabatic piece: $h_s = \varsigma(v_{\|}) H(1-\lambda) K_s(\psi, v, \lambda) + h_s^{even}$
  - K_s is independent of θ
  - Dimensionless magnetic moment: $\lambda = \sin^2 \chi B_{max}(\psi)/B(\psi, \theta)$
- DKE reduced to a solvability condition

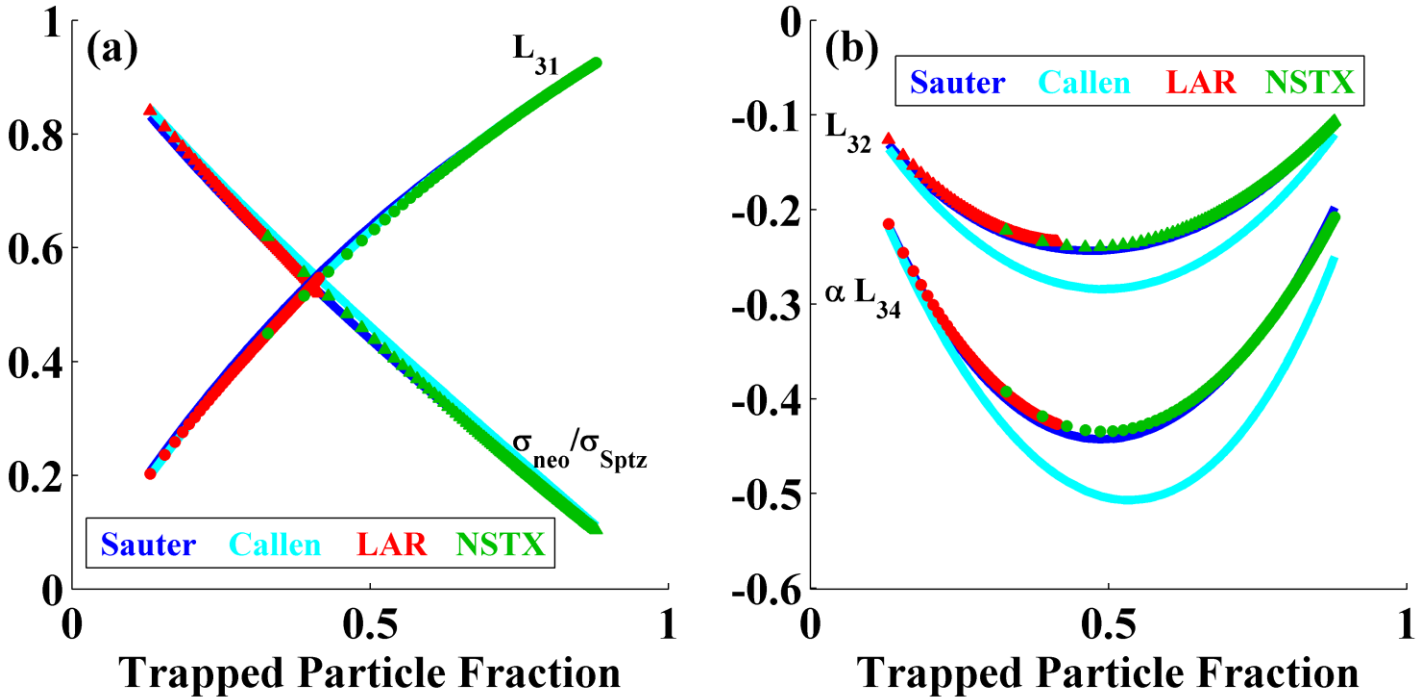
$$\oint_{\psi,v,\lambda} \frac{dl}{v_{\|}} C_s\left[\varsigma H K_s\right] = -\oint_{\psi,v,\lambda} dl S_s$$

  - Ion source contains ion temperature gradient drive
  - Electron source contains ohmic drive, interaction with ion flow, and pressure and electron temperature gradient bootstrap drive
- Need only be solved for passing particles

NIES computational methods

- Expansions
  - Expand Rosenbluth Potentials in Legendre polynomials in $y = \cos\chi$, Fourier series in $\cos\theta$, and piecewise linear finite elements in v
  - Then expand K_s in piecewise linear finite elements in v and λ
- Solvability condition and the Rosenbluth Potential Poisson equations solved simultaneously on one flux surface at a time
  - Inner products taken with finite elements, Legendre polynomials, and Fourier modes
  - Matrix equation is block tridiagonal in v, allowing for efficient solution method
  - Simple moment of K_s gives ohmic and bootstrap current

Benchmarks



$$\langle \mathbf{J}\cdot\mathbf{B}\rangle = \sigma_{neo}\langle \mathbf{E}\cdot\mathbf{B}\rangle - cI\left[\mathcal{L}_{31}\frac{dp}{d\psi} + \mathcal{L}_{32}n_e\frac{dT_e}{d\psi} + \mathcal{L}_{34}\alpha n_i\frac{dT_i}{d\psi}\right]$$

- Sauter analytic fits: O. Sauter, et al., Contrib. Plasma Phys. **34**, 2/3, pp 169-174 (1994).
- Callen analytic fomulae: J.D. Callen, et al., Phys. Plasmas **17**, 056113 (2010).
- Our results: B.C. Lyons, S.C. Jardin, and J.J. Ramos, Phys. Plasmas **19**, 082515 (2012).

Hybrid Simulations

Axisymmetric case

Overview of next step

- Retain axisymmetric geometry for now
- Want to solve the full Ramos DKE without further expansions in collisionality
  - Extends result to first-order in collisionality
  - Allows solution to vary poloidally
- Will couple directly to reduced MHD equations

Reduced MHD equations

- Use the two field representation

$$\mathbf{B} = \nabla\psi \times \nabla\zeta + I_0 \nabla\zeta \qquad \mu_0 \mathbf{J} = -\Delta^* \psi \nabla\zeta \qquad \mathbf{u} = R^2 \nabla U \times \nabla\zeta$$

- Combining Faraday's Law and Ohm's law gives

$$\dot{\psi} + R^2 \left[\psi, U\right] = \frac{R^2}{en} \mathbf{F}_e^{coll} \cdot \nabla\zeta$$

  - Collisional friction force:

$$\mathbf{F}_e^{coll} = \frac{2 m_e \nu_e}{3\sqrt{2\pi} e} \mathbf{J} - 2\pi m_e \nu_e v_{the}^3 \int_0^\infty dv' \int_0^\pi d\chi \sin\chi \cos\chi f_{NMe} \mathbf{b}.$$

- Momentum equation gives

$$\nabla \cdot \left(R^2 \nabla \dot{U} \right) + \frac{1}{2} \left[R^2 \left(U, U\right), R^2 \right] + \left[R^4 \nabla^2 U, U \right] = \frac{1}{n m_i} \left[\Delta^* \psi, \psi \right]$$

Note that: $(a, b) = \nabla a \cdot \nabla b \qquad [a, b] = \nabla\zeta \cdot (\nabla a \times \nabla b)$

$$\Delta^* a = R^2 \nabla_\perp \cdot \left(R^{-2} \nabla a \right)$$

Reduced DKE

- To start, assume:
  - Flat and stationary temperature & density profiles
  - Zero pressure anisotropy
  - Large aspect ratio
  - Equal ion and electron temperatures
- DKE simplifies to

$$\frac{\partial f_{NMe}}{\partial t} + v\cos\chi \mathbf{b}\cdot\nabla f_{NMe} + \frac{1}{2}v\sin\chi \mathbf{b}\cdot\nabla \ln B \frac{\partial f_{NMe}}{\partial \chi} = -\frac{v\cos\chi}{nT_e}\mathbf{b}\cdot\mathbf{F}_e^{coll} f_{Me} + \langle C_{ee} + C_{ei}\rangle$$

$$\langle C_{ee} + C_{ei}\rangle = \nu_{De}(v)\mathcal{L}[f_{NMe}] + \frac{\nu_e v_{the}^3}{v^2}\frac{\partial}{\partial v}\left\{\xi_e\left[v\frac{\partial f_{NMe}}{\partial v} + \frac{v^2}{v_{the}^2}f_{NMe}\right] + \xi_i\left[v\frac{\partial f_{NMe}}{\partial v} + \frac{m_e v^2}{m_i v_{thi}^2}f_{NMe}\right]\right.$$
$$\left. + \frac{\nu_e v_{the}}{n}f_{Me}\left(4\pi v_{the}^2 f_{NMe} - \Phi_e[f_{NMe}] + \frac{v^2}{v_{the}^2}\frac{\partial^2 \Psi_e[f_{NMe}]}{\partial v^2}\right) + \nu_e f_{Me}\frac{v_{the}}{v_{thi}^2}\frac{\mathbf{b}\cdot\mathbf{J}}{en}\xi_i y\right.$$

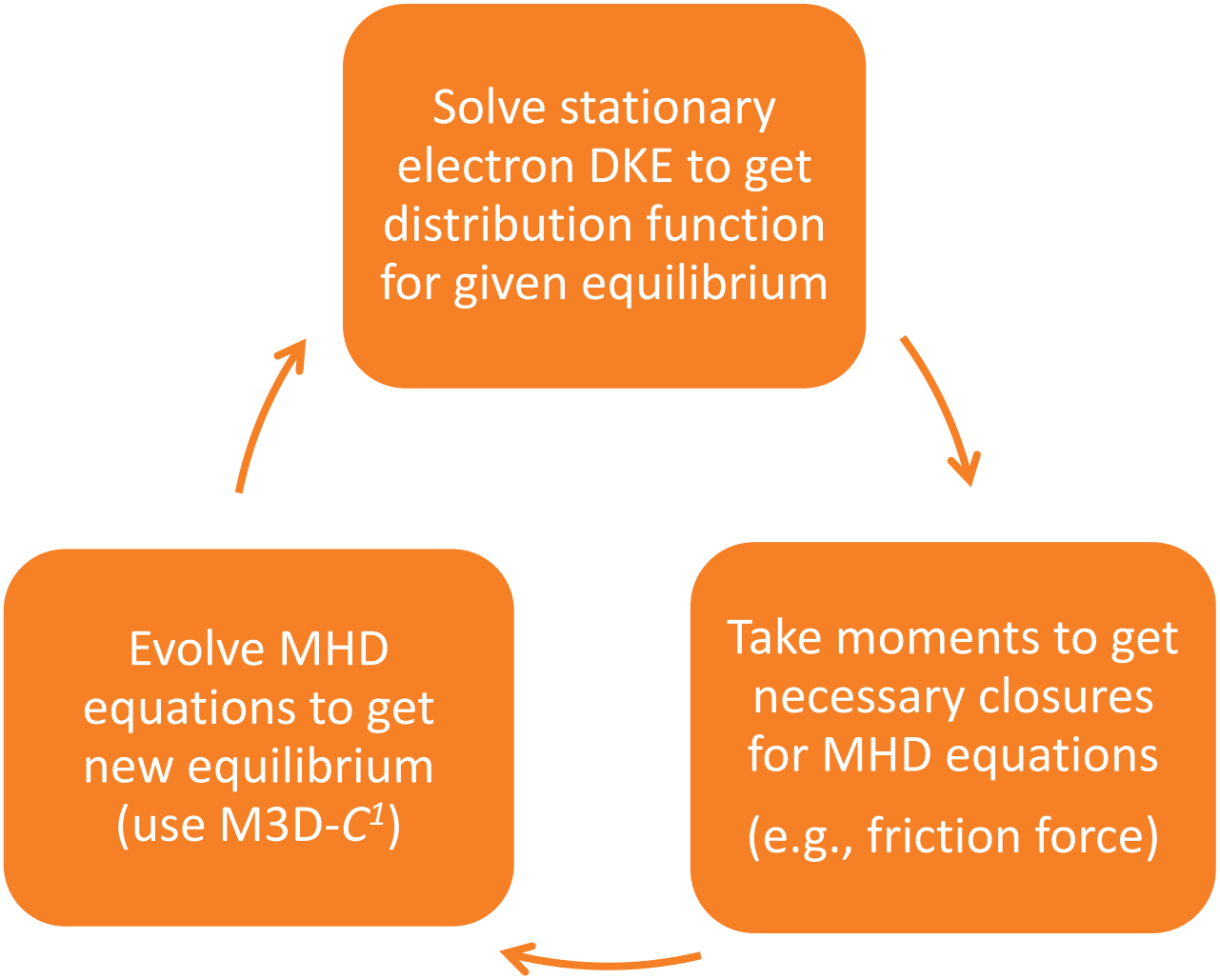
Timescales

Machine	n (m^{-3})	T (keV)	B (T)	a (m)	R (m)
LTX	3.15×10^{19}	0.2	0.34	0.26	0.4
NSTX	9.04×10^{19}	1	0.45	0.65	0.85
DIII-D	1.13×10^{20}	5	2.1	0.65	1.67
ITER	1.19×10^{20}	20	5.3	2.0	6.2

Machine	τ_{Alfven} (s)	$\tau_{e,conv}$ (s)	$\tau_{i,conv}$ (s)	$\tau_{e,coll}$ (s)	$\tau_{i,coll}$ (s)	$\tau_{resistive}$ (s)
LTX	3.0×10^{-7}	6.7×10^{-8}	2.9×10^{-6}	5.8×10^{-7}	2.5×10^{-5}	3.3×10^{-1}
NSTX	8.2×10^{-7}	6.4×10^{-8}	2.7×10^{-6}	2.0×10^{-6}	8.6×10^{-5}	2.0×10^{1}
DIII-D	3.9×10^{-7}	5.6×10^{-8}	2.4×10^{-6}	1.6×10^{-5}	6.7×10^{-4}	2.0×10^{2}
ITER	5.9×10^{-7}	1.0×10^{-7}	4.5×10^{-6}	1.1×10^{-4}	4.6×10^{-3}	1.3×10^{4}

- Difficult to consider DKE time dependently
  - In DKE, collision time 10-10^3 longer than convective time
  - MHD resistive time 10^6-10^8 longer than collision time
- Reasonable to assume that distribution function evolves to steady state on the resistive timescale

Proposed solution iteration

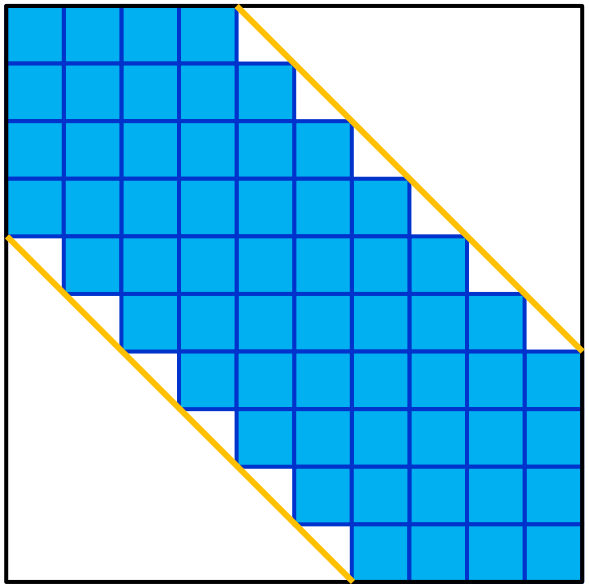


Proposed expansions in DKE

- Velocity
  - Cubic B-spline finite elements for v
- Pitch angle
  - λ no longer a good expansion variable
  - Use $y = \cos\chi$ instead
    - Legendre polynomials
    - Finite elements
- Configuration Space
  - (ψ, θ) - only need to expand poloidal angle
    - Fourier modes
    - Finite elements
  - (R, Z) - finite elements

Proposed Solution Method

- Cubic B-spline finite elements create block septadiagonal matrix in v



- Each block contains information on y and θ derivatives
- Solve as a sparse banded matrix using ScaLAPACK

Test problem

- Diffusion of current into a toroidal plasma due to a loop voltage at its edge
- Current evolves self-consistently with equilibrium
- Should observe neoclassical conductivity reduction
  - Trapped particles carry no net current
  - Can benchmark to theoretical and numerical results
- Code currently in production

Extensions to axisymmetric code

- Eventually want to solve time-independent, axisymmetric form of the full Ramos DKE
  - Allow separate ion and electron temperatures
  - Relax constraints on density and temperature profiles
  - Relax large aspect ratio equilibrium assumption
- Will have to solve separate, but similar, ion DKE
- Will allow for simulations of the inductive formation of the bootstrap current
- Use full six-field MHD model with M3D-C^1 to self-consistently evolve pressure as well

Conclusion

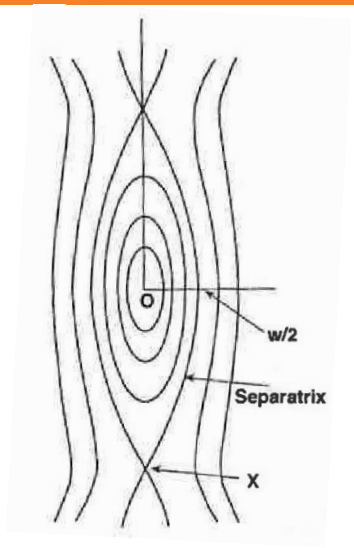
Summary

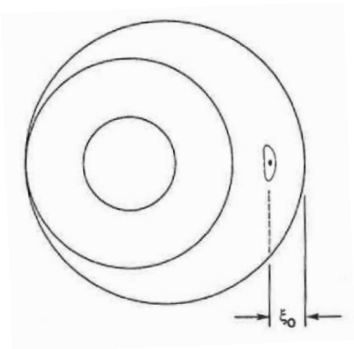
- The operation of ITER and other future MCF experiments requires predictive capabilities for core plasma instabilities (e.g., Sawtooths, NTMs)
- To date, no neoclassical code exists that is well-suited for such simulations
- We are creating such a code based on the Ramos drift-kinetic formulation
  - Axisymmetric hybrid code currently in development
  - Work on nonaxisymmetric code should be underway by next year's APS
    - See my poster (#112 Tuesday afternoon) for more info

Additional Slides

Neoclassical tearing mode (NTMs)

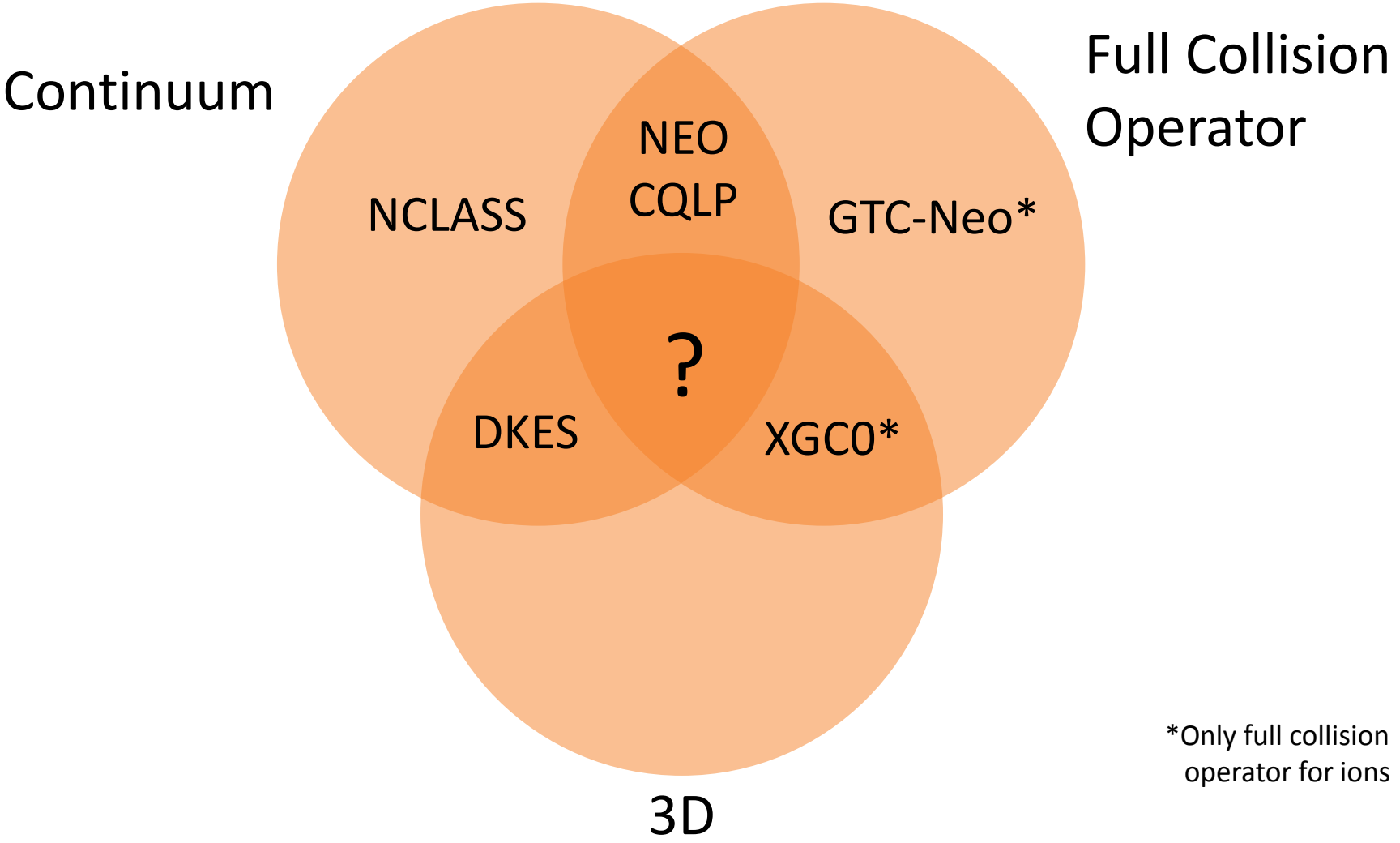
- Density, temperature, pressure, etc. tend to equilibrate across an island width
- Difference in current at O-point and X-point can drive island growth
  - Without these gradients, there can be no bootstrap current within the island
  - Bootstrap current at the X-point can drive island growth
- Large islands allow hot, dense plasma near core to be transported outward, reducing confinement
- Modifications to magnetic topology can result in macroscopic instability and disruption





Images taken from The Theory of Toroidally Confined Plasmas by R. White, 2006

Neoclassical code characteristics



*Only full collision operator for ions

Ramos DKE

$$\frac{\partial f_{NMe}}{\partial t}+\cos\chi\left(v\mathbf{b}\cdot\nabla f_{NMe}+v_{the}^{2}\mathbf{b}\cdot\nabla\ln n\frac{\partial f_{NMe}}{\partial v}\right)-\frac{\sin\chi}{v}\left(v_{the}^{2}\mathbf{b}\cdot\nabla\ln n-\frac{v^{2}}{2}\mathbf{b}\cdot\nabla\ln B\right)\frac{\partial f_{NMe}}{\partial\chi}$$
$$=\left\{\cos\chi\frac{v}{2T_{e}}\left(5-\frac{v^{2}}{v_{the}^{2}}\right)\mathbf{b}\cdot\nabla T_{e}+\cos\chi\frac{v}{nT_{e}}\mathbf{b}\cdot\left[\frac{2}{3}\nabla\left(p_{e\parallel}-p_{e\perp}\right)-\left(p_{e\parallel}-p_{e\perp}\right)\nabla\ln B-\mathbf{F}_{e}^{coll}\right]\right.$$
$$+P_{2}\left(\cos\chi\right)\frac{v^{2}}{3v_{the}^{2}}\left(\nabla\cdot\mathbf{u}_{e}-3\mathbf{b}\cdot\left[\mathbf{b}\cdot\nabla\mathbf{u}_{e}\right]\right)+\frac{1}{3nT_{e}}\left(\frac{v^{2}}{v_{the}^{2}}-3\right)\left[\nabla\cdot\left(q_{e\parallel}\mathbf{b}\right)-G_{e}^{coll}\right]$$
$$+\frac{1}{6eB}\left[2P_{2}\left(\cos\chi\right)\frac{v^{2}}{v_{the}^{2}}\left(\frac{v^{2}}{v_{the}^{2}}-5\right)+\frac{v^{4}}{v_{the}^{4}}-10\frac{v^{2}}{v_{the}^{2}}+15\right]\left(\mathbf{b}\times\boldsymbol{\kappa}\right)\cdot\nabla T_{e}$$
$$+\frac{1}{6eB}\left[-P_{2}\left(\cos\chi\right)\frac{v^{2}}{v_{the}^{2}}\left(\frac{v^{2}}{v_{the}^{2}}-5\right)+\frac{v^{4}}{v_{the}^{4}}-10\frac{v^{2}}{v_{the}^{2}}+15\right]\left(\mathbf{b}\times\nabla\ln B\right)\cdot\nabla T_{e}$$
$$\left.+P_{2}\left(\cos\chi\right)\frac{v^{2}}{3eBv_{the}^{2}}\left(\mathbf{b}\times\nabla\ln n\right)\cdot\nabla T_{e}\right\}f_{Me}+\left\langle C_{ee}+C_{ei}\right\rangle .$$

with full Fokker-Planck-Landau collision operator

$$\left\langle C_{ee}+C_{ei}\right\rangle =\nu_{De}(v)\mathcal{L}[f_{NMe}]+\frac{\nu_{e}v_{the}^{3}}{v^{2}}\frac{\partial}{\partial v}\left\{\xi_{e}\left[v\frac{\partial f_{NMe}}{\partial v}+\frac{v^{2}}{v_{the}^{2}}f_{NMe}\right]+\xi_{i}\left[v\frac{\partial f_{NMe}}{\partial v}+\frac{m_{e}v^{2}}{m_{i}v_{thi}^{2}}f_{NMe}\right]\right\}$$
$$+\frac{\nu_{e}v_{the}}{n}f_{Me}\left(4\pi v_{the}^{2}f_{NMe}-\Phi_{e}[f_{NMe}]+\frac{v^{2}}{v_{the}^{2}}\frac{\partial^{2}\Psi_{e}[f_{NMe}]}{\partial v^{2}}\right)$$
$$+\nu_{e}v_{the}f_{Me}\left(\frac{T_{e}}{T_{i}}-1\right)\left[\frac{4\pi v_{thi}^{2}}{n}f_{Mi}-\frac{v}{v_{the}^{2}}\xi_{i}\right]+\nu_{e}f_{Me}\frac{v_{the}}{v_{thi}^{2}}\frac{\mathbf{b}\cdot\mathbf{J}}{en}\xi_{i}\cos\chi,$$

Nonaxisymmetric hybrid simulations

- Toroidal angle could be expanded in Fourier modes or finite elements
- No concentrically-nested flux surfaces
  - Flux surfaces still exist in most of torus, however, both inside and outside islands
  - May be possible to parameterize field lines or surfaces with something like (ψ, θ)
  - May require (R, Z) representation of poloidal plane

Possible nonaxisymmetric test case

- Begin with a stable equilibrium with a seed magnetic island
- Evolve equilibrium with neoclassical effects due to nonaxisymmetric code
- Explore how bootstrap current can destabilize the seed island, leading to neoclassical tearing mode