

Nonlinear Calculations of Soft and Hard Beta Limits in NSTX

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Acknowledgements also to: G. Fu, S. Hudson, W. Park, H. Strauss, L. Sugiyama.

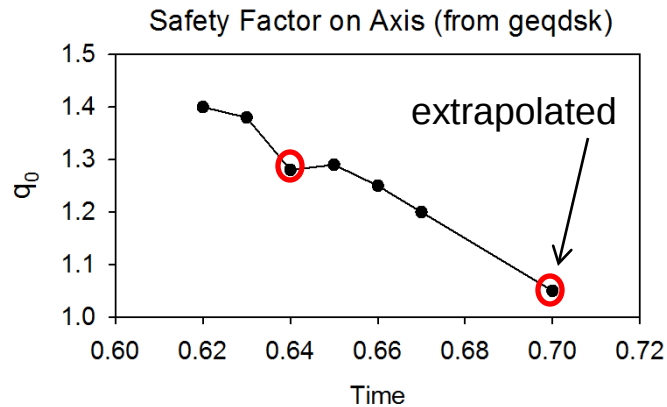
B. Lyons and J. Ramos are incorporating neoclassical effects into M3D-C¹

motivation

- What causes a plasma to disrupt?
- Linear stability to all global modes for all time will ensure disruption-free operation.
- However, the converse is not true.
 - Linear instability does not necessarily imply a disruption.
- Can we use a non-linear MHD code to identify nonlinear events that lead to a disruption (or not)?

NSTX pressure driven modes with $q_0 \geq 1$

Series of geqdsk equilibrium for shot 124379 generated by S. Gerhardt for 2011 Breslau, et al NF paper.



$$\beta_P \approx 0.8$$

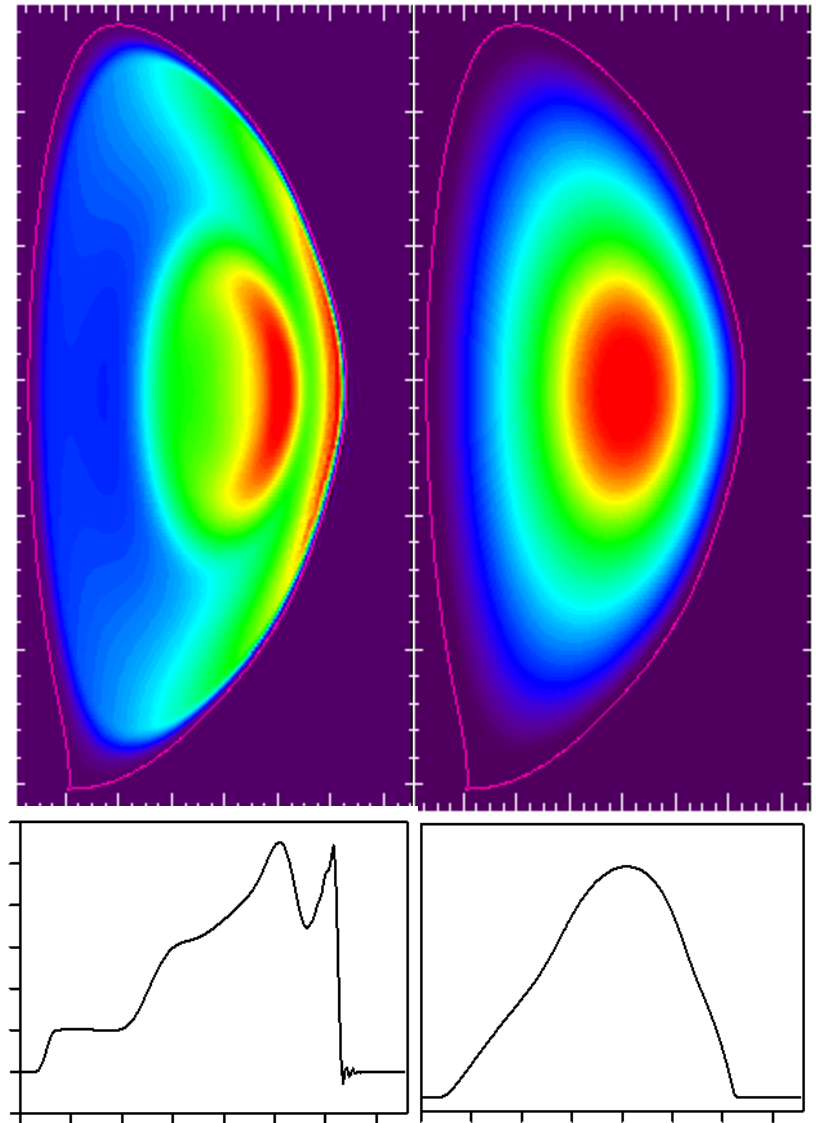
$$\beta_T \approx 7 + \%$$

$$I_P \approx 1 \text{ MA}$$

Midplane
values →

Toroidal current

pressure



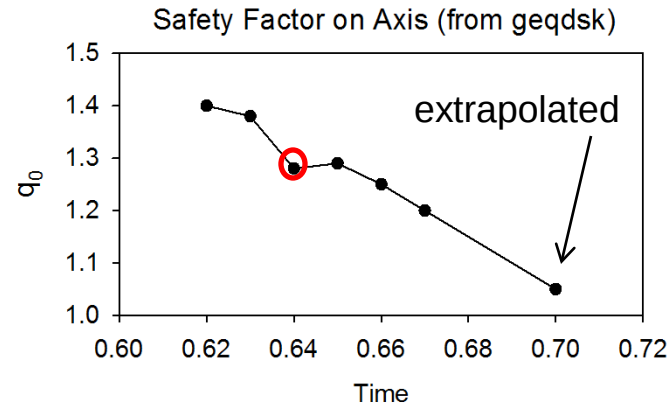
Possible mechanism for soft beta limit

Shot 124379

Time .640

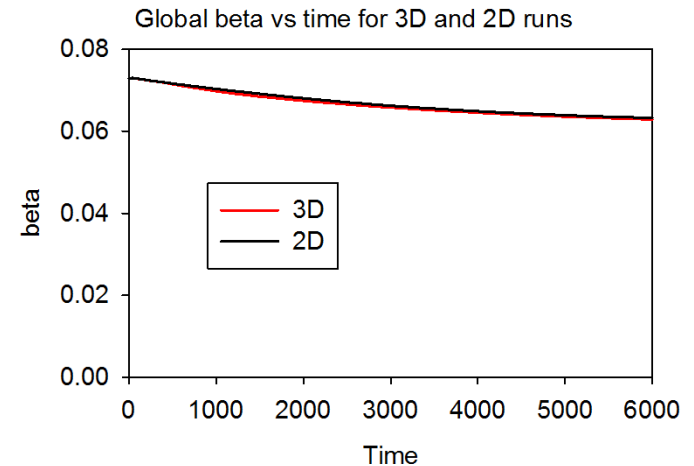
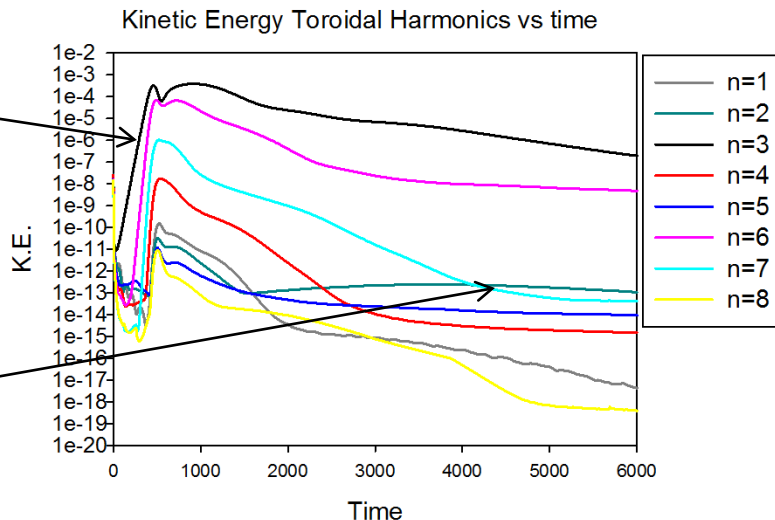
$q_0 = 1.28$

No toroidal rotation



Initially, only $n=3$ is unstable

All modes saturate with K.E. decreasing with time



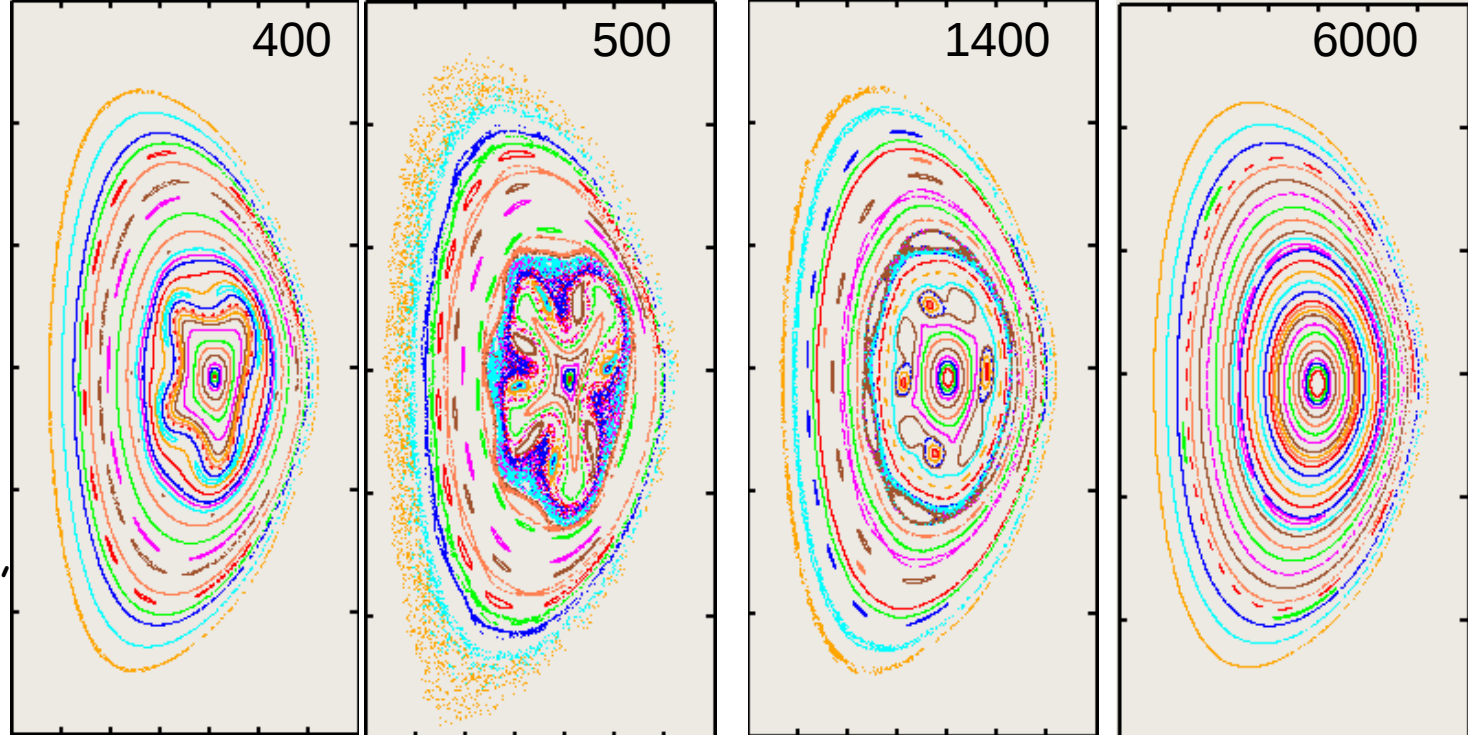
β decreases slightly in time, but no more than in an 2D run with same transport model 4

Soft beta limit

$$q_0 = 1.28$$

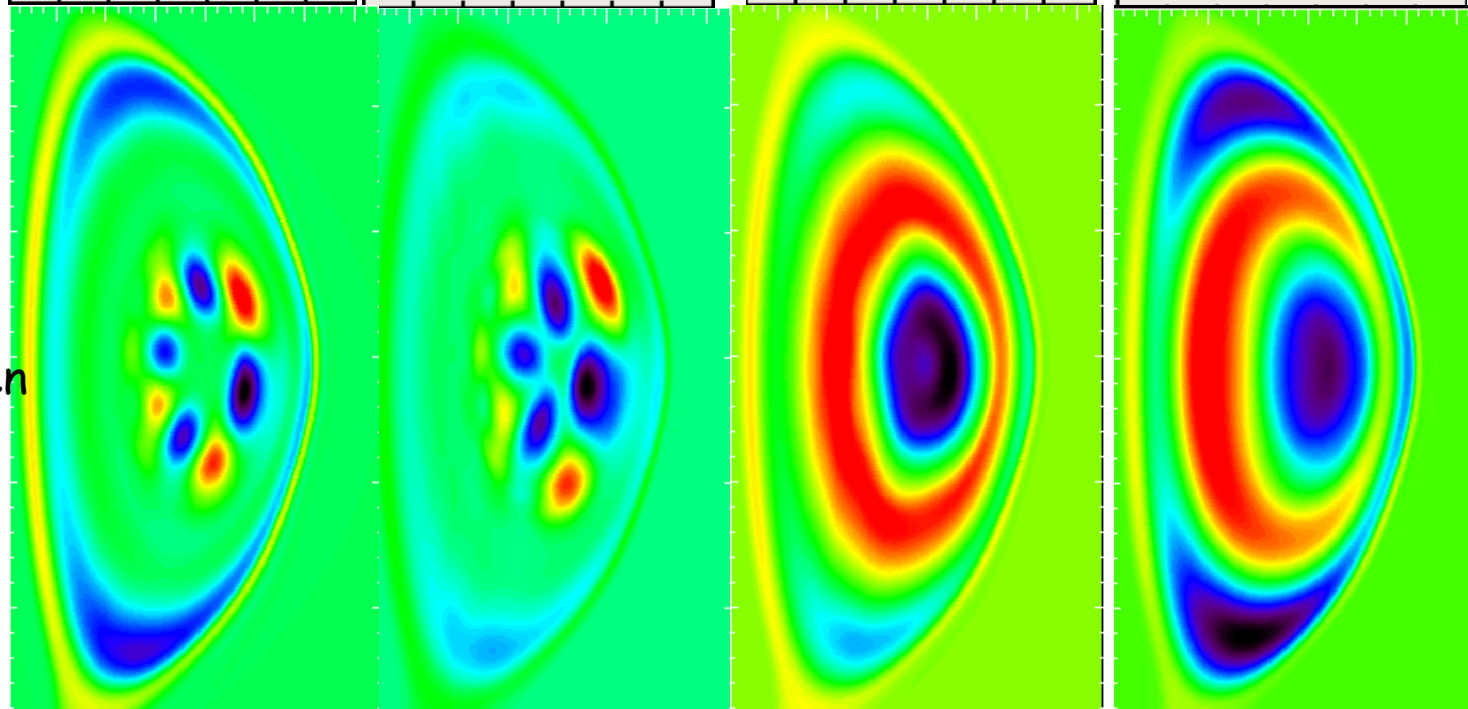
Poincare plots $\rightarrow$

Surfaces deform,
become stochastic,
& completely heal.

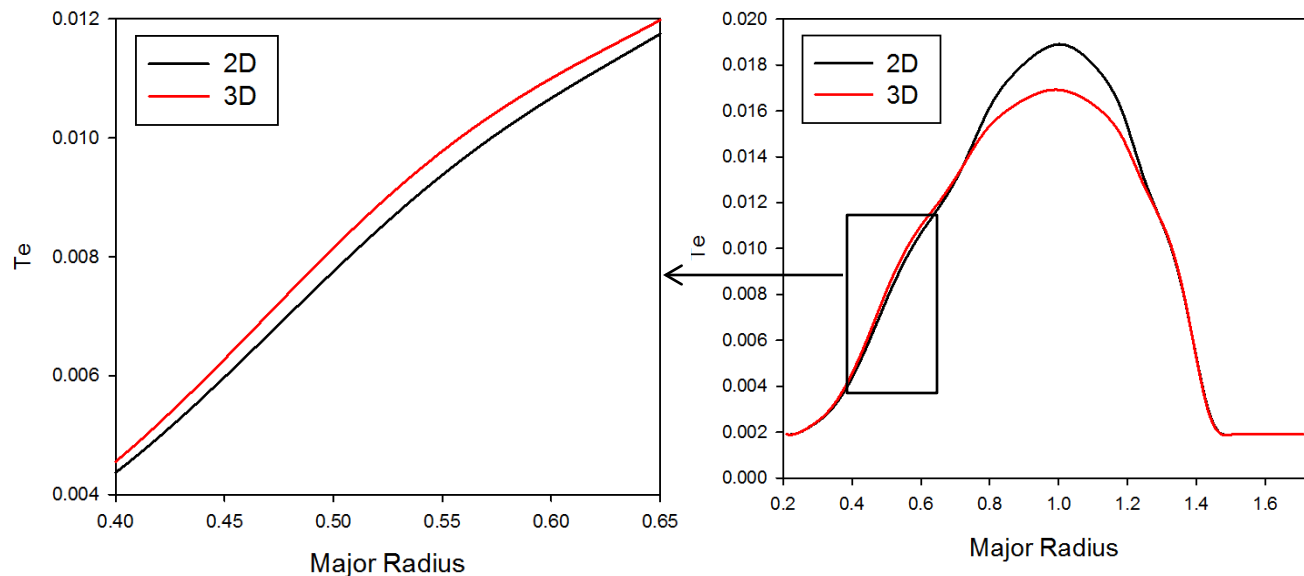


$\Delta T_e \rightarrow$

First pure $n=3$, then
nonlinear, finally
axisymmetric
annulus



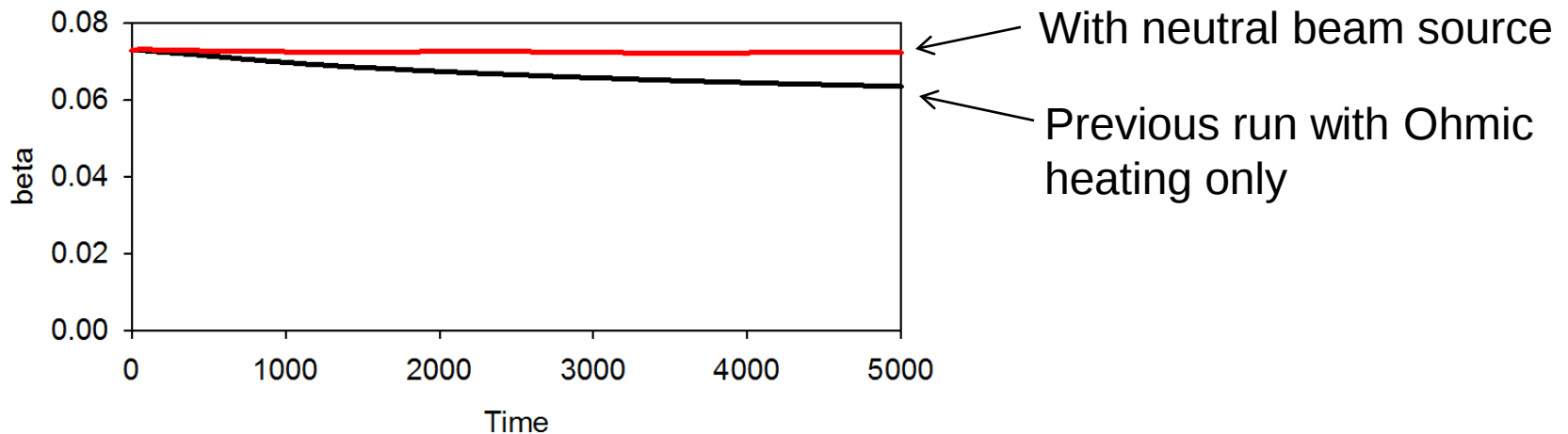
soft beta limit -- continued



- Comparison of 3D run at $t=6000$ with 2D run with identical transport coeffs. shows thermal energy has been redistributed.
- Central T_e differs by 10%, beta differs by only 0.6 %

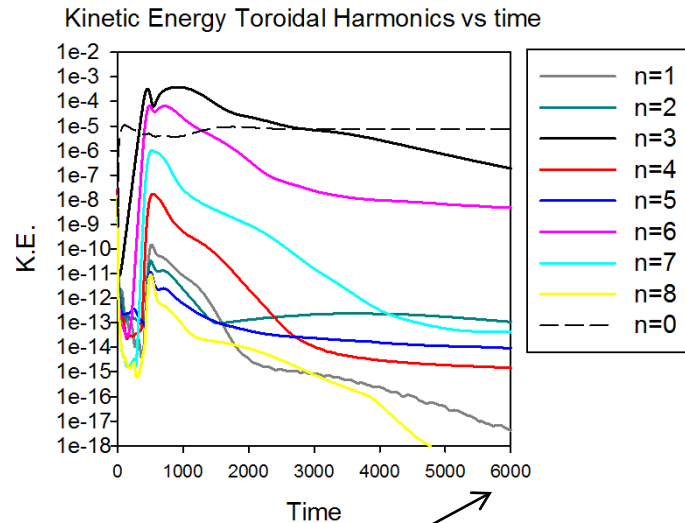
dependence on heating source

- Previous run had beta decreasing in time, even in 2D case, because there was no heating source (except Ohmic).
- Now add *neutral beam source* to keep beta constant and to drive sheared toroidal rotation

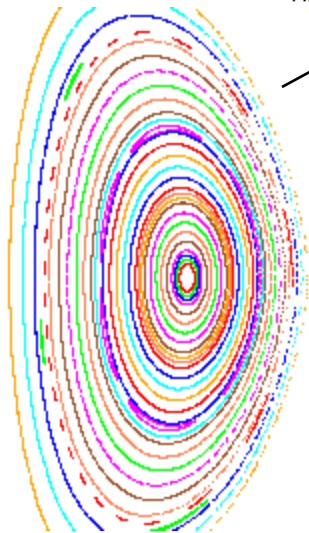
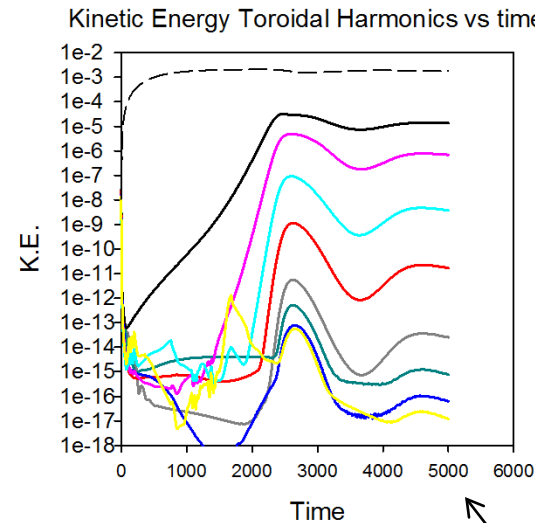


dependence on heating source-cont.

Ohmic heating only

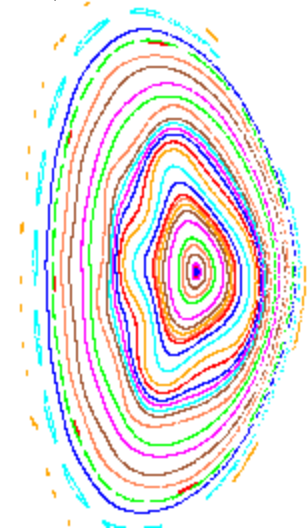


With neutral beam source

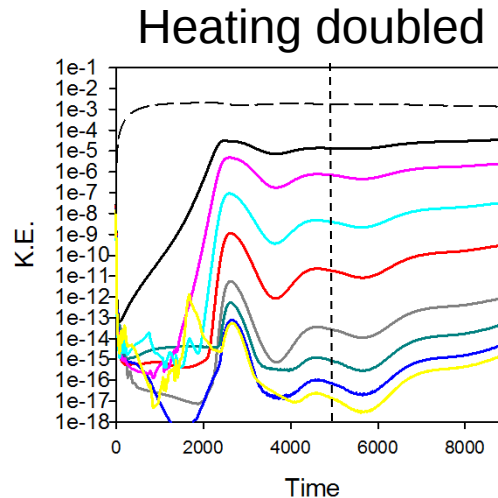
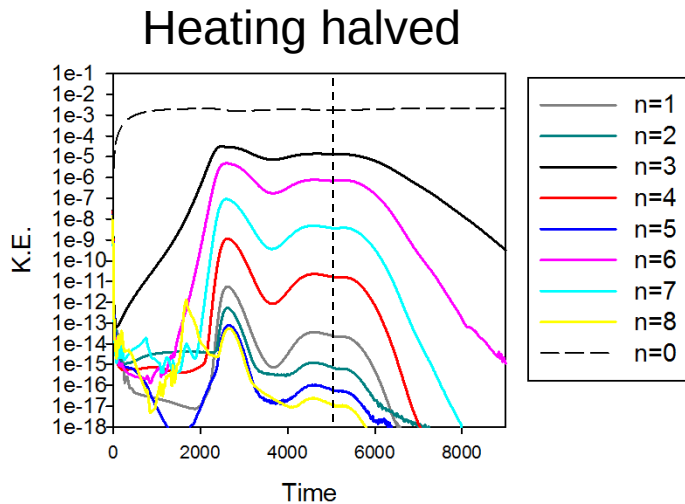
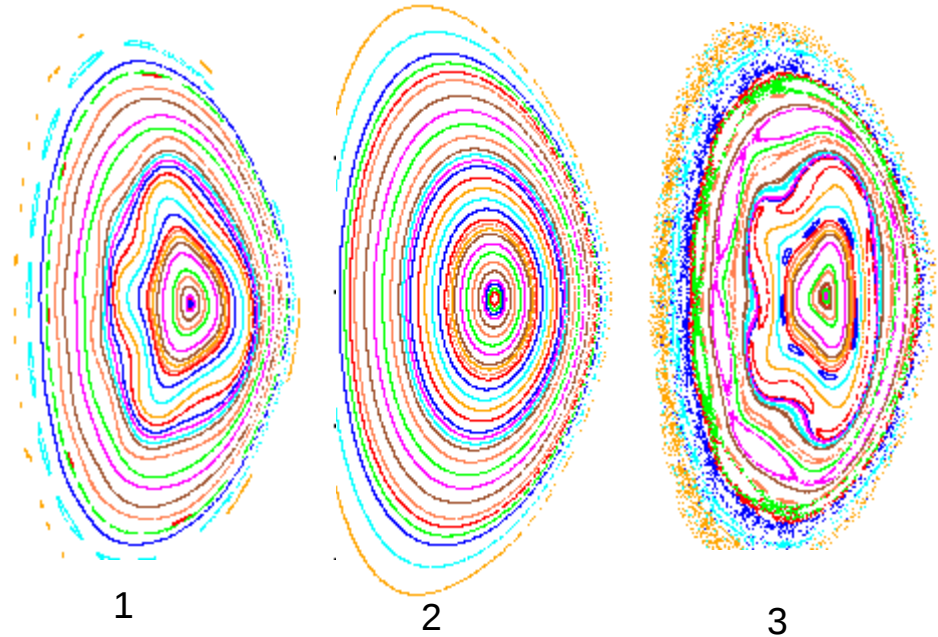
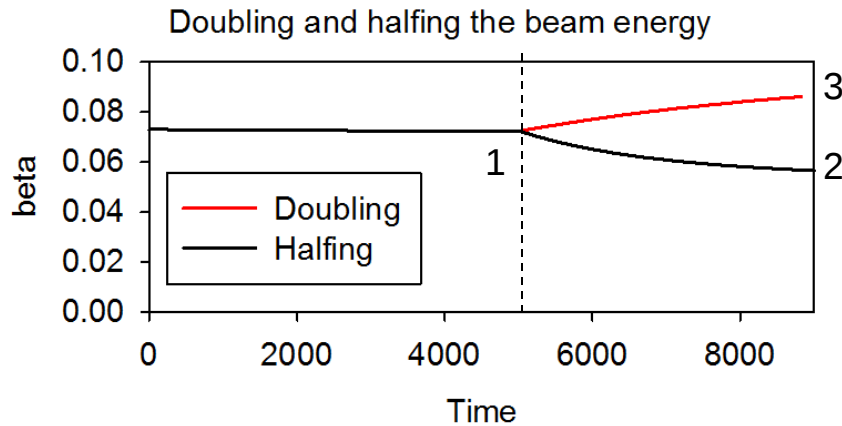


With heating and momentum source:
(constant beta and sheared rotation)

- Initial linear growth of $n=3$ mode much slower
- $n=3$ and higher harmonics do not decay away: surfaces distort



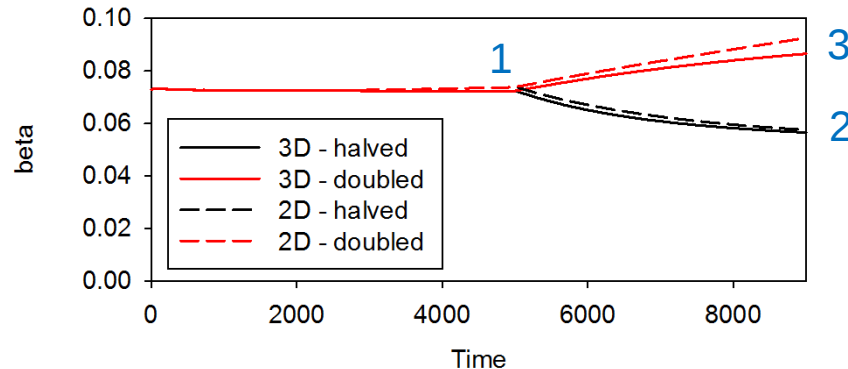
effect of increasing (decreasing) heating



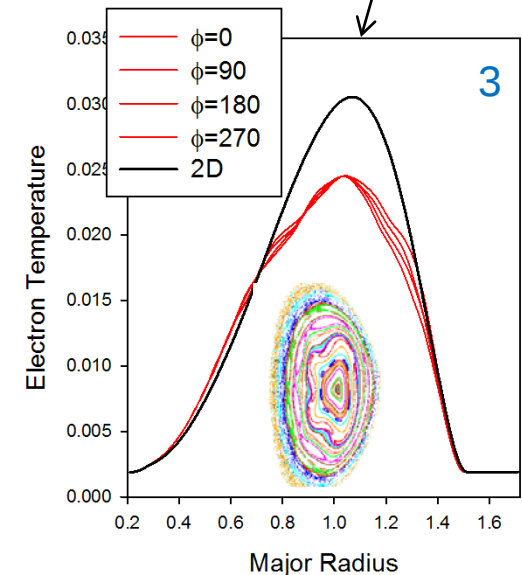
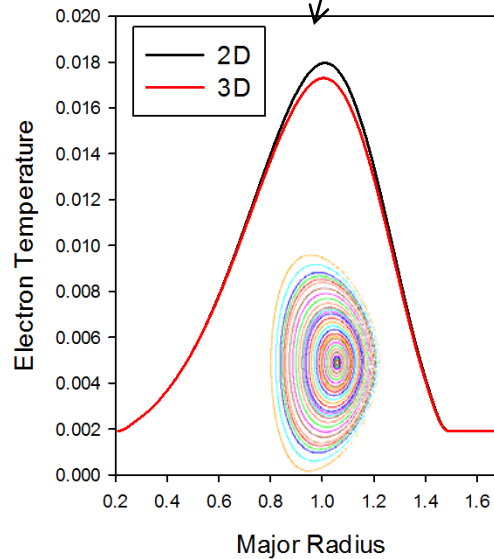
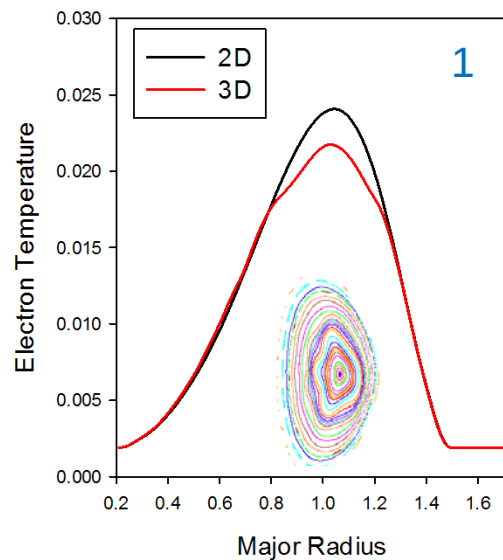
- With heating *reduced*, plasma returns to an axis-symmetric state (2)
- With heating *increased*, surfaces become more distorted, but still exhibits confinement (3)

effect of increasing (decreasing) heating

Comparison of 2D and 3D runs

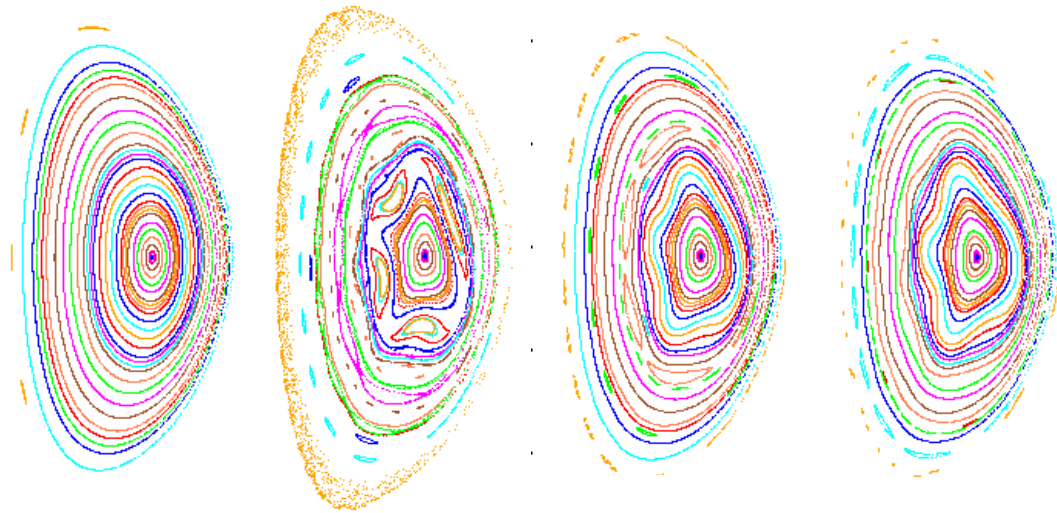


- at low heating power, T_e profiles from 2D and 3D agree
- at higher heating powers, they differ considerably

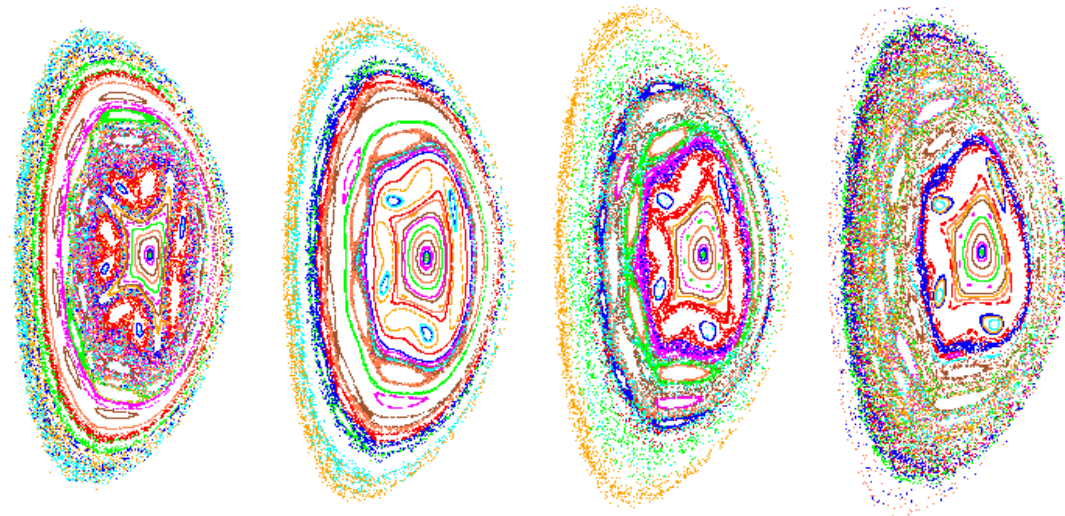


importance of sheared rotation

With
heating and
momentum
input
(sheared
rotation)



With
heating only
(no rotation)



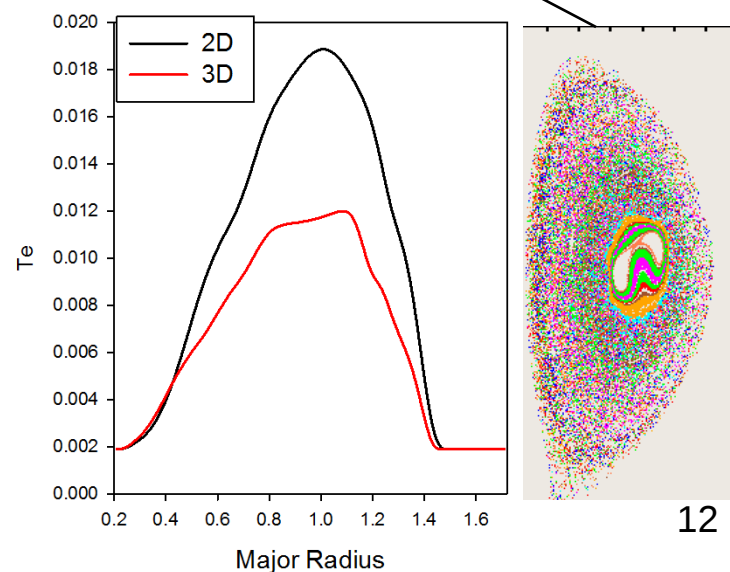
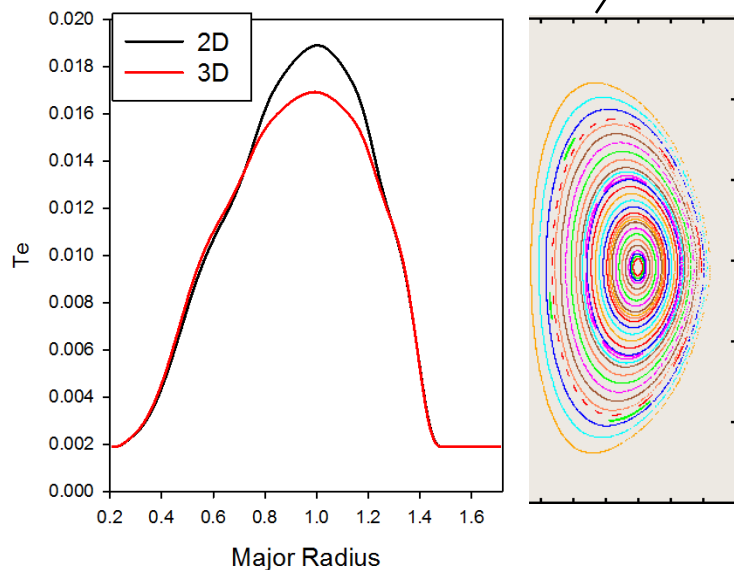
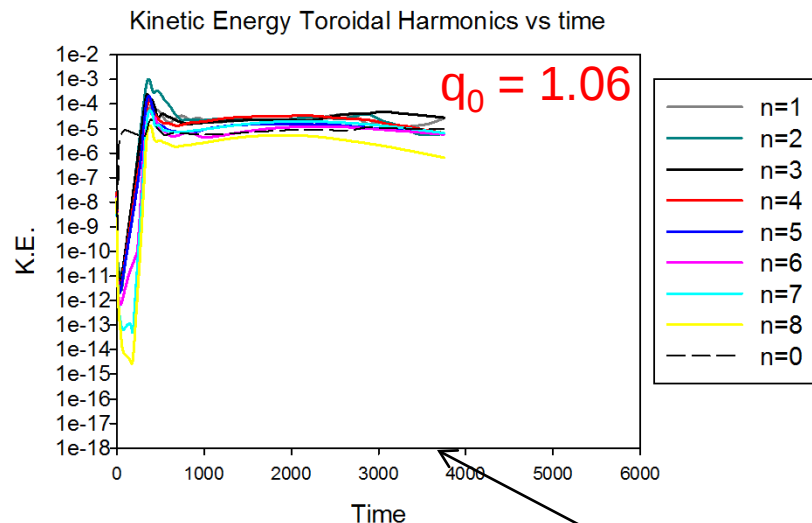
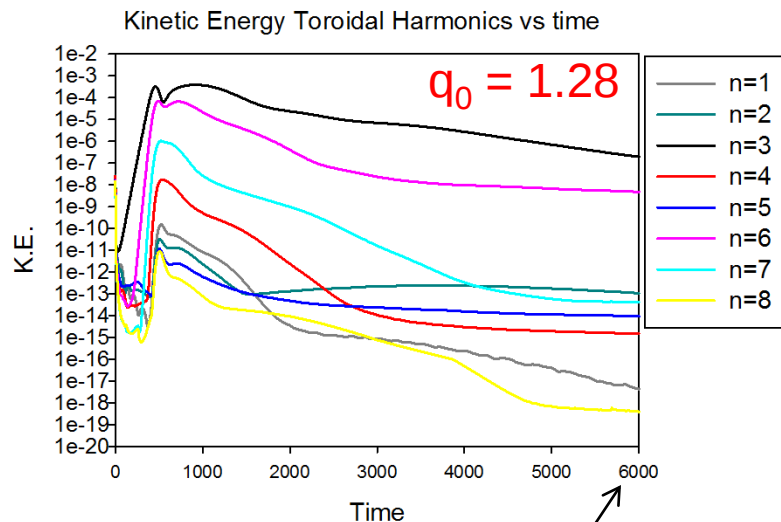
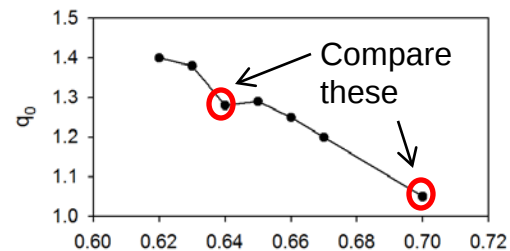
t=2000

t=3000

t=4000

t=5000

equilibrium with lower q_0 shows thermal collapse

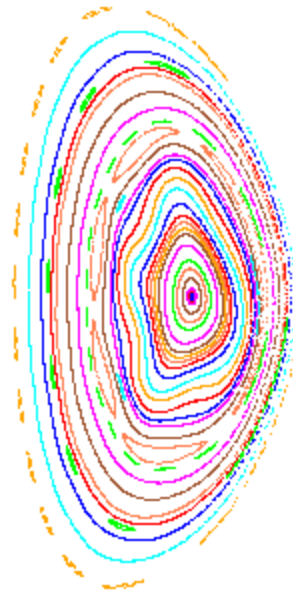
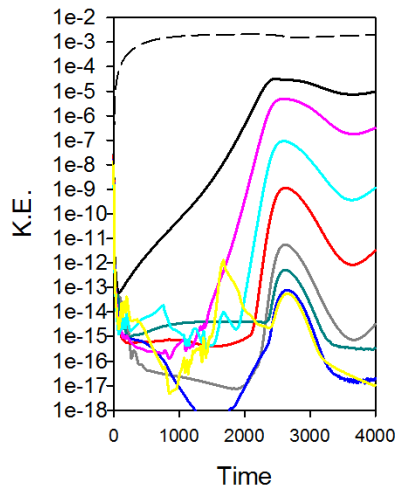


numerical convergence study

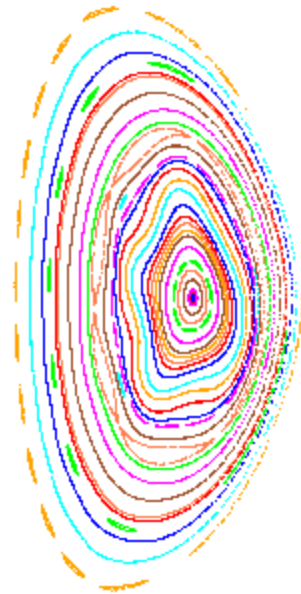
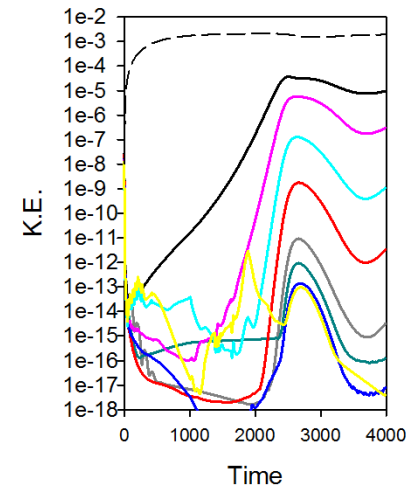
Original constant β run

With double the poloidal zones

Kinetic Energy Toroidal Harmonics vs time



Kinetic Energy Toroidal Harmonics vs time



summary

- M3D- C^1 working nonlinearly in production mode
- Studying nonlinear consequences of exceeding beta limits in NSTX for $q_0 > 1$.
- Mechanism for soft beta limit identified.
- Sheared rotation is stabilizing
- More violent behavior expected as $q_0 \rightarrow 1$
- Convergence studies underway

Exploratory Studies of Tokamak Sawteeth using M3D-C¹

Presented by S. C. Jardin

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Princeton Plasma Physics Laboratory

¹General Atomics

²IPP, Garching

Reconnection Workshop
Max-Planck/Princeton Center
Oct 28-29, 2013
Princeton NJ

Several types of long-time behavior have been found

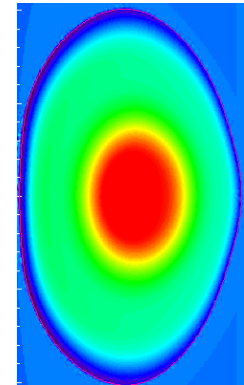
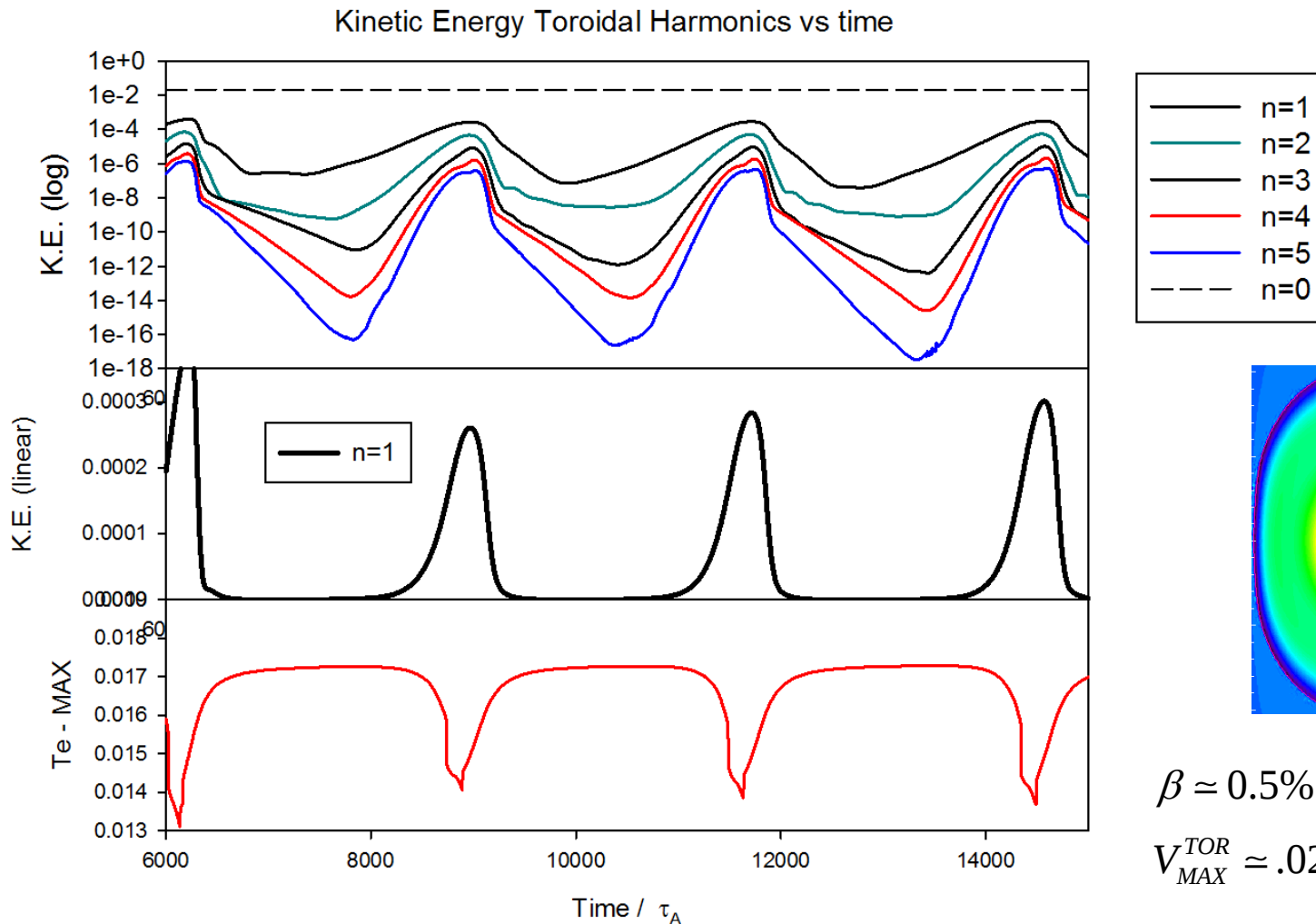
We are using M3D- $\mathbf{C}^1$ to solve the MHD equations to compute the self-consistent long-time (transport timescale) behavior of a tokamak discharge subject to:

- loop voltage (I_p controller)
- density source (n_e controller)
- heating source (NB)
- momentum source (NB)
- shaping fields
- resistivity
- viscosity
- thermal conductivity $\kappa_{\parallel}$ & $\kappa_{\perp}$
- particle diffusivity
- ion-skin depth c/ω_{pi}

Depending on these physical parameters, we find two types of long-term behavior of the system:

- periodic oscillations (sawteeth)
- stationary helical states with flow

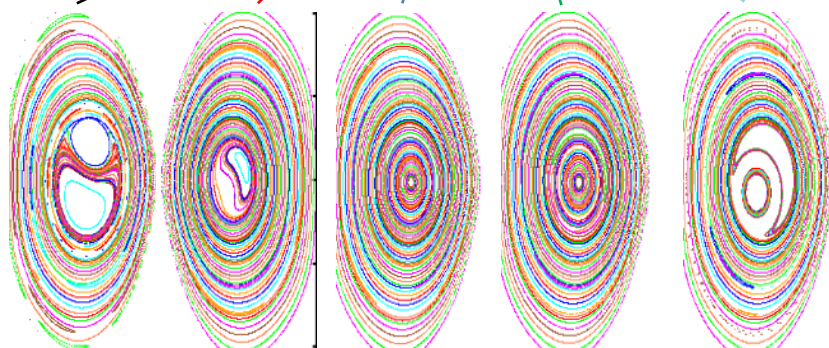
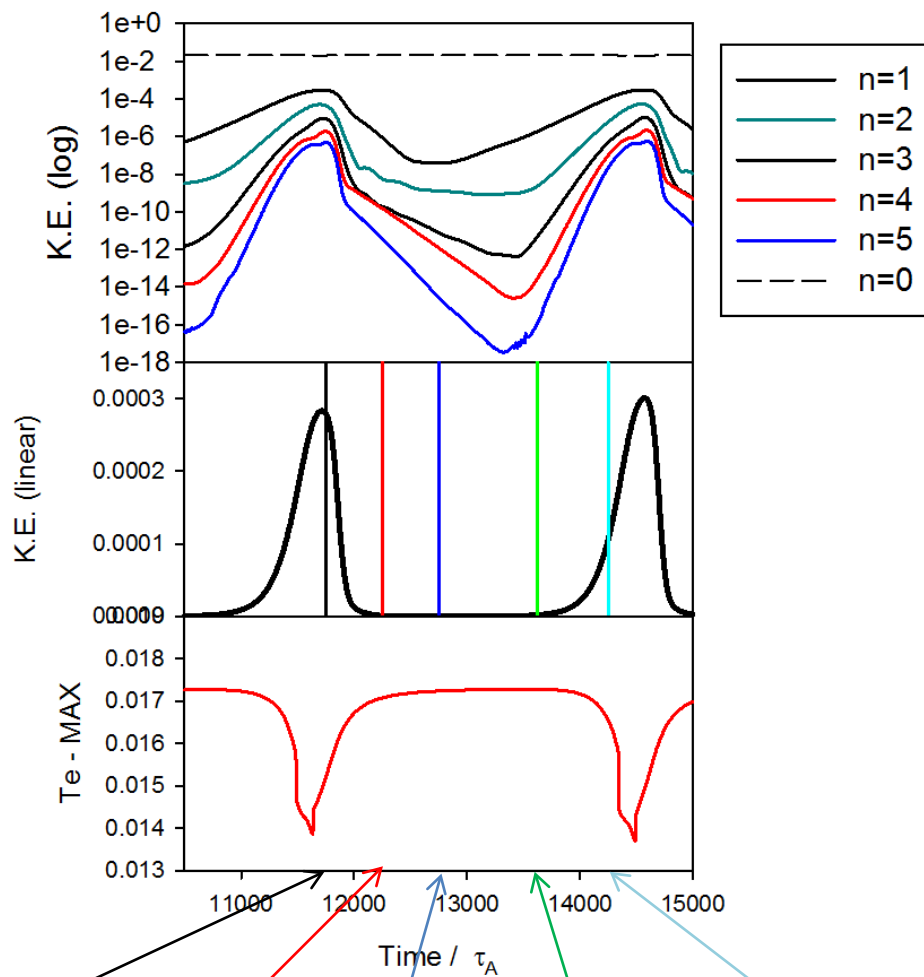
Canonical periodic oscillating discharge



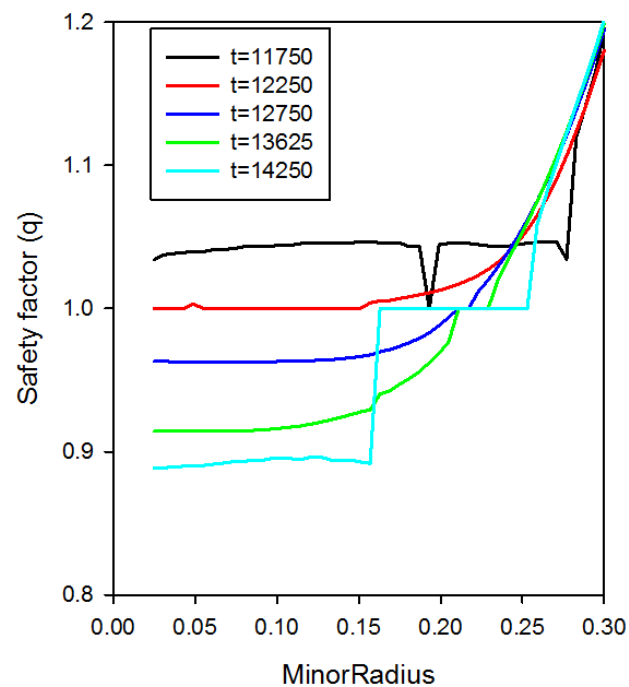
$$\beta \approx 0.5\% \quad q_{edge} \approx 5.5$$

$$V_{MAX}^{TOR} \approx .02 V_{Alfven} \quad S \approx 10^6$$

Kinetic Energy Toroidal Harmonics vs time

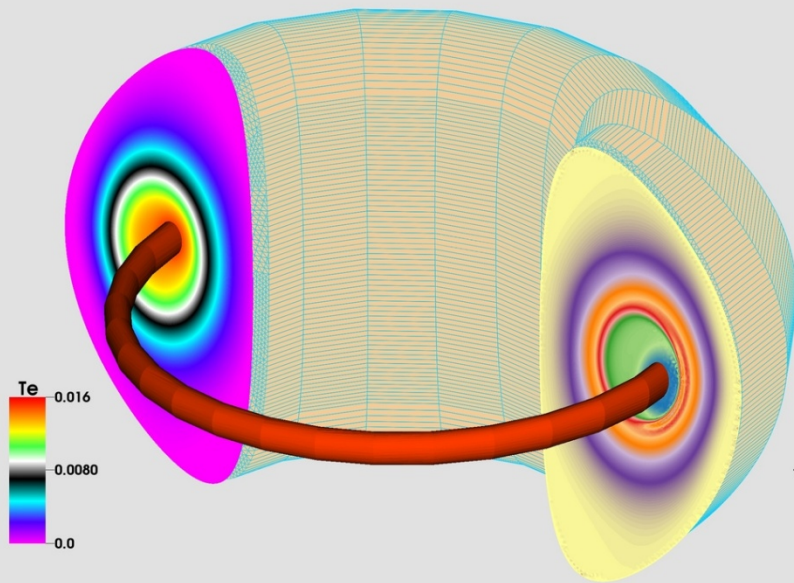


q-profile evolution
during single
sawtooth cycle

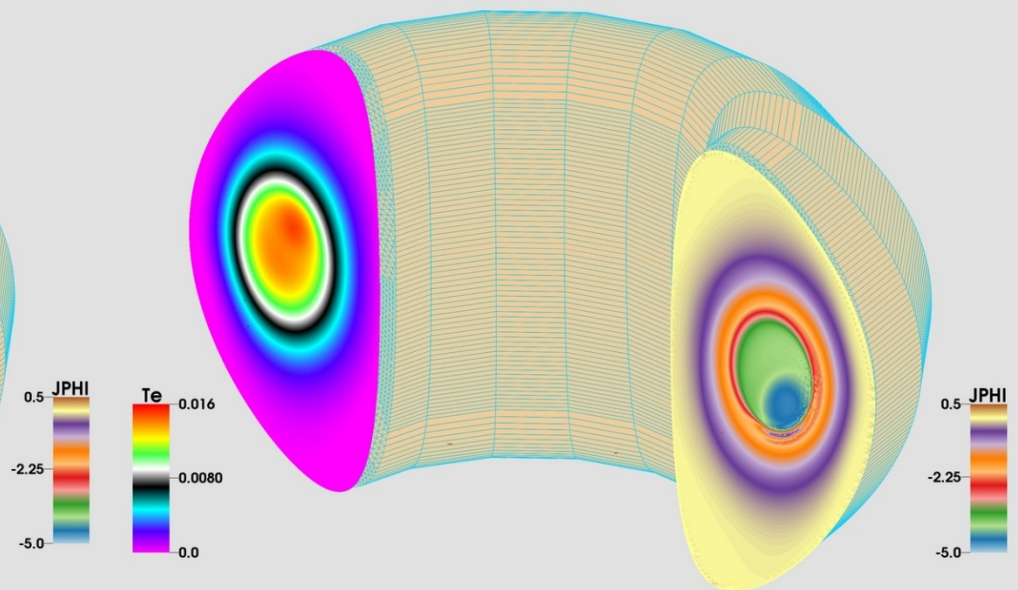


Sawtooth Movie

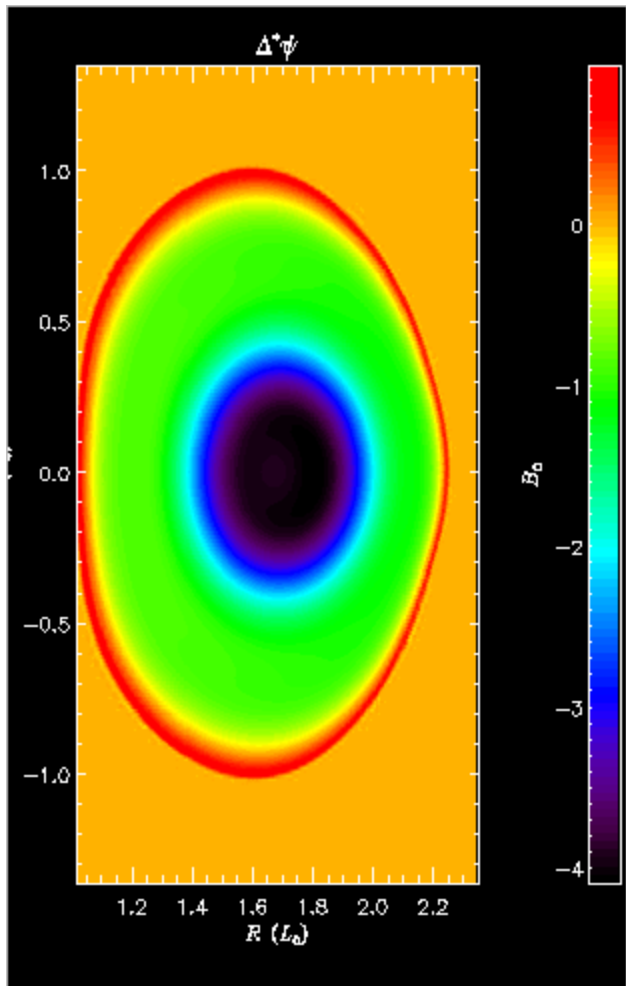
DB: Job45P
Cycle: 571 Time:14375



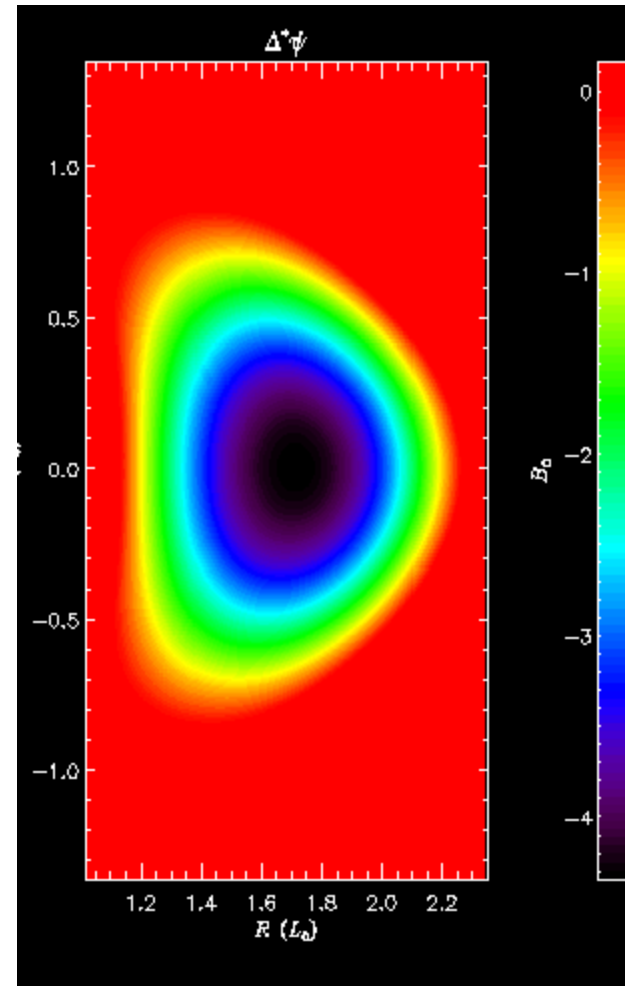
DB: Job45P
Cycle: 582 Time:14430



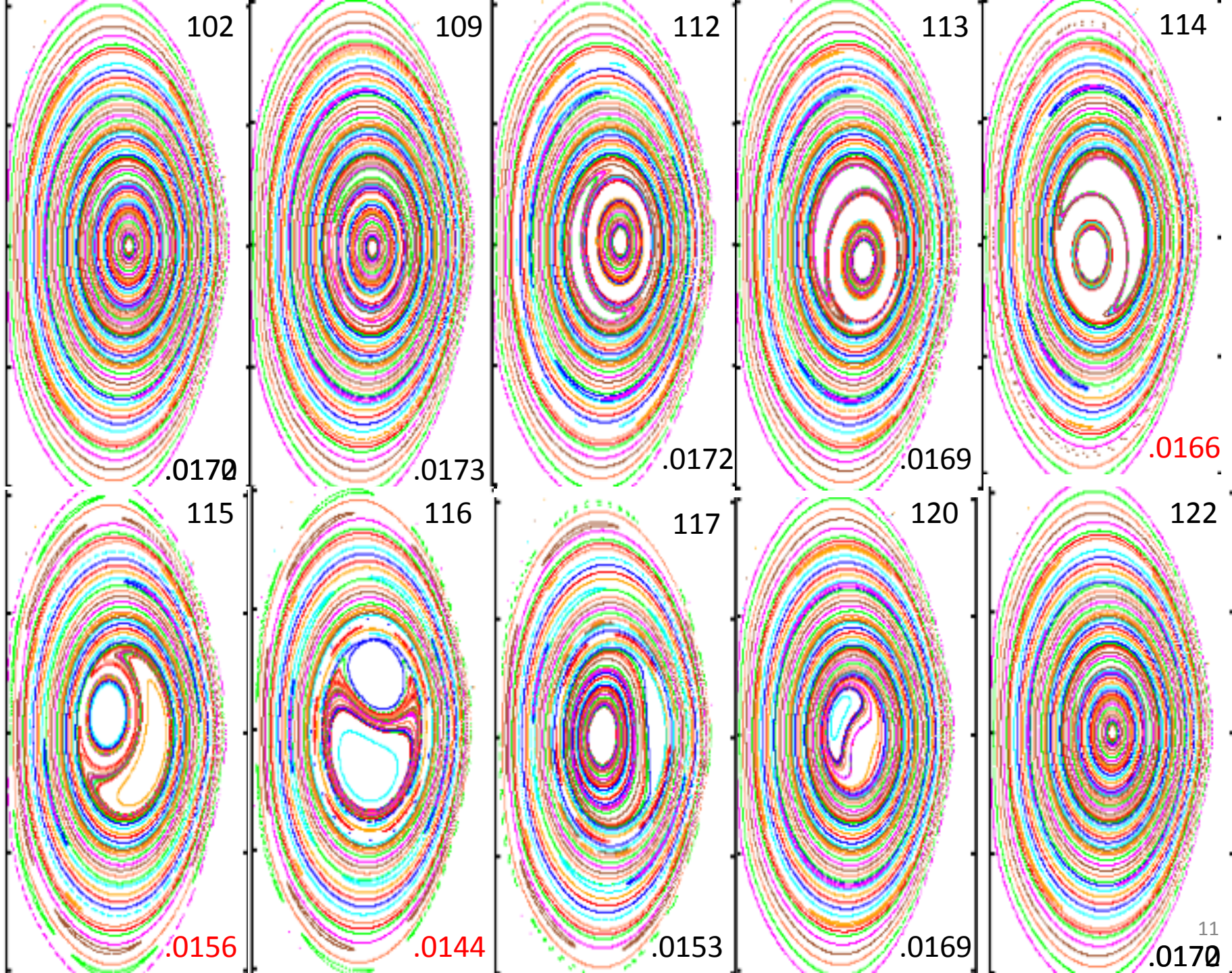
Differences in sawtooth behavior for bean-shaped and elliptical-shaped plasmas has been well documented experimentally (Lazarus, Tobias, ...)

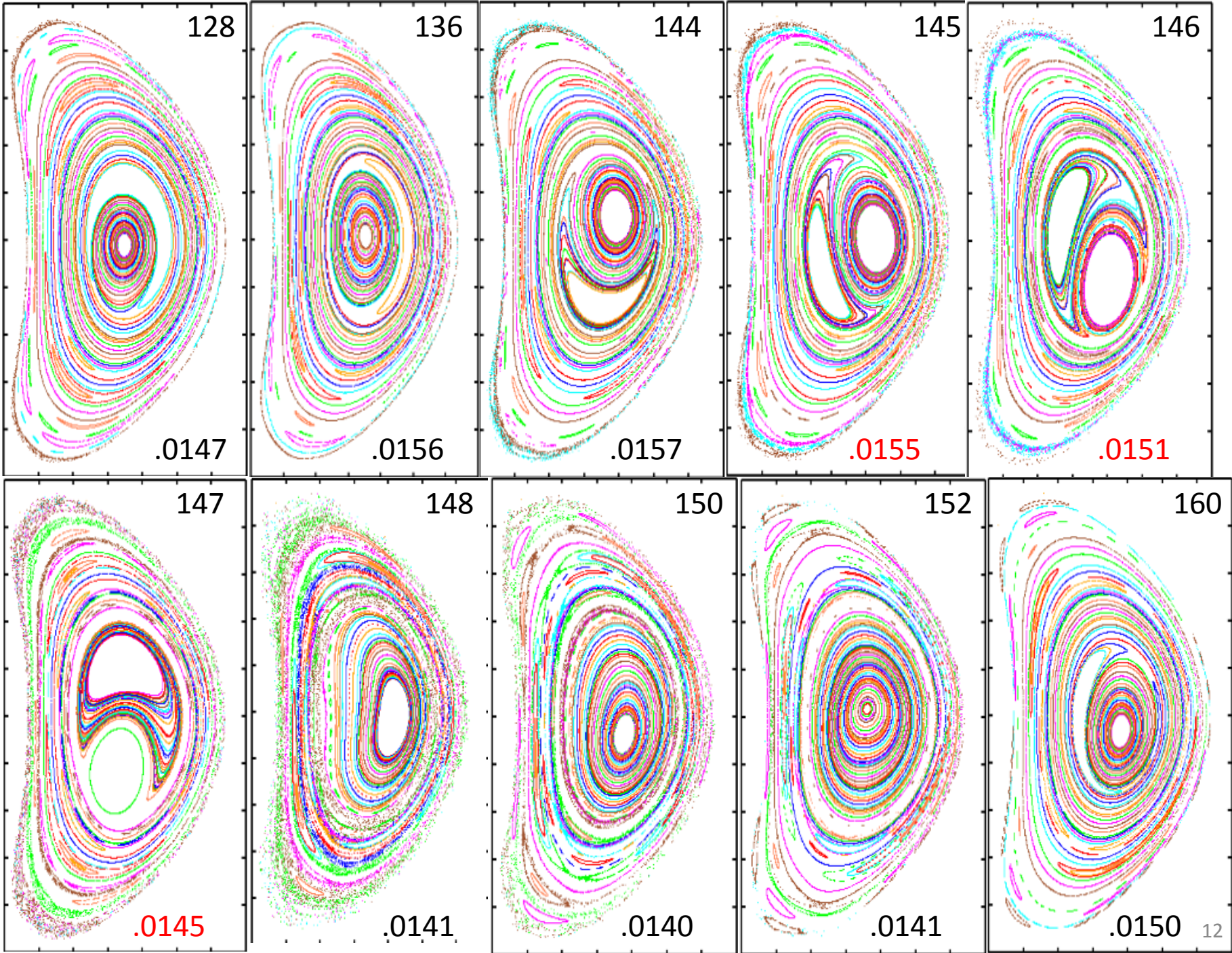


DIII-D shot 118164



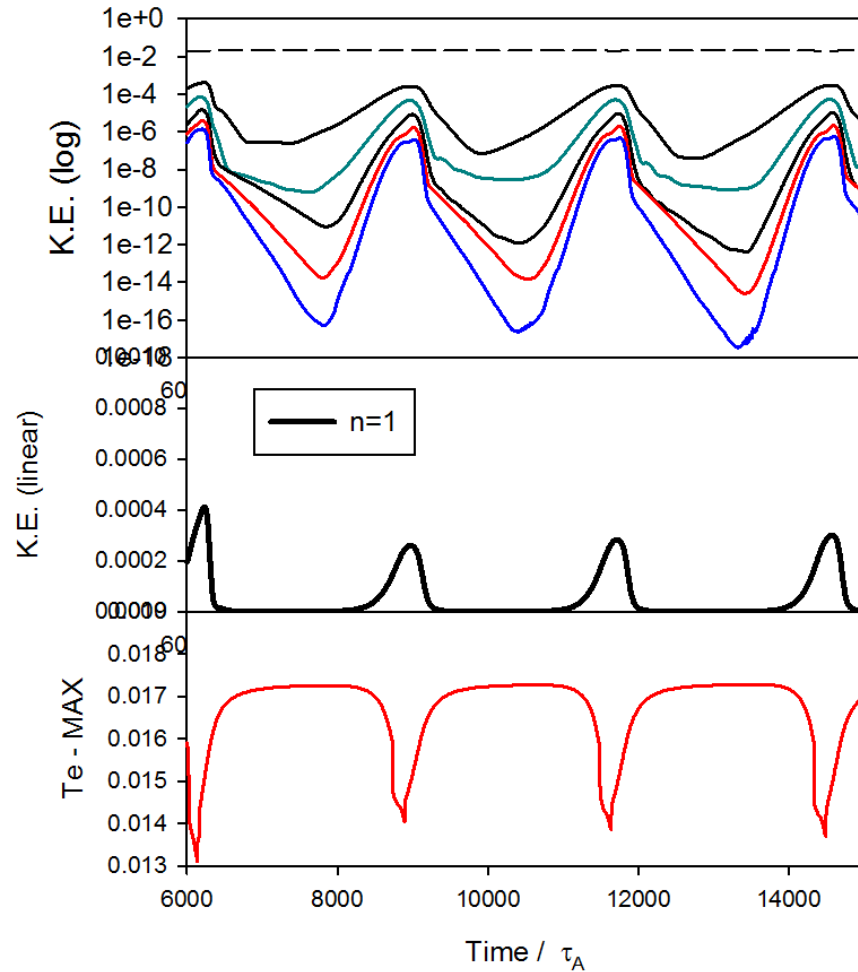
DIII-D shot 118162



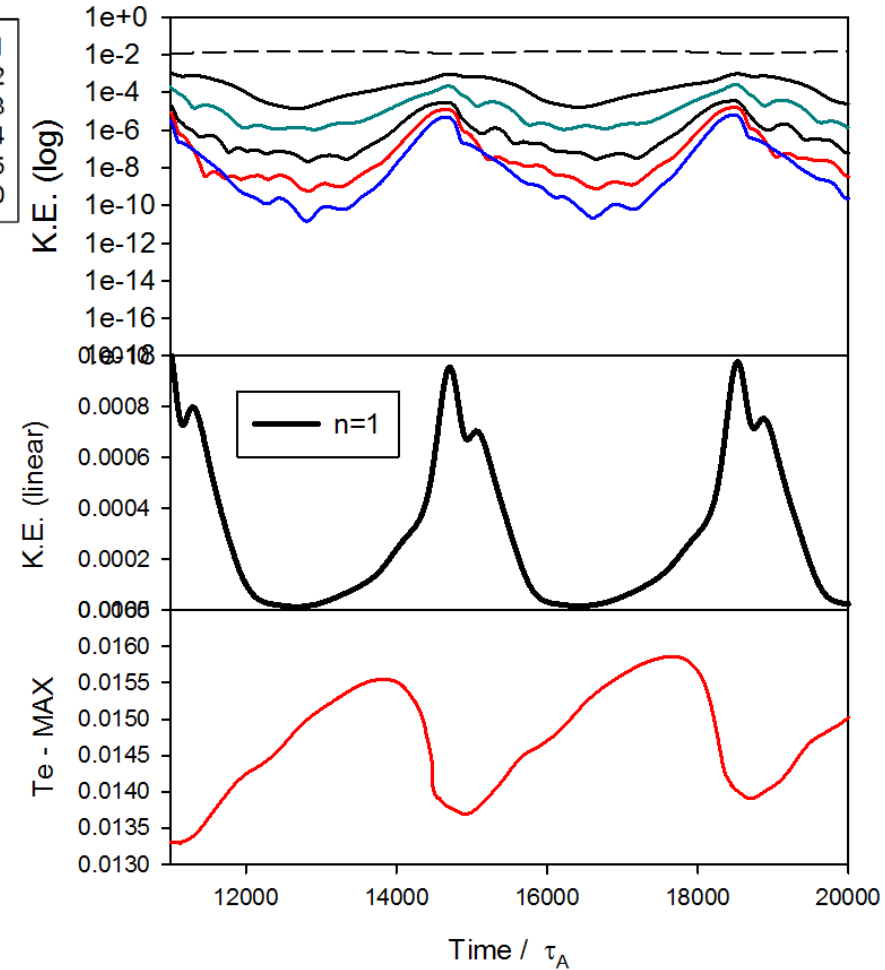


Comparison of Ellipse and Bean

Ellipse K.E. Toroidal Harmonics vs time

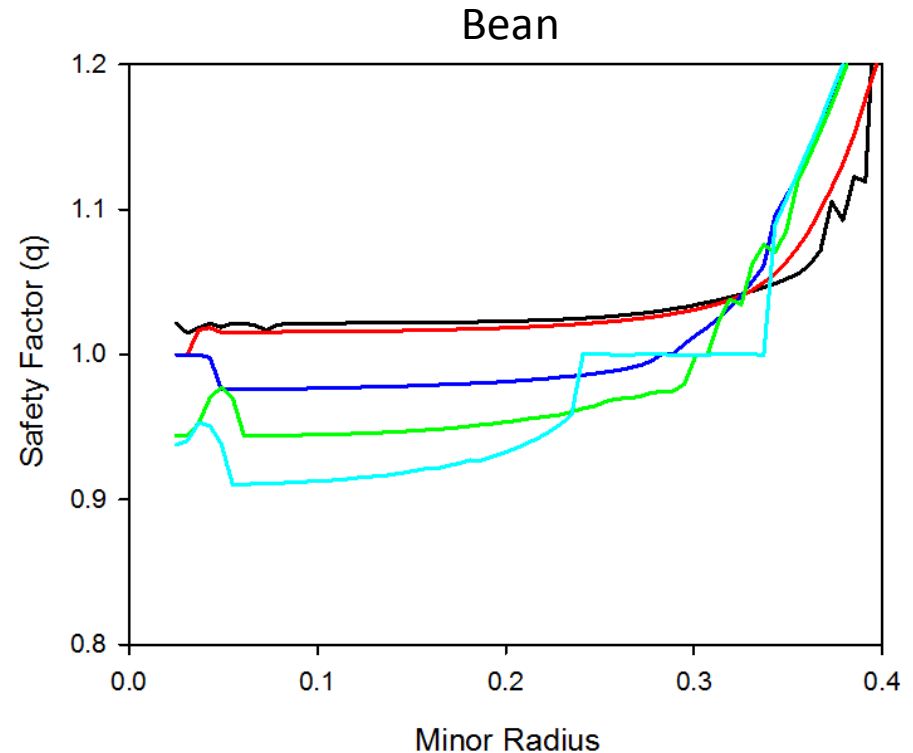
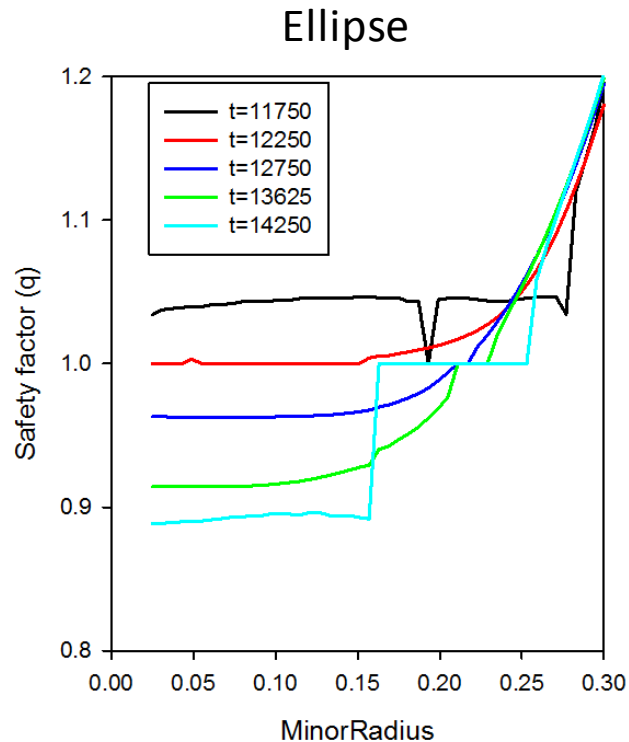


Bean K.E. Toroidal Harmonics vs time



Bean has shorter period, larger amplitude $n=1$, less decay in energy harmonics between ST.

Comparison of Ellipse and Bean

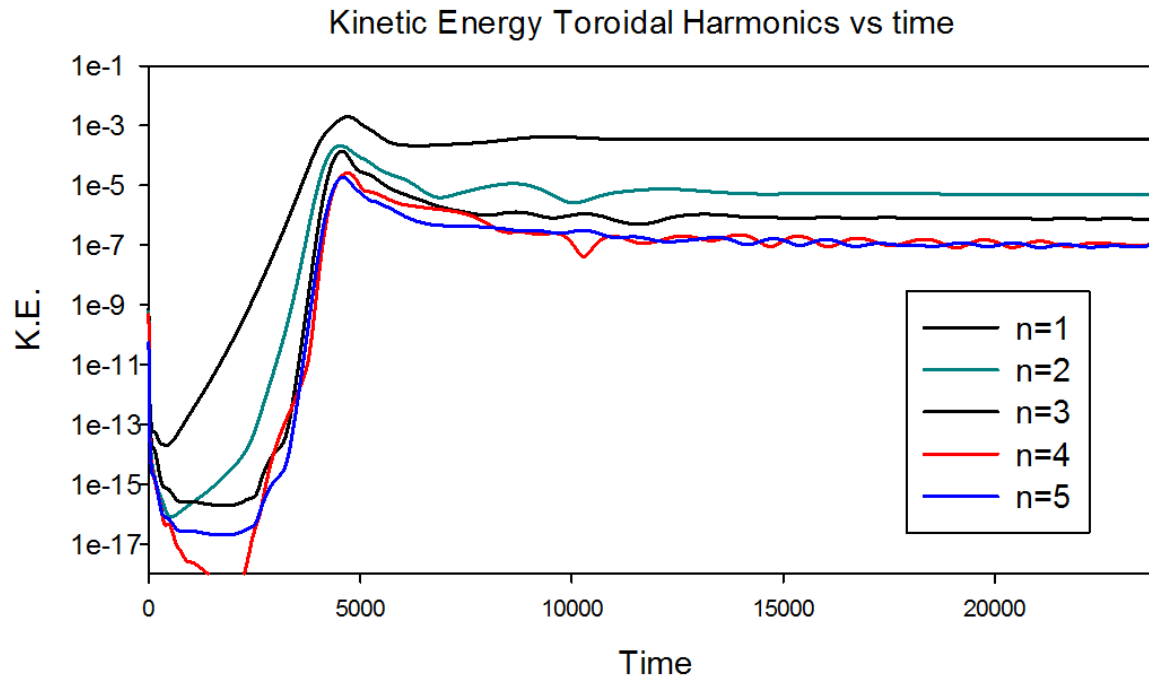


In Bean:

- $q=1$ surface extends to a larger radius
- $q(0)$ does not vary as much during sawtooth cycle

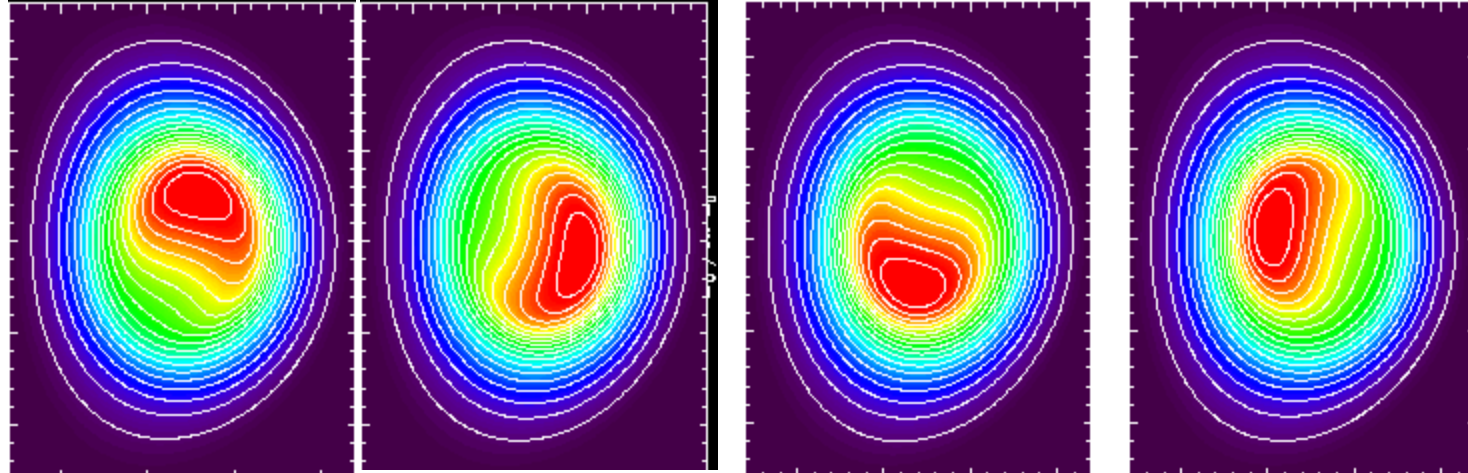
Stationary Helical State with Flow

In some cases, the sawteeth die out, and the system becomes stationary on all timescales.



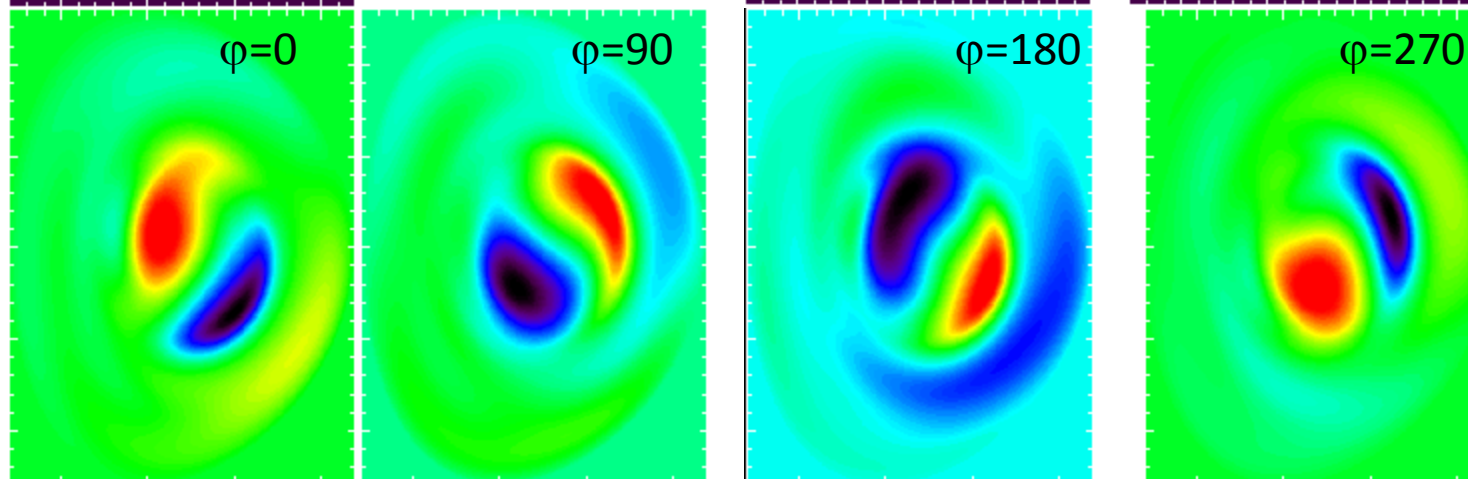
T=24000

Electron
Temperature



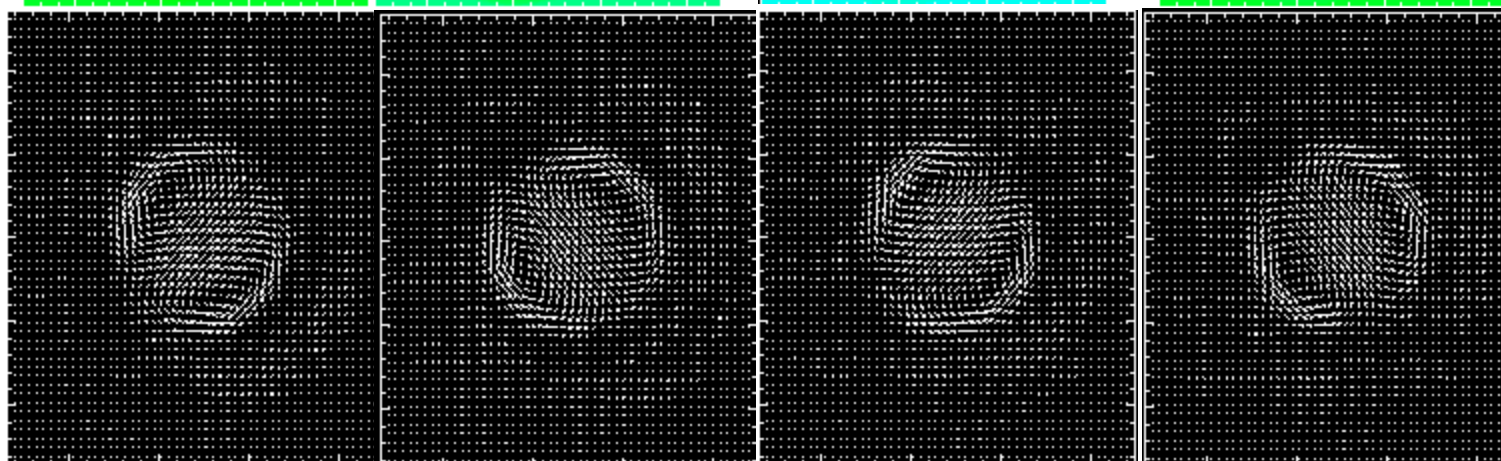
Toroidal
Velocity

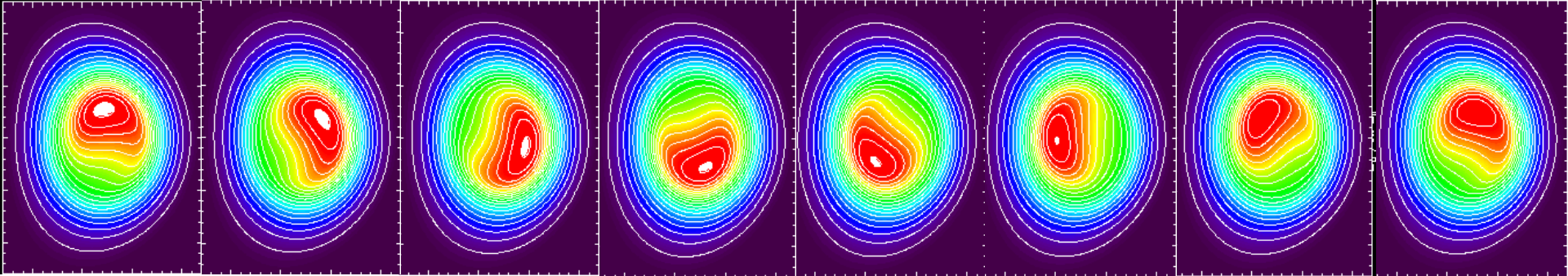
(Max= .004)



Poloidal
Velocity

max=0.00064





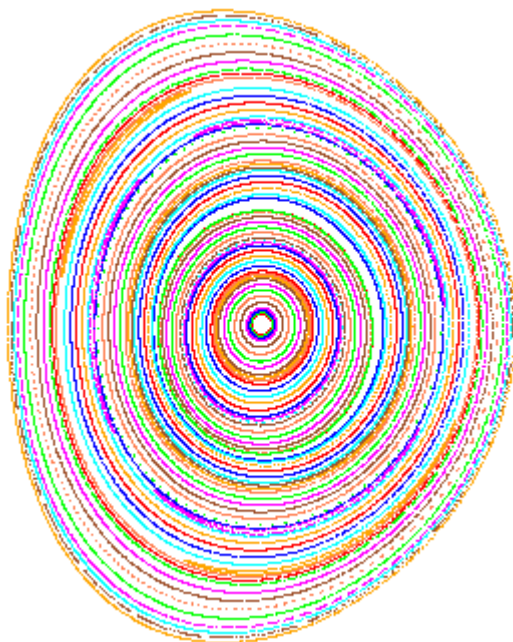
T=17000

T=20000

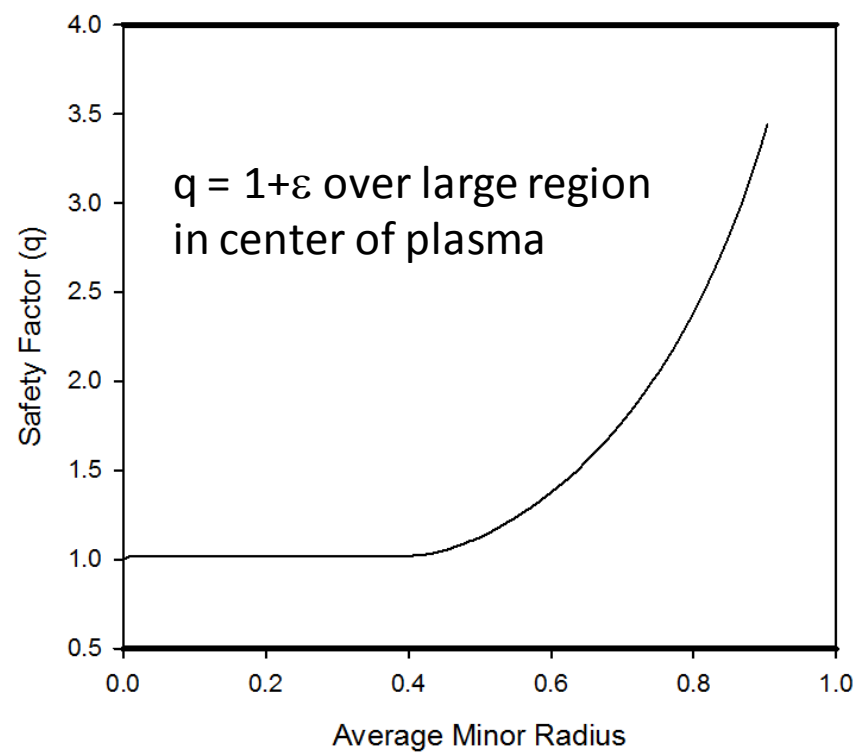
T=24000

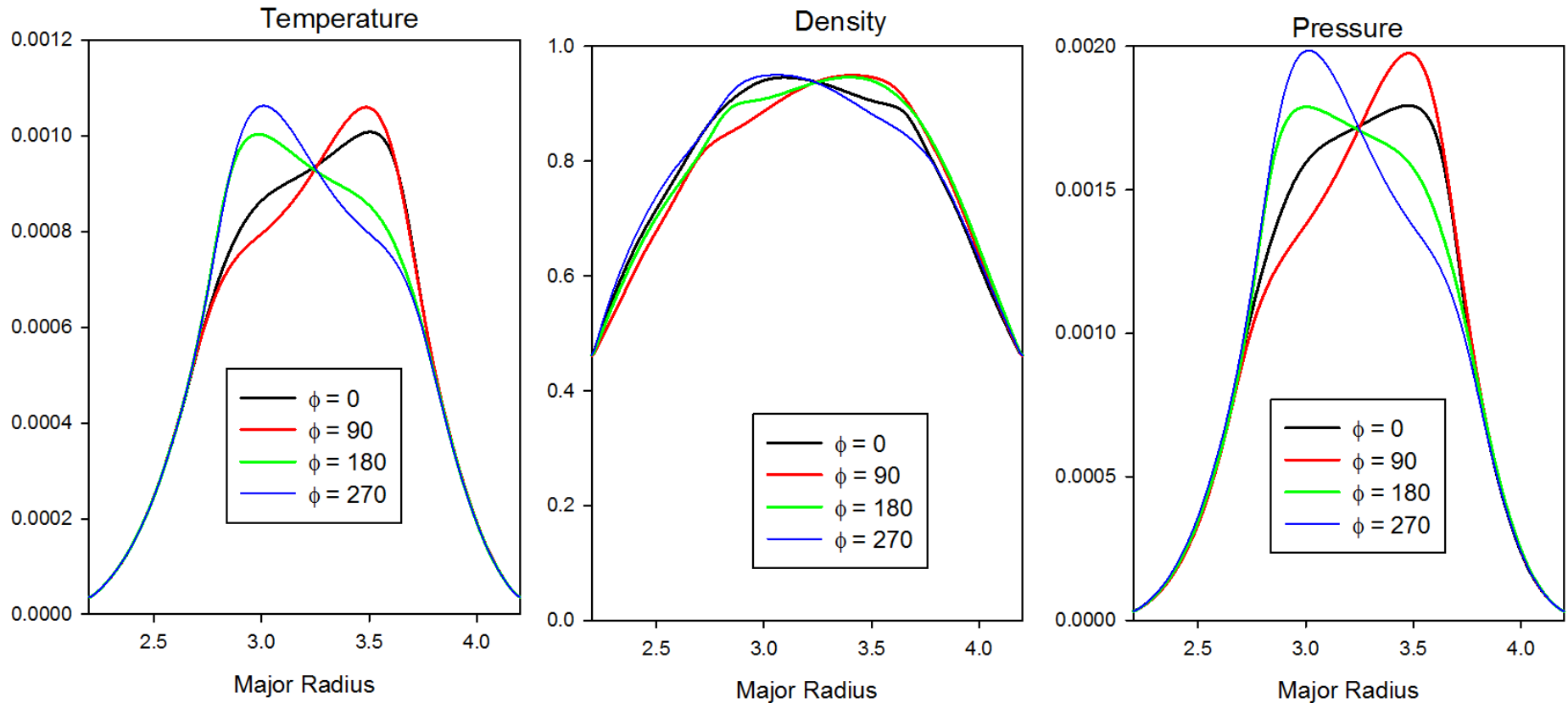
Te at same poloidal location as time evolves $\rightarrow$

Mode is
rotating
every
 $7000 \tau_A$



Surfaces are almost
axisymmetric with just
a small $m=1$ island





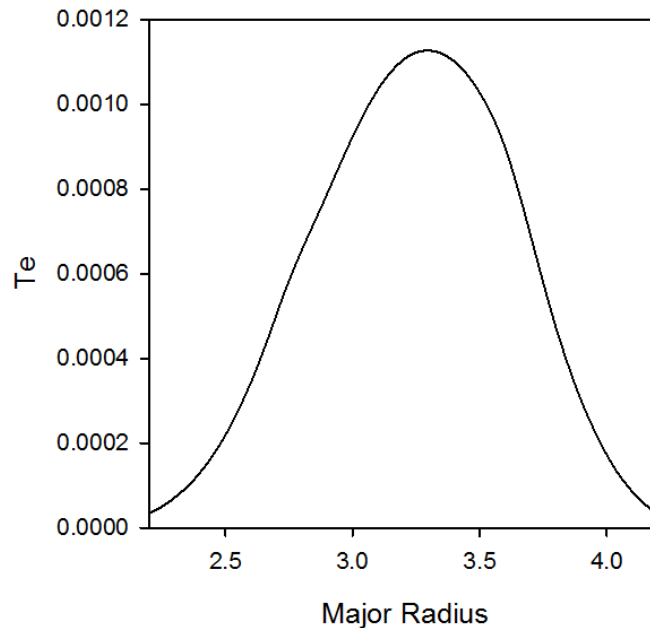
Interior to the region where $q=1+\varepsilon$, p , n , and T profiles are not constant on the magnetic surfaces. i.e., $p \neq p(\psi)$

Exterior to the $q=1$ surface, they are constant on surfaces $p=p(\psi)$

Terms in temperature equation

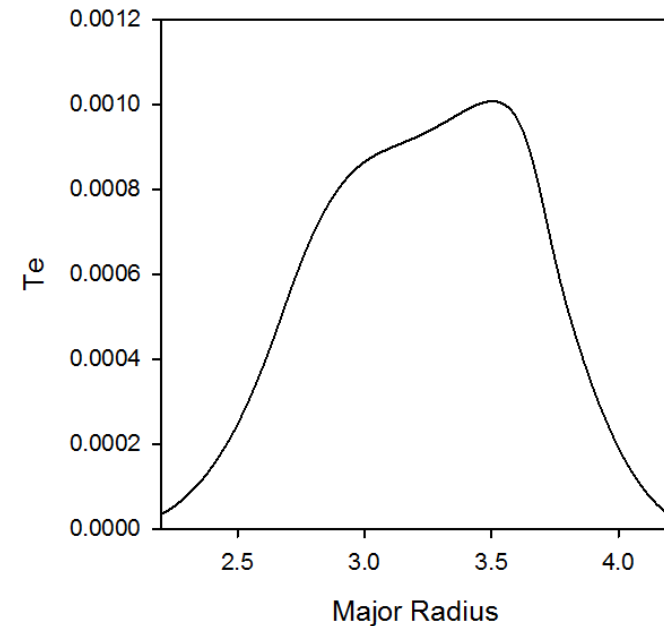
$$u \cdot \nabla T_e = \eta J^2 + \nabla \cdot \kappa_{\perp} \nabla T_e$$

Axisymmetric state (before)



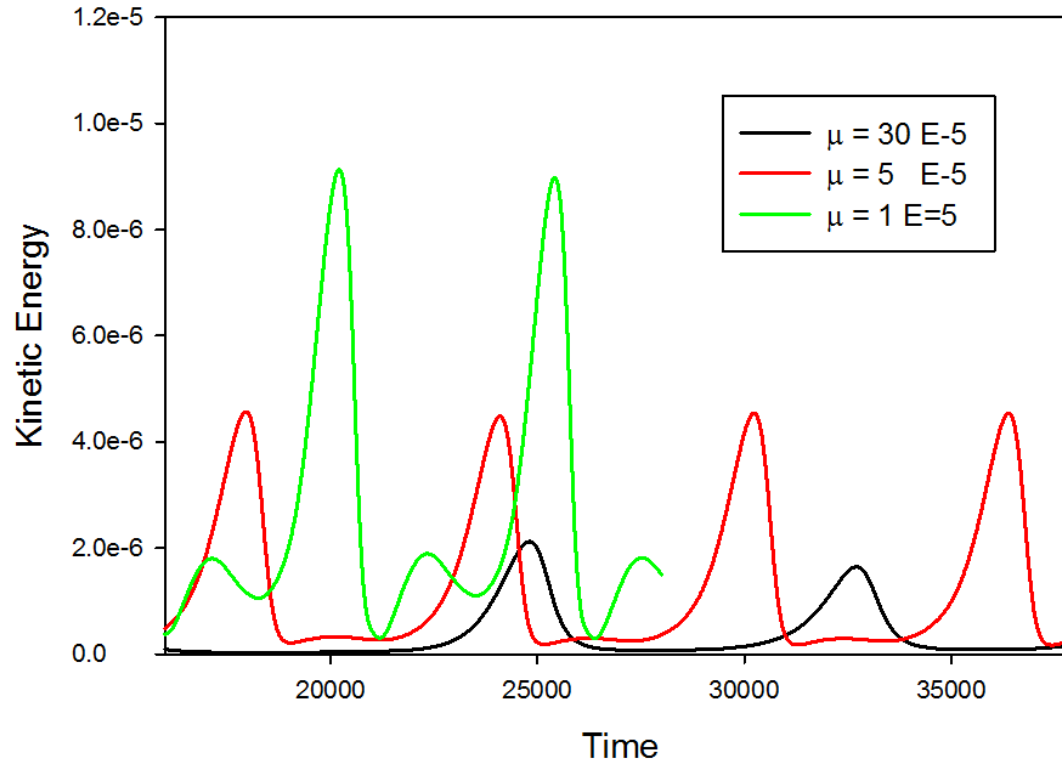
$$1.0 \times 10^{-8} \approx 3.7 \times 10^{-7} - 3.7 \times 10^{-7}$$

Non-axisymmetric state (after)



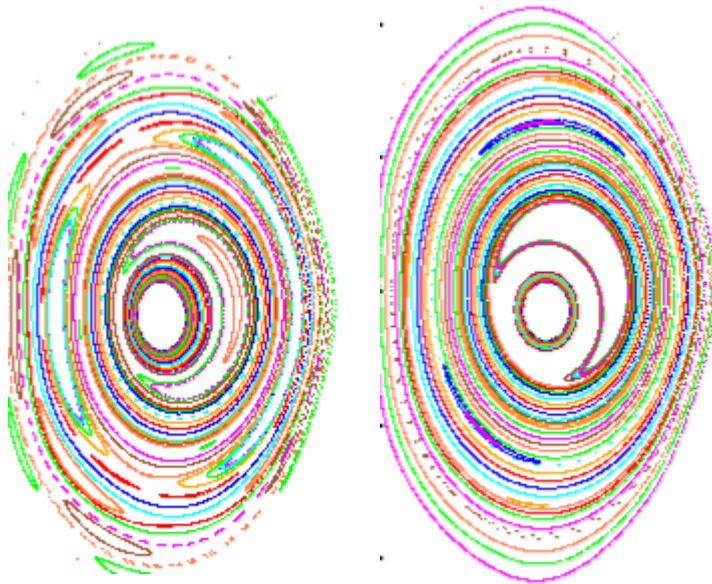
$$3.4 \times 10^{-7} \approx 3.7 \times 10^{-7} - 3.0 \times 10^{-8}$$

Viscosity Scan



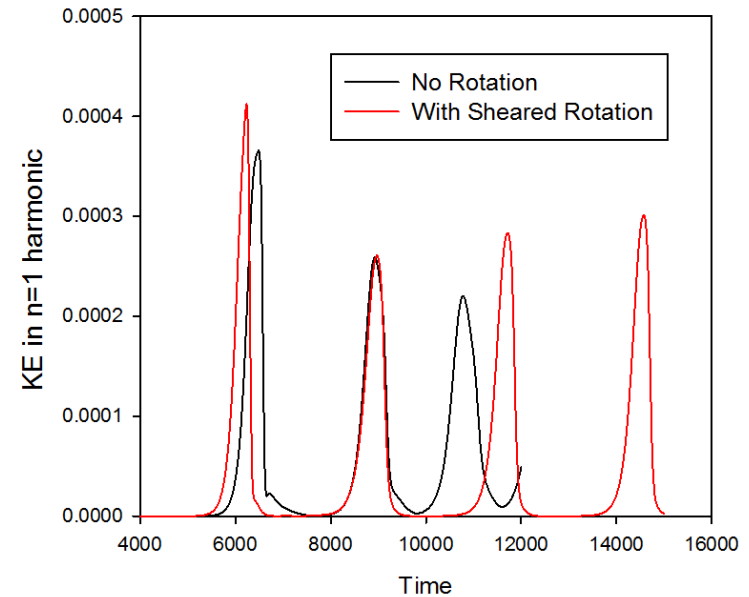
- Max KE amplitude increases with μ^{-1} (to a point)
- Period increases with μ
- Lowest μ can have more complex behavior

Effect of Sheared Rotation



Without rotation

With sheared rotation



Without sheared rotation in ellipse:

- sawteeth tend to die out
- large magnetic islands form

Two-Fluid Effects

$$n\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V}\right) + \nabla p = \mathbf{J} \times \mathbf{B} - \nabla \cdot \boldsymbol{\Pi}_{GV} + \mu \nabla^2 \mathbf{V}$$

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{J} + d_i \left(\mathbf{J} \times \mathbf{B} - \nabla p_e \right)$$

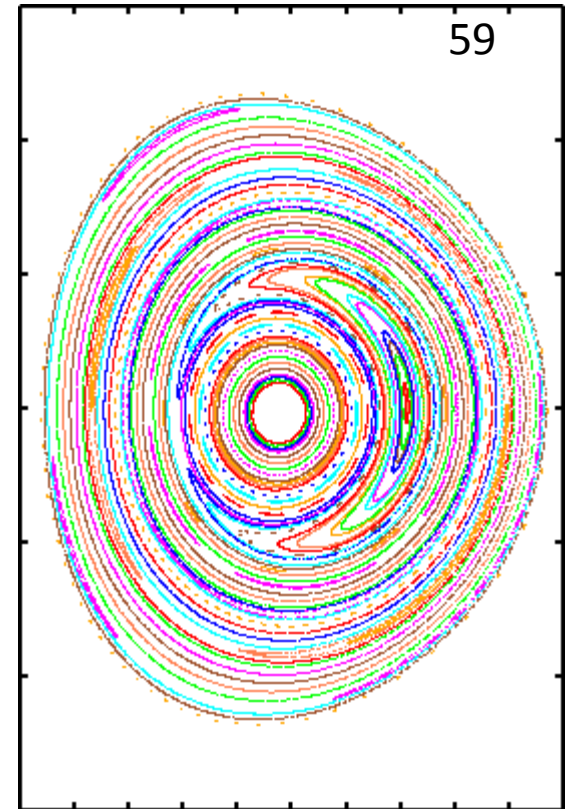
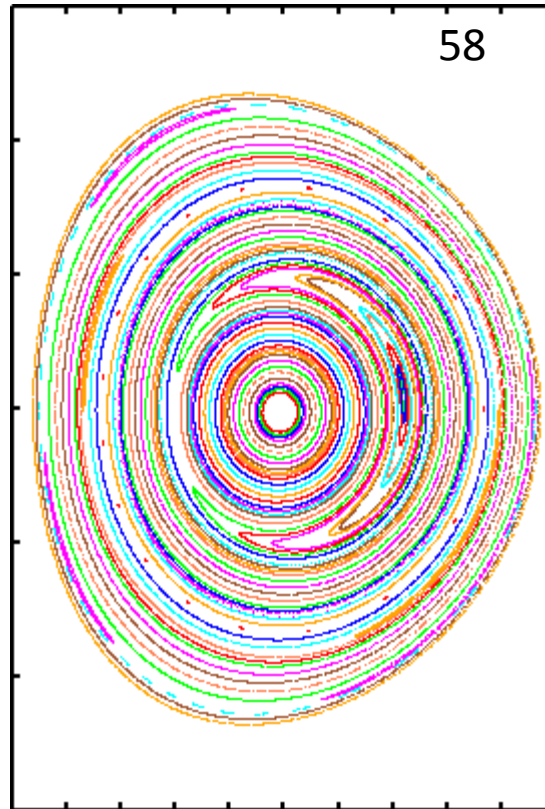
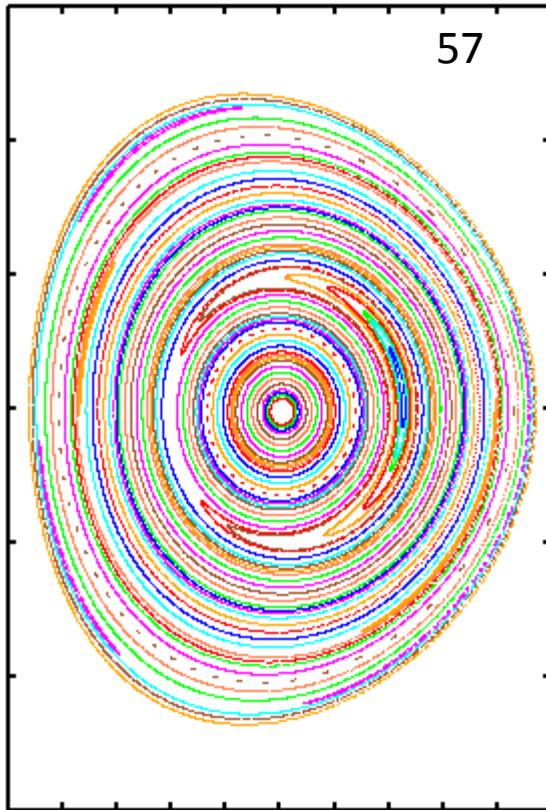
- Full Braginskii gyroviscous tensor

- Keep $\mathbf{J} \times \mathbf{B}$ term in Ohm's law

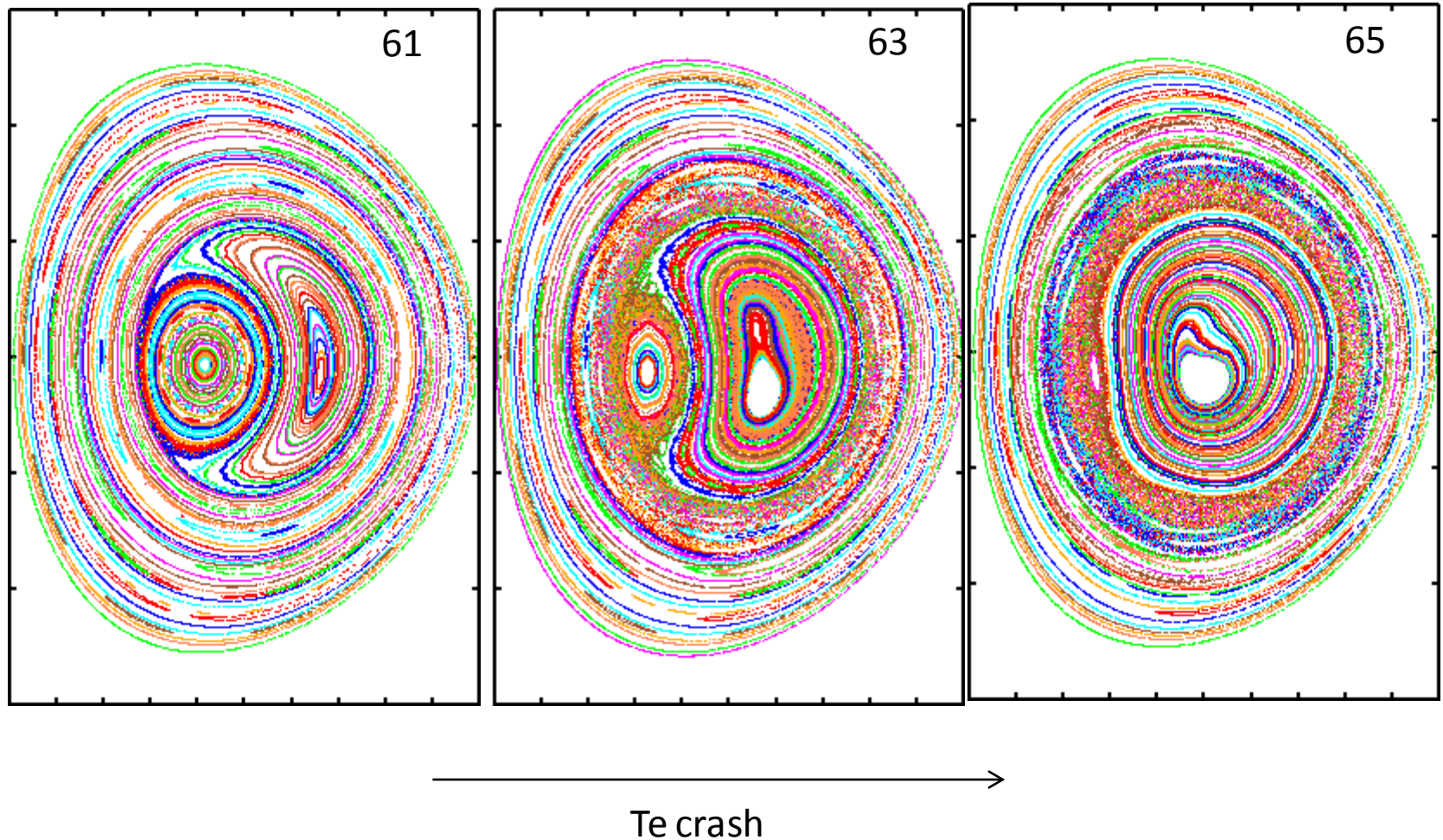
- 8 scalar variables advanced in time

- Ion skin depth: $d_i = \frac{1}{\ell_0} \left[\frac{M_i}{\mu_0 n_0 e^2} \right]^{1/2} = .0227 \frac{1}{\ell_0 [m]} [n_0 [20]]^{-1/2}$

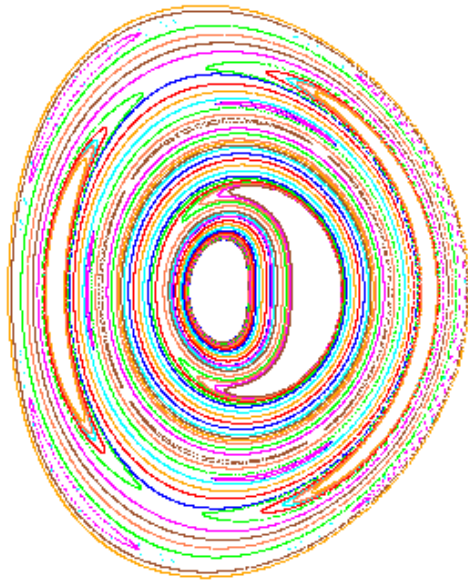
2F sawtooth shows distinctive shape



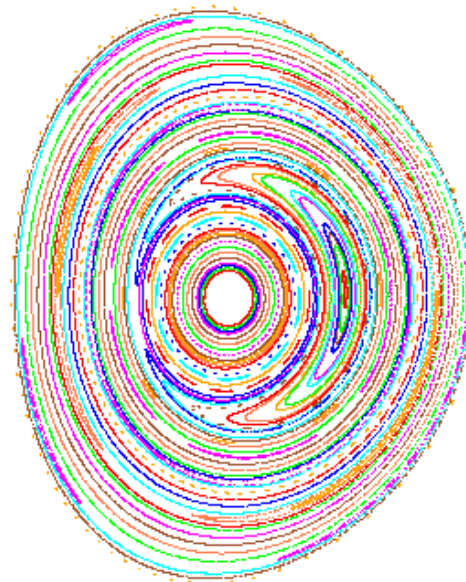
Surfaces are destroyed in reconnection region
during temperature crash



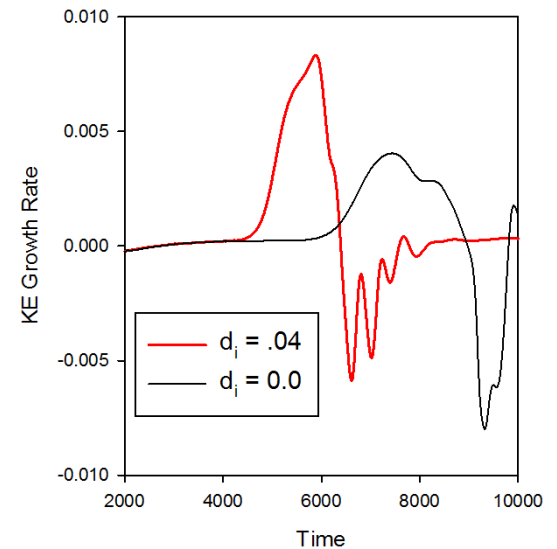
Comparison of surfaces for resistive and two-fluid MHD at similar stage in cycle shows reconnection layer is shorter in two-fluid. Rate increases by ~ 2



$d_i = 0$



$d_i = 0.04$



Summary

- M3D-C1 has gone through many verification tests
- Two types of long time behavior: periodic or stationary states
- Comparison of ellipse and bean shaped X-section has begun
- Stationary states can have pressure and temperature variation within surfaces
- Growth rate almost independent of viscosity, but peak k_e differs
- Sheared rotation promotes periodic behavior and good surfaces
- Two fluid terms lead to more circular interior surfaces, shorter reconnection layer, and faster reconnection times, however stochastic layer forms at late times