Nonlinear Reduced Magnetohydrodynamic Simulations of Edge-Localized Modes in Tokamak Plasmas

Isabel Krebs

M. Hoelzl, K. Lackner, S. C. Jardin, S. Günter

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$$\frac{\partial (\rho T)}{\partial t} = -\mathbf{v} \cdot \nabla(\rho T) - \gamma \rho T \nabla \cdot \mathbf{v} + \nabla \cdot \left(K_\perp \nabla_\perp T + K_{||} \nabla_{||} T\right) + S_T$$

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Definitions: $\mathbf{B} \equiv \frac{F_0}{R}\mathbf{e}_\phi + \frac{1}{R}\nabla\Psi \times \mathbf{e}_\phi$ and $\mathbf{v} \equiv -R\nabla u \times \mathbf{e}_\phi + v_{||}\mathbf{B}$

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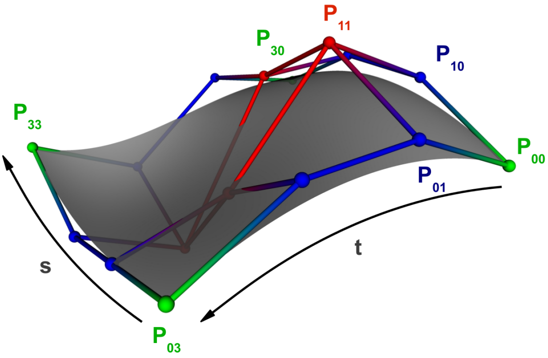
Variables: Ψ, u, $v_{||}$, ρ, T, j, ω

Discretization

- **poloidal plane:** 2D Bézier finite elements

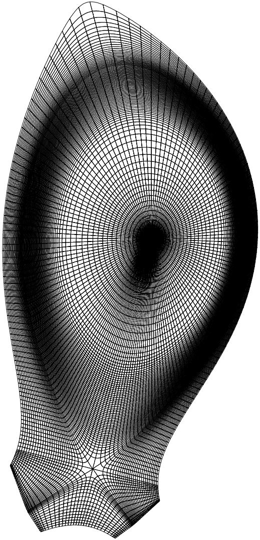
$$\mathbf{P}(s,t) = \sum_{i=0}^{3}\sum_{j=0}^{3} \mathbf{P}_{ij} B_i(s) B_j(t)$$

- **toroidal direction:** Fourier decomposition
- fully implicit time stepping



Grid generation

- equilibrium is computed on initial polar grid
- flux surface aligned X-point grid is generated
- grid can be refined in the regions of interest



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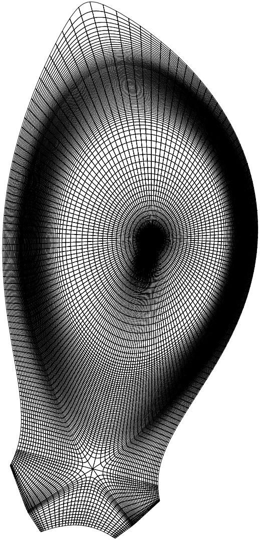
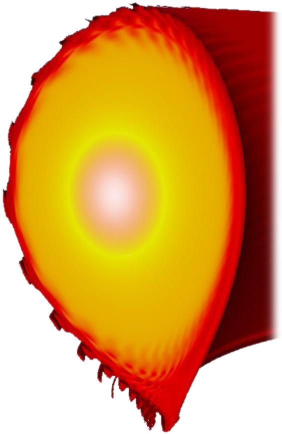
Boundary conditions

- ideally conducting wall and modified Bohm



- relaxation-oscillation instability at edge of H-mode plasmas
- driven by large edge pressure gradient & edge current density
- eject energy & particles from plasma
- relevant for future fusion devices
    - ⊕ help to control particle & impurity content
    - ⊖ high heat fluxes can damage plasma facing components



→ theoretical comprehension of ELMs is crucial to predict and control ELM properties

- **linear theory:** intermediate toroidal mode numbers are most unstable

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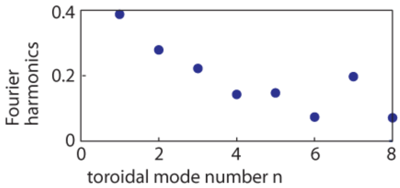
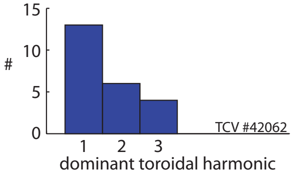
- **recent experimental observations (TCV):** toroidal mode structure often dominated by low-n components



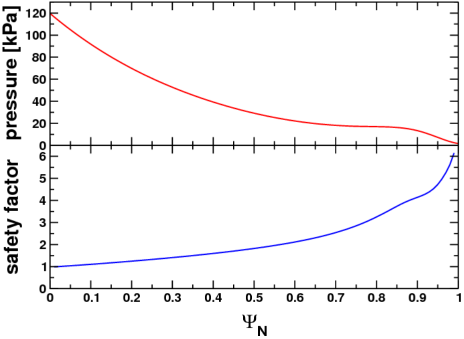
Example of measured ELM Fourier spectrum



Dominant toroidal components in ELMy discharge

[R.P. Wenninger, H. Zohm et al, Nucl. Fusion 2013]

- simulations are based on typical type-I ELMy ASDEX Upgrade discharge
  - plasma parameters based on ASDEX Upgrade, but larger resistivity ($S \approx 10^5$)
  - ASDEX Upgrade geometry including separatrix, X-point and open field lines



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    - plasma parameters based on ASDEX Upgrade, but larger resistivity ($S \approx 10^5$)
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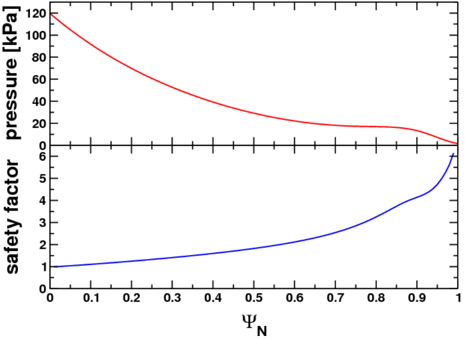
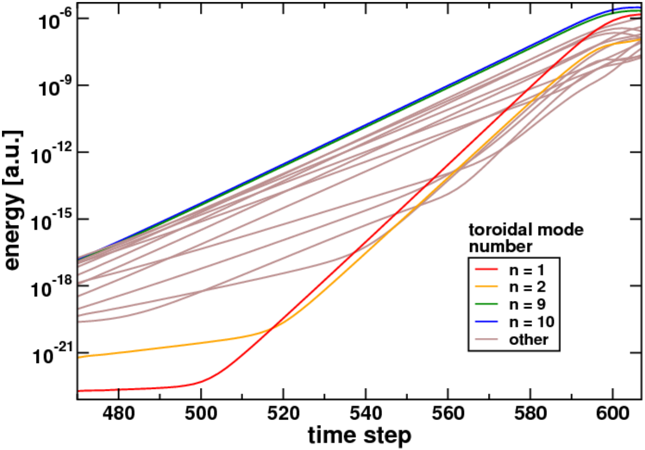
- large set of included toroidal Fourier harmonics ($n = 1, 2, ..., 16$)



Evolution of the perturbation



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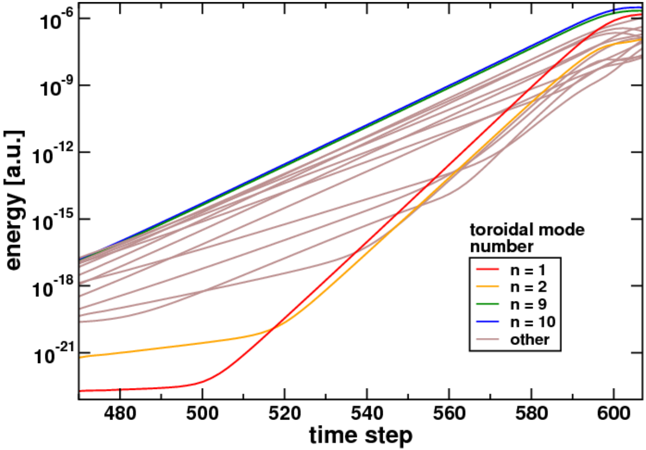
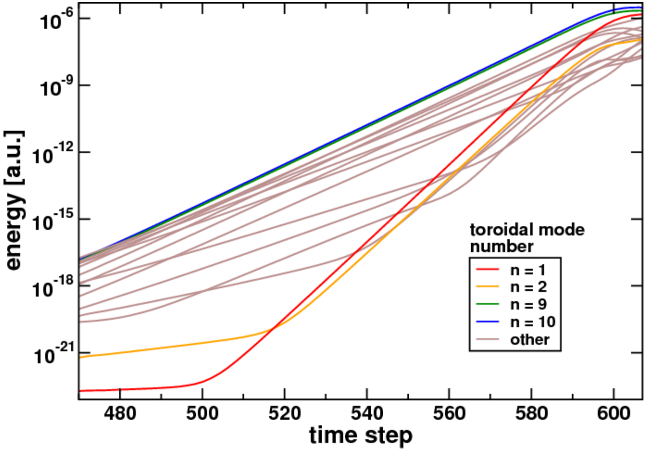
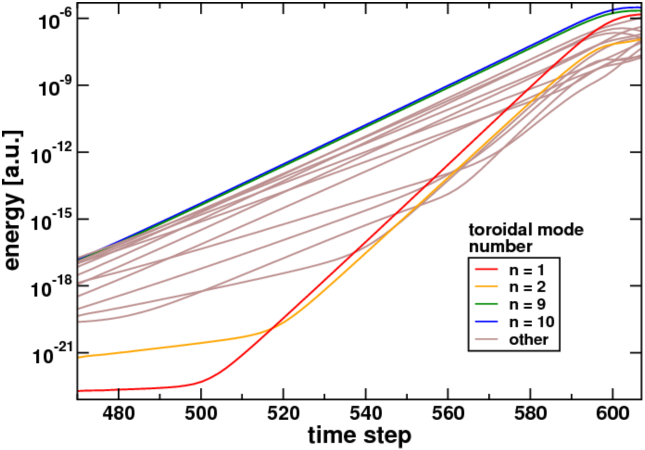


linear phase



linear phase $\longrightarrow$ early nonlinear phase



linear phase $\longrightarrow$ early nonlinear phase $\longrightarrow$ saturation

Simple quadratic coupling model

Idea: ”sum & difference mode number generation”

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$$\frac{\partial A_i}{\partial t} = \underbrace{\gamma_i A_i}_{\text{linear growth}} + \underbrace{\gamma^i_{jk} A_j A_k}_{\text{coupling}}$$

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γ^i_{jk} : coupling constant

$\hookrightarrow$ constant $\Rightarrow$ mode rigidity assumed

$\Downarrow$ for a set of harmonics $i = 1, 2, ..., 16$

$$\frac{\partial A_i}{\partial t} = \gamma_i A_i + \sum_{j=1}^{16} \sum_{k=1}^{16} \gamma_{jk}^{i} A_j A_k \delta(i \pm j \pm k)$$

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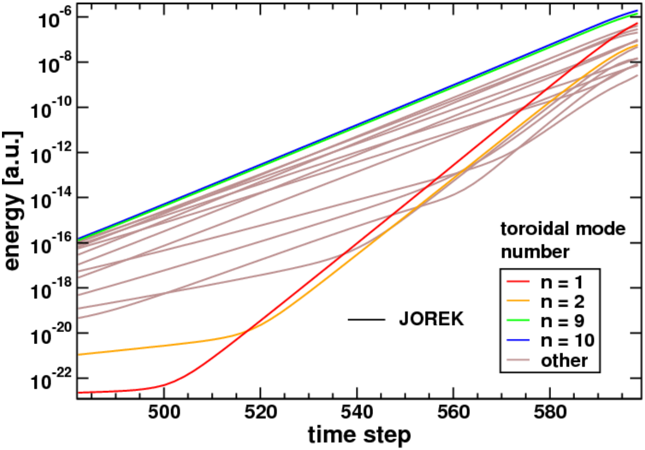
- set of coupled nonlinear differential equations reproduces evolution of toroidal Fourier spectrum in JOREK simulations
- relevant coupling constants: $\gamma^1_{9,10}, \gamma^2_{8,10}, \gamma^3_{7,10}, \gamma^4_{6,10}, \gamma^{15}_{7,8}, \gamma^{16}_{7,9}$

Results of simple model



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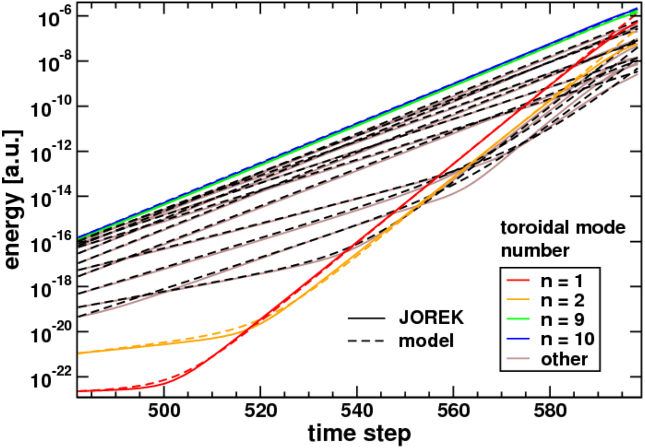

- simple quadratic coupling model reproduces JOREK results in early nonlinear phase
- model gives explanation for strong low-n components in experiments

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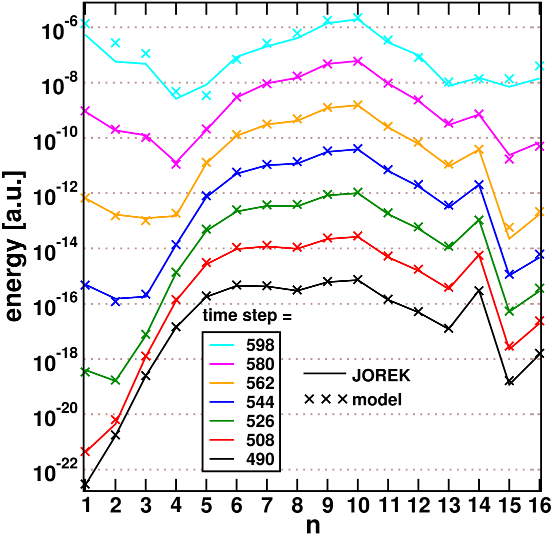
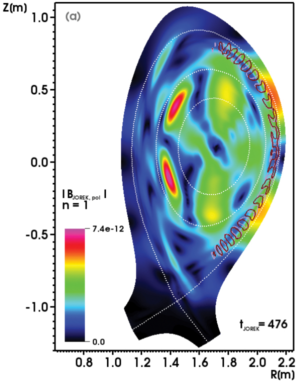


Localization of driven harmonics



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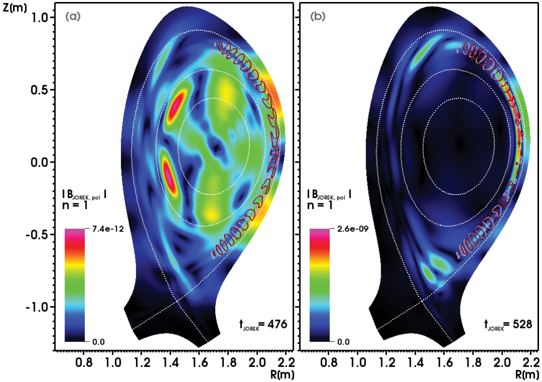

- linearly unstable $n = 1$ extends over a large part of the plasma core
- nonlinearly driven $n = 1$ is localized at plasma edge (where driving harmonics are maximal and in phase)

Summary…

- nonlinear reduced MHD **ELM simulations** based on ASDEX Upgrade
- large set of included toroidal harmonics
- **subdominant low-n harmonics** become important due to **nonlinear drive**
- $n = 1$ reaches energies comparable to linearly dominant harmonics
- **correspondence to experimental observations** of dominant low-n components
- **simple quadratic interaction model** reproduces and explains early nonlinear evolution of toroidal harmonics in JOREK simulations
- spatial structure of $n = 1$ becomes localized at edge when nonlinearly driven

…and Outlook

- enable more realistic resistivity
- analyze how nonlinear interaction of toroidal harmonics is influenced by
  - diamagnetic drift effects
  - sheared toroidal plasma rotation

Thank you for your attention!

References

Simulations

I. Krebs, M. Hoelzl et al, Phys. Plasmas 20, 082506 (2013)

M. Hoelzl, S. Günter et al, Phys. Plasmas 19, 082505 (2012)

Experiment

R.P. Wenninger, H. Zohm et al, Nucl. Fusion 53, 113004 (2013)

JOREK

G.T.A. Huysmans and O. Czarny, Nucl. Fusion 47, 659 (2007)

O. Czarny and G.T.A. Huysmans, J. of Comput. Phys. 227, 7423 (2008)

Acknowledgements

R. P. Wenninger

ASDEX Upgrade Team

Max-Planck/Princeton Center for Plasma Physics

HELIOS at IFERC-CSC

$$\frac{\partial A_i}{\partial t} = \gamma_i A_i + \sum_{j=1}^{16} \sum_{k=1}^{16} \gamma^i_{jk} A_j A_k \delta(i \pm j \pm k) \qquad \text{for } i = 1, 2, ..., 16$$

- linear terms $\rightarrow$ influx of energy
- nonlinear terms $\rightarrow$ exchange of energy between different harmonics (total energy should be conserved)

$$\Rightarrow 0 \overset{!}{=} \frac{\partial E_{tot}}{\partial t} \propto \frac{\partial}{\partial t} \sum_i A_i^2 \qquad \text{(only nonlinear terms)}$$

$\Rightarrow$ additional constraints for the coupling constants (12 free parameters remain)

Energy conservation