Self-Organized Stationary States in Inductively Driven Tokamaks

Presented by S. C. Jardin[1]
N. Ferraro[2], I. Krebs[1,3], J. Chen[1]

[1]Princeton Plasma Physics Laboratory
[2]General Atomics
[3]Max Plank Institute for Plasma Physics, Garching, Germany

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Summary

- For certain parameters, regardless of initial state, plasma will go into a "self-organized" state with q = 1 + ε in a central volume

- This large shear-free region is unstable to interchange modes for any pressure gradient and the instability will drive a strong (1,1) helical flow.

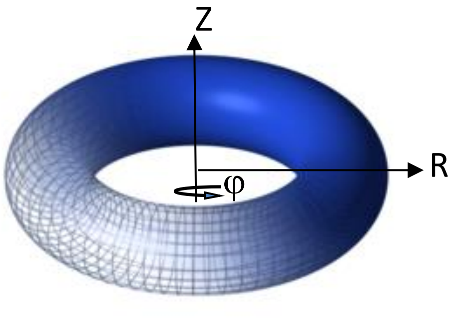
- This flow does not affect the magnetic field evolution since it has the property that :

$$\mathbf{V}\times\mathbf{B} = -\nabla\Phi \qquad \Rightarrow \qquad \frac{\partial \mathbf{B}}{\partial t} = \nabla\times\left(\mathbf{V}\times\mathbf{B}+\cdots\right) = 0$$

- However, the driven flow is a dominant term in the Temperature evolution equation and dominates over the thermal conductivity in the center of the discharge where q is flat.

- The net effect is to keep the central temperature (and resistivity) flat so that the resistive steady state is such as to preserve the self-organized state with q = 1 + ε in a central volume.

3D Extended MHD Equations



$$\frac{\partial n}{\partial t}+\nabla\bullet(n\mathbf{V})=S_n$$

$$\frac{\partial \mathbf{B}}{\partial t}=-\nabla\times\mathbf{E} \qquad \mathbf{B}=\nabla\times\mathbf{A} \qquad \mathbf{J}=\nabla\times\mathbf{B}$$

$$nM_i\left(\frac{\partial \mathbf{V}}{\partial t}+\mathbf{V}\bullet\nabla\mathbf{V}\right)+\nabla p=\mathbf{J}\times\mathbf{B}-\nabla\bullet\mathbf{\Pi}_i+\mathbf{S}_m$$

$$\mathbf{E}+\mathbf{V}\times\mathbf{B}=\frac{1}{ne}\left(\mathbf{R_c}+\mathbf{J}\times\mathbf{B}-\nabla p_e-\nabla\bullet\mathbf{\Pi}_e\right)-\frac{m_e}{e}\left(\frac{\partial \mathbf{V}_e}{\partial t}+\mathbf{V}_e\bullet\nabla\mathbf{V}_e\right)+\mathbf{S}_{CD}$$

$\frac{3}{2}\left[\frac{\partial p_e}{\partial t}+\nabla\right.$ [redacted]

$$\frac{3}{2}\left[\frac{\partial p_i}{\partial t}+\nabla\bullet\left(p_i\mathbf{V}\right)\right]=-p_i\nabla\bullet\mathbf{V}-\mathbf{\Pi}_i:\nabla\mathbf{V}-\nabla\bullet\mathbf{q}_i-Q_\Delta+S_{iE} \qquad \mathbf{V}_e=\mathbf{V}-\mathbf{J}/ne$$

$$\mathbf{R}_c=\eta ne\mathbf{J}, \qquad \mathbf{\Pi}_i=-\mu\left[\nabla\mathbf{V}+\nabla\mathbf{V}^\dagger\right]-2(\mu_c-\mu)(\nabla\bullet\mathbf{V})\mathbf{I}+\mathbf{\Pi}_i^{GV} \qquad \mathbf{q}_{e,i}=-\kappa_{e,i}\nabla T_{e,i}-\kappa_\parallel\nabla_\parallel T_{e,i}$$

$$\mathbf{\Pi}_e=(\mathbf{B}/B^2)\nabla\bullet\left[\lambda_h\nabla\left(\mathbf{J}\bullet\mathbf{B}/B^2\right)\right], \qquad Q_\Delta=3m_e(p_i-p_e)/\left(M_i\tau_e\right)$$

Kinetic closures extend these to include neo-classical, energetic particle, and turbulence effects.

We are using M3D-C^1 to solve the MHD equations to compute the self-consistent long-time (transport timescale) behavior of a tokamak discharge subject to:

- loop voltage (I_P controller)
- density source (n_e controller)
- heating source (NB)
- momentum source (NB)
- shaping fields

- resistivity η
- viscosity ν
- thermal conductivity $\kappa_{\|}$ & $\kappa_{\perp}$
- particle diffusivity D
- ion-skin depth $d_i = c/\omega_{pi}$

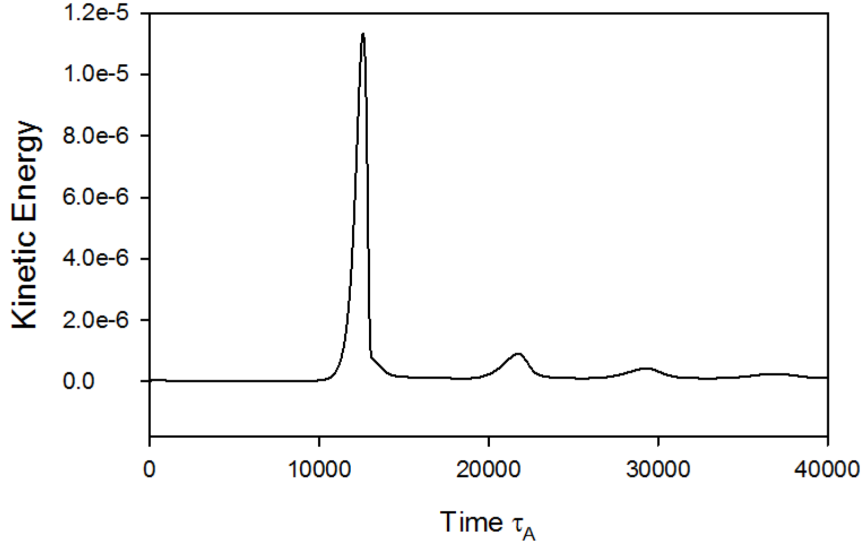
Standard transport model:

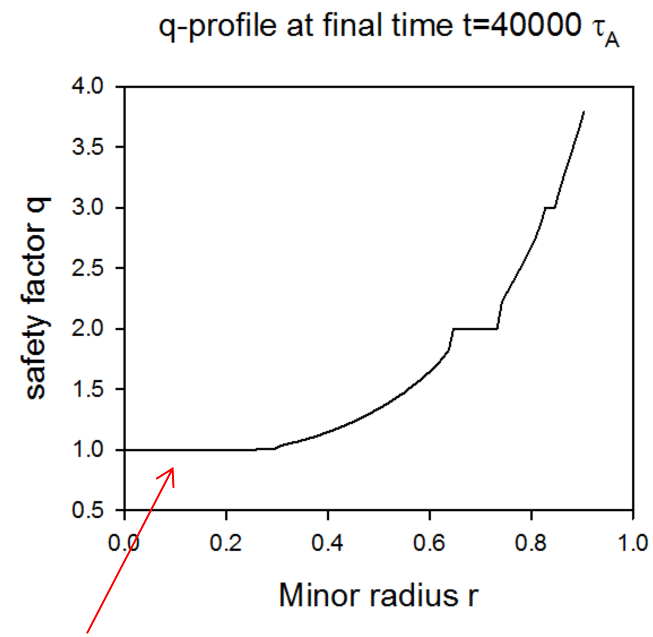
$$\eta = \eta_0 \left(\frac{T_e}{T_{e0}} \right)^{-3/2} \qquad \nu, D = \text{const} \qquad \kappa_{\perp} = \kappa_0 \left[1 + \alpha \left| \nabla T_e^2 \right| \right] \left(\frac{p}{p_0} \right)^{-1/2} \qquad \kappa_{\|} \simeq 10^5 \kappa_0$$

Initial conditions have $q_0 < 1$, so one sawtooth always occurs.

When does the tokamak go into a stationary state and what are its properties? What is the relation to sawteeth?

Example: β=2%   S=10^6
-- oscillations die out to form stationary state



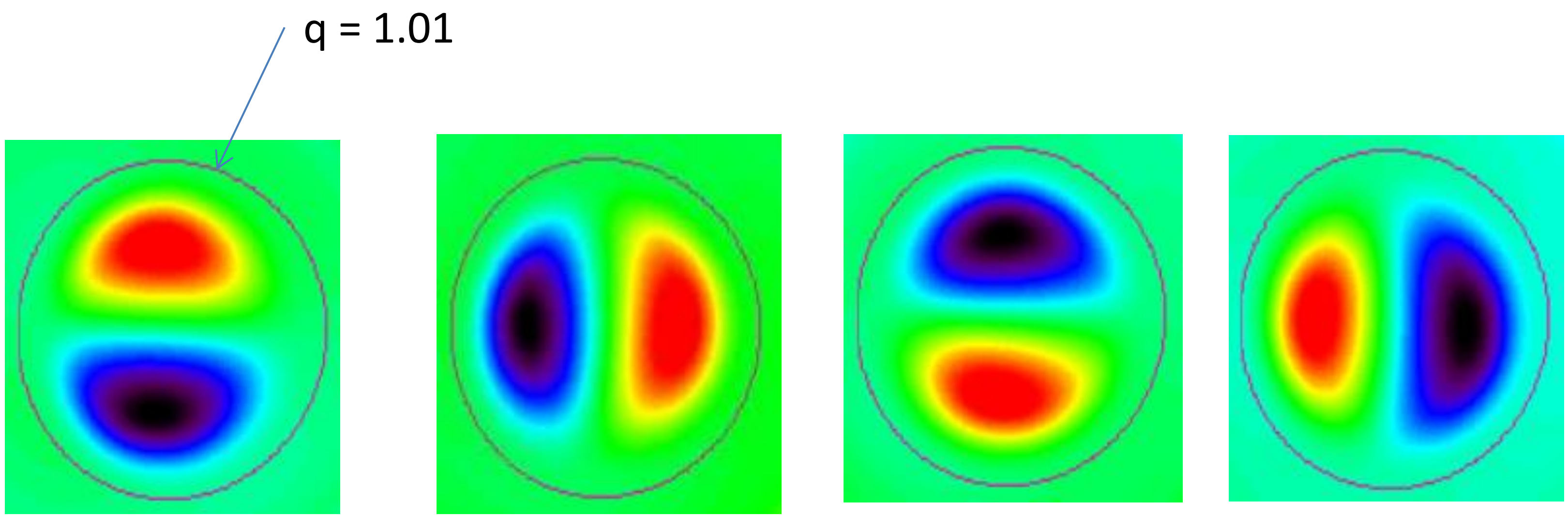


Large region in center with q = 1 + ε

CMOD15 μ=10
Also, see
CMOD02
CMOD11
CMOD35

At higher values of β , periodic oscillations die out and a stationary interchange mode develops with q just above 1 in a large volume near the axis   (S = 10^6)

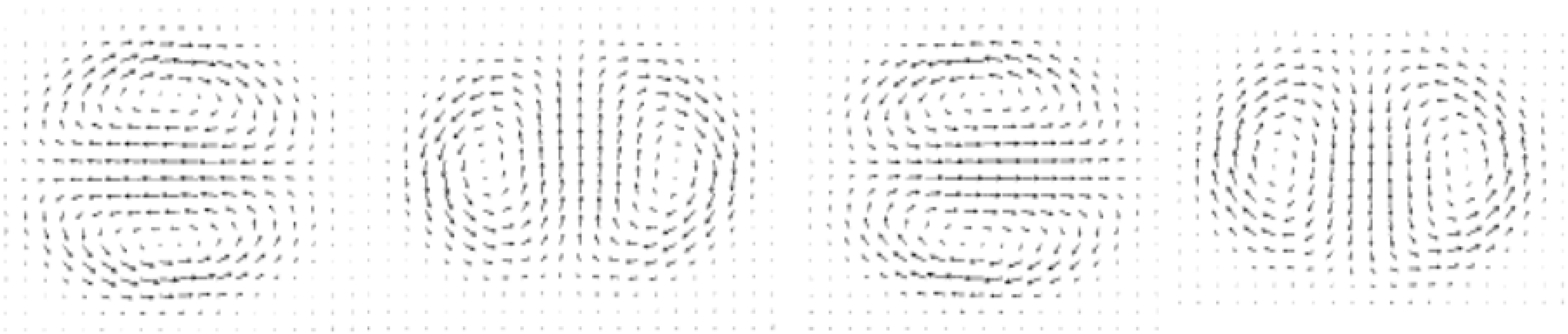
Central poloidal flow flattens current profile



$\varphi = 0^0$ $\varphi = 90^0$ $\varphi = 180^0$ $\varphi = 270^0$

Contours of poloidal velocity stream function U at final time shows a clear (1,1) structure that is stationary in time.

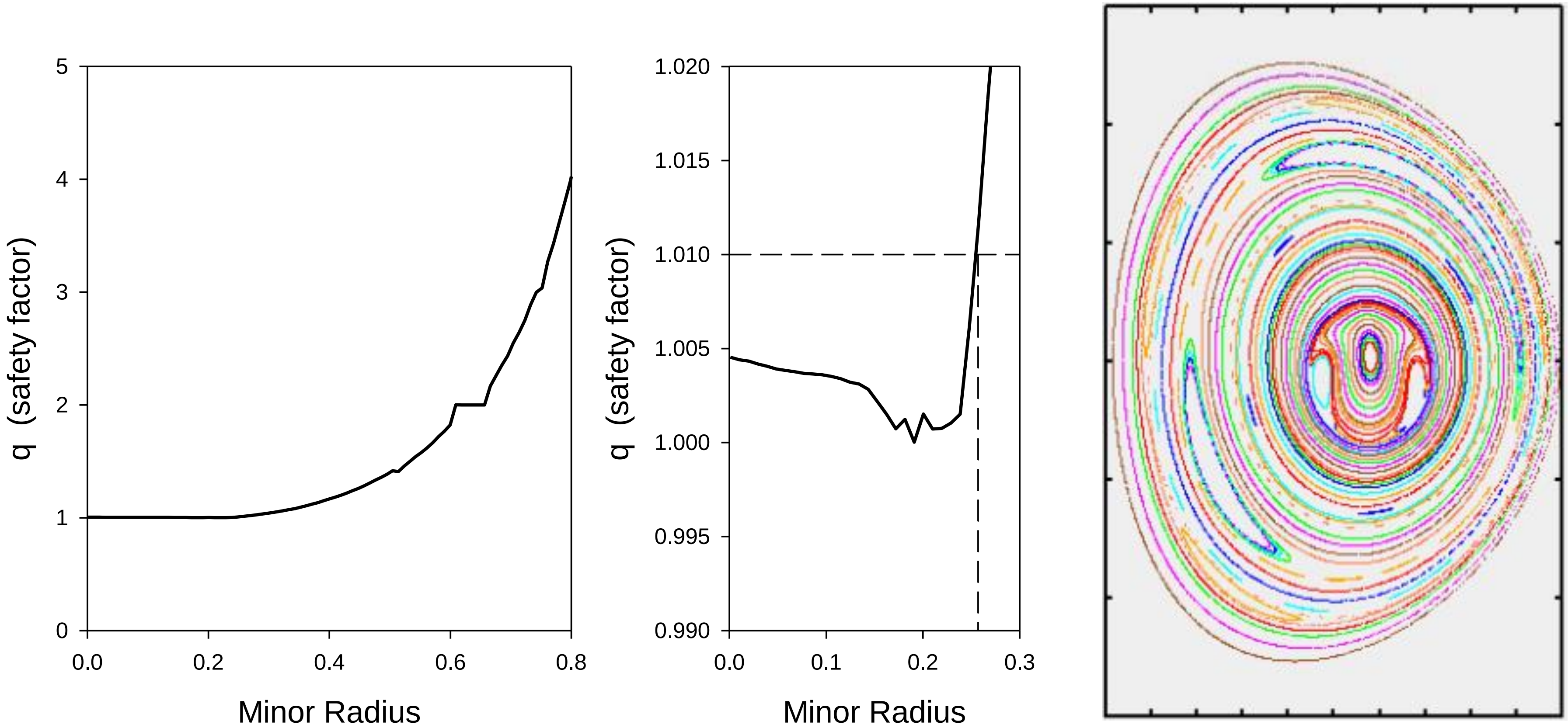
Hill’s vortex like flow pattern in center



$\varphi = 0^0$ $\varphi = 90^0$ $\varphi = 180^0$ $\varphi = 270^0$

This stationary flow pattern is being driven by the interchange instability. It is also acting to flatten the temperature and current profiles to keep the central q=1 region stationary in time.

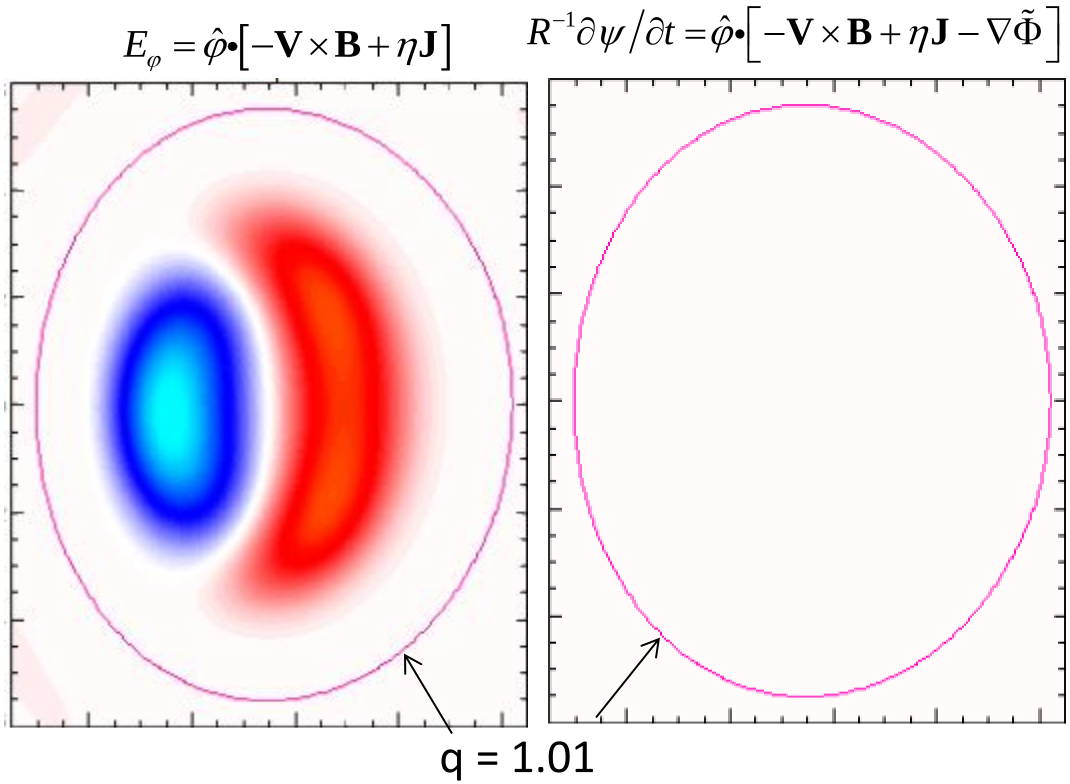
Large shear-free region near axis

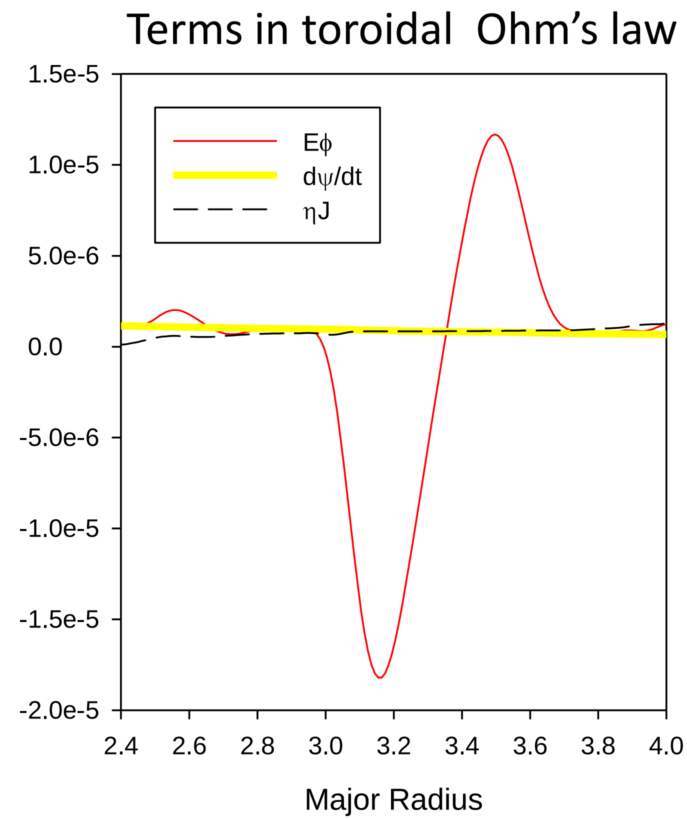


$1.00 < q < 1.01$ in inner 1/3 of minor radius (1/9[th] of volume)

q_{min} = 1.000 slight reversed shear near axis

Term's in Ohm's law for stationary state with q = 1 + ε





$$\mathbf{A} = R^2\nabla\varphi\times\nabla f + \psi\nabla\varphi - F_0 \ln R\hat{Z}$$

$$\mathbf{B} = \nabla\psi\times\nabla\varphi - \nabla_\perp f' + F\nabla\varphi$$

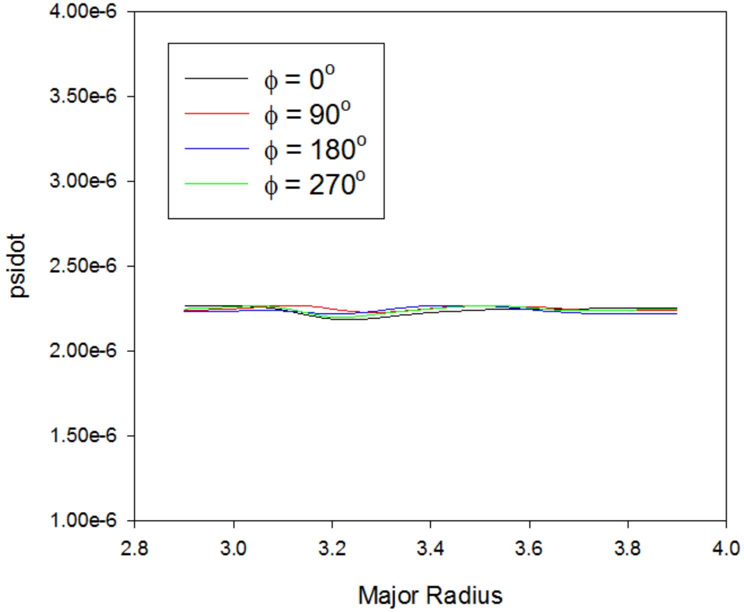
$$F \equiv F_0 + R^2\nabla\bullet\nabla_\perp f$$

Large **V x B** flow generated by interchange instability is canceled in Ohm's law equation by the gradient of the scalar potential $\nabla\tilde{\Phi}$

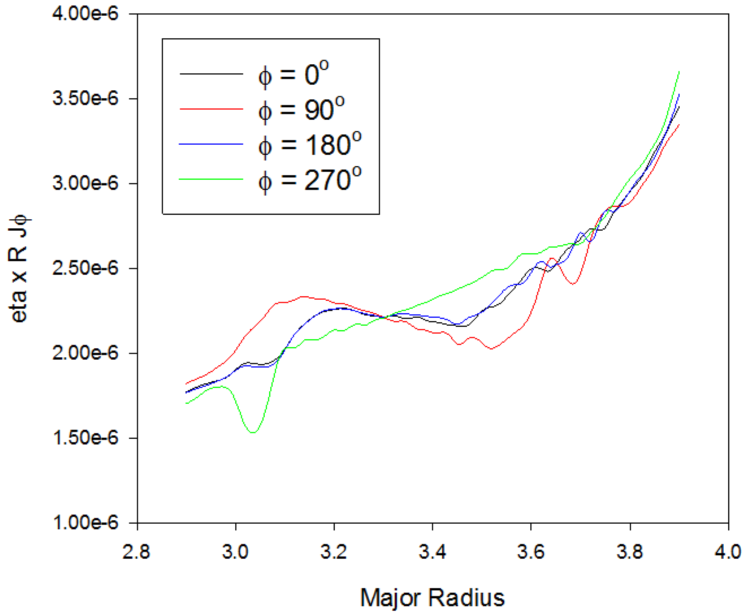
Run35

Magnified view of mid-plane values of terms in Ohm's law

$$\partial\psi/\partial t = R\hat{\varphi}\bullet\left[-\mathbf{V}\times\mathbf{B}+\eta\mathbf{J}-\nabla\tilde{\Phi}\right]$$



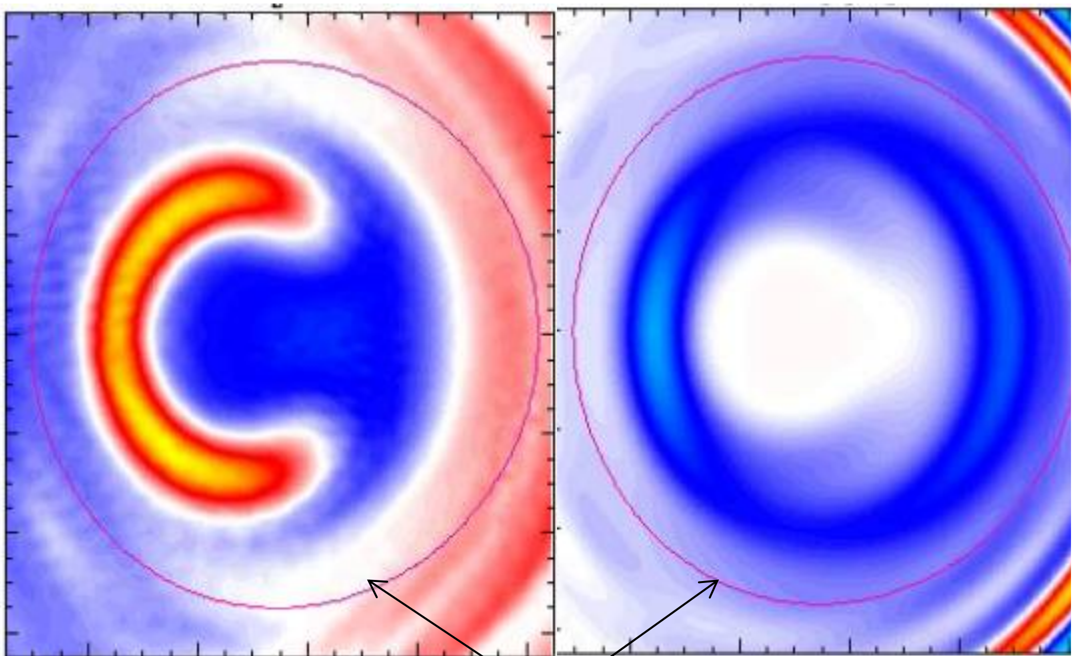
$$\partial\psi/\partial t = R\hat{\varphi}\bullet\left[\eta\mathbf{J}\right]$$

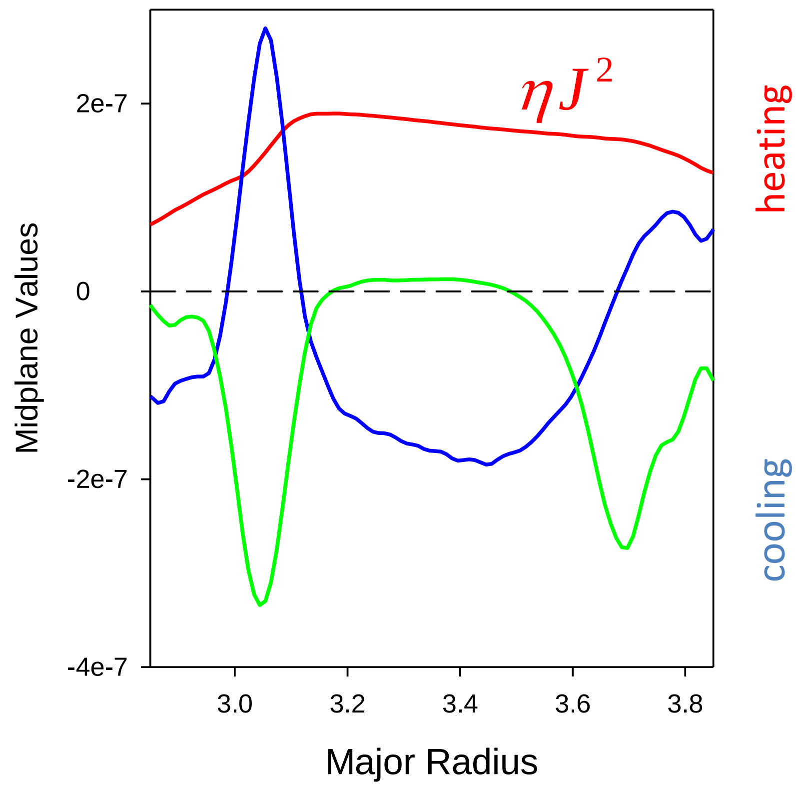


While $\eta\mathbf{J}$ has some spatial variation, the other terms combine to make the time derivative of ψ a spatial constant (stationary state).

Strong helical flow velocity field from interchange instability is dominant transport loss in center.

$n\mathbf{V}\bullet\nabla T + n(\gamma-1)T\nabla\bullet\mathbf{V}$ $\quad$ $(\gamma-1)\nabla\bullet\mathbf{q}_{\perp}$

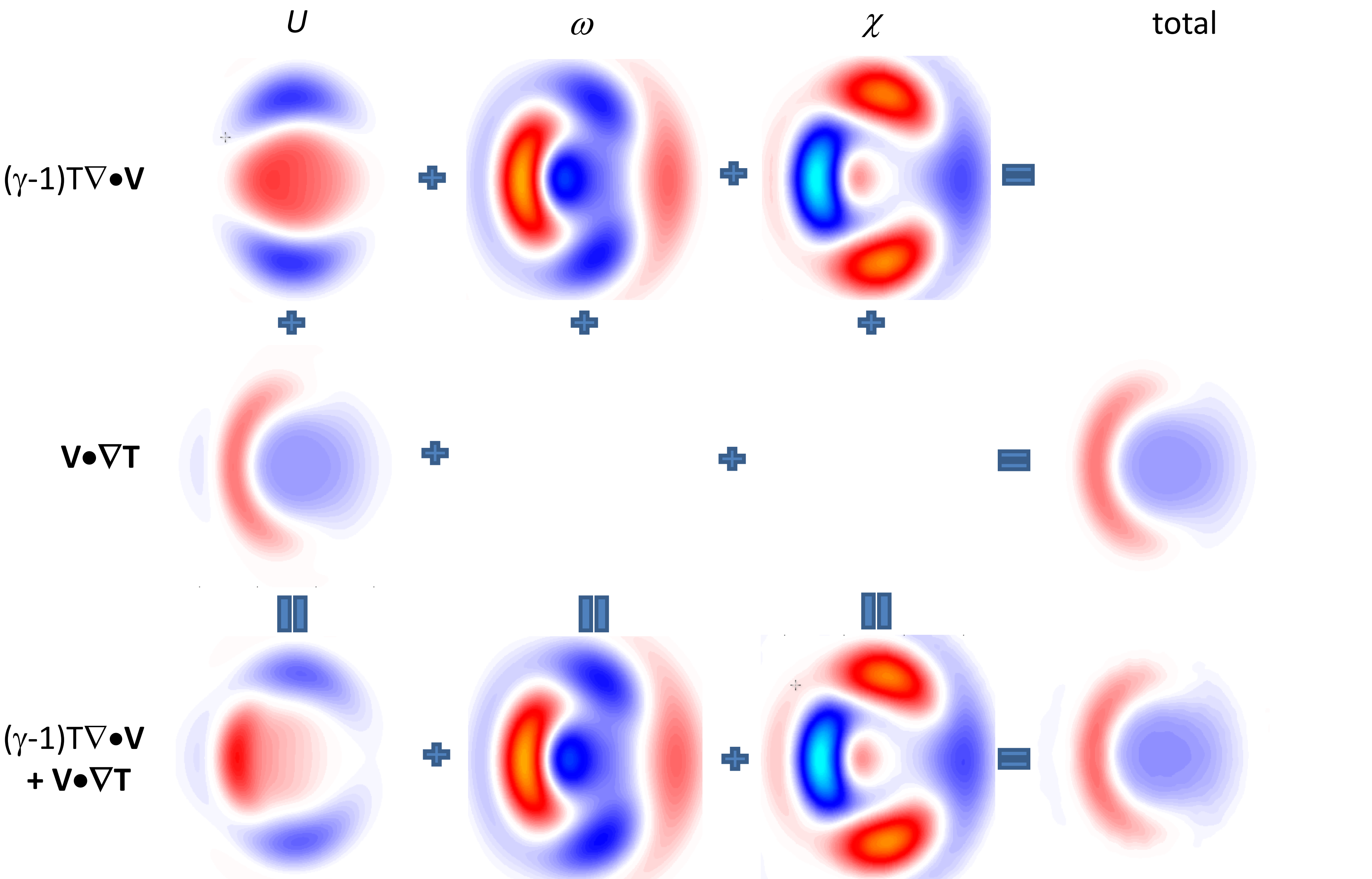




$$\frac{\partial T}{\partial t} + n\mathbf{V}\bullet\nabla T + n(\gamma-1)T\nabla\bullet\mathbf{V} + (\gamma-1)\nabla\bullet\mathbf{q}_{\perp} + (\gamma-1)\nabla\bullet\mathbf{q}_{\parallel} = \eta J^2$$

S01-slice118-60º

Decomposition of velocity field in M3D-C1: 3 components but ∇•V=0



$$\mathbf{V} = R^2 \nabla U \times \nabla \varphi + R^2 \omega \nabla \varphi + R^{-2} \nabla_\perp \chi$$

Run35RRe,51,phi=335,rrange=[2.925,3.7],zrange=[-.425,.425], +-6.1e-6

Analysis of terms in temperature equation near magnetic axis

$$n\mathbf{v}^{(1)}\bullet\nabla T^{(0)} + \frac{2}{3} n T^{(0)} \nabla\bullet\mathbf{v}^{(1)} = \frac{2}{3}\nabla\cdot\left[\left(\kappa\mathbf{I} + \kappa_{\|}\hat{\mathbf{b}}\hat{\mathbf{b}}\right)\bullet\nabla T^{(1)}\right]$$ n=1 (1)

$$n\mathbf{v}^{(1)}\bullet\nabla T^{(1)} + \frac{2}{3} n T^{(1)} \nabla\bullet\mathbf{v}^{(1)} = \frac{2}{3}\nabla\cdot\left[\left(\kappa\mathbf{I} + \kappa_{\|}\hat{\mathbf{b}}\hat{\mathbf{b}}\right)\bullet\nabla T^{(0)}\right] + \frac{2}{3}\left[\eta\mathbf{J}^2 + S_e\right]^{(0)}$$ n=0 (2)

Take the stream function to be the (cylindrical) unstable eigenmode found in [1]:

$$U(r,\theta,\varphi) = U_0 r[1-(r/r_1)^2]\sin(\theta-\varphi)$$

$$V_r = U_0\left[1-\left(r/r_1\right)^2\right]\cos\left(\theta-\varphi\right), \qquad V_\theta = -U_0\left[1-3\left(r/r_1\right)^2\right]\sin\left(\theta-\varphi\right)$$

q(r) is flat interior to radius r_1

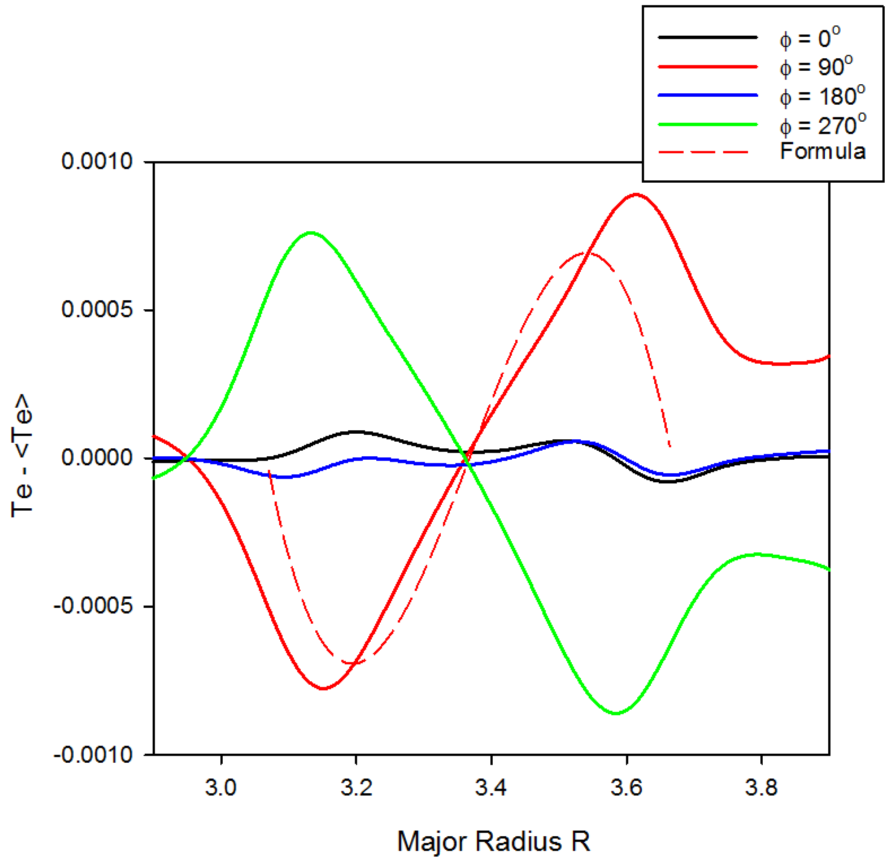
Assuming constant source, and balancing the first and last terms in (2)

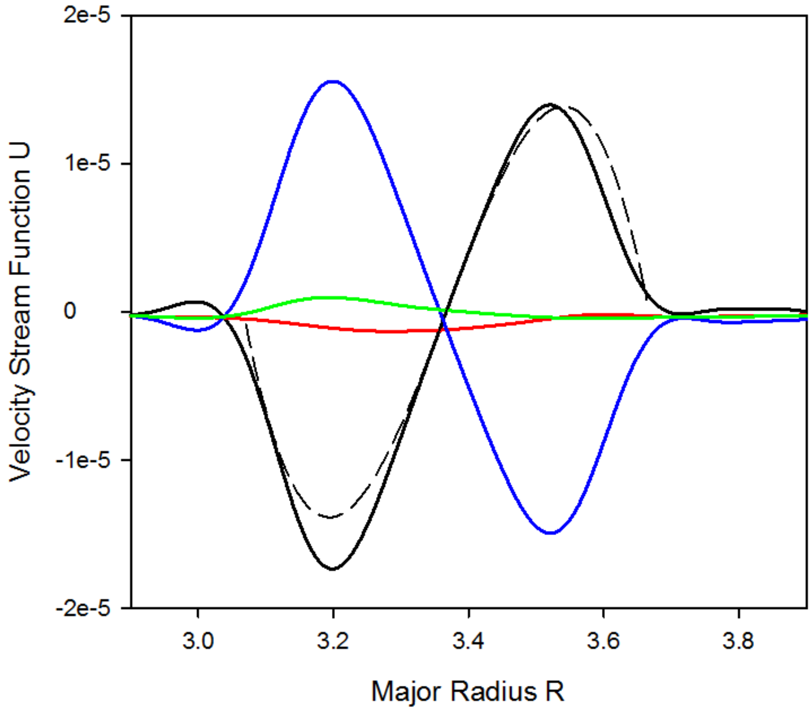
$$T^{(1)} = \frac{2}{3}\frac{S^0}{n_0 U_0} r\left[1-(r/r_1)^2\right]\cos(\theta-\varphi)$$

$$n_0\mathbf{V}\bullet\nabla T^{(1)} = \frac{2}{3} S\left[1-\left(r/r_1\right)^2\right]\left[1-3\left(r/r_1\right)^2\right]$$

[1] Hastie and Hender, NF 28 (1988) p. 585 “Toroidal internal kink stability in tokamaks with ultra flat q profiles”

Analytic formula give reasonable agreement with code results





$$T^{(1)} = \frac{2}{3}\frac{S^0}{n_0 U_0} r\left[1-(r/r_1)^2\right]\cos(\theta-\varphi)$$

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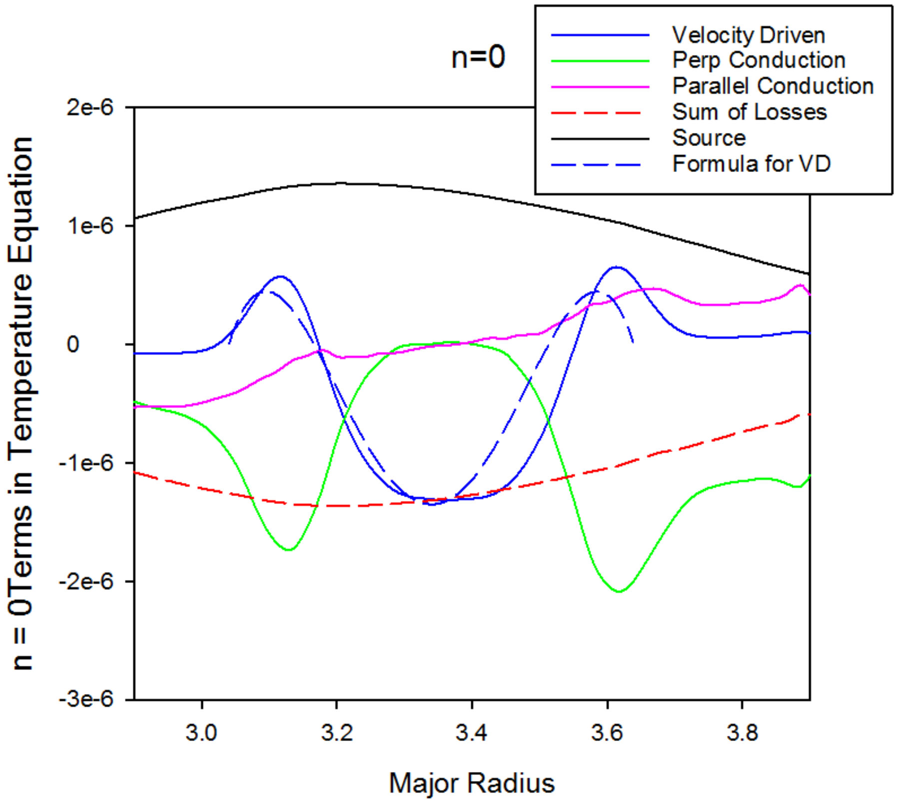
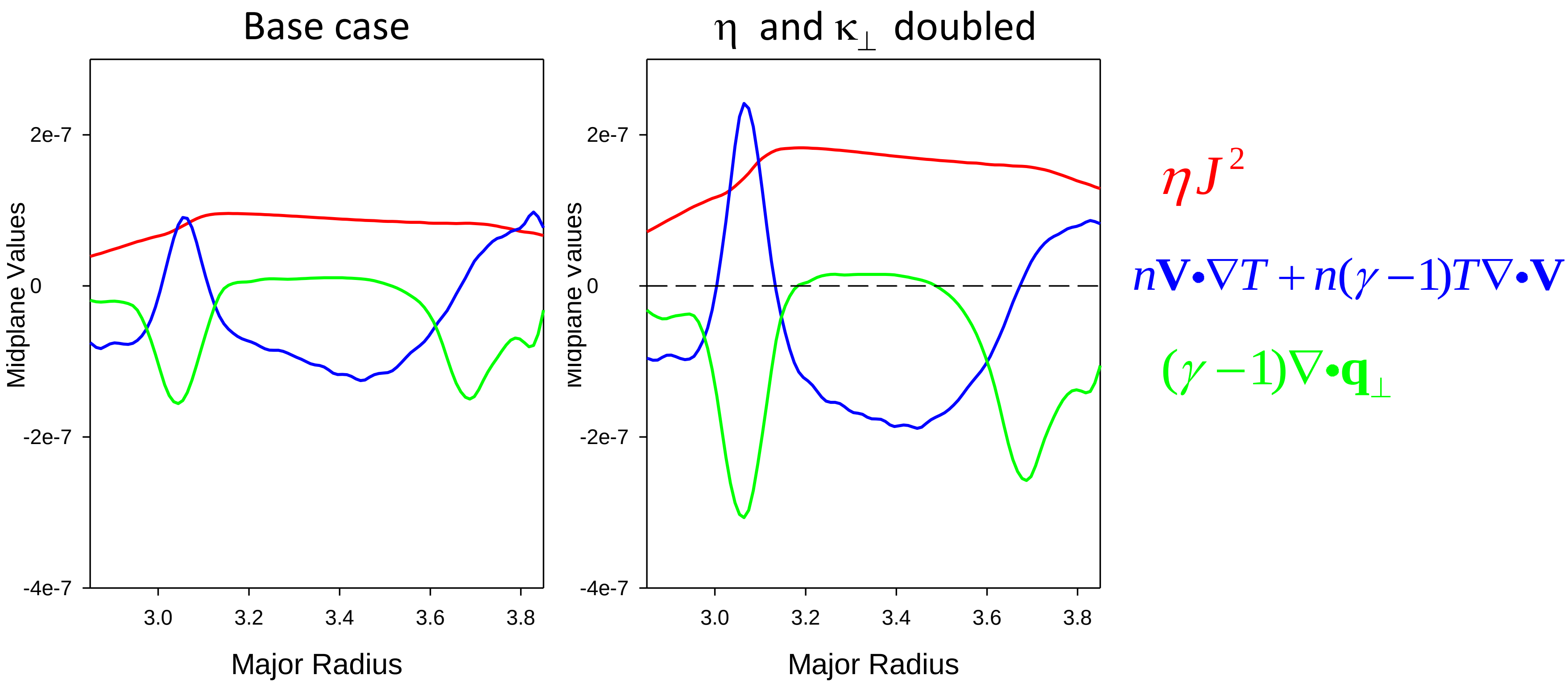


$$n\mathbf{V}\bullet\nabla T + n(\gamma-1)T\nabla\bullet\mathbf{V} + (\gamma-1)\nabla\bullet\mathbf{q}_\perp + (\gamma-1)\nabla\bullet\mathbf{q}_\parallel = \eta J^2$$

$$n_0\mathbf{V}^{(1)}\bullet\nabla T^{(1)} = \frac{2}{3}S\left[1-\left(r/r_1\right)^2\right]\left[1-3\left(r/r_1\right)^2\right]$$

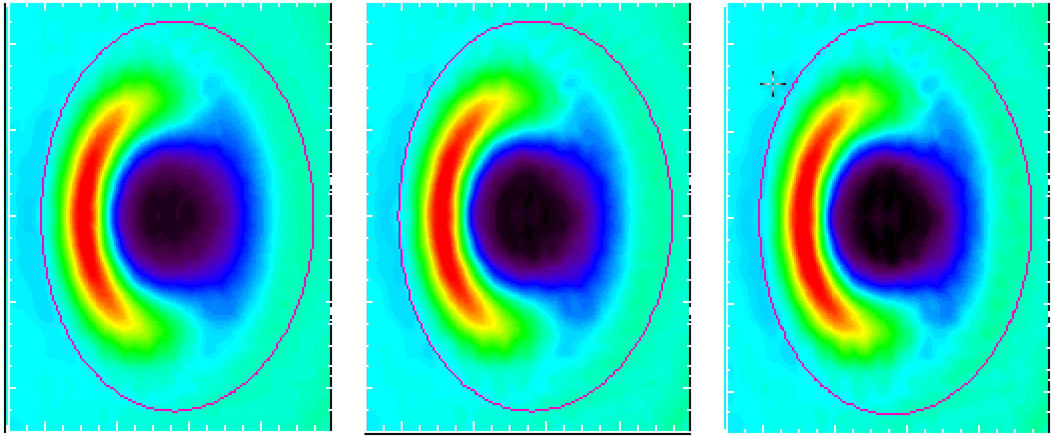
Scaling of driven flow with source and sink terms



Electrostatic potential and flow velocity scale with the size of the source and sink terms in the temperature equation in this regime. Need to extend to more extreme regimes.

Scaling of driven flow with viscosity

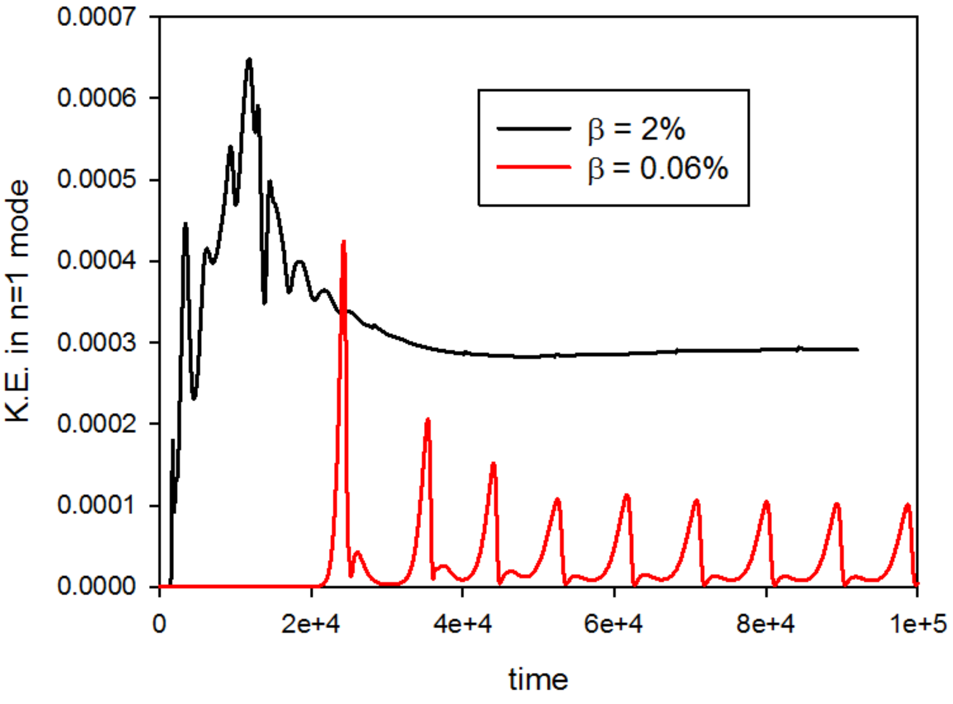
Contours of **V** •∇T in stationary state



$\nu = 10 \times 10^{-6}$ $\nu = 5 \times 10^{-6}$ $\nu = 2.5 \times 10^{-6}$

Final state has very little dependence on value of viscosity (same color scale in the 3 plots)

Scaling of driven flow with beta



The kinetic energy in the driven flow does depend on the plasma beta.
For small enough beta, the system exhibits periodic oscillations (sawteeth).

M3D-C1 has two options for advancing the poloidal field. Results were essentially identical for the two modes.

JADV=0:
Time advance poloidal flux and solve elliptic equation for electro-static potential

$$\begin{cases} \dot{\psi} = R^2[U,\psi] - R^2(U,f') - R^{-2}(\chi,\psi) - [\chi,f'] - \Phi' + \eta\Delta^*\psi + 2F \\ \nabla_\perp \bullet \frac{1}{R^2}\nabla\Phi = \nabla_\perp \cdot \left[-\frac{F}{R^2}\nabla_\perp U + \frac{\omega}{R^2}\nabla_\perp\psi + \omega\nabla_\perp f' \times \nabla\varphi + \frac{F}{R^4}\nabla_\perp\chi \times \nabla\varphi \right] \\ \quad -\nabla_\perp \cdot \eta \left[\frac{1}{R^2}\nabla F \times \nabla\varphi + \frac{1}{R^2}\nabla f'' \times \nabla\varphi + \frac{1}{R^4}\nabla_\perp \psi' \right] + 2F \end{cases}$$

JADV=1:
Time advance toroidal current density

$$\begin{cases} \nabla_\perp \bullet \frac{1}{R^2}\nabla\dot{\psi} = \nabla_\perp \bullet \frac{1}{R^2}\nabla R^2[U,\psi] - \nabla_\perp \bullet \frac{1}{R^2}\nabla R^2(U,f') + \nabla_\perp \cdot \left[\frac{F}{R^2}\nabla_\perp U \right]' \\ \quad -\nabla_\perp \cdot \left[\frac{\omega}{R^2}\nabla_\perp\psi + \omega\nabla_\perp f' \times \nabla\varphi \right]' - \nabla_\perp \bullet \frac{1}{R^2}\nabla R^{-2}(\chi,\psi) \\ \quad -\nabla_\perp \bullet \frac{1}{R^2}\nabla[\chi,f'] - \nabla_\perp \cdot \left[\frac{F}{R^4}\nabla_\perp\chi \times \nabla\varphi \right]' \\ \quad +\nabla_\perp \bullet \frac{1}{R^2}\nabla\eta\Delta^*\psi + \nabla_\perp \cdot \left[\frac{\eta}{R^2}\nabla F^* \times \nabla\varphi + \frac{\eta}{R^4}\nabla_\perp\psi' \right]' \end{cases}$$

Definitions:

(R,φ,Z) are cylindrical coordinates

$$\mathbf{A} = R^2\nabla\varphi \times \nabla f + \psi\nabla\varphi - F_0 \ln R\hat{Z}$$

$$\mathbf{V} = R^2\nabla U \times \nabla\varphi + \omega R^2\nabla\varphi + R^{-2}\nabla_\perp\chi$$

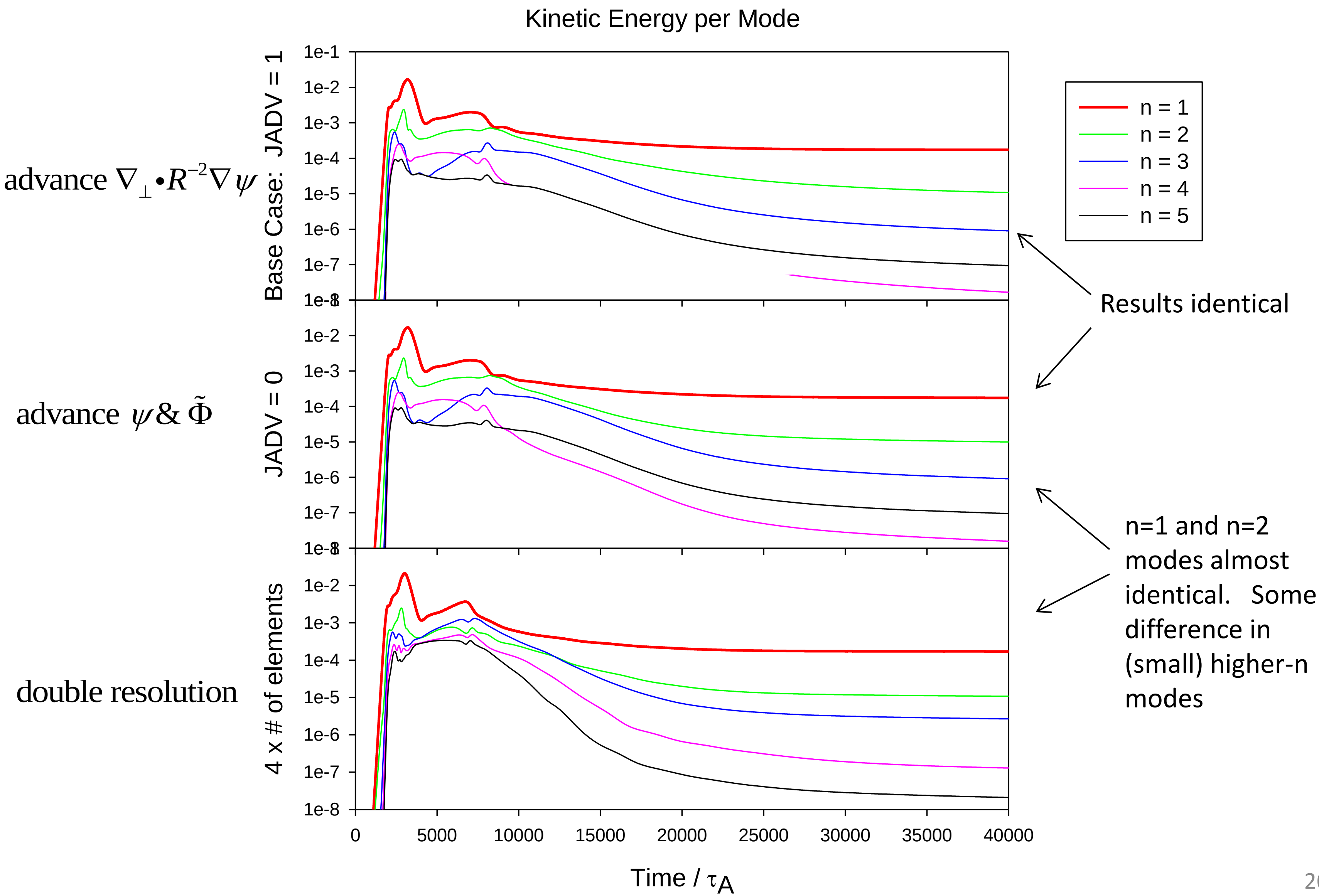
$$F \equiv F_0 + R^2\nabla \bullet \nabla_\perp f$$

$f' \equiv \partial f/\partial\varphi \qquad \dot{\psi} \equiv \partial\psi/\partial t$

$$[a,b] = [\nabla a \times \nabla b \bullet \nabla\varphi] = \frac{1}{R}(a_Z b_R - a_R b_Z)$$

$$(a,b) = \nabla a \bullet \nabla b = a_R b_R + a_Z b_Z$$

Convergence Tests



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- However, the driven flow is the dominant term in the Temperature evolution equation and dominates over the thermal conductivity.

- The net effect is to keep the central temperature (and resistivity) flat so that the resistive steady state is such as to preserve the self-organized state with q = 1 + ε in a central volume.

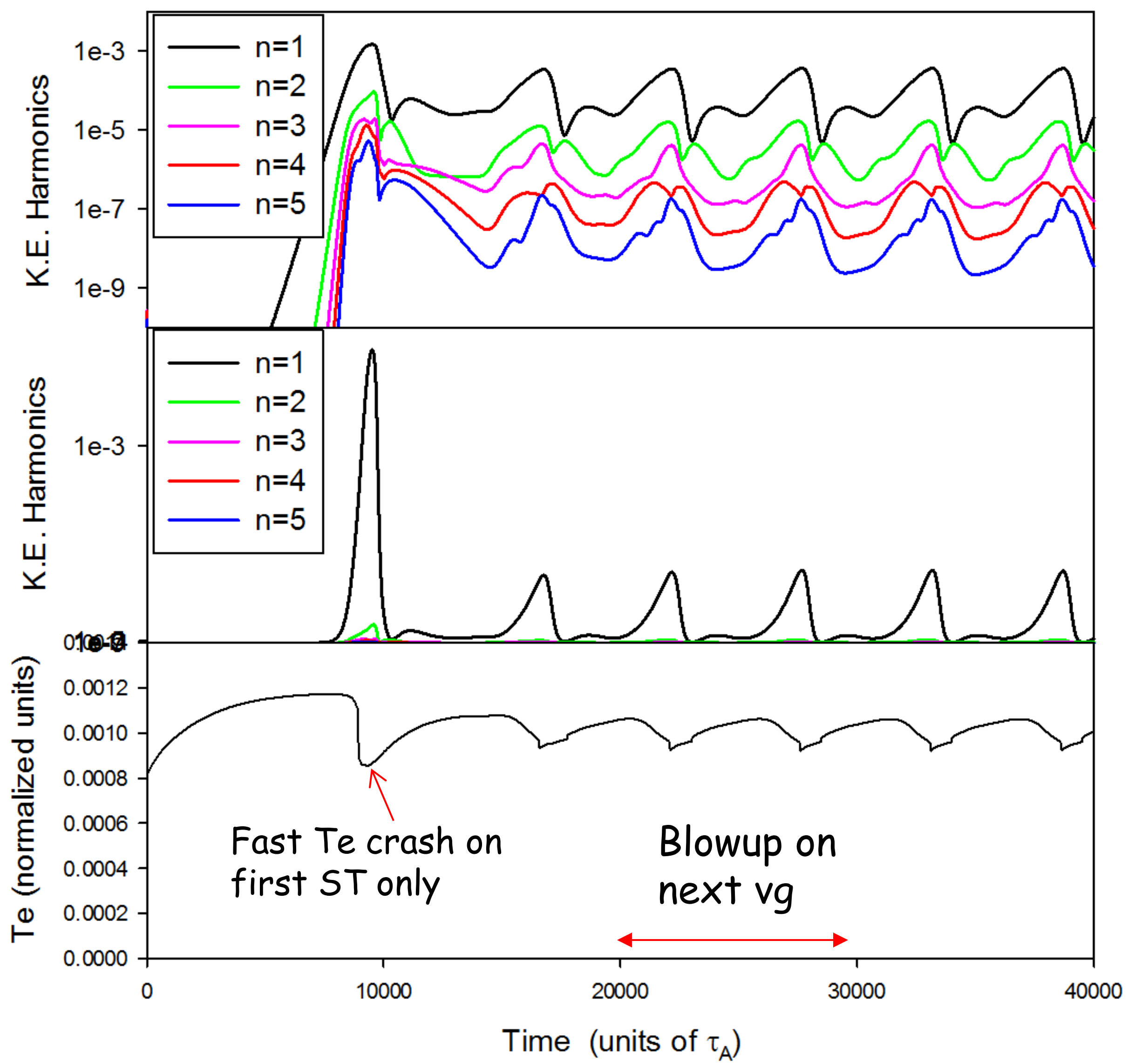
To do

- dependence on resistivity and neoclassical effects
- dependence on form of thermal conductivity profile $\kappa_\perp$ and heating source *S*
- dependence on beta (more systematic)
- dependence on 2F terms
- dependence on size of $\kappa_{||}$
- dependence on sheared rotation
- dependence on error fields

- relation to hybrid modes in DIII-D and ASDEX-U?
- can we combine transport and stability analysis?

Extra Viewgraphs

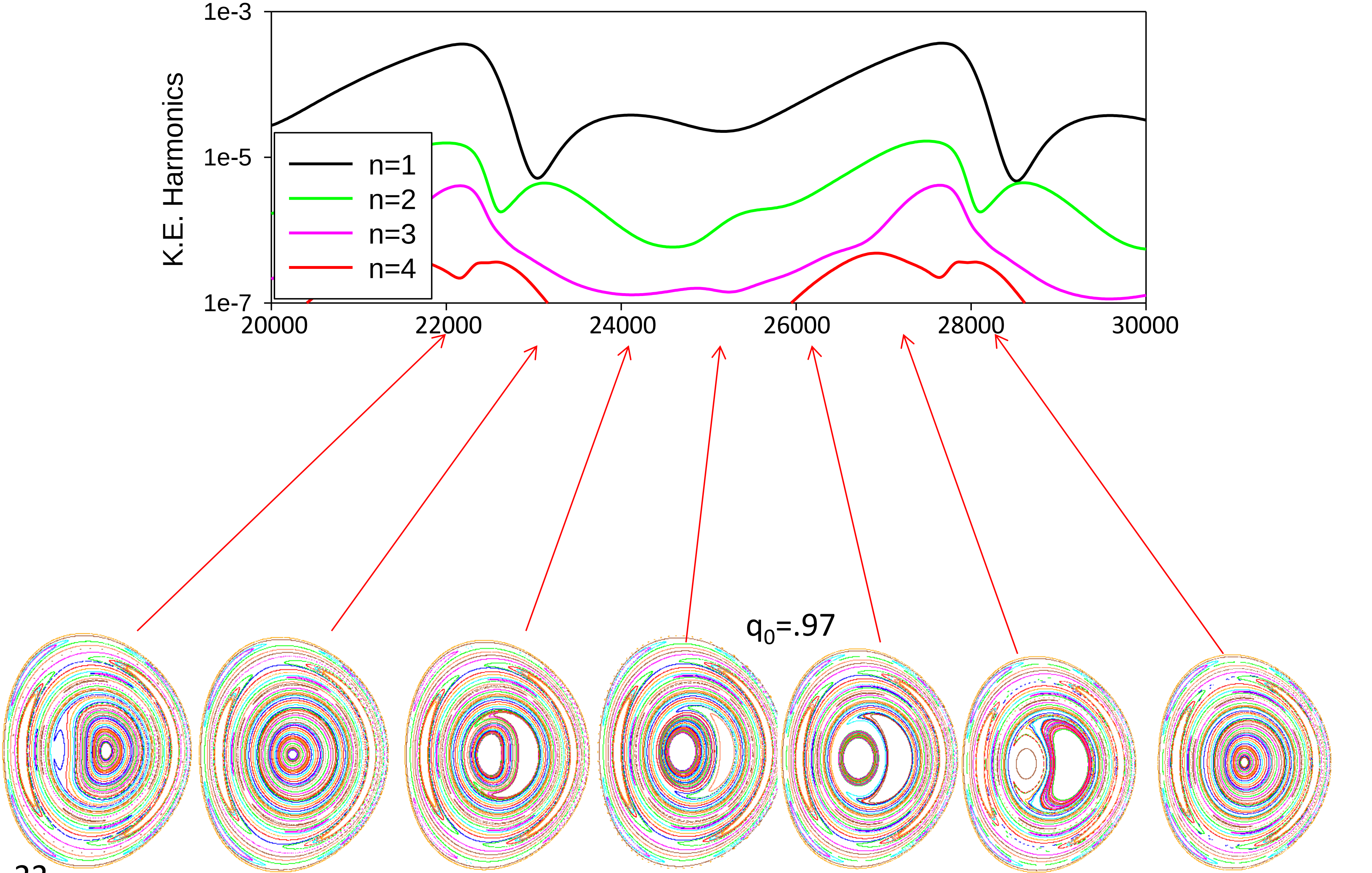
Typical periodic oscillations $S=10^6$, $\beta=.001$



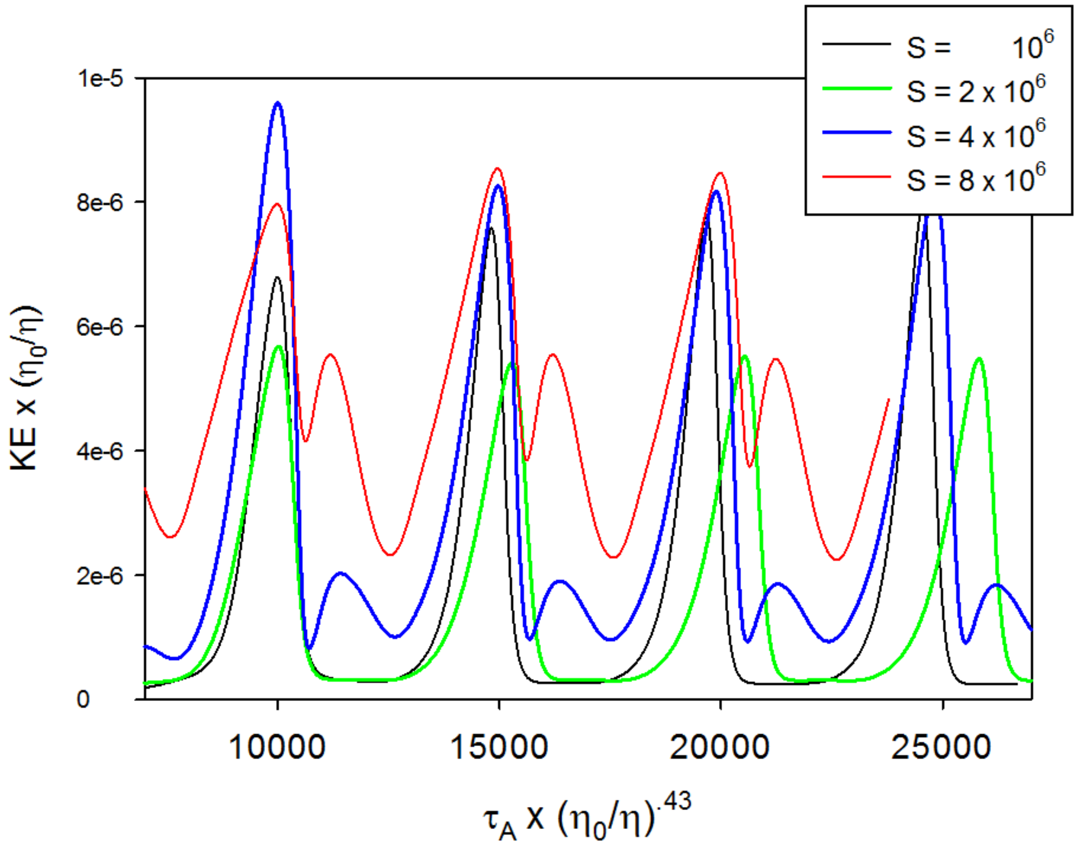
At low β, low S, resistive MHD plasma exhibits periodic oscillations, but does not show repeating fast Te crashes

CMOD 16G

Kadomsev complete reconnection $S=10^6$ β=.001



Resistivity scan: $\beta = .001$, no rotation



- period gets longer as η gets smaller as $\eta^{-0.43}$
- kinetic energy per event decreases as η

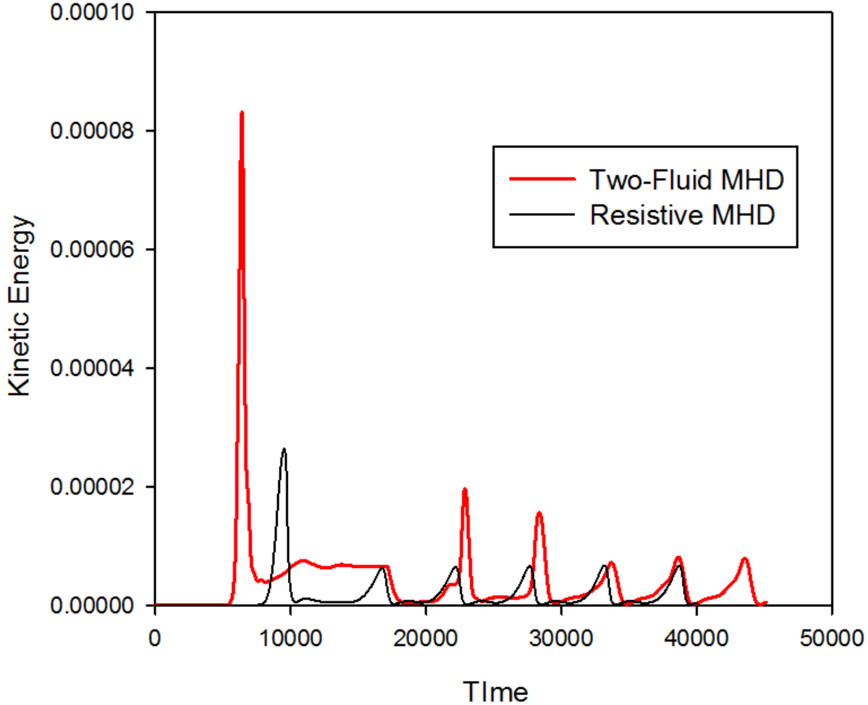
- Δq_0 decreases from 0.05 to < 0.01 as η decreases :

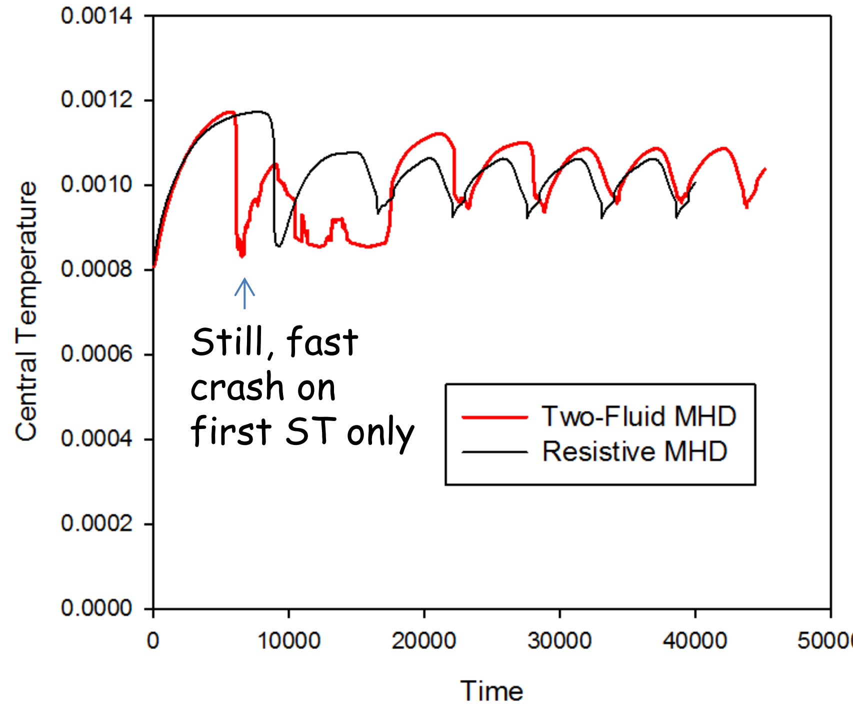
CMOD07 μ = 10 E-5
CMOE09 μ = 10
CMOD29 μ = 2.5
CMOD1E μ = 1.0

- Complete Kadomsev reconnection does not occur at high S
- Evidence of incomplete reconnection but fast Te crash at high S (2F)

Comparison of resistive MHD and 2F MHD

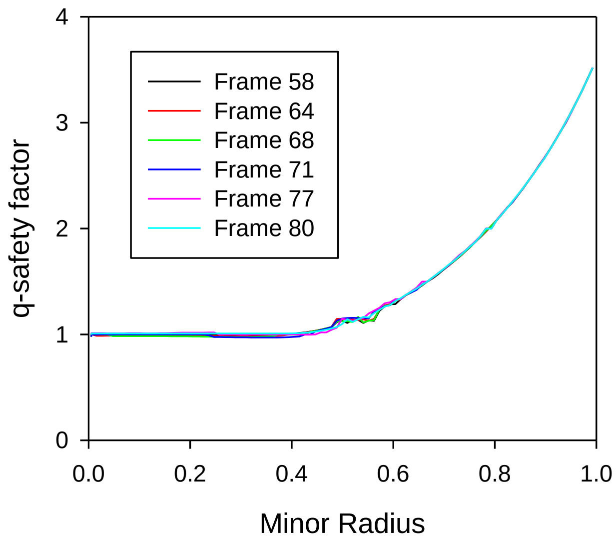
Two simulations with same β=.001 and S=10^6: with and without 2F terms

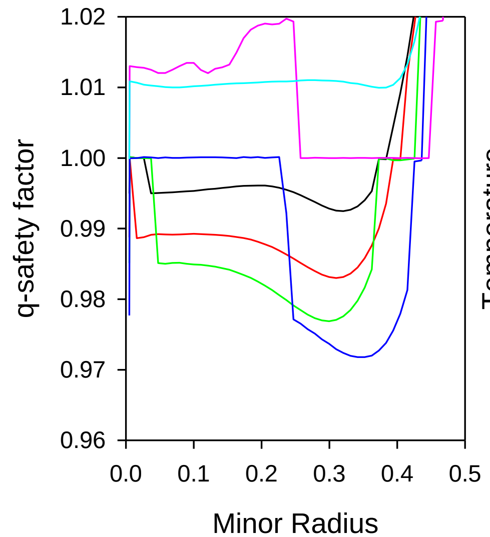


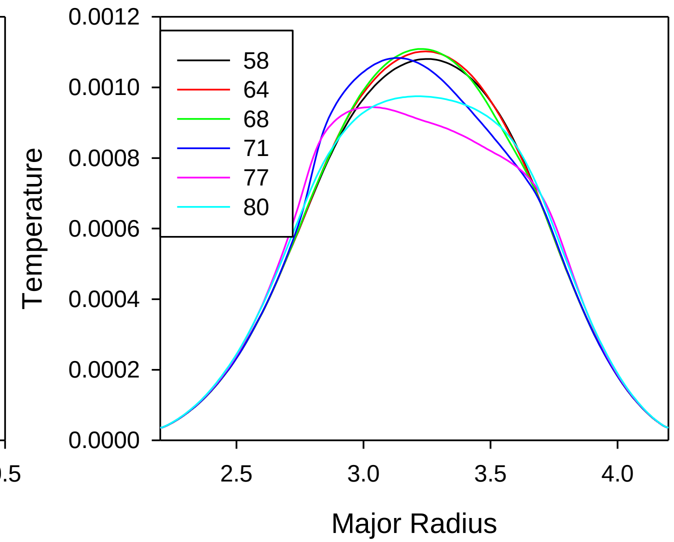


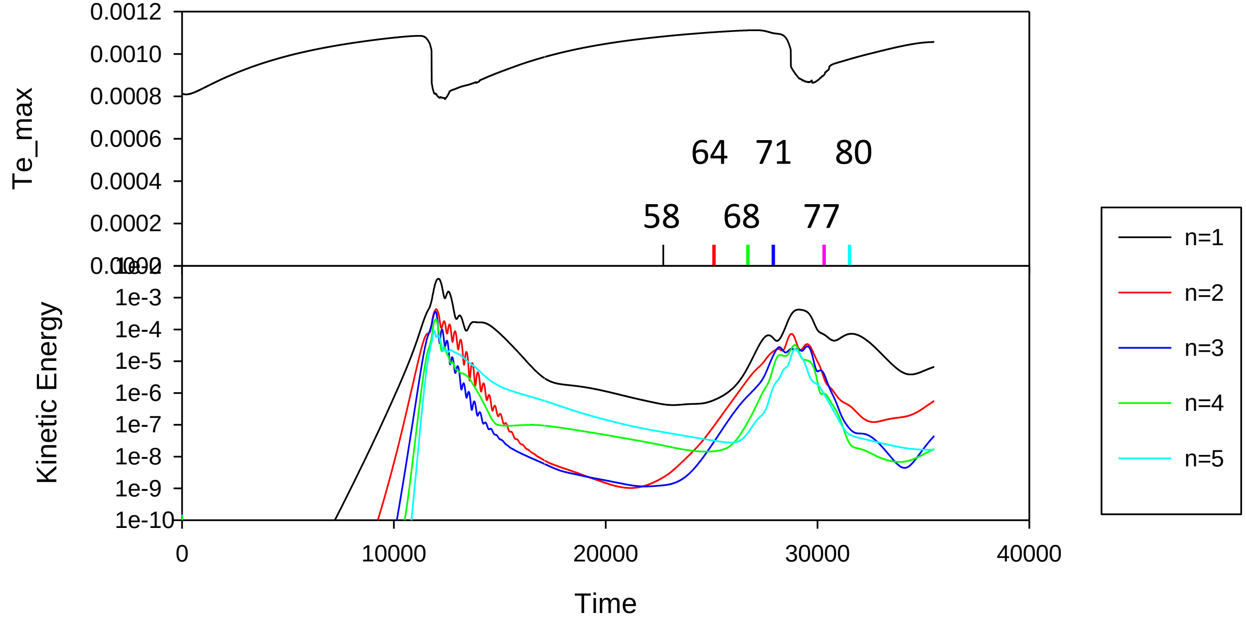
Two-fluid terms change the initial behavior, but not the long-time behavior of repeating sawteeth at low β and low S=10^6.

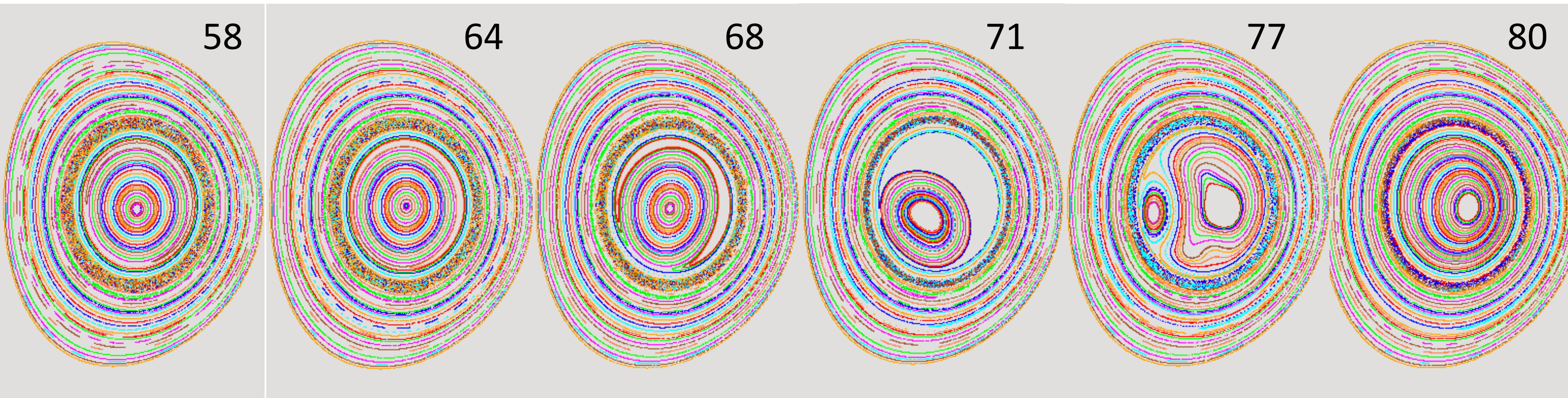
CMOD16G
CMOD25G

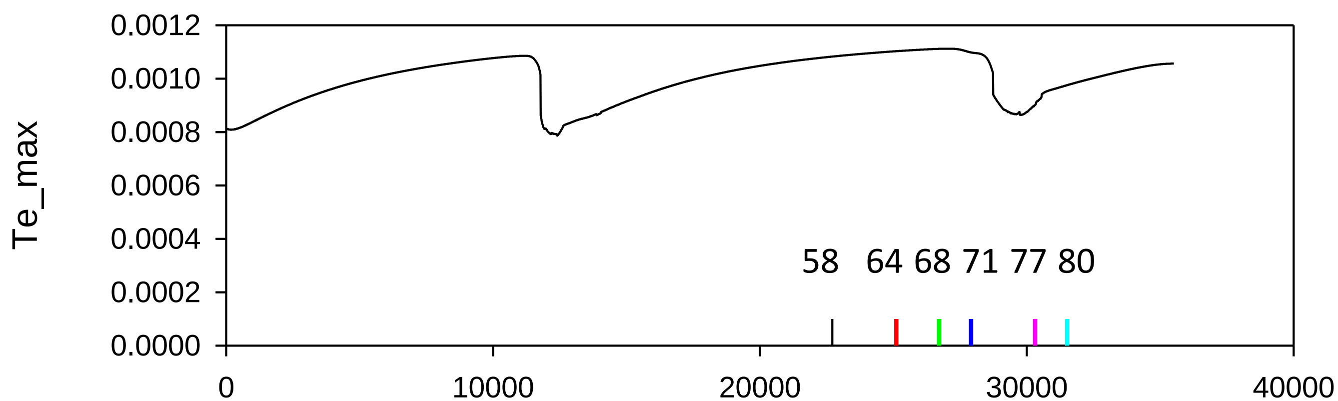

q-safety factor
Minor Radius
Frame 58
Frame 64
Frame 68
Frame 71
Frame 77
Frame 80
0
1
2
3
4
0.0
0.2
0.4
0.6
0.8
1.0

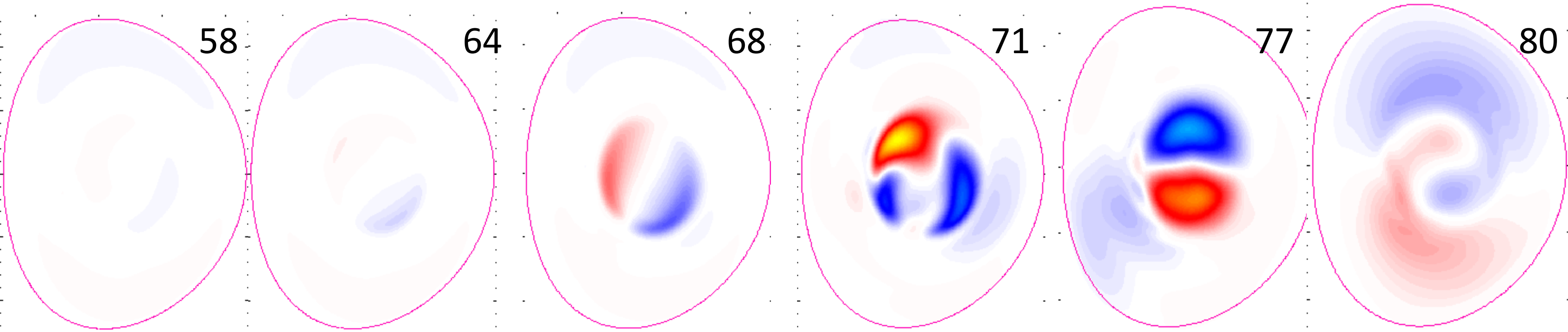

q-safety factor
Minor Radius
0.96
0.97
0.98
0.99
1.00
1.01
1.02
0.0
0.1
0.2
0.3
0.4
0.5


Temperature
Major Radius
58
64
68
71
77
80
0.0000
0.0002
0.0004
0.0006
0.0008
0.0010
0.0012
2.5
3.0
3.5
4.0


Te_max
Kinetic Energy
Time
64
71
80
58
68
77
0.0012
0.0010
0.0008
0.0006
0.0004
0.0002
1e-3
1e-4
1e-5
1e-6
1e-7
1e-8
1e-9
1e-10
0
10000
20000
30000
40000
n=1
n=2
n=3
n=4
n=5


58
64
68
71
77
80


Te_max
0.0012
0.0010
0.0008
0.0006
0.0004
0.0002
0.0000
58 64 68 71 77 80
0
10000
20000
30000
40000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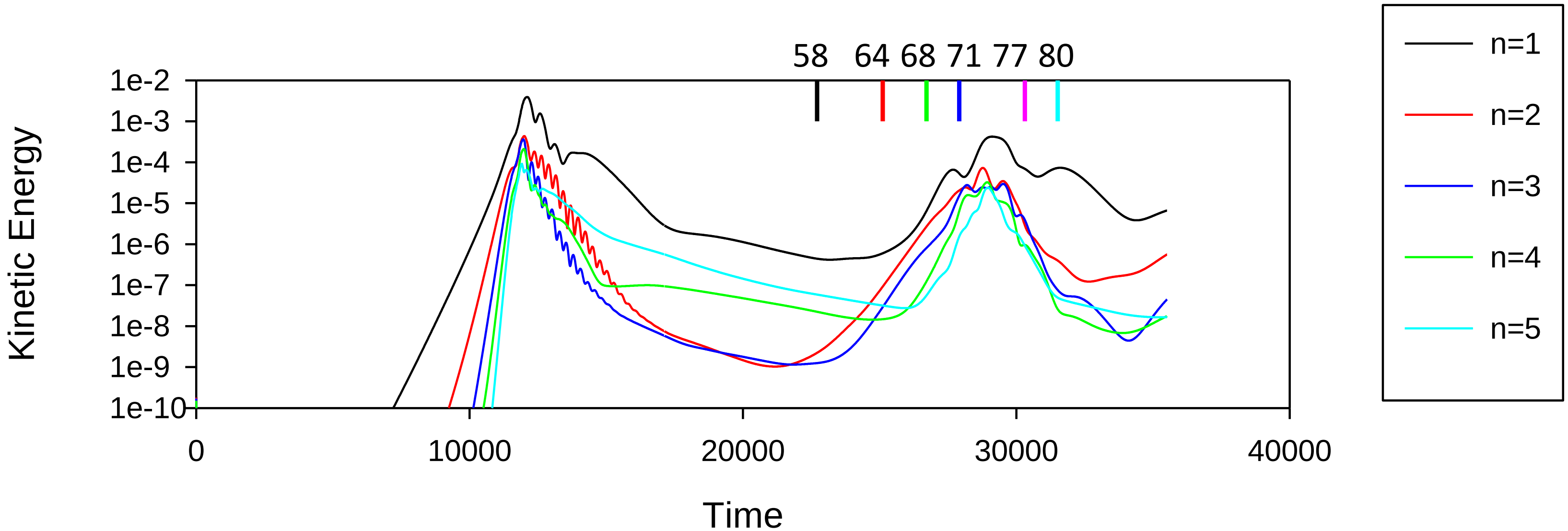
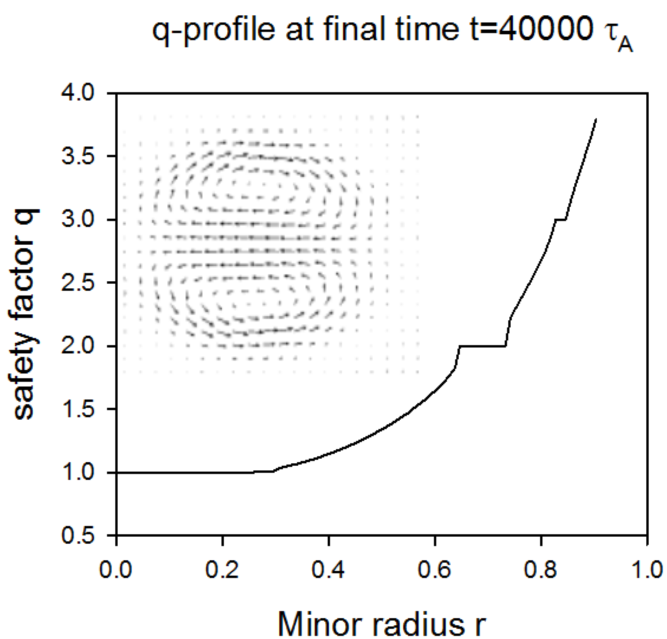
64
68
71
77
80


58 64 68 71 77 80
1e-2
1e-3
1e-4
1e-5
1e-6
1e-7
1e-8
1e-9
1e-10
Kinetic Energy
0
10000
20000
30000
40000
Time
n=1
n=2
n=3
n=4
n=5


q-profile at final time t=40000 τ_A
safety factor q
Minor radius r
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.2
0.4
0.6
0.8
1.0