Forced Magnetic Reconnection In Tokamak Plasmas – a new CEMM paradigm problem?

J.D. Callen, University of Wisconsin, Madison, WI 53706-1609 USA

Talk at CEMM meeting, Savannah, GA, November 15, 2015

Questions to be addressed:

1) Forced magnetic reconnection (FMR) issues for tokamaks?

2) Why are they important at present and in ITER?, and

3) What could CEMM do to resolve these issues?

Outline:

- **Forced magnetic reconnection status, issues for tokamaks**
- **Resonant 3-D effects—reconnection, penetration, islands, transport**
- **Examples of resonant 3-D effects in DIII-D:**

  FE — penetration of $\delta\vec{B}^{3D}$, spontaneous island forms, locked mode grows;

  NTMs — require seed island, then island grows on resistive time scale;

  RMPs — penetrate, transport increases at pedestal top without large islands.
- **Suggested paradigm problems (FEs, RMPs), issues for CEMM**

Present Theory Of Forced Magnetic Reconnection (FMR)

- **The forced magnetic reconnection problem involves study of the**
  response of a magnetized plasma to an externally-imposed resonant field,
  reconnection of magnetic field lines at the resonant surface, and
  resultant possible bifurcation into a magnetic island topology, mode locking.
- **Original theory[1] of dynamics of "Taylor problem" of forced magnetic reconnection was developed for sheared slab magnetic field:**
  current sheet forms at resonant surface on shear Alfvén time scale τ_A,
  resistive reconnection forms magnetic island on $\tau_A^{2/3}\tau_R^{1/3} = \tau_A\, S^{1/3}$ time scale,
  magnetic island grows to width determined by fully penetrated resonant field.
- **Determining effect of forced magnetic reconnection by a 2/1 field error (FE) on an ohmic tokamak plasma uses a cylindrical model:[2]**
  flow-screening prevents reconnection at rational surface,[2]
  strong field error bifurcates solution to magnetic island topology,
  diamagnetic flow effects influence error field required for bifurcation,[3]
  critical flow in tokamaks is the electron flow in $\perp$ (bi-normal) direction.[4,5]

---

[1] T.S. Hahm and R.M. Kulsrud, "Forced magnetic reconnection," Phys. Fluids **28**, 2412 (1985).

[2] R. Fitpatrick, "Interaction of tearing modes with external structures in cylindrical geometry," Nucl. Fusion **33**, 1049 (1993).

[3] A.J. Cole and R. Fitzpatrick, "Drift-magnetohyrodynamical model of error-field penetration in tokamak plasmas," Phys. Plasmas **13**, 032503 (2006).

[4] E. Nardon, P. Tamain, M. Bécoulet, G. Huysmans and F.L. Waelbroeck, "Quasi-linear MHD modelling of H-mode plasma response to resonant magnetic perturbations," Nucl. Fusion **50**, 034002 (2010).

[5] F.L. Waelbroeck, I. Joseph, E. Nardon, M. Bècoulet and R. Fitzpatrick, "Role of singular layers in the plasma response to resonant magnetic perturbations," Nucl. Fusion **52**, 074004 (2012).

FMR Studies Are More Complicated In Tokamak Plasmas

ANALYTIC THEORY:

- **Dynamical theory is needed to address temporal development and dynamical accessibility — not just time-asymptotic states.[2,5]**
- **Both electron and ion diamagnetic equilibrium flows are needed.**
- **Full tokamak geometry is needed, particularly in edge pedestal region where resonant magnetic perturbations (RMPs) are applied.**
- **Singular resistive reconnection layer widths $\boldsymbol{\delta_\eta}$ are much smaller.**
- **Toroidal torques competing with resonant field induced torques are different at edge — ion orbit and c-x losses, different transport.**

EXTENDED MHD CODE MODELING:

- **M3D-C1 and NIMROD mainly calculate linear response $\boldsymbol{\delta B^{\rho}_{\mathrm{n}}}$ now.**
- **FMR needs toroidal and poloidal flows, nonlinear evolution.**

Logic Of FMR Theory Involves Some Key Elements

- **Equilibrium axisymmetric magnetic field is basis of geometry.**

- **Local helical field geometry is useful near rational surface.**

- **Faraday's law with two-fluid Ohm's law for $\vec{E}$ yields equation for radial magnetic field perturbation induced by a single RMP.**

- **Magnetostatic two-fluid momentum equation that takes account of compressional Alfvén wave constraints is useful.**

- **Linear and nonlinear (island) responses to RMPs are important.**

- **Comprehensive plasma toroidal torque balance is needed.**

Faraday's Law Yields Equation For Radial Field Induced By Single 3-D Resonant Perturbation

- A two-fluid Ohm's law will be used for electric field $\vec{E} = \vec{E}_0 + \delta\vec{E}$:

$$\vec{E}_0 = -\vec{V}_0 \times \vec{B}_0 + \left[\vec{R}_{e0} + \vec{J}_0 \times \vec{B}_0 - \vec{\nabla} p_{e0} - \vec{\nabla}\cdot \overleftrightarrow{\pi}_{e0}\right]/n_{e0}e = -\vec{\nabla}\Phi_0(\psi_{\rm p}), \tag{1}$$

$$\begin{aligned}\delta\vec{E} = &- \vec{V}_0 \times \delta\vec{B} - \delta\vec{V} \times \vec{B}_0 + \delta(\vec{R}_e/n_e e) + \left[\delta\vec{J} \times \vec{B}_0 + \vec{J}_0 \times \delta\vec{B} - \vec{\nabla}\delta p_e - \vec{\nabla}\cdot \delta\overleftrightarrow{\pi}_e\right]/n_{e0}e \\ &- (\delta n_e/n_{e0})\left[\vec{J}_0 \times \vec{B}_0 - \vec{\nabla} p_{e0} - \vec{\nabla}\cdot \overleftrightarrow{\pi}_{e0}\right]/n_{e0}e.\end{aligned} \tag{2}$$

- Neglecting $\mathcal{O}\{|xq'/q|\}$ corrections, helical component of $\delta\vec{E}$ is

$$\vec{B}_{\rm hel}\cdot\delta\vec{E} = -\,\Omega_e^\alpha\,\psi_{\rm p}'(\vec{\nabla}\rho\cdot\delta\vec{B}) + (q-m/n)\frac{I}{qR^2}\psi_{\rm p}'(\vec{\nabla}\rho\cdot\delta\vec{V}_e) + \eta\,B_0\,\delta J_\| - \frac{\vec{B}_0\cdot[\vec{\nabla}\delta p_e + \vec{\nabla}\cdot\delta\overleftrightarrow{\pi}_e]}{n_{e0}e}. \tag{3}$$

- The cross helical electron flow rotation frequency here is
  [$\alpha \equiv \zeta - (m/n)\,\theta$ is helical angle coordinate and $d\psi_{\rm p}/d\rho = RB_{\rm p}$]

$$\boxed{\Omega_e^\alpha \equiv \vec{\nabla}\alpha\cdot\vec{V}_{e0} = -\left(\frac{d\Phi_0}{d\psi_{\rm p}} - \frac{1}{n_{e0}e}\frac{dp_{e0}}{d\psi_{\rm p}} - \frac{0.71}{e}\frac{dT_{e0}}{d\psi_{\rm p}}\right) = \frac{1}{RB_{\rm p}}\left(E_\rho + \frac{1}{n_{e0}e}\frac{dp_{e0}}{d\rho} + \frac{0.71}{e}\frac{dT_{e0}}{d\rho}\right),} \tag{4}$$

  which is more complete than the electron $\perp$ flow frequency $\omega_{\perp e}$ that is usually cited[5] and used in forced magnetic reconnection studies, because this Ω_e^α includes the effects of the electron thermal force induced by the usually neglected radial electron temperature gradient in $\vec{R}_e \equiv n_e e\eta\vec{J} - 0.71 n_e \vec{\nabla}_\| T_e$.

Faraday's Law Yields Equation For Radial Field Induced By Single Resonant 3-D Perturbation (cont'd)

- **Final evolution equation for radial field perturbation $\delta\hat{B}^\rho(\rho,t)$ is**

$$\boxed{\left.\frac{\partial}{\partial t}\right|_{\psi_{\rm p}} \delta\hat{B}^\rho - in\,\Omega_e^\alpha\,\delta\hat{B}^\rho - \frac{\eta}{\mu_0}\,\overline{\nabla}^2\delta\hat{B}^\rho \simeq\; ik_\|(x)\,B_{\rm t0}\,\delta\hat{V}_e{}^\rho,} \qquad \boxed{\delta\hat{B}^\rho \equiv -\,\frac{i\,k_\theta\,\delta\hat{\psi}}{R_0},} \qquad (5)$$

which is an inhomogeneous parabolic (diffusive) partial differential equation in ρ, t for either the FSA radial magnetic field $\delta\hat{B}^\rho$ or associated flux $\delta\hat{\psi}$.

- **The magnetic field diffusion is caused by the plasma resistivity:**

$$\eta B_{\rm t0}\,\overline{\delta J}_\| \equiv \left\langle e^{in\alpha}\frac{B_0\,\eta\,\delta J_\|}{\psi_{\rm p}'}\right\rangle \simeq \frac{\eta}{\mu_0}\,\frac{B_{\rm t0}^2}{I\psi_{\rm p}'}\left[\frac{1}{\varepsilon}\frac{\partial}{\partial\rho}\left(\varepsilon\,\overline{g}^{\rho\rho}\,\frac{\partial\,\delta\hat{\psi}}{\partial\rho}\right) - m^2\,\overline{g}^{\theta\theta}\,\delta\hat{\psi}\right] \equiv \frac{\eta}{\mu_0}\,\frac{q\,\overline{\nabla}^2\delta\hat{\psi}}{\rho R_0}. \qquad (6)$$

- **Right side of Eq. (5) represents advection of $\delta\vec{B}$ with flow $\delta\vec{V}$:**

$$i\,k_\|(x)\,B_{\rm t0}\,\delta\hat{V}_e^\rho \;\equiv\; \left\langle e^{in\alpha}\,(\vec{B}_0\cdot\vec{\nabla})\,(\vec{\nabla}\rho\cdot\delta\hat{\vec{V}}_e)\right\rangle. \qquad (7)$$

- **Solutions of Eq. (5) for $\delta\hat{B}^\rho(\rho,t)$ have the following properties:**

  1) away from the rational surface, advection of $\delta\hat{B}^\rho$ with $\delta\hat{V}^\rho$ to lowest order;

  2) when Ω_e^α is small, magnetic reconnection occurs in singular layer of width δ_η;

  3) some radial diffusion of $\delta\hat{B}^\rho$ is induced at all $\rho \Longrightarrow \overline{\delta J}_\|$, flutter transport.

Magnetostatic Two-fluid Momentum Equation Is Useful

- **Magnetostatic quasi-equilibrium of tokamak plasma is governed by**

  MHD force balance after compressional Alfvén waves have been eliminated; the appropriate annihilator is $\vec{B}\cdot\vec{\nabla}\times$ which yields a vorticity equation.

- **General magnetostatic MHD quasi-static force balance equation is**

$$\boxed{-\vec{\nabla}\cdot\left[\vec{B}\times\frac{\rho_m}{B^2}\left(\frac{\partial\vec{V}}{\partial t}+(\vec{V}\cdot\vec{\nabla})\vec{V}+\frac{\vec{\nabla}\cdot\overleftrightarrow{\Pi}}{\rho_m}\right)\right]=\vec{B}\cdot\vec{\nabla}\left(\frac{J_\parallel}{B}\right)+\vec{\nabla}\cdot\left(\frac{\vec{B}\times\vec{\nabla}P}{B^2}\right).} \quad (8)$$

- **Using $\boxed{\delta\vec{V}\equiv(1/B^2)\vec{B}\times\vec{\nabla}\delta\phi}$ and the gyroviscous cancellation due to $\vec{\nabla}\cdot\overleftrightarrow{\Pi}_\wedge$, the lowest order linearized vorticity equation is**[6]

$$\boxed{\vec{\nabla}\cdot\left[\frac{\rho_m}{B_0^2}\left(\frac{\partial}{\partial t}+\Omega_E^\alpha\frac{\partial}{\partial\alpha}\right)\vec{\nabla}_\perp\delta\phi\right]=\vec{B}_0\cdot\vec{\nabla}\left(\frac{\delta J_\parallel}{B_0}\right)+\delta\vec{B}\cdot\vec{\nabla}\left(\frac{J_{\parallel 0}}{B_0}\right)+\vec{\nabla}\cdot\left(\frac{\vec{B}_0\times\vec{\nabla}\delta P}{B_0^2}\right),} \quad (9)$$

  $\Omega_E^\alpha\equiv\vec{\nabla}\alpha\cdot\vec{E}_0\times\vec{B}_0/B_0^2\simeq-d\Phi_0/d\psi_{\rm p}=E_\rho/RB_{\rm p}$ is $\vec{E}_0\times\vec{B}_0\perp$ rotation frequency.

- **Eq. (9) can be used to determine usual**[7,8] **Δ', plus $\delta\vec{V}^\rho$, $\Delta'_{\rm layer}$, $\Delta'_{\rm ext}$.**

---

[6] See Eq. (19) in S.E. Kruger, C.C. Hegna, and J.D. Callen, "Generalized reduced magnetohydrodynamic equations," Phys. Plasmas **5**, 4169 (1998).
[7] H.P. Furth, J. Killeen and M.N. Rosenbluth, "Finite-Resistivity Instabilities of a Sheet Pinch," Phys. Fluids **6**, 459 (1963).
[8] C.C. Hegna and J.D. Callen, "Stability of tearing modes in tokamak plasmas," Phys. Plasmas **1**, 2308 (1994).

Response To Imposed Resonant $\delta \hat{B}^\rho \to \delta B_\rho$ Is Dynamic

- **3-D magnetic perturbation near a rational surface is governed by**

  $$\frac{\partial \delta B_\rho}{\partial t} - i\,\Omega_e^\alpha \delta B_\rho + \frac{\eta}{\mu_0}\overline{\nabla}^2 \delta B_\rho = (\vec{B}_0\cdot\vec{\nabla})\delta V_e^\rho;$$

  in $\tau_{\rm Ap} \sim 10^{-6}$s sheet current forms at $\rho_{m/n}$, with minimal reconnection if Ω_e^α is large, but if $|\Omega_e^\alpha| \lesssim 10^4$/s, at $t \sim 10^{-4} - 10^{-3}$ s δB_ρ "penetrates" in a resistive layer δ_η.

- **When $\mathrm{w} \equiv 4(\delta B_\rho L_S/k_\theta B_0)^{1/2} > \delta_\eta$, the island width w is governed by modified Rutherford eq. (MRE):**

  $$\frac{d\mathrm{w}}{dt} \simeq \frac{\eta}{\mu_0}\left[\Delta' + \frac{\sqrt{\epsilon}\,\beta_{\rm p}}{\mathrm{w}}\frac{L_S}{L_P} + \Delta'_{\rm ext} - \frac{\mathrm{p}\,\delta J_\|}{\mathrm{w}^3}\right];$$

  "drives" are $\sim \beta_{\rm p}/L_P$ for NTMs or $\Delta'_{\rm ext} \propto 2/L_{\delta B} > 0$ from applied RMPs, but damped by $\Delta' \simeq -2m$ and FLR, FBW ion polarization currents ($\mathrm{p}\,\delta J_\|$).

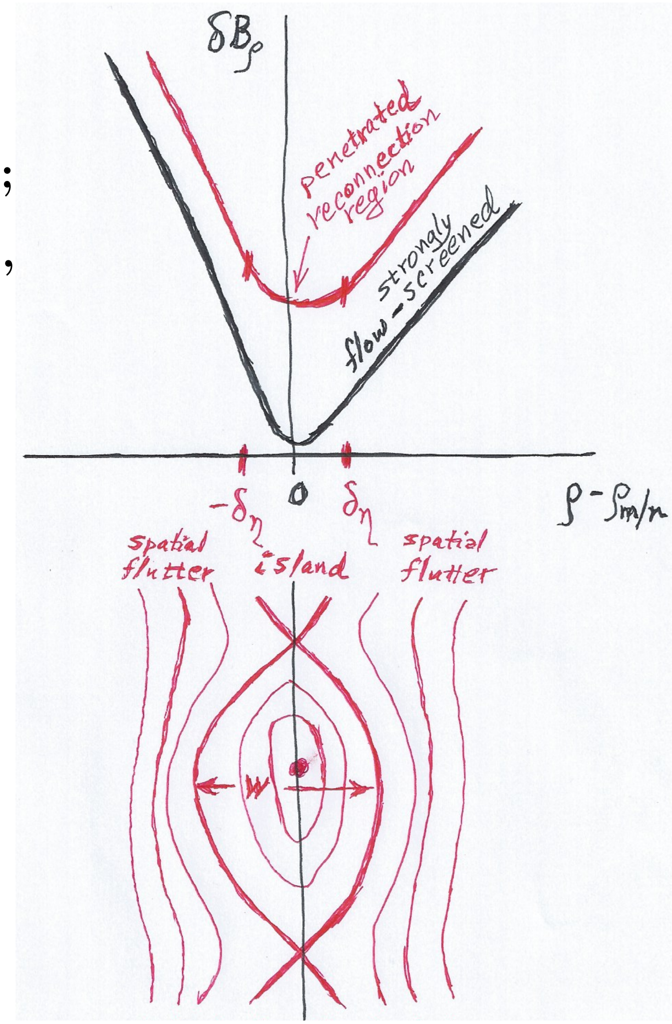

Figure 1: **Schematic of $\delta B^{\rm plasma}_{\rho\, m/n}$ and field lines in vicinity of rational surface.**

3-D Fields $\delta\vec{B}^{3D}$ Introduce Toroidal Torques On Plasma

- **Toroidally symmetric magnetic fields in tokamaks $\Longrightarrow$ no torques:**
  NBI etc. torques are balanced by Reynolds stress torque from fluctuations.[9]

- **3-D fields break symmetry and introduce toroidal torques:**[9]
  toroidal variation of $|\vec{B}|$ produces ion neoclassical toroidal viscosity (NTV)[10,11]
  $\Longrightarrow$ counter-current torque $\langle R\,\hat{\vec{e}}_\zeta\cdot\vec{\nabla}\cdot\overleftrightarrow{\pi}_{\|i}\rangle$ due to ripple, field errors, RMPs;
  3-D fields resonant on $q = m/n$ rational surfaces induce Maxwell stress
  $\Longrightarrow$ co-current torque $\langle R\,\hat{\vec{e}}_\zeta\cdot\overline{\delta J_\| \times \delta B_\rho}\rangle$ due to 3-D electron collision effects;[12]
  changes in fluctuation-induced Reynolds stress torques due to 3-D fields are usually most influenced by changes in $\vec{E}\times\vec{B}$ shear flow on zonal flows.[13]

- **Approximate toroidal torque balance in pedestal is thus**[9.11]

$$I_\Omega \frac{\partial\,\Omega_t}{\partial t} = -\underbrace{\langle R\,\hat{\vec{e}}_\zeta\cdot\vec{\nabla}\cdot\overleftrightarrow{\pi}_{\|i}\rangle}_{\text{NTV}} + \underbrace{\langle R\,\hat{\vec{e}}_\zeta\cdot\overline{\delta J_\| \times \delta B_\rho}\rangle}_{\text{Maxwell stress}} - \underbrace{\frac{1}{V'}\frac{\partial}{\partial\rho}(V'\Pi_{i\rho\zeta})}_{\text{Reynolds stress}} + \underbrace{\langle R\,\hat{\vec{e}}_\zeta\cdot\vec{S}_{\rm p}\rangle}_{\text{mom. sources}},$$

$I_\Omega \equiv m_i n_i \langle R^2\rangle$ is moment of inertia and Ω_t is rotation frequency of plasma.

[9] J.D. Callen, A.J. Cole, & C.C. Hegna, "Toroidal flow and radial particle flux in tok. plasmas," Phys. Pl. **16**, 082504 (2009); Errat. **20**, 069901 (2013).
[10] K.C. Shaing, "Magnetohydrodynamic-activity-induced toroidal momentum dissipation," Phys. Plasmas **10**, 1443 (2005).
[11] J.D. Callen, topical review paper on "Effects of 3D Magnetic Perturbations on Toroidal Plasmas," Nucl. Fusion **51**, 094026 (2011).
[12] J.D. Callen, A.J. Cole & C.C. Hegna, "Resonant-magnetic-perturbation-induced plasma transport in H-mode pedestals," Phys. Pl. **19**, 112505 (2012).
[13] M. Leconte et al., "Drive of a mesocale Vortex-Flow pattern by coupling to Zonal-Flows in presence of RMPs," H-mode 2015 workshop, Garching.

Plasma Toroidal Rotation Frequency $\Omega_{\rm t}$ Depends On E_ρ

- Radial force balance of tokamak plasma yields definition of $\Omega_{\rm t}$:

$$0 = \vec{\nabla}\rho \cdot [n_{i0}q_i(\vec{V}_i \times \vec{B}_0 - \vec{\nabla}\Phi_0) - \vec{\nabla}p_{i0}] \implies \boxed{\Omega_{\rm t} \equiv R\,\hat{\vec{e}}_\zeta \cdot \vec{V}_i \simeq \frac{1}{RB_{\rm p}}\left[E_\rho - \frac{1}{(n_{i0}e)}\frac{dp_{i0}}{d\rho}\right] + qV_i \cdot \vec{\nabla}\theta.}$$

- Since $\Omega_{\rm t}$ depends on the radial electric field $E_\rho \equiv -\, d\Phi_0/d\rho$, the torque balance is equivalently an equation for E_ρ and net torques $T_{s\zeta} \equiv R\,\hat{\vec{e}}_\zeta \cdot \vec{F}_{{\rm orce}\,s} = -\,RB_{\rm p}n_sq_sV_{s\rho}$ are due to non-ambipolar fluxes:

$$I_\Omega \frac{\partial\,\Omega_{\rm t}}{\partial t} = \textstyle\sum_s T_{s\zeta}(\Omega_{\rm t}) = -\,RB_{\rm p}\sum_s q_s\Gamma_s^{\rm na}(E_\rho) \quad \implies 0 \text{ yields ambipolar } E_\rho\,.$$

- The primary non-ambipolar (na) fluxes in the pedestal are[9,11]

  ion due to NTV $T_{i\zeta}^{\rm NTV} \equiv -\,\langle R\hat{\vec{e}}_\zeta \cdot \vec{\nabla} \cdot \overleftrightarrow{\pi}_{\|i}\rangle \simeq -\,I_\Omega\,\mu_{i\|}(\delta B_\|/B_0)^2(\Omega_{\rm t} - \Omega_{*i})$,

  electrons, $T_{e\zeta}^{\rm Maxwell} \equiv \langle R\hat{\vec{e}}_\zeta \cdot \overline{\delta J_\| \times \delta B_\rho}\,\rangle \simeq -\,I_\Omega\,\mu_{e\zeta}(\delta B_\rho/B_0)^2\omega_{\perp e}, \quad \omega_{\perp e} \equiv \vec{V}_{e0} \cdot \vec{\nabla}\alpha$,

  ion flux due to fluctuation-induced Reynolds stress $\implies$ radial diffusion of $\Omega_{\rm t}$,

  ion flux due to ion orbit losses near separatrix $\langle R\hat{\vec{e}}_\zeta \cdot \vec{S}_{\rm p}\rangle \simeq -\,RB_{\rm p}J_r^{\rm orbit\,loss}$.

- Ion na fluxes decrease $\Omega_{\rm t}$, E_ρ, while electron fluxes increase them.

- Net ambipolar density flux is[9] $\Gamma \equiv \Gamma_e^{\rm amb} + \Gamma_e^{\rm na}(E_\rho) = \Gamma_i^{\rm amb} + \Gamma_i^{\rm na}(E_\rho)$.

Non-ambipolar Ion & Electron Fluxes Are Equal At $E_\rho^{\rm amb}$

- E_ρ needed[14,15] for ambipolar density flux depends on $\kappa \propto \langle \delta \hat{B}_\rho \rangle^2$:

$\kappa \ll 1 \;\to$ ion root, $\quad \kappa \sim 1 \;\to$ ambipolar root, $\quad \kappa \gg 1 \;\to$ electron root.

[14] J.D. Callen, C.C. Hegna and A.J. Cole, "Magnetic-flutter-induced pedestal plasma transport," Nucl. Fusion **53**, 113015 (2013).
[15] J.D. Callen, "Pedestal Structure without and with 3D Fields," Contrib. Plasma Phys. **54**, 484 (2014).

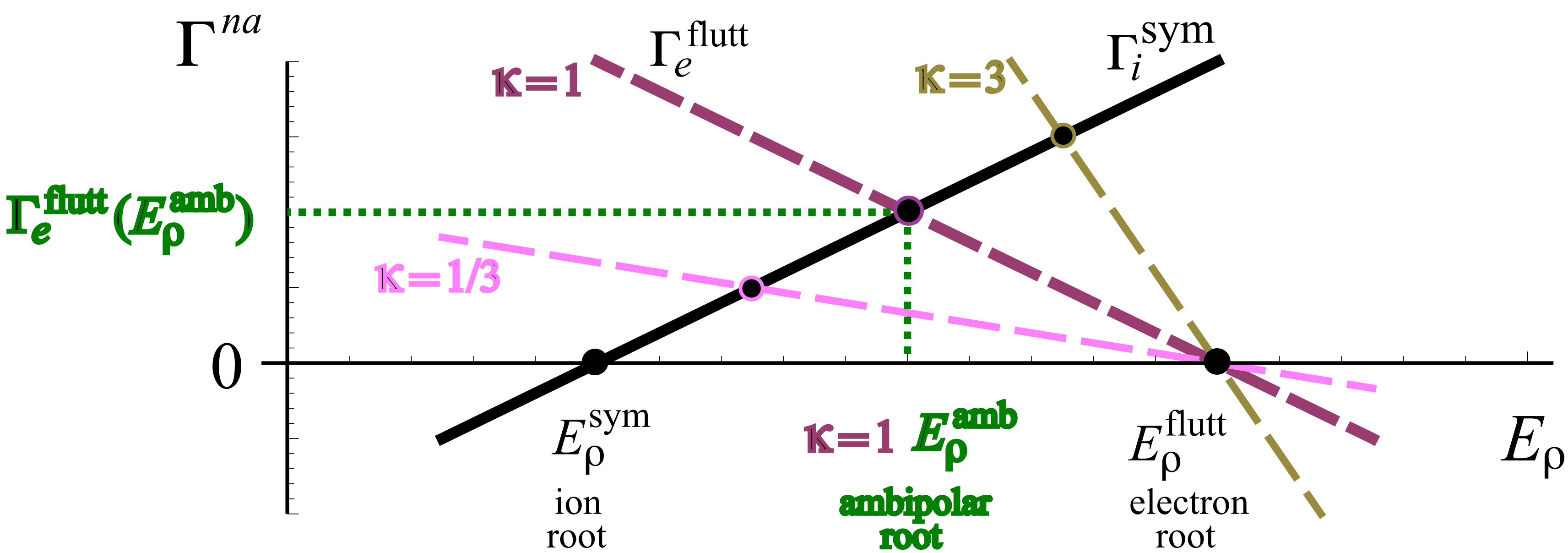

Figure 2: Dependence of electron and ion non-ambipolar density fluxes on the radial electric field. The dotted lines indicate the radial electric field and ambipolar density flux $\Gamma_e^{\rm flutt}(E_\rho^{\rm amb})$ at ambipolar root $E_\rho^{\rm amb}$ for[14] $\kappa \equiv (T_i/T_e) D_{et}^{\rm flutt}/D_i^{\rm na} = 1$.

Torque Balance Yields Bifurcation To 3-D $\delta\vec{B}$ Penetration

- Toroidal torques are exerted on the plasma by all non-ambipolar ($\Gamma_s^{\rm na}$) density fluxes,[9] including inertial ones ($\psi'_{\rm p} \equiv d\psi_{\rm p}/d\rho = RB_{\rm p}$):

$$m_i n_i \langle R^2\rangle \frac{\partial \Omega_{\rm t}}{\partial t} = e\psi'_{\rm p}\big[\Gamma_e^{\rm na}(E_\rho) - \Gamma_i^{\rm na}(E_\rho)\big] + \cdots, \quad \Gamma_e^{\rm 3D}(E_\rho) \propto \omega_{\perp e}\,|\delta B^{\rm plasma}_{\rho\, m/n}|^2 \sim \frac{\omega_{\perp e}\,|\delta B^{\rm vac}_{\rho\, m/n}|^2}{\Delta'^{\,2} + (\omega_{\perp e}\tau_\delta)^2}.$$

- Torque balance is in equilibrium when non-ambipolar fluxes of electrons $\Gamma_e^{\rm na}(E_\rho)$ and ions $\Gamma_i^{\rm na}(E_\rho)$ are equal $\Longrightarrow$ ambipolar E_ρ.

- Time scale for $\Omega_{\rm t}(E_\rho)$ to reach equilibrium is estimated by taking account of the radial force balance equation, $\Omega_{\rm t} = E_\rho/RB_{\rm p} + \cdots$:

$$m_i n_i \langle R^2\rangle \frac{\partial \Omega_{\rm t}}{\partial t} = e\,\Gamma_e^{\rm 3D}(E_\rho)\,\psi'_{\rm p} + \cdots = -\,m_i n_i\,\mu_{e\zeta}^{\rm 3D}\langle R^2\rangle\,\omega_{\perp e} + \cdots,$$ in which (flutter model[12])

$\mu_{e\zeta}^{\rm 3D} \equiv D_{\rm et}^{3D}/\varrho_{S\rm p}^2 \sim |\delta B^{\rm plasma}_{\rho\, m/n}|^2$, $\varrho_{S\rm p} \equiv c_S/\omega_{cip}$ is the ion sound gyroradius in poloidal field $B_{\rm p}$,

$\omega_{\perp e} \equiv [E_\rho + (dp_e/d\rho + 0.71\,n_e\,dT_e/d\rho)/n_e e]/RB_{\rm p}$ is perpendicular electron flow frequency.[5]

For 3-D RMP effects[12] $D_{\rm et}^{\rm 3D} \gtrsim 0.2\,{\rm m^2/s}$, $\varrho_{S\rm p} \sim 0.02$ m $\Longrightarrow \tau_{e\zeta}^{\rm 3D} \sim 1/\mu_{e\zeta}^{\rm 3D} \lesssim 2$ ms.

- Thus, requirements and time scale for 3-D field penetration are:

for penetration of 3-D field at rational surface $|\Gamma_e^{\rm na}(E_\rho)| > |\Gamma_i^{\rm na}(E_\rho)|$, and

time scale for bifurcation to small $|\omega_{\perp e}|$ state may be $\tau_{e\zeta}^{\rm 3D} \sim 1/\mu_{e\zeta}^{\rm 3D} \lesssim$ ms.

Theory: What Can Happen After 3-D Field Penetration?

- Reconnection: Penetrated magnetic field lines in thin resistive layer[12] $\delta_\eta \simeq c_t L_S / k_\theta \lambda_e$ ($\simeq 2$ mm, $c_t \simeq 3$, RMPs) form a nascent magnetic island of width $w \sim \delta_\eta$ around the rational surface.

- Does this island grow? There are two possibilities (next viewgraph):

  if δ_η or initial "seed island" of width $w_{\rm init} \simeq 4\sqrt{\delta B^{\rm plasma}_{\rho\, m/n}(\rho_{m/n}) L_S / k_\theta B_{t0}}$ is larger than the ion banana width parameter $w_{\rm ib} \equiv \sqrt{\epsilon}\, \varrho_{\theta i}$, an island can grow, BUT,

  if $\delta_\eta < w_{\rm ib}$, island width is limited to $\sim \delta_\eta$ ($\Delta' < 0$, ion polarization currents damp), and $\delta B^{\rm plasma}_{\rho\, m/n}$ perturbation decays unless it is driven continuously.

- Evolution and transport: Then, m/n magnetic field perturbation $\delta B^{\rm plasma}_{\rho\, m/n}$ expands radially away from the initial $\sim \delta_\eta$ or $w_{\rm init}$ width:

  growing island ($\max\{\delta_\eta, w_{\rm init}\} > w_{\rm crit}$) — width grows on resistive time scale, and radial transport within expanding island region is effectively infinite, which causes the T_e profile to be flat within the island;

  limited island ($w \sim \delta_\eta < w_{\rm crit}$) — driven $\delta B^{\rm plasma}_{\rho\, m/n}$ remains constant at $q = m/n$, but may spread radially from δ_η region, and induce additional transport.

Island Growth Requires Layer Width δ_η OR Initial Island Width $w_{init} >$ banana width parameter w_{ib}[16,17]

[16] R.J. La Haye, R.J. G.L. Jackson, T.C. Luce, K.E.J. Olofsson, W.M. Solomon and F. Turco, "Insights Into m/n=2/1 Tearing Mode Stability Based on Initial Island Growth Rate in DIII-D ITER Baseline Scenario Discharges," paper O5.134 at 41st EPS Conference Berlin 2014.
[17] R.J. La Haye, review paper on "Neoclassical tearing modes and their control," Phys. Plasmas **13**, 055501 (2006).

- **Island growth rate dw/dt**

  is governed by the Modified Rutherford Equation (MRE) $dw/dt = \cdots$, which is

  negative (damping) if island width $w < w_{crit} \simeq 1.3\, w_{ib}$ due to $\Delta' < 0$ and FLR, FBW polarization current effects,

  but can be positive (growing) for $\Delta' > 0$ tearing modes or NTMs if $w > w_{crit} \simeq 1.3\, w_{ib}$.

- **Growth of w occurs if**

  layer width $\delta_\eta \gtrsim w_{crit}$ OR

  initial width $w_{init} \gtrsim w_{crit}$.

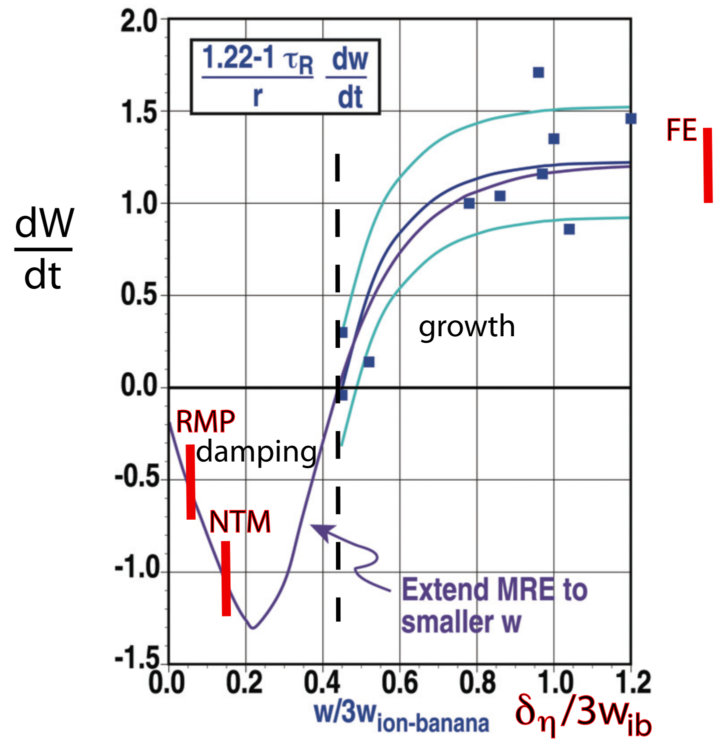

Figure 3: **MRE dw/dt indicates island growth for[14] $w \gtrsim w_{crit} \simeq 0.43 \times 3\, w_{ib} \simeq 1.3\, w_{ib}$, otherwise damping. Red bars are normalized layer widths $\delta_\eta / 3\, w_{ib}$ for DIII-D 3-D effects.**

Resonant Field Error (FE)[18] Can Grow Out Of Noise

- **Low n_e threshold for $\delta B_{\rho\,2/1}$ penetration, $|\Gamma_e^{\rm 3D}(E_\rho)| > |\Gamma_i^{\rm equil}(E_\rho)|$.**
- **2/1 mode "grows out of noise" because $\delta_\eta \simeq 3.3$ cm $>$ $\mathrm{w_{crit}} \simeq 1$ cm.**
- **2/1 locked mode $\delta B_{\rho\,2/1}$ grows on the resistive time scale.**

[18] R.J. La Haye, C. Paz-Soldan and E.J. Strait, "Lack of dependence on resonant error field of locked mode island size in ohmic plasmas in DIII-D," Nucl. Fusion **55**, 023011 (2015).

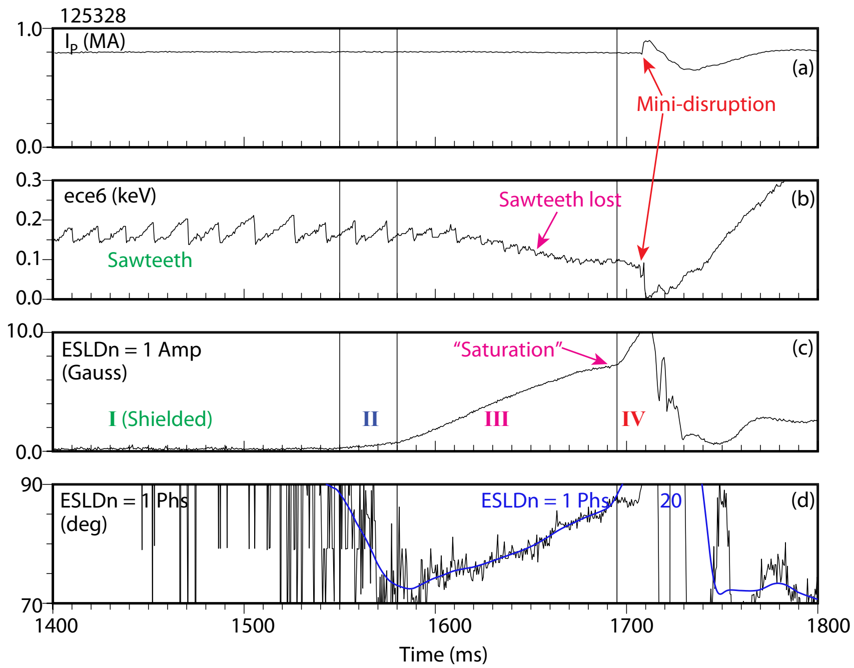

Figure 4: **Locked mode (III: detected by edge saddle loops, ESL) is induced by decreasing n_e, then grows out of noise spontaneously on resistive time scale.**

Neoclassical Tearing Mode (NTM)[19,17] Needs Big Seed

- **Plasma is metastable; a seed island is required[17,19] to excite NTM.**
- **If seed is too small, it decays because $\delta_\eta \sim 0.5$ cm $< \mathrm{w_{crit}} \sim 1.4$ cm; but if large enough (i.e., $\mathrm{w_{init}} > \mathrm{w_{crit}}$), it induces a growing island.**
- **NTM-island-induced $\delta B_{2/1}$ grows on the resistive time scale.[19]**

[19] Z. Chang , J.D. Callen, E.D. Fredrickson, R.V. Budny, C.C. Hegna, K.M. McGuire, M.C. Zarnstorff, and TFTR group, "Observation of Nonlinear Pressure-Gradient-Driven Tearing Modes," Phys. Rev. Lett. **74**, 4663 (1995).

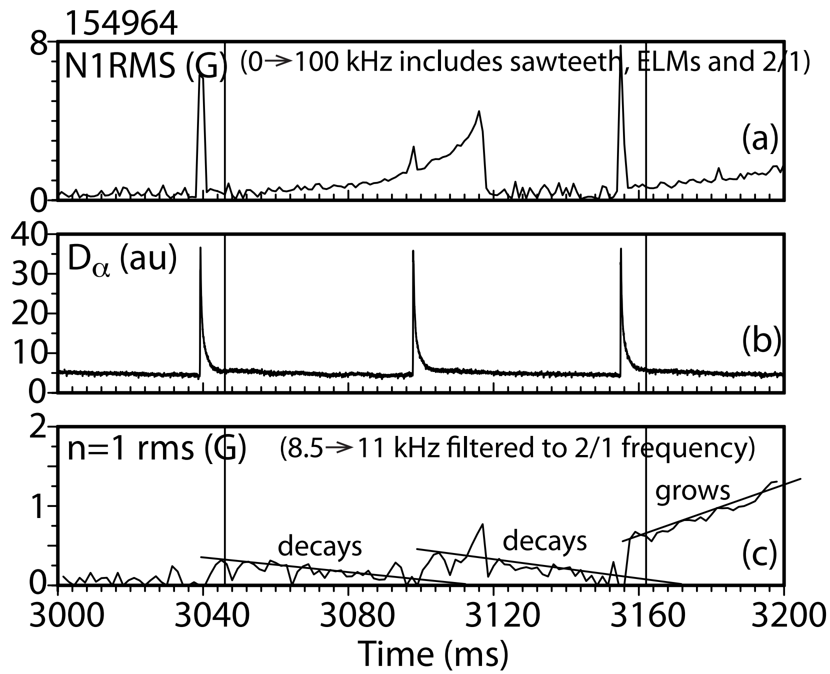

Figure 5: **First two ELM seeds are too small, last one causes growing NTM.**

RMPs Bifurcate Pedestal Into ELM-Suppressed State[20,21]

[20] C. Paz-Soldan et al., "Observation of a Multimode Plasma Response and its Relationship to Density Pumpout and Edge-Localized Mode Suppression," Phys. Rev. Lett. **114**, 105001 (2015).
[21] R. Nazikian et al., "Pedestal Bifurcation and Resonant Field Penetration at the Threshold of Edge-Localized Mode Suppression in the DIII-D Tokamak," Phys. Rev. Lett. **114**, 105002 (2015).

- **At $\gtrsim$ 4707 ms**

  inner wall magnetic resonant field $\delta\vec{B}^{3D}_{pol}$ jumps up, and "simultaneously" the CER-inferred ($\Delta t \simeq 5$ ms) edge rotation increases, because electric field E_ρ increases in response to non-ambipolar electron flux caused by increased $\delta\vec{B}^{3D}_{pol}$.

- **From 4730–4810 ms**

  rotation, $\delta\vec{B}^{3D}_{pol}$ and T_e gradient are about constant, but no magnetic islands form with widths >0.5 cm.

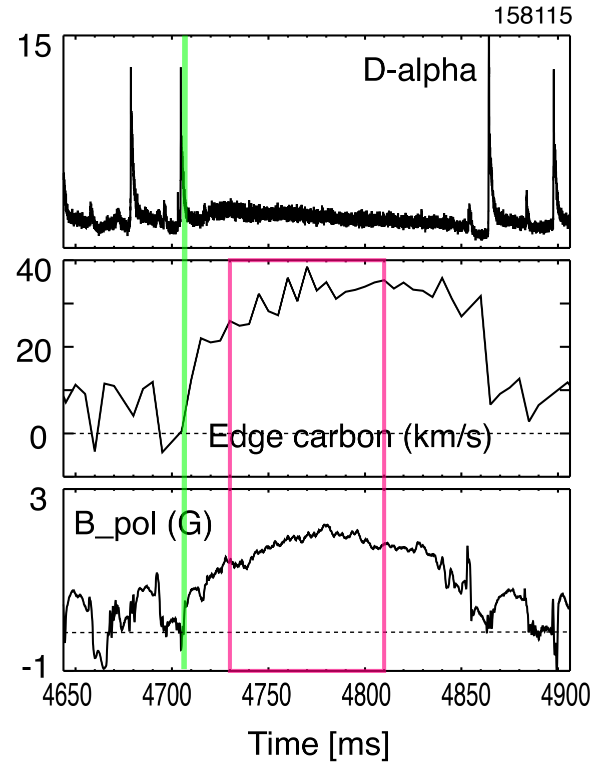

Figure 6: **Edge rotation and resonant $n = 2$ RMP-induced $\delta\vec{B}^{3D}_{pol}$ bifurcate into ELM-suppressed state at $\gtrsim$ 4707 ms for Fig. 2 case in Ref. 21.**

Possible Interpretation Of 3-D Resonant Field Effects

- Field lines reconnect in thin δ_η layers at rational surfaces, and lead to density pump-out throughout pedestal $\propto (\delta B^{\rm plasma}_{\rho\, m/n})^2$.
- Strong penetration occurs for large $\delta B^{\rm vac}_{\rho\, m/n}$, small $\omega_{\perp e}$ at $q = m/n$.
- Bifurcation to penetrated state can occur in $\tau^{\rm 3D}_{e\zeta} \sim 1/\mu^{\rm 3D}_{e\zeta} \lesssim$ ms.
- Induced nascent magnetic island can be unstable and grow if

  $\delta_\eta \gtrsim {\rm w}_{\rm crit} \simeq 1.3\, {\rm w}_{\rm ib}$ — large enough resistive layer width, or

  ${\rm w}_{\rm init} \gtrsim {\rm w}_{\rm crit} \simeq 1.3\, {\rm w}_{\rm ib}$ — large enough seed island,

  BUT, if ${\rm w}_{\rm init} \sim \delta_\eta < {\rm w}_{\rm crit}$, RMPs just continuously drive stable ${\rm w} \sim \delta_\eta$ islands.
- Region affected can expand radially away from $\delta_\eta, {\rm w}_{\rm init}$ at $q = m/n$

  with growing $\delta B^{\rm plasma}_{\rho\, m/n} \propto {\rm w}(t)^2$ if island is growing, but

  with $\sim$ constant $\delta B^{\rm plasma}_{\rho\, m/n}$ on rational surface if driven $\max\{{\rm w}\} \sim \delta_\eta$.
- Radial plasma transport in possibly radially expanding region is

  effectively infinite within growing island region which causes flat T_e profile, but may be caused by flutter, 3-D or $\omega_{\rm ExB}$ affected transport if $\max\{{\rm w}\} \sim \delta_\eta$.

FMR Is Important Process For Tokamak Plasmas

- **Major programmatic thrust is disruption control, which requires**
  understanding forced magnetic reconnection (FMR) processes that lead to locked modes via field errors (FEs), NTMs, and ELM suppression via RMPs.

- **Analytic-based theory is being developed; it needs to be tested and work with M3D-C1 and NIMROD studies of FMR processes.**

- **FMR studies are logical next steps for extended MHD codes:**

  study evolution from linear δB^{ρ} studies into nonlinear island states,

  begin coping with poloidal and toroidal flow evolution,

  figure out how to couple extended MHD, kinetic, transport for 3-D effects,

  provide a target case for unified extended MHD, kinetic, transport models.

Field Errors And RMPs Are Good Paradigm Problems

- $m/n = 2/1$ **field errors (FEs) are good focus for initial FMR studies:**

  low T_e ($\sim$ 250 eV) ohmic (OH) plasmas where $S \lesssim 10^7$ where causes δ_η to be larger than FLR, FBW effects and modes can grow out of noise, and

  since 2/1 modes are resonant at about the half radius, the mode coupling effects are likely to be small,

  plasma pressure is small for OH plasmas so finite β effects are likely small and

  plasma response to slowly increasing δB^ρ (or decreasing n_e) is good test.

- **Ultimate tests will be provided by pedestal responses to RMPs:**

  due to significant geometry effects in pedestal near separatrix,

  finite mode coupling and $\beta'_{\rm p}$ effects,

  significant FLR and FBW effects,

  multiple m resonant modes present simultaneously,

  toroidal plasma rotation that varies strongly in radius, and

  challenge of predicting δB_ρ and q_{95} needed for ELM suppression and why no significant magnetic islands are produced.

Some Developments Are Needed For Extended MHD

- **General:**

  identify good cylindrical code case for benchmarking M3D-C1 and NIMROD,

  identify experiment-based test case and compare to FE experimental results,

  begin exploring developments needed for modeling RMP effects.

- **Theory:**

  finish developing theory for single resonant magnetic perturbation,

  begin developing theory of mode coupling and finite β effects on δB^{ρ}.

- **M3D-C1 and NIMROD:**

  begin cylindrical benchmarking case,

  begin including poloidal and toroidal flow effects,

  explore how to couple extended MHD, kinetic and transport effects.