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KINETIC MAGNETOHYDRODYNAMICS WITH COLLISIONAL AND TWO-FLUID EFFECTS*

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- A K-MHD SYSTEM THAT IS INTRINSICALLY QUASINEUTRAL AND CONSISTENT WITH MOMENTUM AND ENERGY CONSERVATION IS DISCUSSED HERE
- THIS SYSTEM INCLUDES COLLISIONAL AND TWO-FLUID EFFECTS, AS WELL AS EQUILIBRIUM ROTATION
- IN SUCH K-MHD SYSTEM, THE LINEARIZED DRIFT-KINETIC EQUATION ABOUT AN AXISYMMETRIC EQUILIBRIUM WITH FAST TOROIDAL FLOW IS SIMILAR TO THE TIME-DEPENDENT DRIFT-KINETIC EQUATION IMPLEMENTED IN THE DK4D CODE [Lyons, Jardin and Ramos, PoP 2015]

TWO-FLUID, KINETIC-MHD MODEL

- SINGLE ION SPECIES OF UNIT CHARGE ($e_i = -e_e = e$)
- QUASINEUTRAL PLASMA ($n_i = n_e = n$)
- ZERO LARMOR RADII LIMIT BUT FINITE ION SKIN DEPTH (LOW-β_i)
- NEGLIGIBLE ELECTRON INERTIA
- MEAN FLOW VELOCITY OF THE ORDER OF THE SOUND SPEED
- LOW BUT NOT NEGLIGIBLE COLLISIONALITY

ZERO-LARMOR-RADIUS, TWO-FLUID, EXTENDED-MHD SYSTEM

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \,, \qquad \mathbf{j} = \nabla \times \mathbf{B}$$

$$\mathbf{u}_i \equiv \mathbf{u} \,, \qquad \mathbf{u}_e = \mathbf{u} - \frac{\mathbf{j}}{en} \,, \qquad \frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = 0$$

$$\mathbf{E} = -\mathbf{u}_e \times \mathbf{B} + \frac{1}{en}\left(\mathbf{F}_e^{coll} - \nabla \cdot \mathbf{P}_e^{CGL}\right)$$

$$m_i n \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u}\right] - \mathbf{j} \times \mathbf{B} + \sum_{s=i,e} \nabla \cdot \mathbf{P}_s^{CGL} = 0$$

$$\mathbf{P}_s^{CGL} = p_{s\|}\mathbf{bb} + p_{s\perp}(\mathbf{I} - \mathbf{bb})$$

ZERO-LARMOR-RADIUS DRIFT-KINETIC EQUATIONS AND FLUID CLOSURES

$$\bar{f}_s(w_\parallel, w_\perp, \mathbf{x}, t) \;=\; (2\pi)^{-1} \int_0^{2\pi} d\alpha \;\, f_s(\mathbf{w}, \mathbf{x}, t)$$

with $\quad \mathbf{w} \;=\; \mathbf{v} \;-\; \mathbf{u}_s(\mathbf{x}, t) \;=\; w_\parallel \; \mathbf{b}(\mathbf{x}, t) \;+\; w_\perp \; [\cos\alpha \;\, \mathbf{e}_1(\mathbf{x}, t) + \sin\alpha \;\, \mathbf{e}_2(\mathbf{x}, t)]$

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$$\frac{\partial \bar{f}_s}{\partial t} \;+\; \Big(\mathbf{u}_s + w_\parallel \mathbf{b}\Big) \cdot \frac{\partial \bar{f}_s}{\partial \mathbf{x}} \;+\; \left[\frac{\mathbf{b} \cdot \Big(\nabla \cdot \mathbf{P}_s^{CGL} - \mathbf{F}_s^{coll}\Big)}{m_s n} - w_\parallel (\mathbf{bb}) : (\nabla \mathbf{u}_s) - \frac{w_\perp^2}{2} \mathbf{b} \cdot \nabla \ln B\right] \; \frac{\partial \bar{f}_s}{\partial w_\parallel} \;+$$

$$+ \; \frac{w_\perp}{2} \; \Big[(\mathbf{bb} - \mathbf{I}) : (\nabla \mathbf{u}_s) + w_\parallel \mathbf{b} \cdot \nabla \ln B\Big] \; \frac{\partial \bar{f}_s}{\partial w_\perp} \;=\; \sum_{s'} C_{ss'}[\bar{f}_s, \bar{f}_{s'}]$$

which provides the fluid closures

$$p_{s\parallel} \;=\; m_s \int d^3\mathbf{w} \;\, w_\parallel^2 \;\, \bar{f}_s \;, \qquad p_{s\perp} = \frac{m_s}{2} \int d^3\mathbf{w} \;\, w_\perp^2 \;\, \bar{f}_s$$

$$\mathbf{F}_s^{coll} \;=\; -\; e_s n \eta_{cl} \; \mathbf{j}_\perp \;+\; \left(m_s \int d^3\mathbf{w} \;\, w_\parallel \sum_{s'} C_{ss'}[\bar{f}_s, \bar{f}_{s'}]\right) \mathbf{b} \;, \qquad \eta_{cl} \;=\; \frac{2\nu_e m_e}{3(2\pi)^{1/2} e^2 n}$$

- THIS NON-LINEAR, TWO-FLUID, K-MHD SYSTEM IS OF THE "FULL-f" KIND, WITH DISTRIBUTION FUNCTIONS THAT CAN BE ARBITRARILY DIFFERENT FROM MAXWELLIANS. IT WILL BE LINEARIZED ABOUT A MAXWELLIAN NEAR-EQUILIBRIUM

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- **DEFINING** $n_s^{kin} \equiv \int d^3\mathbf{w}\ \bar{f}_s$ **,** $p_s \equiv (m_s/3) \int d^3\mathbf{w}\ w^2\ \bar{f}_s$ **[i.e.** $p_s = (p_{s\|} + 2p_{s\perp})/3$ **],** $q_{s\|} \equiv (m_s/2) \int d^3\mathbf{w}\ w^2\ w_\|\ \bar{f}_s$ **AND** $G_s^{coll} \equiv (m_s/2) \int d^3\mathbf{w}\ w^2 \sum_{s'} C_{ss'}[\bar{f}_s, \bar{f}_{s'}]$ **, THE** 1 **,** $w_\|$ **AND** w^2 **MOMENTS OF THE DRIFT-KINETIC EQUATIONS YIELD**

$$\frac{\partial n_s^{kin}}{\partial t} + \nabla \cdot (n_s^{kin} \mathbf{u}_s) = 0 \qquad \Rightarrow \qquad n_i^{kin} = n_e^{kin} = n$$

$$\int d^3\mathbf{w}\ w_\|\ \bar{f}_s = 0$$

$$\frac{3}{2}\left[\frac{\partial p_s}{\partial t} + \nabla \cdot (p_s \mathbf{u}_s)\right] + \mathbf{P}_s^{CGL} : (\nabla \mathbf{u}_s) + \nabla \cdot (q_{s\|} \mathbf{b}) = G_s^{coll}$$

INITIAL VALUE LINEAR ANALYSIS

- **WRITE THE STATE VECTOR** $[\bar{f}_i, \bar{f}_e, \mathrm{B}, n, \mathbf{u}] \equiv \Psi$ **AS**

$$\Psi(w_\parallel, w_\perp, \mathbf{x}, t) = \Psi_0(w_\parallel, w_\perp, R, Z) + \Psi_1(w_\parallel, w_\perp, \mathbf{x}, t)$$

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- LINEARIZE NEGLECTING TERMS QUADRATIC IN Ψ_1 AND SOLVE THE INITIAL VALUE PROBLEM FOR $\Psi_1(w_\parallel, w_\perp, \mathbf{x}, t)$

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- **LINEARIZE NEGLECTING TERMS QUADRATIC IN Ψ_1 AND SOLVE THE INITIAL VALUE PROBLEM FOR $\Psi_1(w_\parallel, w_\perp, \mathbf{x}, t)$**

- **IDEALLY, $\Psi_0(w_\parallel, w_\perp, R, Z)$ SHOULD BE AN AXISYMMETRIC EQUILIBRIUM OF THE COMPLETE SYSTEM. HOWEVER, IT IS DIFFICULT TO DERIVE SUCH AN EQUILIBRIUM ANALYTICALLY WHEN FAST ROTATION, COLLISIONS AND TWO-FLUID EFFECTS ARE INCLUDED**

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- **IN THIS CASE, WITH THE INITIAL CONDITION**

$$\Psi_1(w_\parallel, w_\perp, \mathbf{x}, 0) \;=\; \hat{\Psi}_{1,\mathrm{n}}(w_\parallel, w_\perp, R, Z)\; e^{i\mathrm{n}\zeta} \;\;,$$

THE TIME EVOLUTION OF THE LINEAR PERTURBATION IS

$$\Psi_1(w_\parallel, w_\perp, \mathbf{x}, t) \;=\; \Psi_{1,0}(w_\parallel, w_\perp, R, Z, t) \;+\; \Psi_{1,\mathrm{n}}(w_\parallel, w_\perp, R, Z, t)\; e^{i\mathrm{n}\zeta}$$

MAXWELLIAN, SINGLE-FLUID COLLISIONLESS EQUILIBRIUM WITH TOROIDAL FLOW

$$\mathbf{B}_0 \;=\; \nabla\psi \times \nabla\zeta \;+\; I(\psi)\;\nabla\zeta\;, \qquad \mathbf{j}_0 \;=\; \frac{dI}{d\psi}\,\nabla\psi \times \nabla\zeta \;-\; \Delta^*\psi\;\nabla\zeta$$

$$\mathbf{u}_0 \;=\; \Omega(\psi)\;R^2\;\nabla\zeta\;, \qquad (\mathbf{u}_0 \cdot \nabla)\mathbf{u}_0 \;=\; -\Omega^2\;R\;\nabla R\;, \qquad \nabla \cdot \mathbf{u}_0 \;=\; (\mathbf{b}_0\mathbf{b}_0) : (\nabla \mathbf{u}_0) \;=\; 0$$

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$$\nabla \times \mathbf{E}_0 \;=\; -\;\nabla \times (\mathbf{u}_0 \times \mathbf{B}_0) \;=\; 0\;, \qquad \nabla \cdot (n_0\mathbf{u}_0) \;=0$$

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$$\nabla\times\mathbf{E}_0 \;=\; -\;\nabla\times(\mathbf{u}_0\times\mathbf{B}_0) \;=\; 0\;, \qquad\qquad \nabla\cdot(n_0\mathbf{u}_0) \;=0$$

$$\bar{f}_{s0} \;=\; f_{Ms0} \;=\; \left(\frac{m_s}{2\pi}\right)^{3/2}\;\frac{n_0}{T_{s0}^{3/2}}\;\exp\left(-\frac{m_s w^2}{2T_{s0}}\right)$$

$$T_{s0} \;=\; T_{s0}(\psi)\;, \qquad n_0 \;=\; n_0(\psi,R) \;=\; N(\psi)\;\exp\left\{\frac{m_i R^2\Omega^2(\psi)}{2\;[T_{i0}(\psi)+T_{e0}(\psi)]}\right\}$$

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$$\mathbf{j}_0\times\mathbf{B}_0 \;=\; m_i n_0\;(\mathbf{u}_0\cdot\nabla)\mathbf{u}_0 \;+\; \nabla[n_0(T_{i0}+T_{e0})] \quad\Rightarrow\quad -\frac{1}{R^2}\left(I\frac{dI}{d\psi}+\Delta^*\psi\right) \;=\; \frac{\partial[n_0(T_{i0}+T_{e0})]}{\partial\psi}\;\Big|_R$$

LINEARIZED MAGNETOFLUID SYSTEM

$$\frac{\partial \mathbf{B}_1}{\partial t} \;=\; -\,\nabla\times\mathbf{E}_1\;, \qquad\qquad \mathbf{j}_1 \;=\; \nabla\times\mathbf{B}_1$$

$$\mathbf{E}_1 \;=\; -\,\mathbf{u}_0\times\mathbf{B}_1 \;-\; \mathbf{u}_1\times\mathbf{B}_0 \;+\; \frac{1}{en_0}\left(1-\frac{n_1}{n_0}\right)\left[\mathbf{j}_0\times\mathbf{B}_0-\nabla(n_0T_{e0})+en_0\eta_{cl0}\;\mathbf{j}_0\right]\;+$$

$$+\;\frac{1}{en_0}\left\{\mathbf{j}_0\times\mathbf{B}_1+\mathbf{j}_1\times\mathbf{B}_0-\nabla p_{e\perp 1}-\nabla\cdot\left[(p_{e\|1}-p_{e\perp 1})\mathbf{b}_0\mathbf{b}_0\right]+\mathbf{F}_{e1}^{coll}\right\}$$

$$\frac{\partial n_1}{\partial t} \;=\; -\,\nabla\cdot(n_0\mathbf{u}_1+n_1\mathbf{u}_0)$$

$$m_in_0\;\left[\frac{\partial \mathbf{u}_1}{\partial t}+(\mathbf{u}_1\cdot\nabla)\mathbf{u}_0+(\mathbf{u}_0\cdot\nabla)\mathbf{u}_1\right]\;+\;m_in_1\;(\mathbf{u}_0\cdot\nabla)\mathbf{u}_0\;=$$

$$=\;\mathbf{j}_0\times\mathbf{B}_1\;+\;\mathbf{j}_1\times\mathbf{B}_0\;-\;\sum_{s=i,e}\left\{\nabla p_{s\perp 1}+\nabla\cdot\left[(p_{s\|1}-p_{s\perp 1})\mathbf{b}_0\mathbf{b}_0\right]\right\}$$

LINEARIZED DRIFT-KINETIC EQUATION

$$\frac{\partial \bar{f}_{s1}}{\partial t} + (\mathbf{u}_0 + w_\parallel\ \mathbf{b}_0) \cdot \frac{\partial \bar{f}_{s1}}{\partial \mathbf{x}} + \frac{\mathbf{b}_0}{m_s} \cdot \left(T_{s0} \nabla \ln n_0 + e_s \eta_{cl0}\ \mathbf{j}_0\right) \frac{\partial \bar{f}_{s1}}{\partial w_\parallel} + \frac{w_\perp}{2} \left(\mathbf{b}_0 \cdot \nabla \ln B_0\right) \left(w_\parallel \frac{\partial \bar{f}_{s1}}{\partial w_\perp} - w_\perp \frac{\partial \bar{f}_{s1}}{\partial w_\parallel} \right) =$$

$$= \Bigg\{ - \left[\mathbf{u}_{s1} + \frac{n_1}{n_0}\ w_\parallel\ \mathbf{b}_0 \right] \cdot \nabla \ln n_0 \ + \ \left[\left(\frac{3}{2} - \frac{m_s w^2}{2T_{s0}} \right) \mathbf{u}_{s1} + \left(\frac{5}{2} - \frac{m_s w^2}{2T_{s0}} \right) \ \frac{w_\parallel}{B_0}\ \mathbf{B}_1 \right] \cdot \nabla \ln T_{s0} \ +$$

$$+ \frac{w_\parallel}{n_0 T_{s0}}\ \mathbf{b}_0 \cdot \left[\nabla p_{s\parallel 1} - \left(p_{s\parallel 1} - p_{s\perp 1} \right) \nabla \ln B_0 + e_s (n_0 - n_1) \eta_{cl0}\ \mathbf{j}_0 - \mathbf{F}^{coll}_{s1} \right] \ + \ \frac{e_s \eta_{cl0} w_\parallel}{B_0 T_{s0}} (\mathbf{B}_1 - B_1 \mathbf{b}_0) \cdot \mathbf{j}_0 \ -$$

$$- \ \frac{m_s}{2T_{s0}} \ \left[w_\perp^2 \nabla \cdot \mathbf{u}_{s1} + \left(2w_\parallel^2 - w_\perp^2 \right) (\mathbf{b}_0 \mathbf{b}_0) : (\nabla \mathbf{u}_{s1}) + \left(\frac{2w_\parallel^2 - w_\perp^2}{B_0} \right) (\mathbf{b}_0 \mathbf{B}_1 + \mathbf{B}_1 \mathbf{b}_0) : (\nabla \mathbf{u}_{s0}) \right] \Bigg\} \ f_{Ms0} \ +$$

$$+ \ \sum_{s'} \left(C_{ss'}[f_{Ms0}, f_{Ms'0}] + C_{ss'}[f_{Ms0}, \bar{f}_{s'1}] + C_{ss'}[\bar{f}_{s1}, f_{Ms'0}] \right)$$

where

$$\mathbf{u}_{i1} \ = \ \mathbf{u}_1 \ , \qquad\qquad \mathbf{u}_{e1} \ = \ \mathbf{u}_1 \ - \ \frac{\mathbf{j}_1}{e n_0} \ + \ \frac{n_1\ \mathbf{j}_0}{e n_0^2}$$

$$p_{s\parallel 1} \ = \ m_s \int d^3\mathbf{w}\ \ w_\parallel^2\ \bar{f}_{s1} \ , \qquad\qquad p_{s\perp 1} = \ \frac{m_s}{2} \int d^3\mathbf{w}\ \ w_\perp^2\ \bar{f}_{s1}$$

$$\mathbf{F}^{coll}_{e1} = -\mathbf{F}^{coll}_{i1} \ = e \ \left(n_1 \eta_{cl0}\ \mathbf{j}_0 + n_0 \eta_{cl1}\ \mathbf{j}_0 + n_0 \eta_{cl0}\ \mathbf{j}_1 \right) + m_e \int d^3\mathbf{w}\ \ w_\parallel C_{ei}[\bar{f}_{e1}, f_{Mi0}]\ \mathbf{b}_0$$

SUMMARY

- A KINETIC-MHD MODEL IS PROPOSED TO ANALIZE THE LINEAR STABILITY OF AXISYMMETRIC EQUILIBRIA WITH FAST TOROIDAL FLOW, INCLUDING COLLISIONAL AND TWO-FLUID EFFECTS

- THE MAGNETOFLUID PART OF THE SYSTEM COMPRISES THE LINEARIZED FORMS OF THE FARADAY-OHM LAWS, THE CONTINUITY EQUATION AND THE MOMENTUM CONSERVATION EQUATION, THAT EVOLVE B_1, n_1 AND $\mathbf{u}_1$

- THE KINETIC PART YIELDS THE FLUID CLOSURES $p_{s\|1}$, $p_{s\perp 1}$ AND $\mathbf{F}_s^{coll}$ AS MOMENTS OF THE GYROPHASE-INDEPENDENT DISTRIBUTION FUNCTIONS $\bar{f}_{s1}$. THESE EVOLVE WITH LINEARIZED DRIFT-KINETIC EQUATIONS THAT ARE CONSISTENT WITH THE FLUID CONSERVATION LAWS