

Effect of rotation zero-crossing on plasma response to 3D magnetic perturbations

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Theory of ELM suppression by 3D fields still incomplete

- **External 3D magnetic perturbations are routinely used to mitigate or to suppress edge-localized modes (ELMs)**
- **Early theoretical work predicted that**
 - Vacuum fields would produce overlapping islands in edge
 - Stochastic transport would inhibit pedestal growth
- **Recent results inconsistent with formation of stochastic layer**
 - Electron temperature gradient not observed to decrease
 - Electron rotation predicted to screen vacuum islands in edge
- **Current theory requires on island opening only at pedestal top**
 - Observed that zero-crossing of electron rotation aligns with rational surface at top of pedestal during suppression
 - Theory predicts
 - Low rotation permits penetration of resonant field
 - Island arrests growth of pedestal height and width

Extended-MHD analysis can inform this theory

- **Comprehensive model needed for pedestal evolution across ELM-suppression bifurcation**
 - Extended MHD
 - Time-dependent (for evolution)
 - Nonlinear (for island saturation)
 - Two-fluid (for electron rotation physics)
 - Appropriate transport model, particularly for the momentum
- **Current research focuses on individual components of model**
- **Here, we explore how rotation profiles affect single-fluid M3D-C1 plasma response, including**
 - Resonant field
 - Non-resonant field
 - Observable quantities
 - Quasilinear electromagnetic torque

M3D-C1 [1] solves the extended MHD equations

- Three-dimensional
- Includes resistivity, density diffusivity, viscosity, & thermal conductivity
- Two-fluid effects (optional)
- Linear and nonlinear modes
- High-order, C^1 continuous finite element representation
- Mesh adapted to input equilibrium

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \vec{v}) = 0$$

$$m_i n_i \left[\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right] = \vec{J} \times \vec{B} - \nabla p - \nabla \cdot \Pi_i$$

$$\frac{\partial \vec{B}^p}{\partial t} = -\nabla \times \vec{E}$$

$$\vec{E} = \eta \vec{J} - \vec{v} \times \vec{B} + \frac{1}{n_e e} (\vec{J} \times \vec{B} - \nabla p_e)$$

$$\Pi = -\mu(\nabla \vec{v} + \nabla \vec{v}^t) + \Pi_i^{\parallel} + \Pi_i^{\wedge}$$

$$\begin{aligned} \frac{\partial p}{\partial t} + \vec{v} \cdot \nabla p + \Gamma p \nabla \cdot \vec{v} &= (\Gamma - 1)(\eta J^2 - \nabla \cdot \vec{q} - \Pi_i : \vec{v}) \\ &+ \frac{1}{n_e e} \vec{J} \cdot \left(\nabla p_e - \Gamma \frac{\nabla n_i}{n_i} p_e \right) \end{aligned}$$

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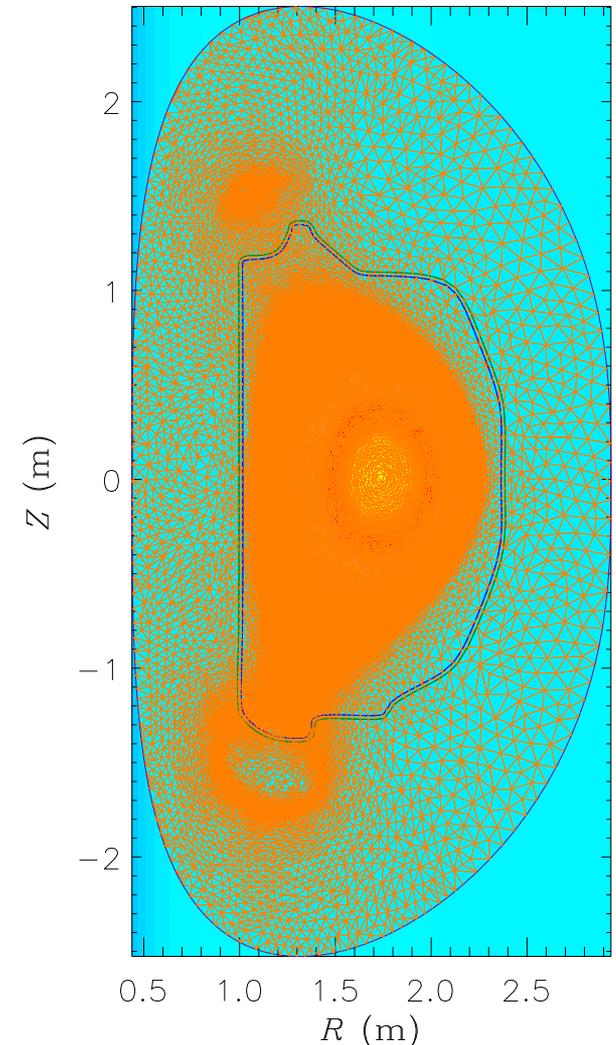
$$\vec{q}_{e,i} = -\kappa \nabla T_e - \kappa_{\parallel} \frac{\vec{B} \vec{B}}{B^2} \cdot \nabla T_e \quad \vec{q} = -\kappa \nabla (T_e + T_i) \quad [2]$$

[1] S. C. Jardin, et al., Comput. Sci. Discovery 5, 014002 (2012).

[2] N.M. Ferraro et al., Phys. Plasmas 23, 056114 (2016)

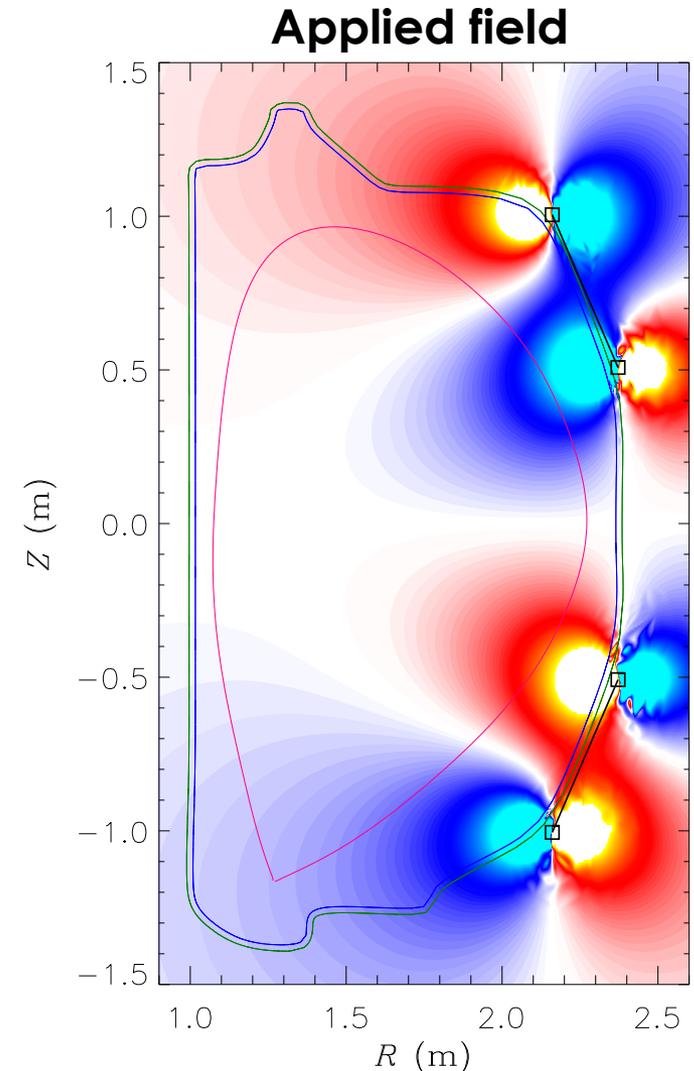
M3D-C1 allows for extended MHD simulations of the plasma response to applied 3D fields

- **Plasma response calculations presented here**
 - Linear $n=2$ (single toroidal mode number)
 - Single-fluid
 - Mesh adapted to equilibrium
 - Resistive wall model allows for free-boundary-like simulations
- Time-independent 3D equilibrium calculated in response to static perturbation field



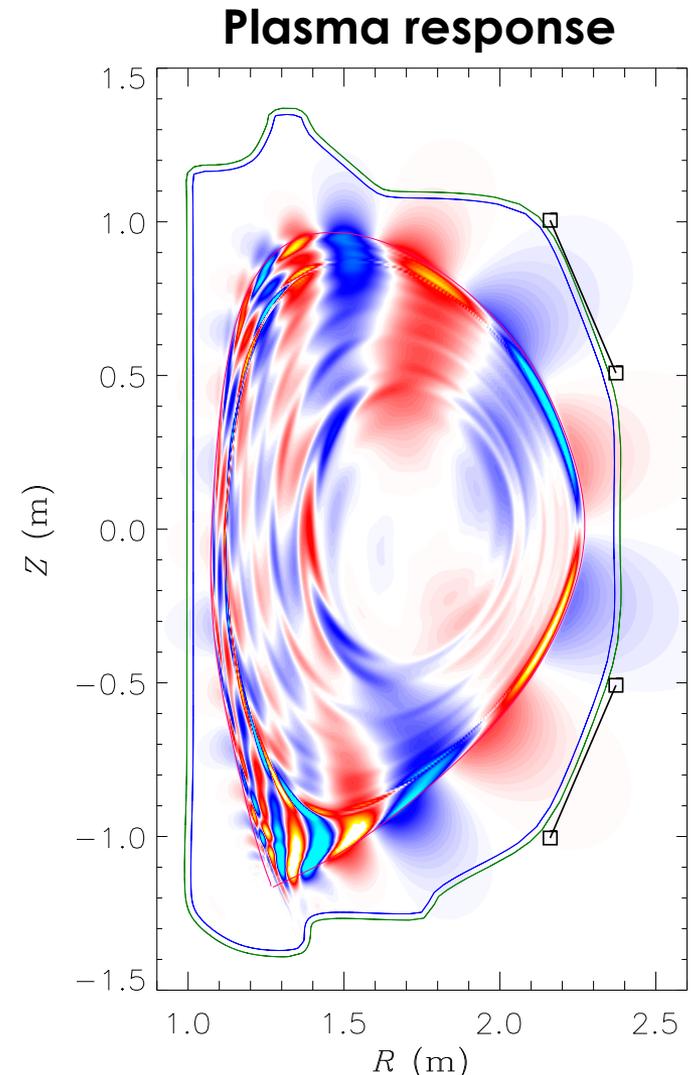
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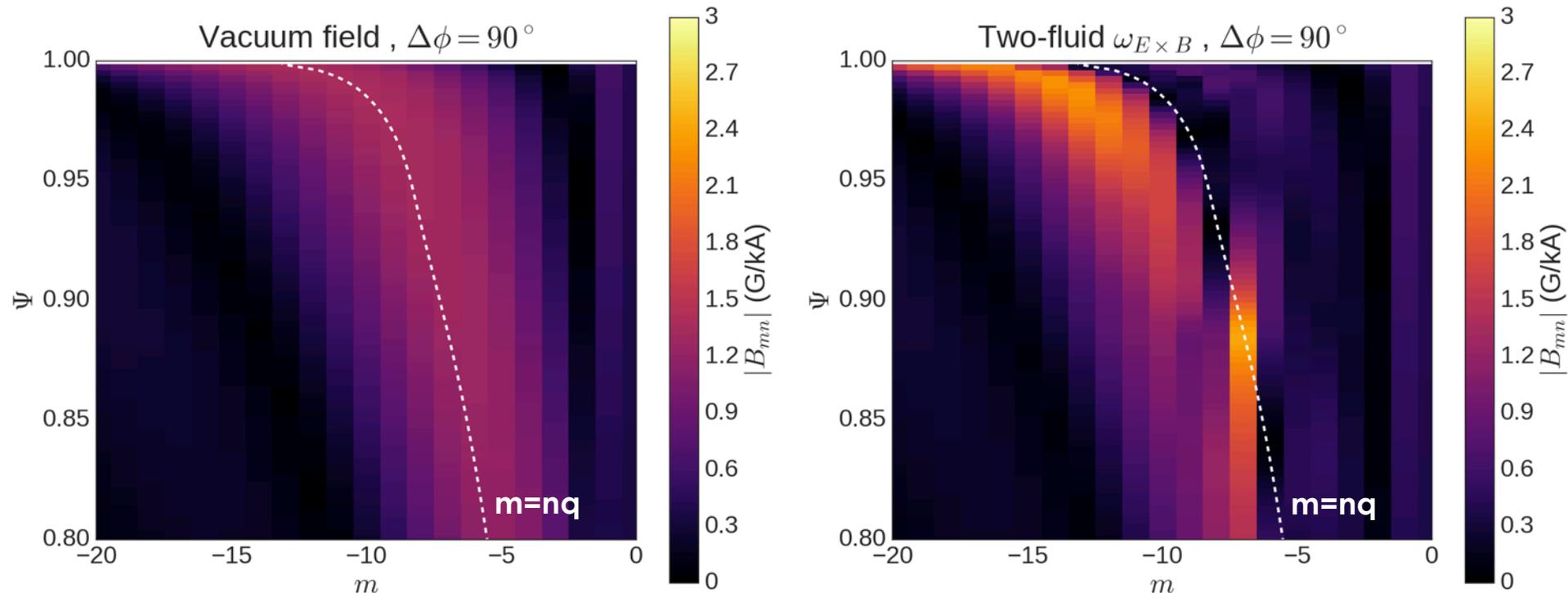
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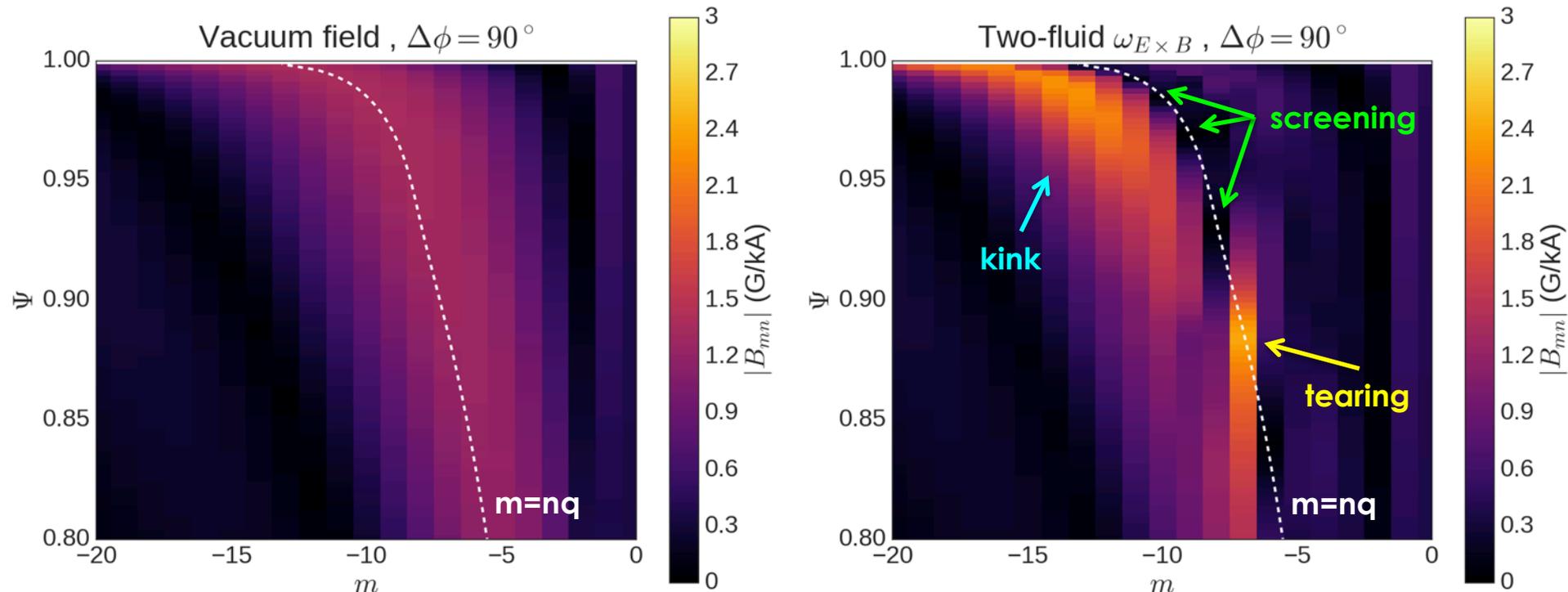
Fourier spectrum provides insight to plasma response

- **Fourier field decomposition:**
$$\delta B_{mn}(\Psi) = \frac{(2\pi)^2}{A} \iint \frac{\delta \mathbf{B} \cdot \nabla \psi}{\mathbf{B} \cdot \nabla \theta} e^{i(m\theta - n\varphi)} d\theta d\varphi$$
- **These SURFMN-like diagrams show magnitude of Fourier components**
 - m - Discrete poloidal harmonic on x-axis
 - Ψ - Continuous normalized poloidal flux on y-axis (radial variable)
 - Resonant $m=nq$ line



Plasma response alters perturbed magnetic spectrum

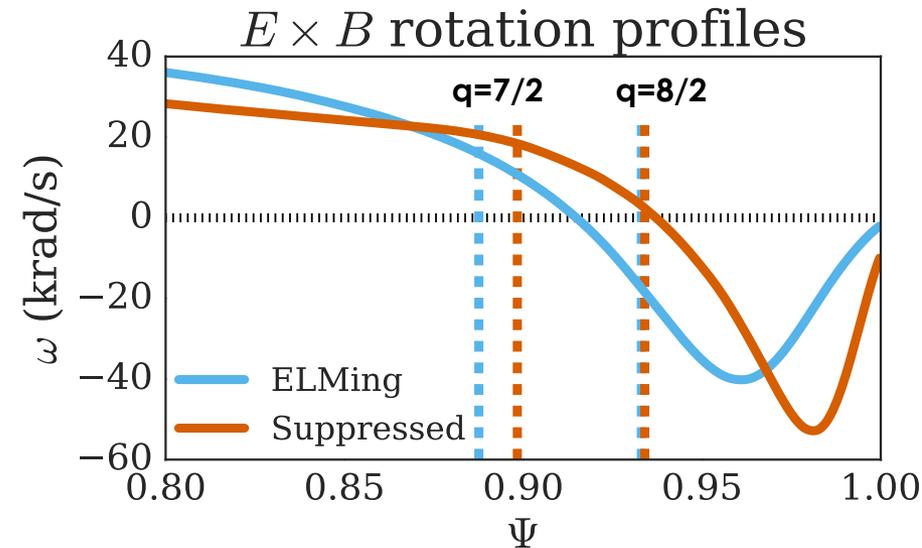
- **Resonant response at rational surfaces ($m=nq$)**
 - Screening suppresses
 - Tearing enhances
- **Kink response amplifies non-resonant fields with $m > nq$**



Rotation scan

Past experiments & simulations motivate rotation scan

- **Rotation profile changes during ELM suppression**
 - Zero-crossing of $E \times B$ and/or electron rotation often aligns with rational surface during ELM suppression
 - Generally leads to increased tearing drive
- **Single-fluid rotation profile affects verification & validation with external magnetics**
 - M3D-C1 w/ $E \times B$ rotation agrees better with data
 - M3D-C1 w/ carbon toroidal rotation agrees better with MARS-F results with same rotation



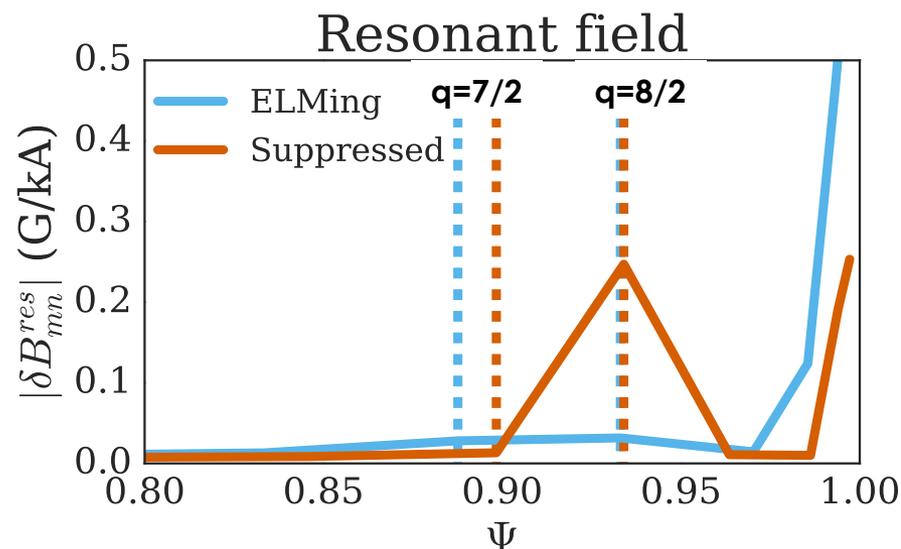
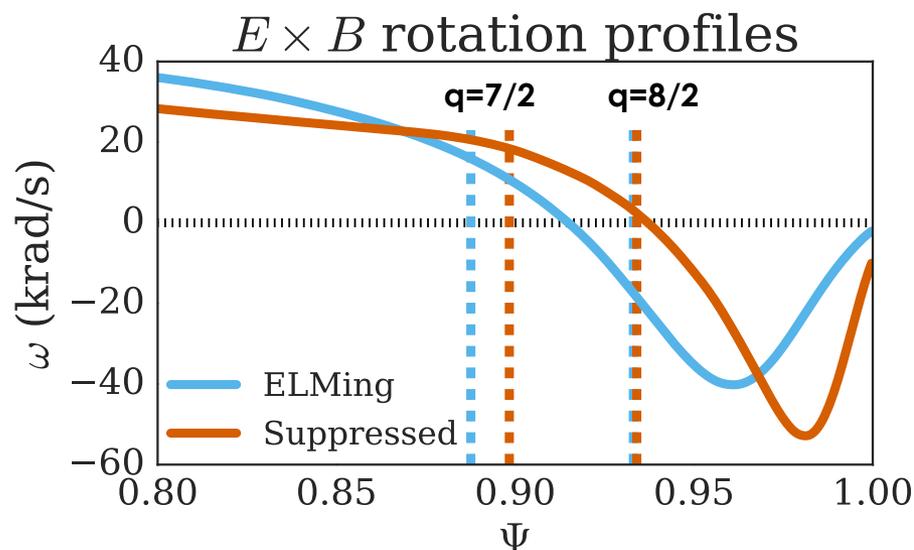
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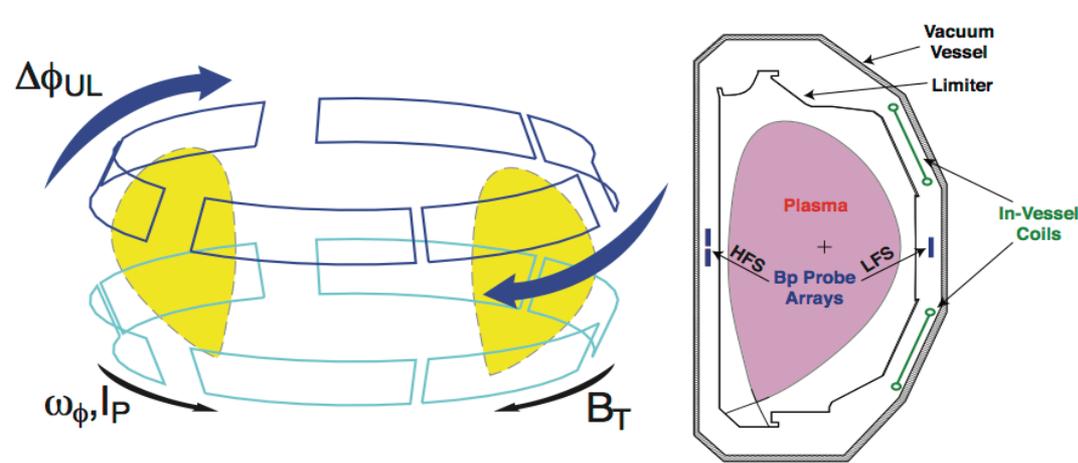
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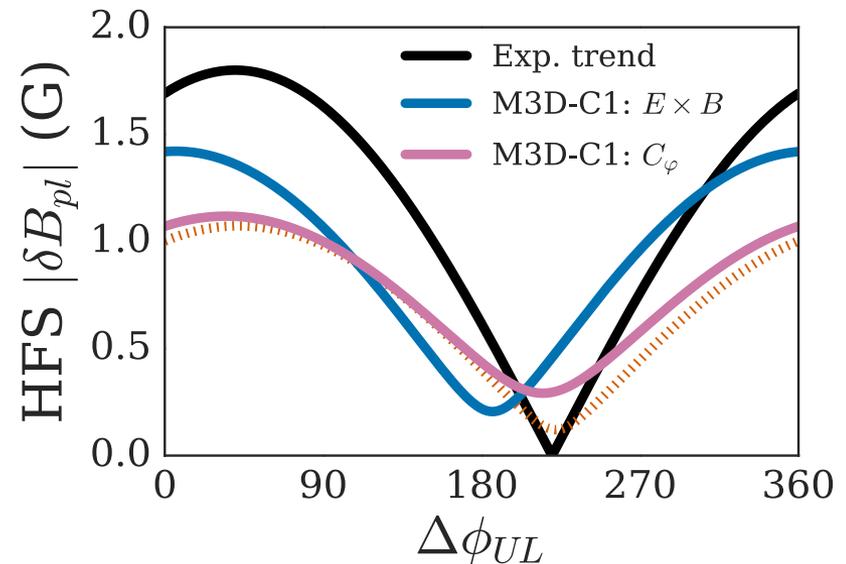
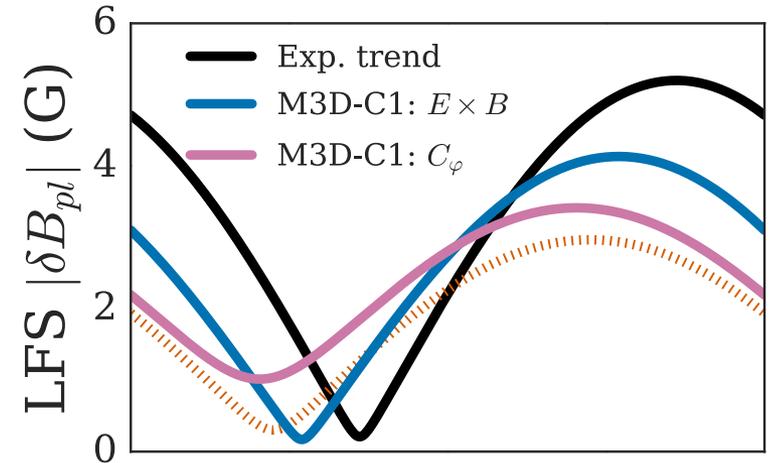


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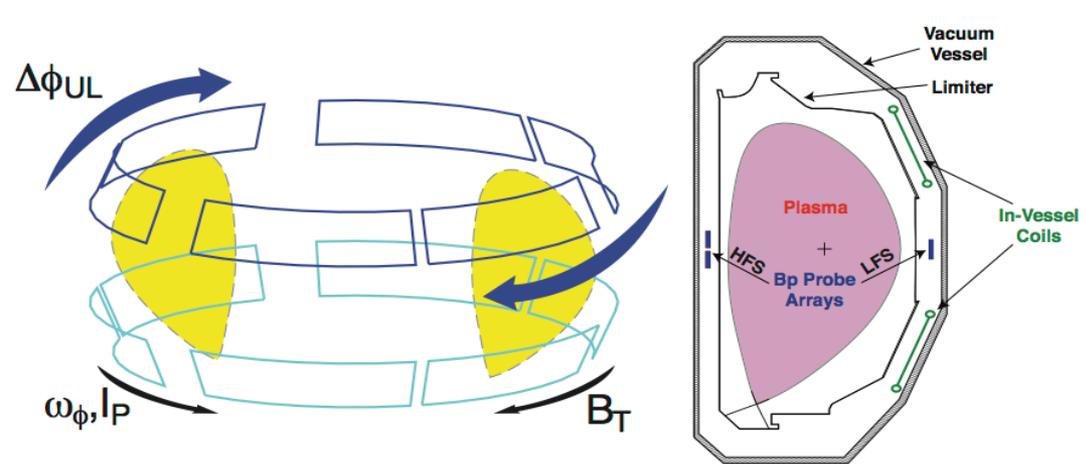


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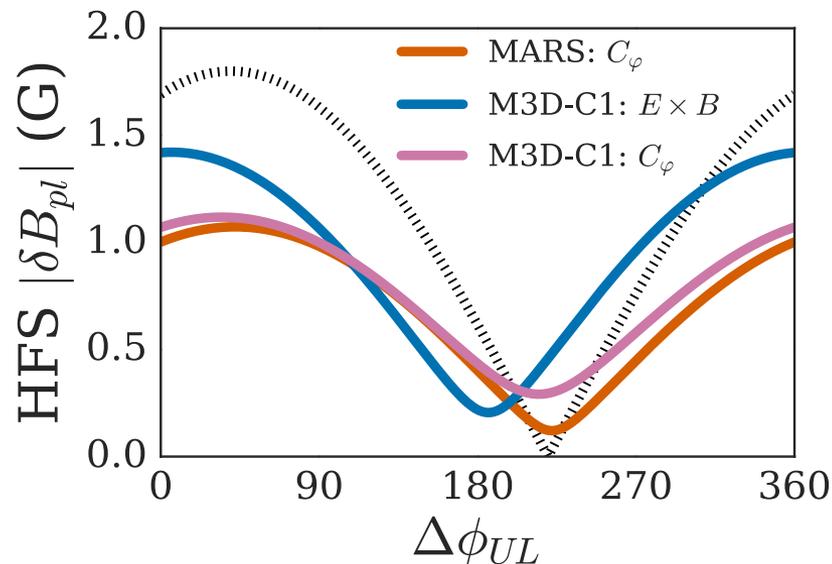
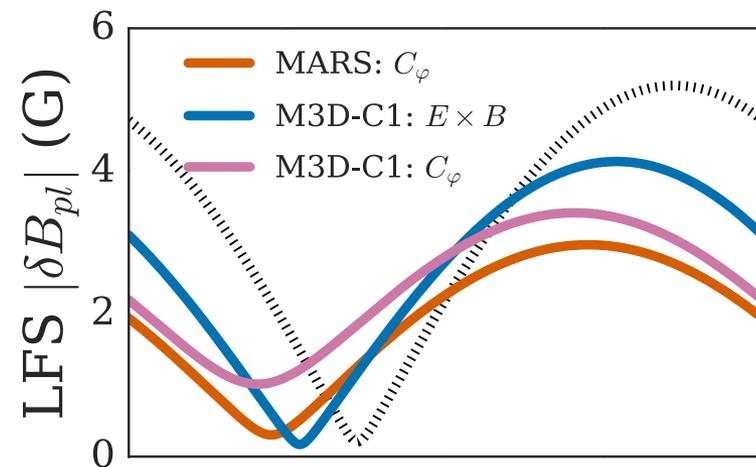


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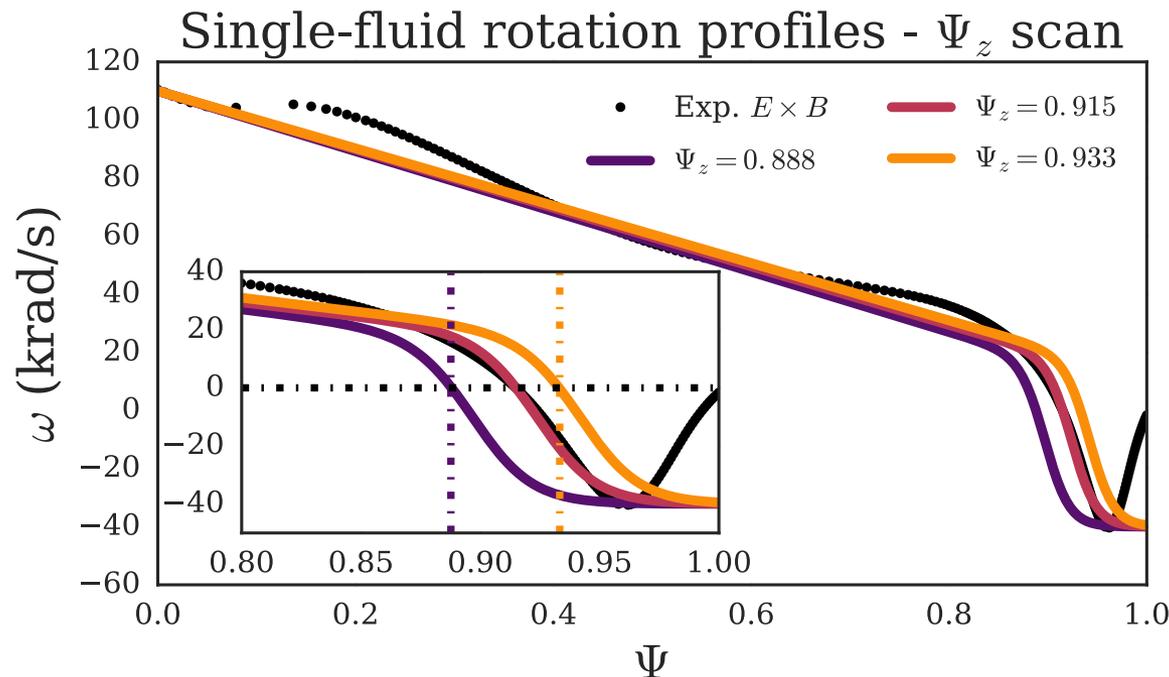
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Effect of rotation zero-crossing can be tested with systematic variation of model rotation profile

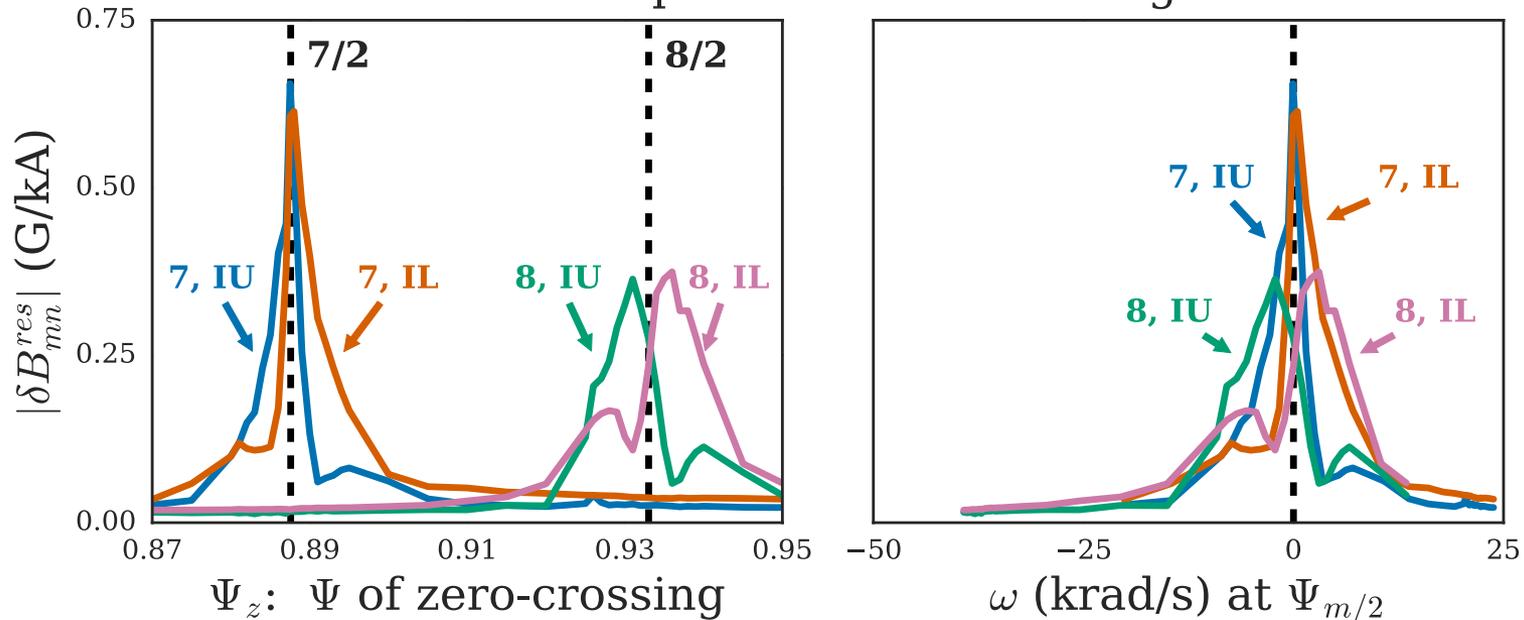
- **DIII-D ITER-similar shape (ISS) equilibrium**
 - Experimental shape and kinetic profiles
- **Model rotation profile with convenient parameterization, including**
 - Zero-crossing: Ψ_z
 - Width of tanh: $\Delta \Psi$ (controls shear)
- **Linear M3D-C1 used to assess effect of rotation on plasma response**



Effect of zero-crossing on plasma response

Low rotation increases resonant response

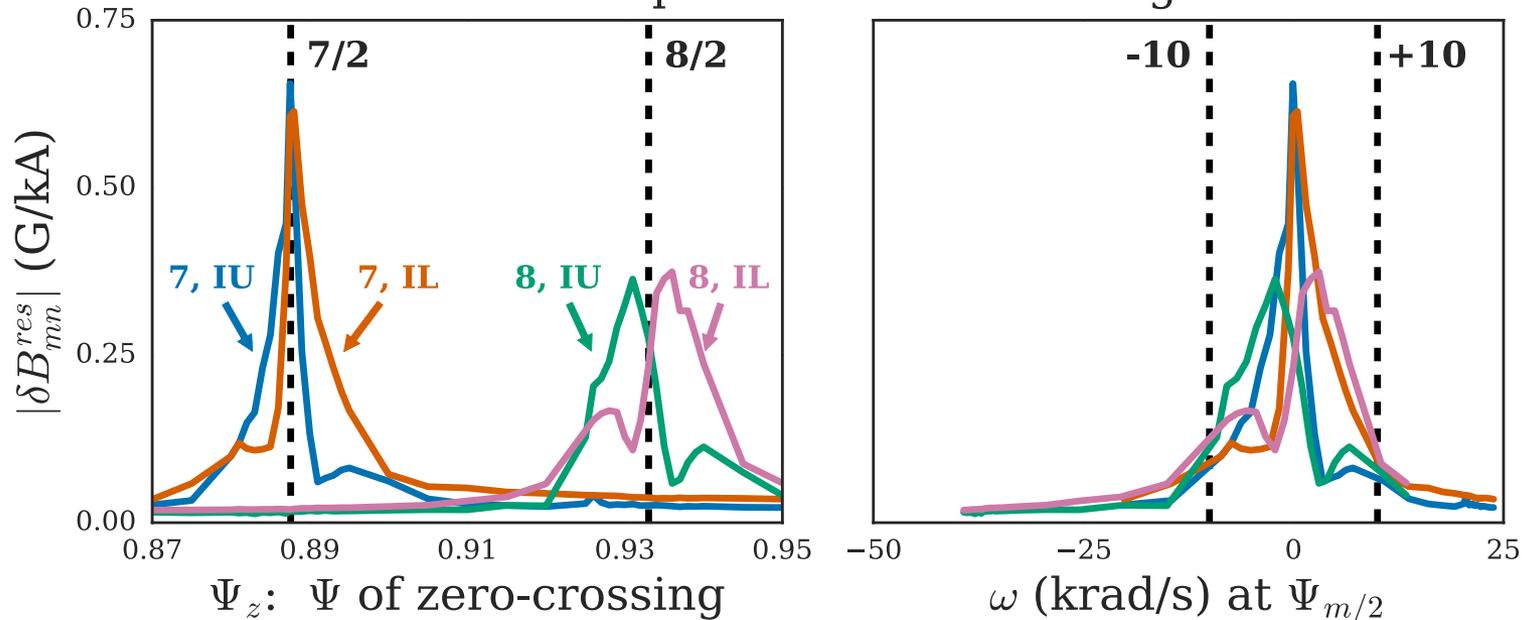
Resonant response as zero-crossing varied



- Resonant field amplified when zero-crossing aligns with rational surface
- Resonant field peaks for $|\omega| < 10$ krad/s at rational surface
- Response almost always screened below vacuum level
- Resonant response exhibits fine structure
 - IU and IL response weighted to opposite sides of surface
 - Multiple peaks clearly visible at $q=8/2$

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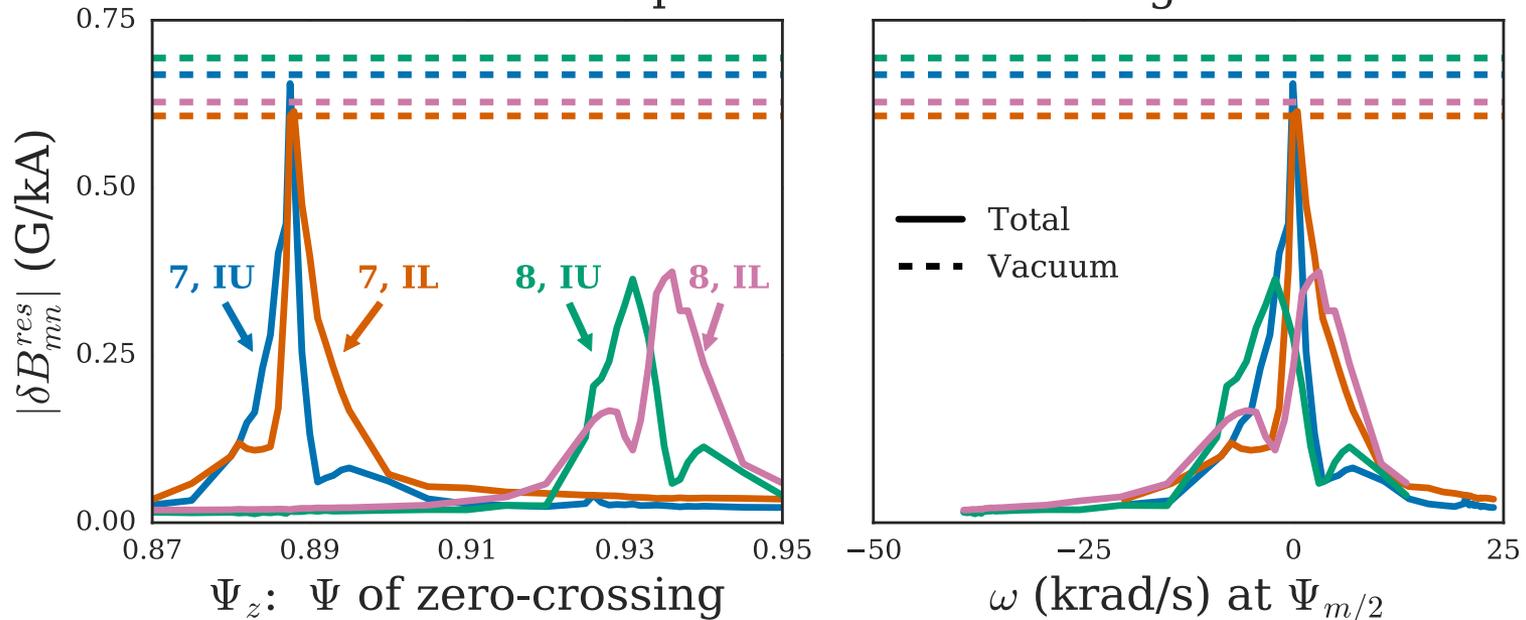
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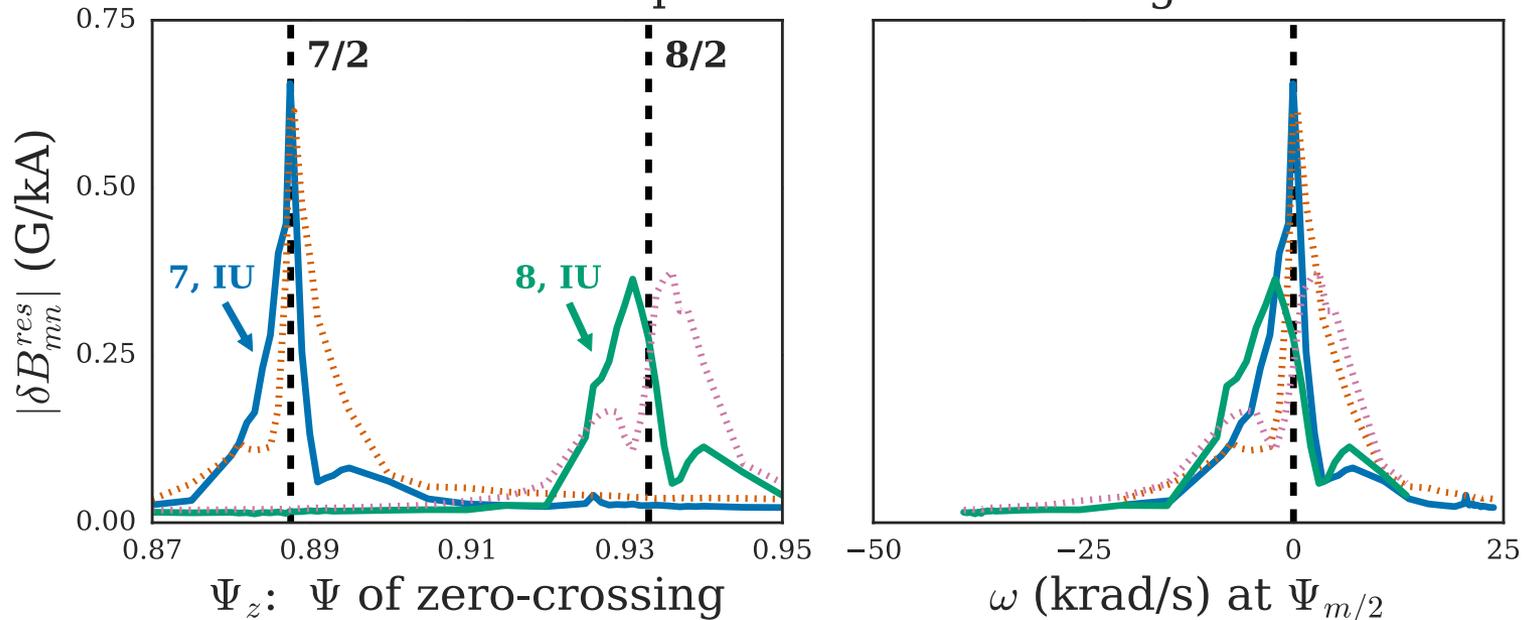
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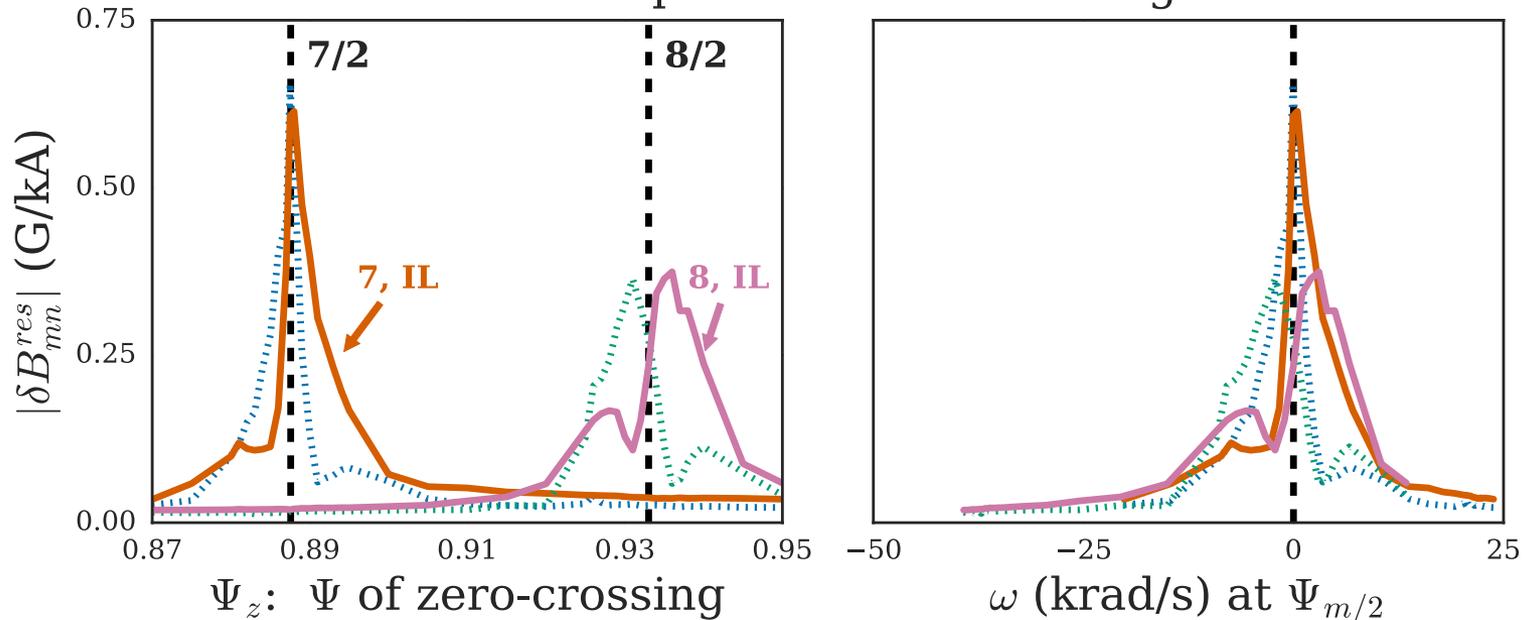
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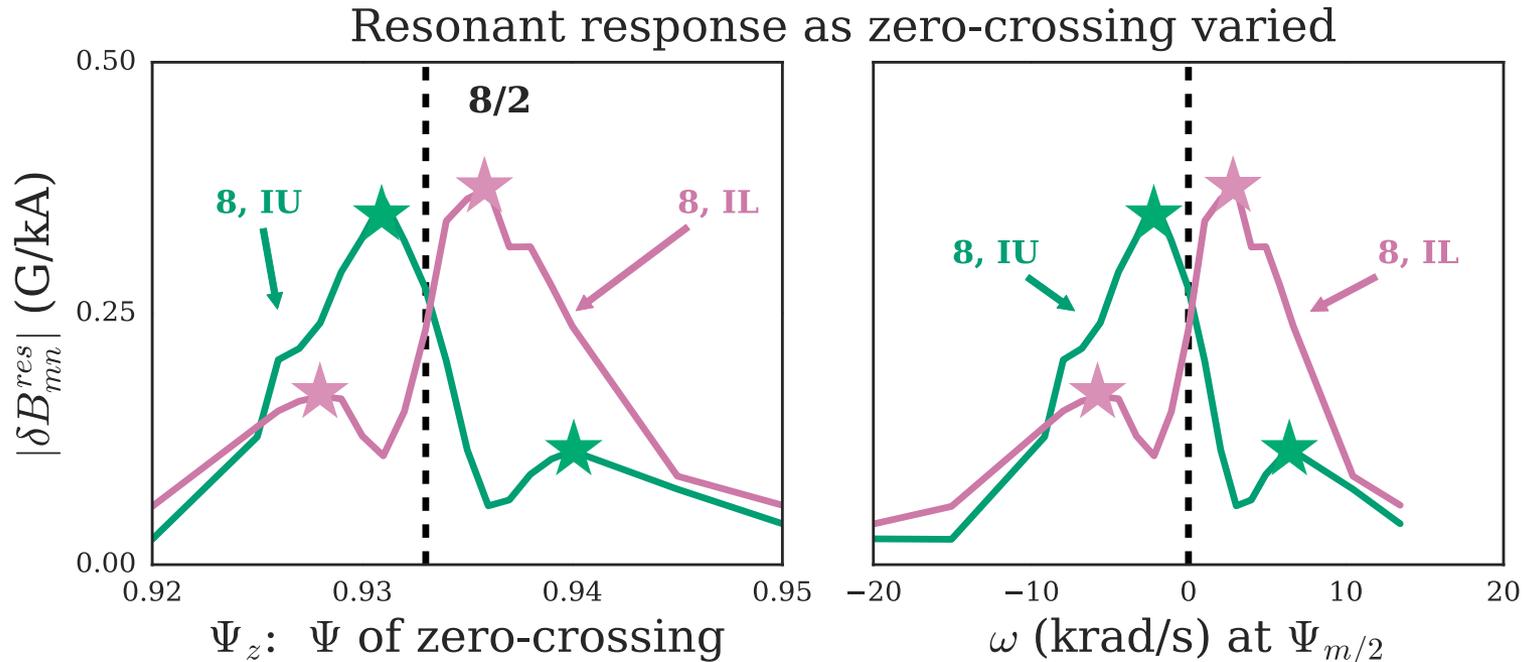
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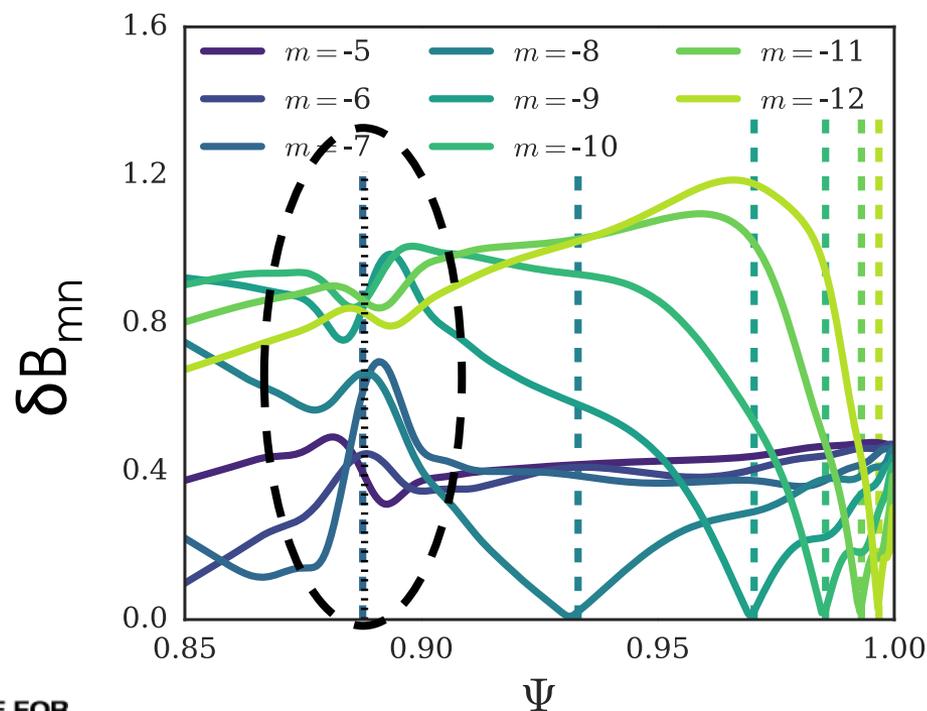


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Zero-crossing induces broad coupling of Fourier components of perturbed magnetic field

- Coupling occurs regardless of whether zero-crossing is on resonant surface or in between
- Near-resonant Fourier components are amplified
- Far-off-resonant Fourier components decrease
- Appears as streak across m in SURFMN-like diagrams

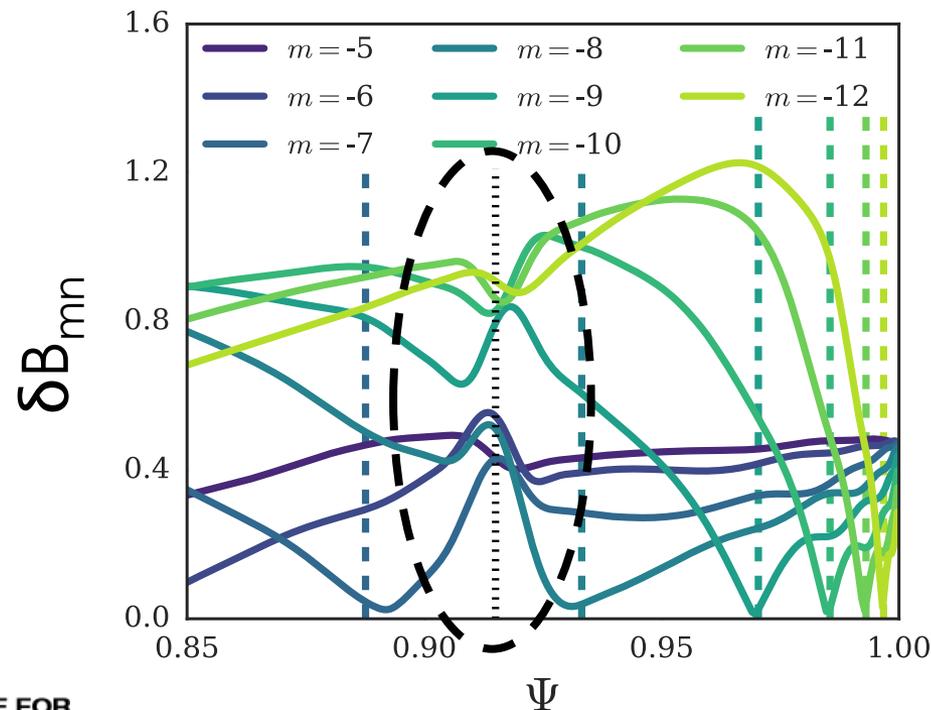
$$\Psi_z = 0.888$$
$$q = 7/2$$



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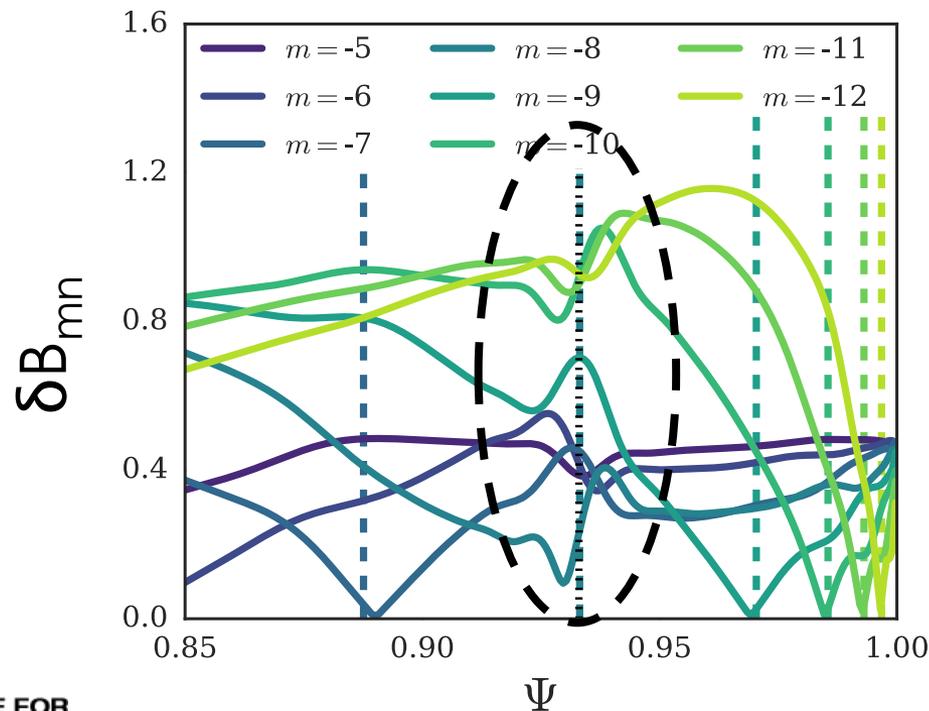
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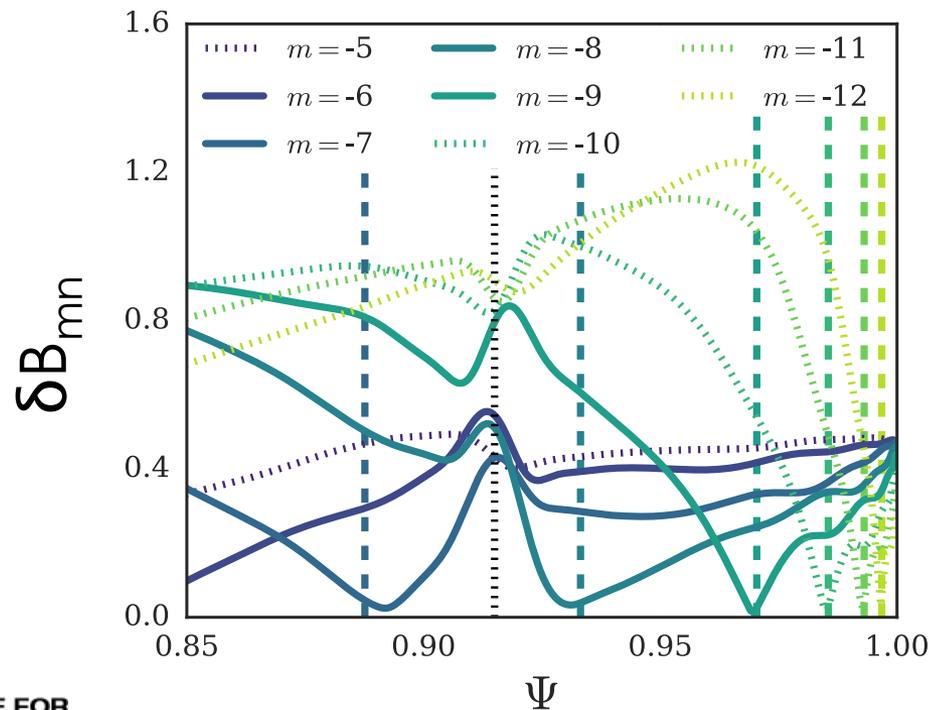
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$$q = 8/2$$



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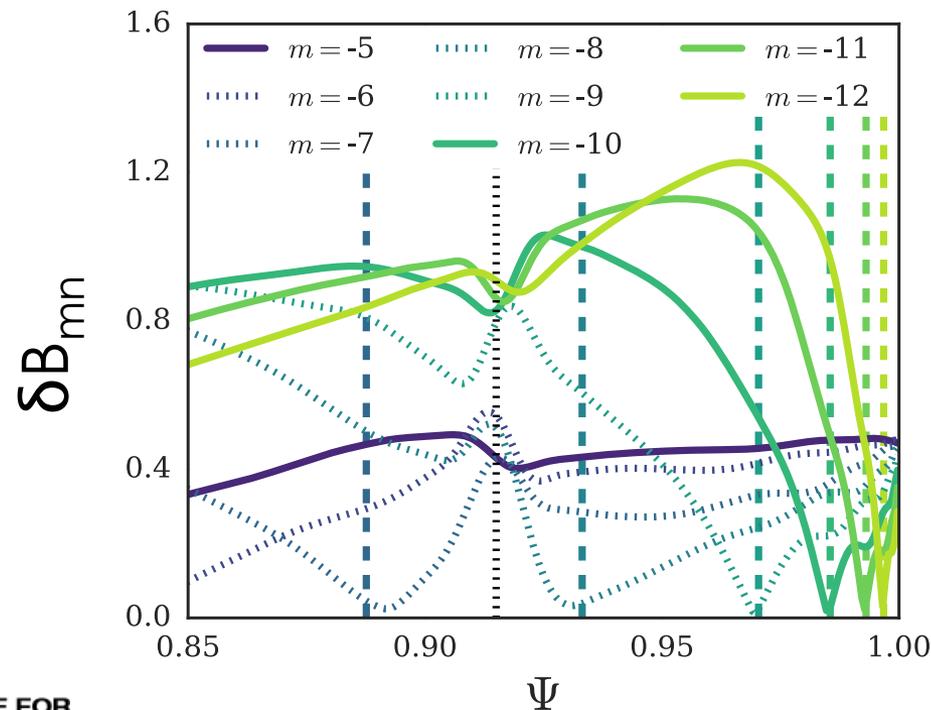
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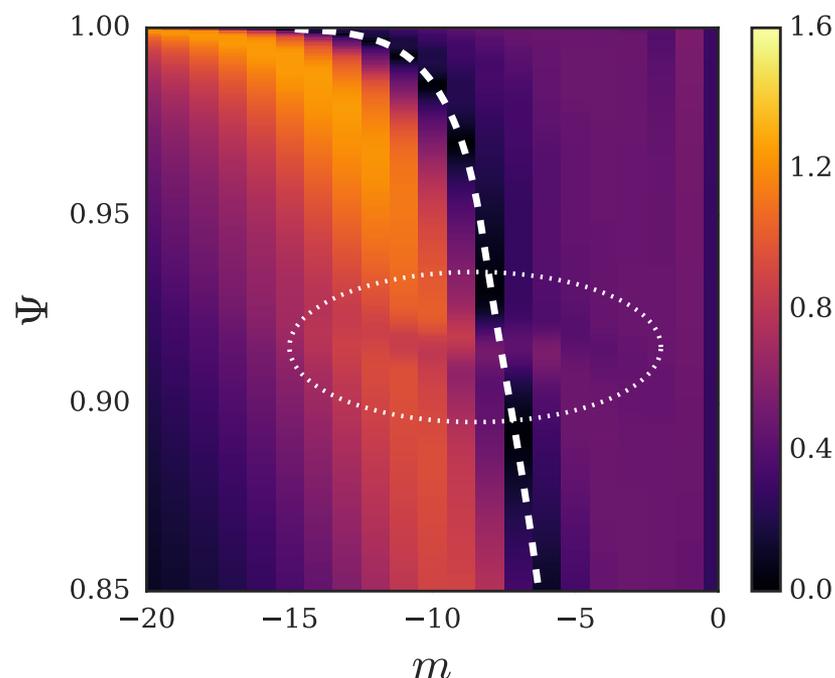
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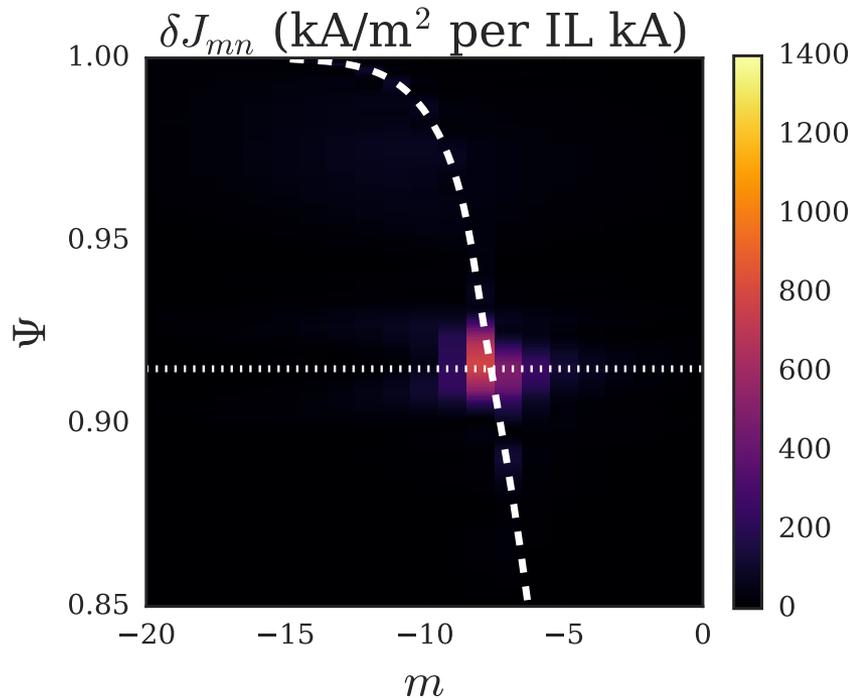
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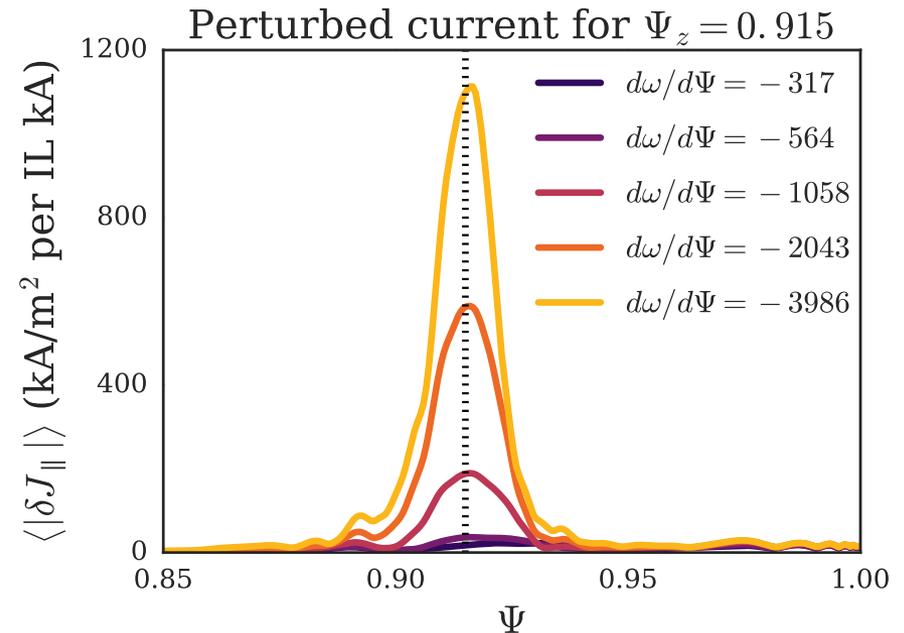


Coupling caused by current induced by mode driven at zero-crossing

Fourier decomposition of parallel current shows near-resonant nature of mode



Significant rotation shear required to drive mode

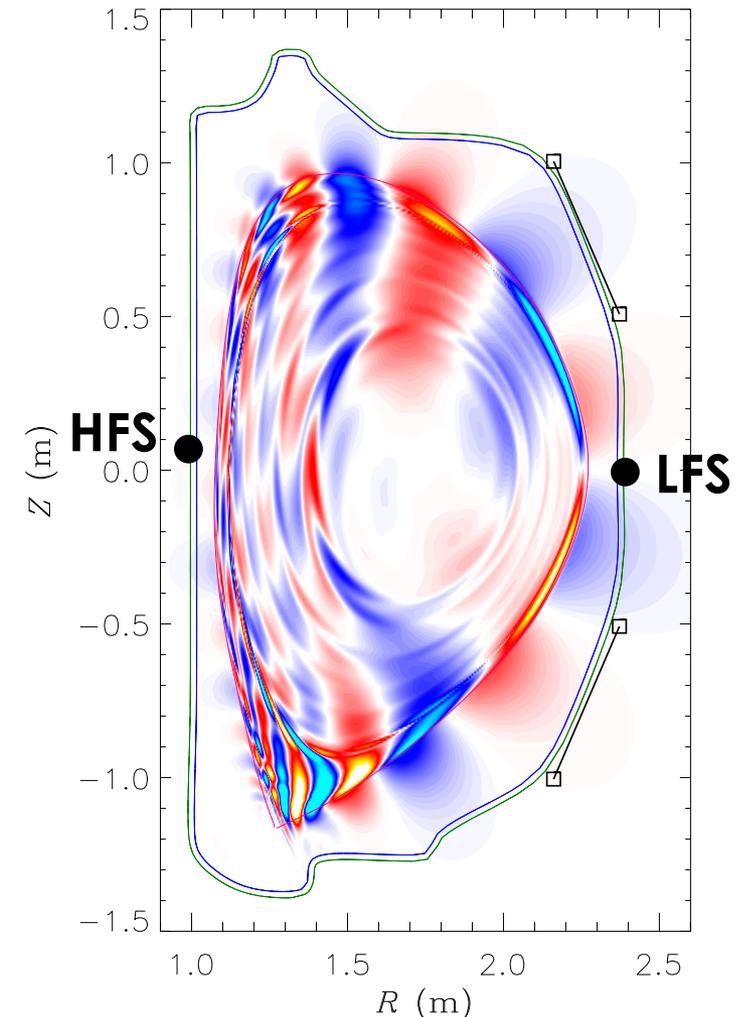


Observability of plasma response

Changes to plasma response observable by high-field side magnetic probes

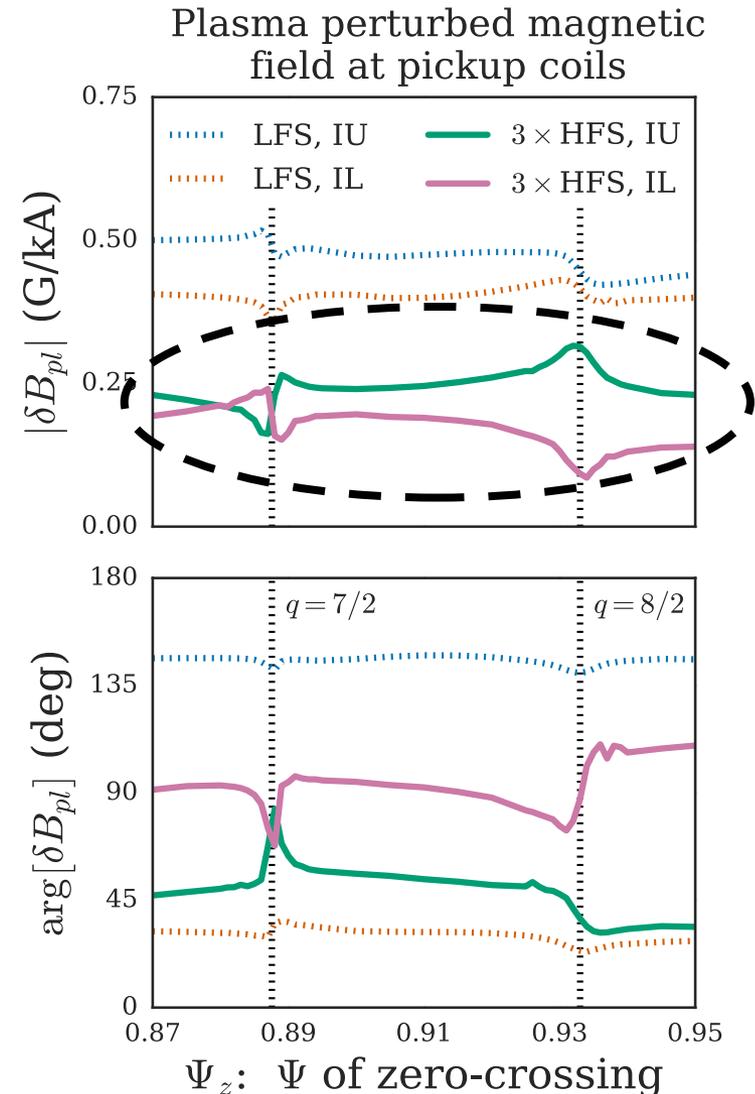
- **Magnetic sensors can measure poloidal field at low-field side (LFS) and high-field side (HFS) midplane**
- **HFS signals show**
 - Up to 50% magnitude change
 - 20° - 45° phase shift
 - Localized around $q=7/2$
 - “Permanent” across $q=8/2$
- **LFS signals show much smaller degree of variation**

δB_z : Even-parity plasma response



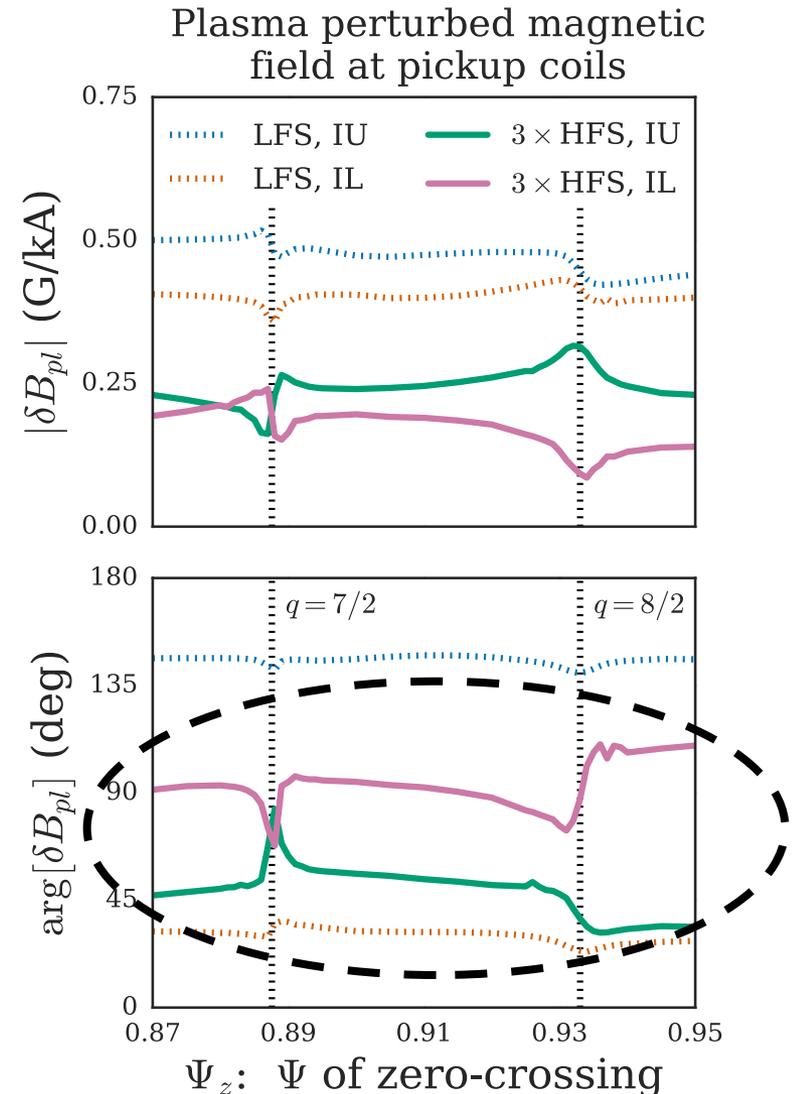
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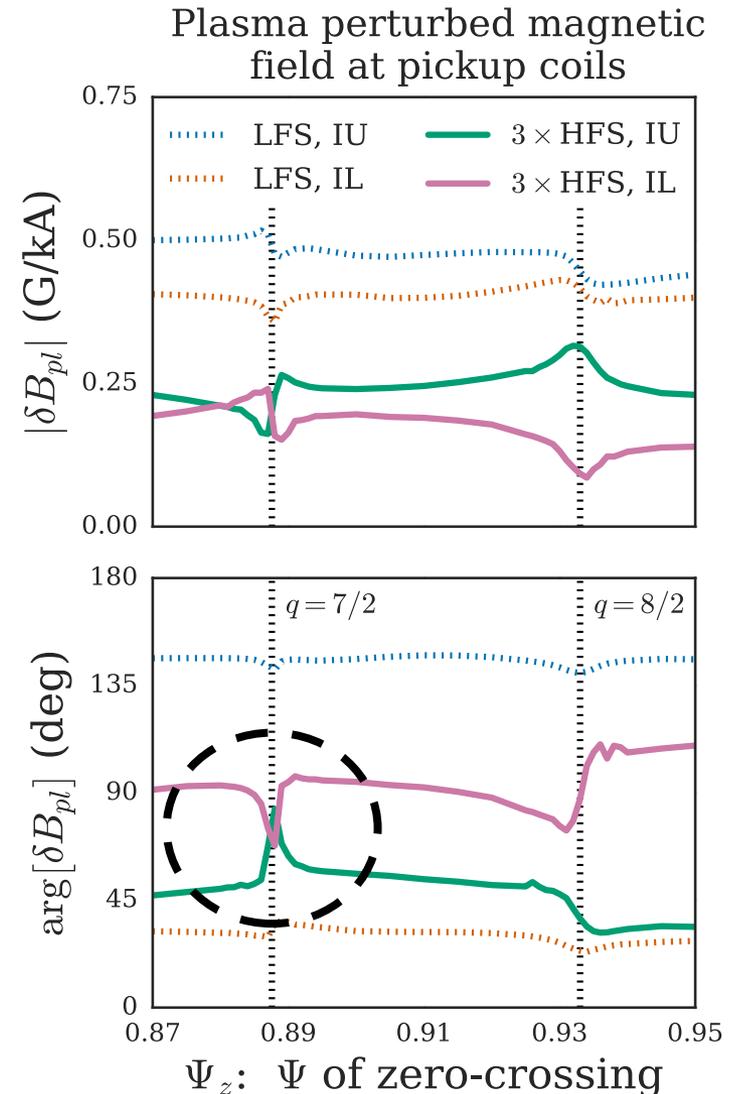
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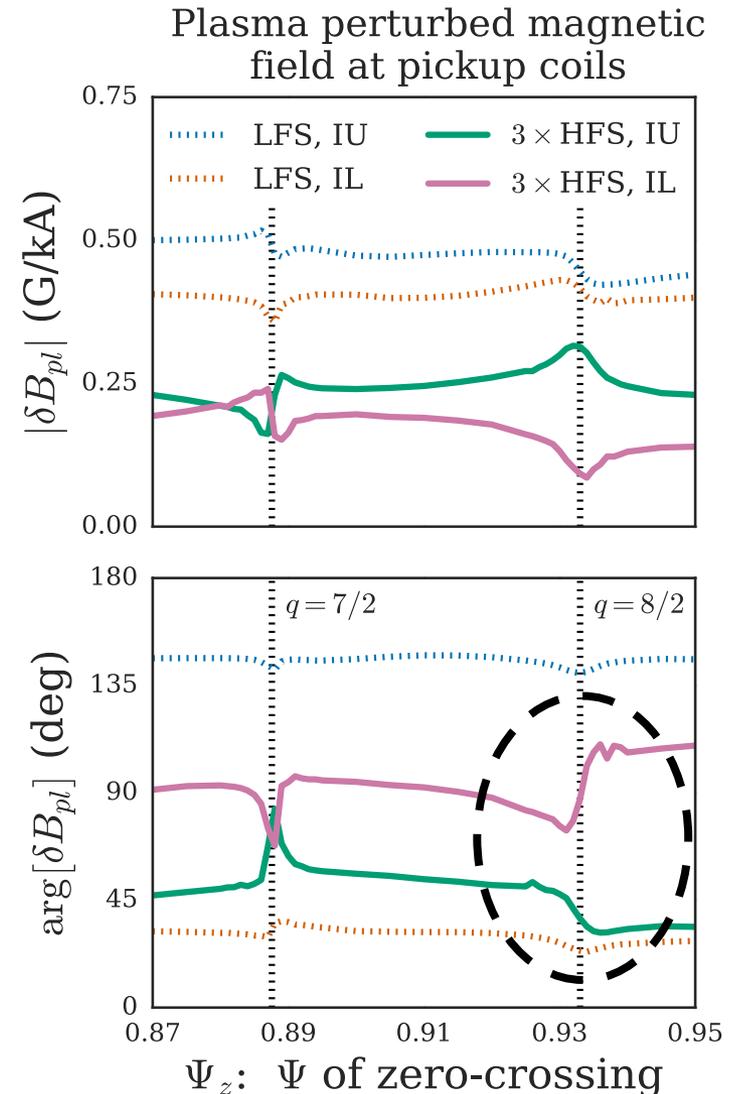
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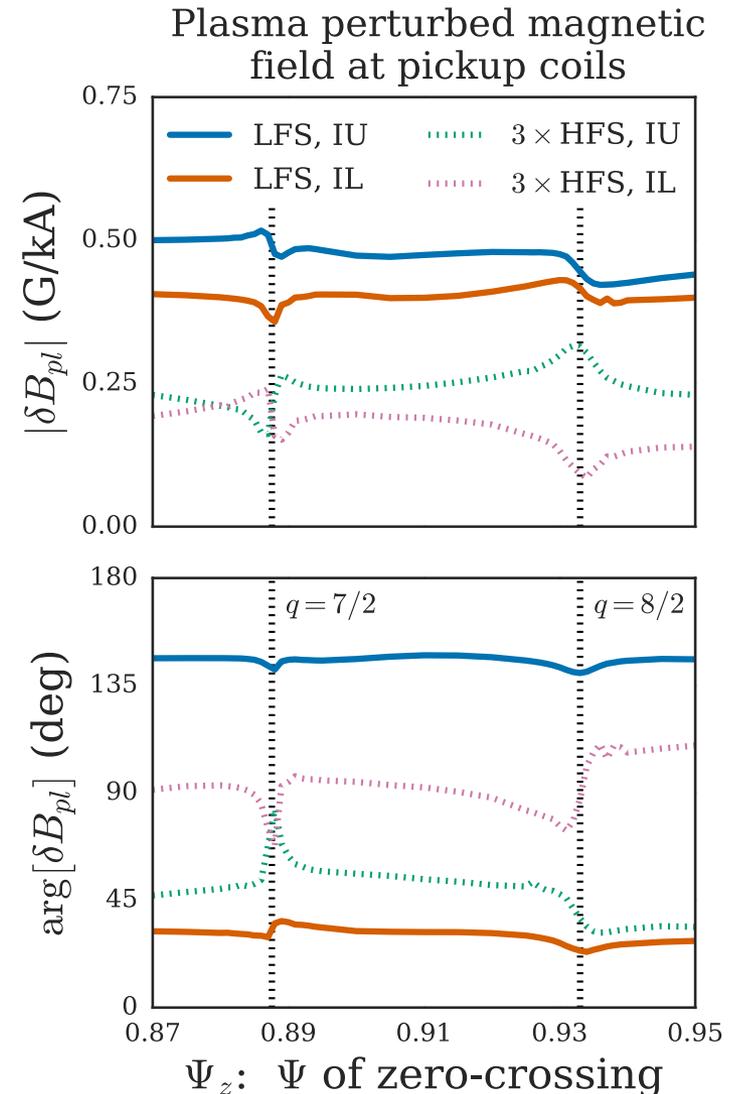
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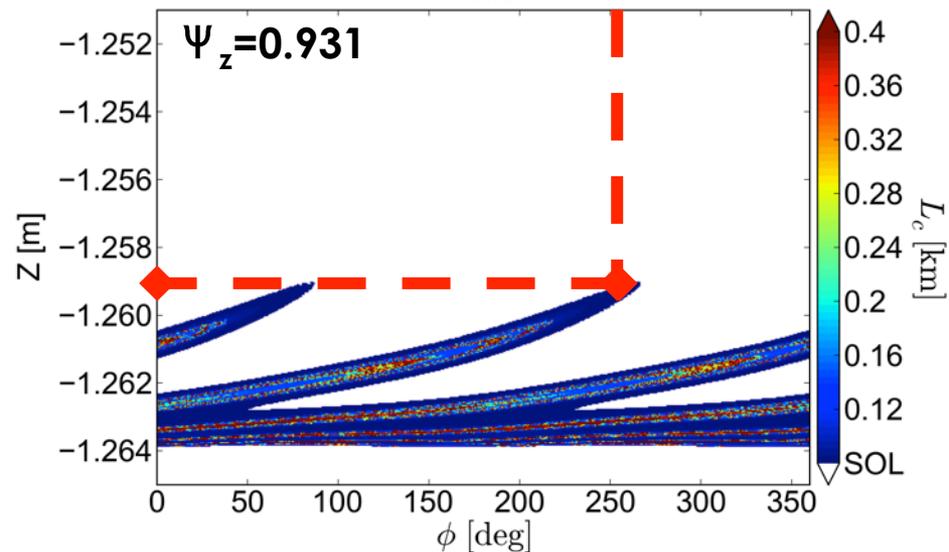
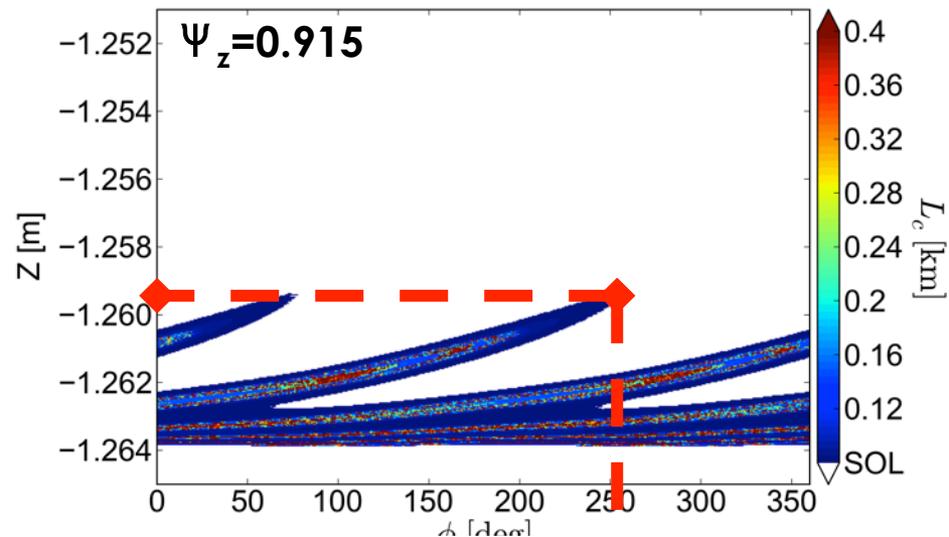
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Divertor footprints are insensitive to these changes

- **Divertor footprint structure calculated with coupled:**
 - TRIP3D (field line integration)
 - MAFOT (invariant manifold)
- **Simulations show little change as zero-crossing is varied**
- **Strike point splitting observed in experiments may be modified by plasma response closer to edge**

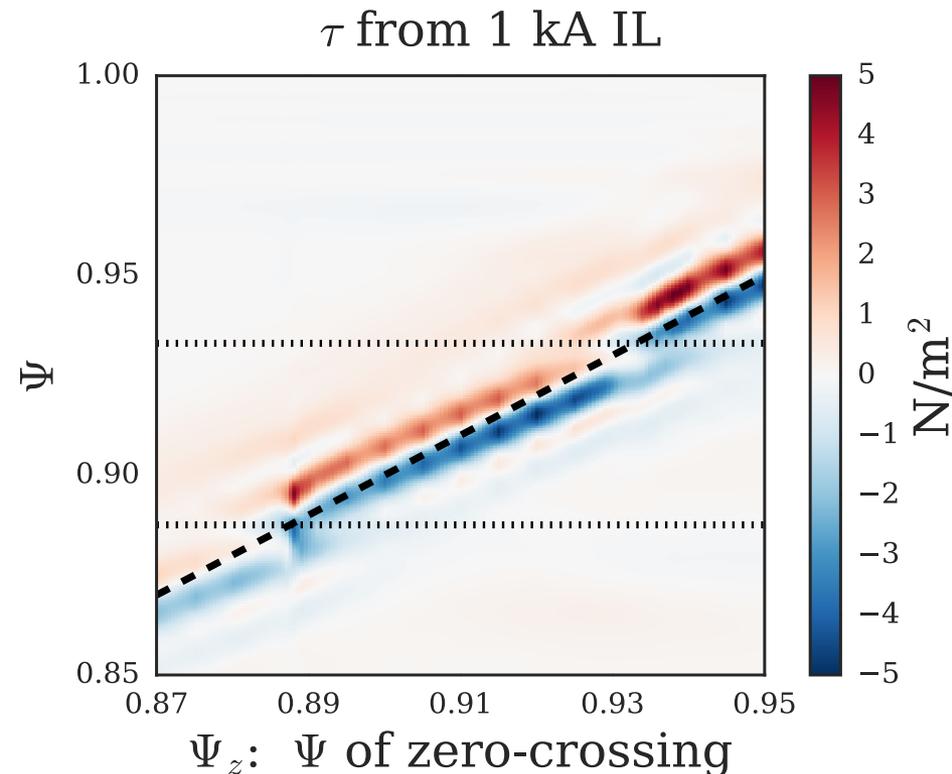


Quasilinear electromagnetic torque

Quasilinear torque density from non-resonant response acts to flatten rotation profile

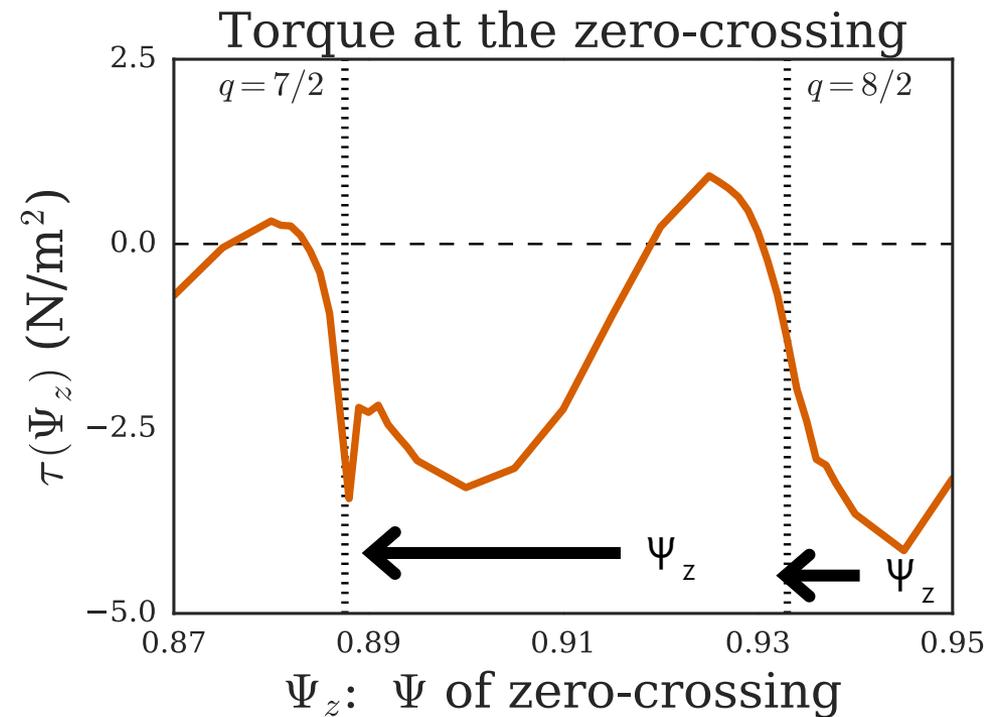
- **Negative torque inside zero-crossing decreases positive rotation**
- **Positive torque outside zero-crossing increases negative rotation**
- **ELM-suppression hypothesis**
 - Reduced shear destabilizes turbulent modes
 - Increased transport arrests growth of pedestal height and width

$$\tau(\Psi) = \langle R^2 \nabla \varphi \cdot (\delta \mathbf{J} \times \delta \mathbf{B}) \rangle$$



Quasilinear torque density from non-resonant response drives zero-crossing toward rational surfaces

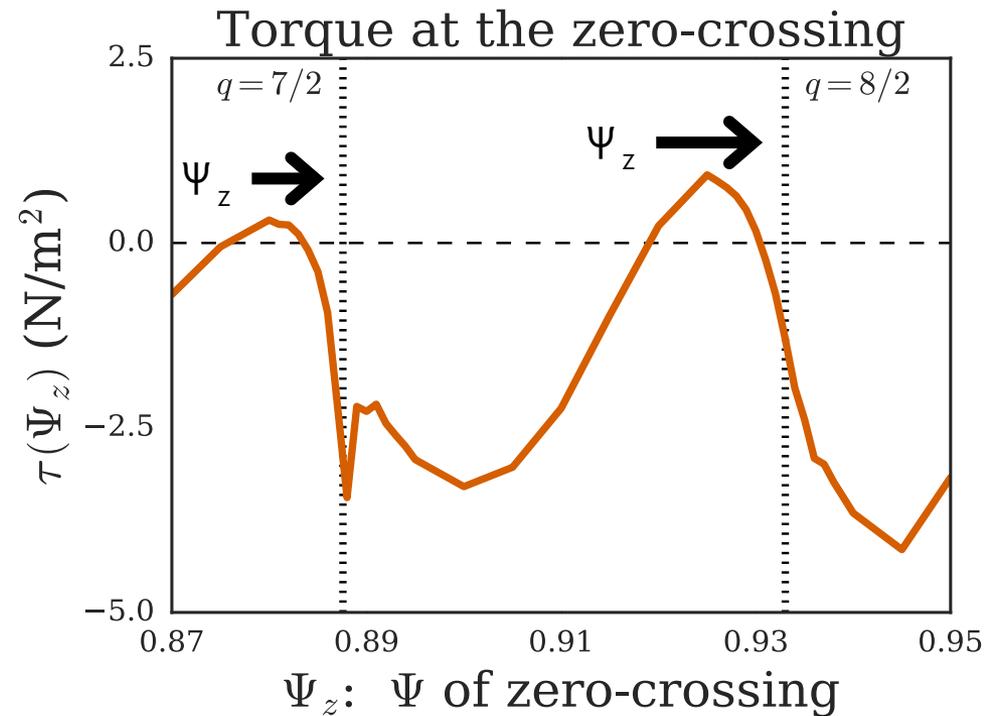
- **Negative torque at Ψ_z**
 - Drives negative rotation
 - Zero-crossing moves inward
- **Positive torque at Ψ_z**
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 - Zero-crossing moves outward
- **“Stable points” exist in vicinity of rational surface**
- **ELM-suppression hypothesis**
 - Torque locks zero-crossing close to rational surface
 - Low rotation permits increased resonant field
 - Island penetration leads to ELM-suppression bifurcation



Torque from IL response

Quasilinear torque density from non-resonant response drives zero-crossing toward rational surfaces

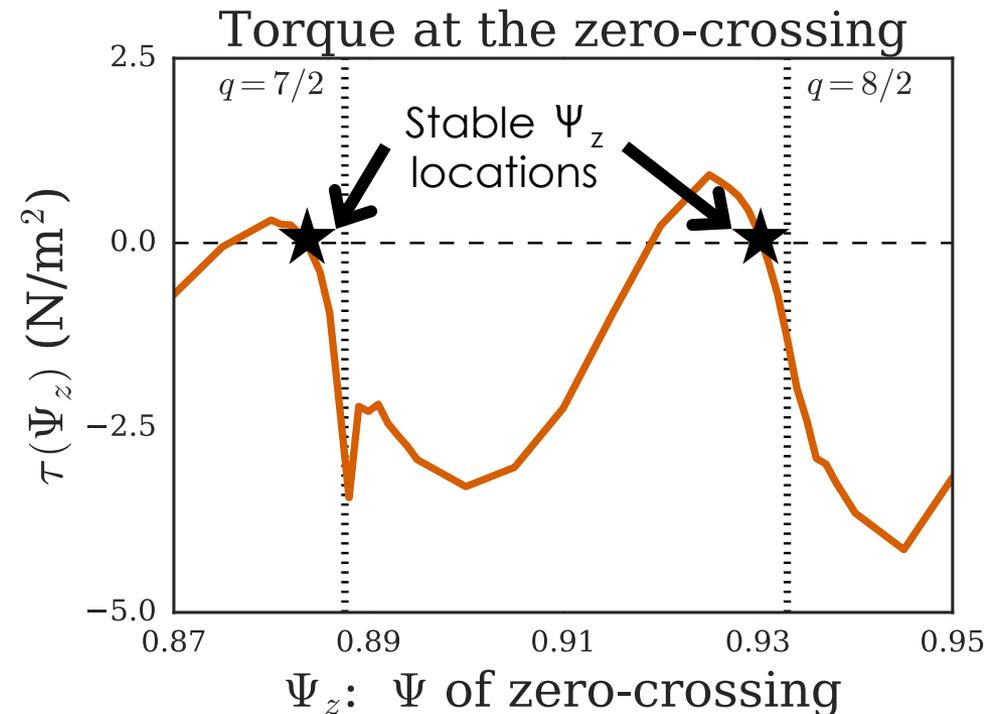
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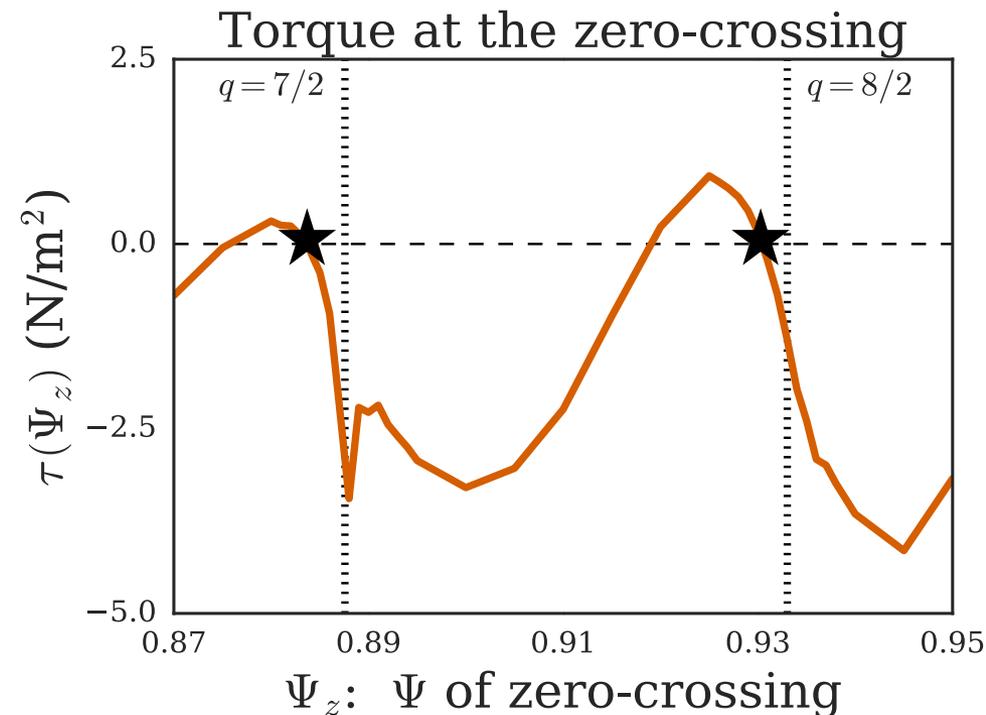
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 - Island penetration leads to ELM-suppression bifurcation



Torque from IL response

Quasilinear torque density from non-resonant response drives zero-crossing toward rational surfaces

- **Negative torque at Ψ_z**
 - Drives negative rotation
 - Zero-crossing moves inward
- **Positive torque at Ψ_z**
 - Drives positive rotation
 - Zero-crossing moves outward
- **“Stable points” exist in vicinity of rational surface**
- **ELM-suppression hypothesis**
 - Torque locks zero-crossing close to rational surface
 - Low rotation permits increased resonant field
 - Island penetration leads to ELM-suppression bifurcation



Torque from IL response

Summary

- **Resonant and non-resonant plasma response sensitive to rotation zero-crossing**
- **Changes to resonant response should be observable by HFS magnetic sensors**
- **Quasilinear torque from near-resonant mode may play an important role in ELM-suppression**
 - Reduced shear causes increased turbulent transport
 - Zero-crossing driven toward rational surface permits island penetration
- **Future work**
 - Further investigation of hypothesized ELM-suppression mechanisms
 - Scan of edge rotation profile
 - Detailed study of two-fluid effects, including rotation scan

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