Center for Simulation of Wave Interactions with MHD (SWIM)

March 23, 2006
Closures workshop

D. B. Batchelor, L. A. Berry, S. P. Hirshman, W. A. Houlberg, E. F. Jaeger, R. Sanchez – ***ORNL Fusion Energy***

D. E. Bernholdt, E. D'Azevedo, W. Elwasif, S. Klasky– ***ORNL Computer Science and Mathematics***

S. C. Jardin, G-Y Fu, D. McCune, J. Chen, L. P Ku, M. Chance, J. Breslau – ***PPPL***

R. Bramley – ***Indiana University,*** **D. Keyes** – ***Columbia University,*** **D. P. Schissel** – ***General Atomics,***

R. W. Harvey – ***CompX,*** **D. Schnack** – ***SAIC,*** **J. Ramos, P. T. Bonoli** – ***MIT***

Unfunded participants:

L. Sugiyama – ***MIT,*** **C. C. Hegna** – ***University of Wisconsin,*** **H. Strauss** – ***New York University,*** **P. Collela** – ***LBNL***

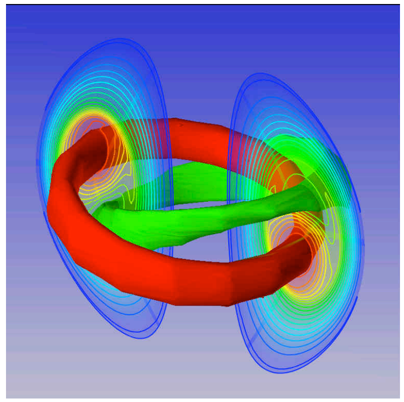
H. St. John – ***General Atomics,*** **G. Bateman, A. Kritz** – ***Lehigh Univ.***

- **Program logic**
- **Some stuff about RF**
- **Theoretical issues for SWIM**

SWIM brings together two mature sub-disciplines of fusion plasma physics, each with a demonstrated code base using the most advanced computers

Extended MHD – CEMM

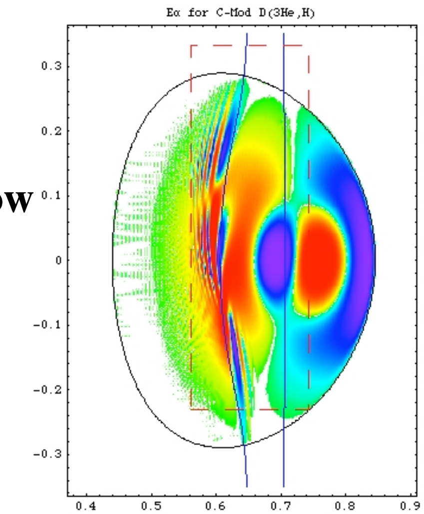
- MHD equilibrium
- Macroscopic fluid instability
- Current and magnetic field evolution



Fluid equations, extended to include non-ideal and kinetic effects
(10^{-5} sec < τ_{MHD} < 10^{-1} sec)

High power wave-plasma interactions – CSWPI

- Plasma heating
- Externally driven current or plasma flow
- Non-Maxwellian particle distributions



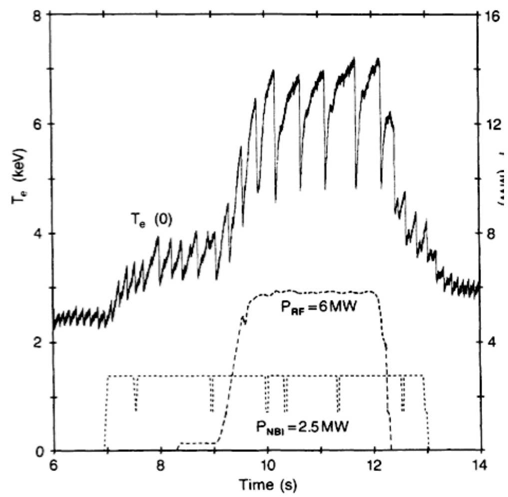
Plasma wave equation (τ_{RF} < 10^{-7} sec), coupled to slow evolution of plasma velocity distribution (τ_{FP} > 10^{-2} sec)

Why couple these particular two disciplines?

- Macroscopic instabilities can limit plasma performance
- RF waves can mitigate and control instabilities

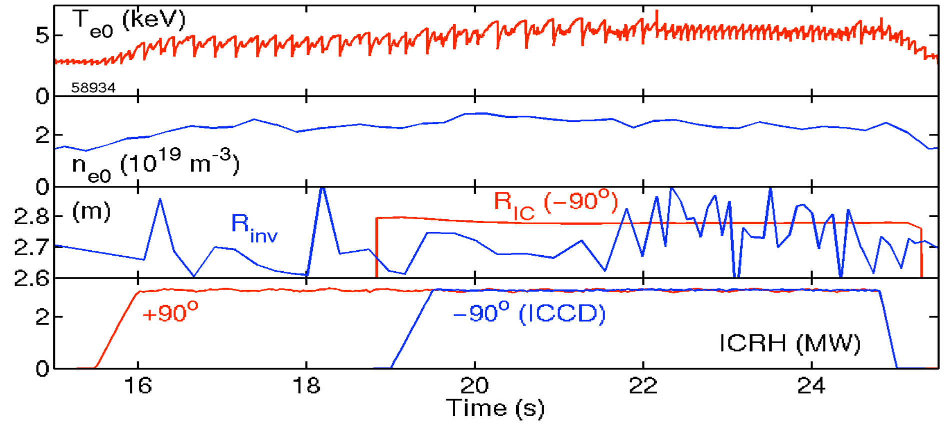
There are several experimentally demonstrated mechanisms by which RF waves can control sawtooth behavior

ICRF stabilization on JET



- ICRF heating can produce “monster” sawteeth – period and amplitude increased
- Likely stabilization mechanism – energetic particle production by RF
- We need an accurate calculation of energetic tail evolution:
  - RF/energetic particle interaction
  - Fokker Planck solution with RF driven radial transport
  - Slow profile evolution consistent with non-Maxwellian distributions

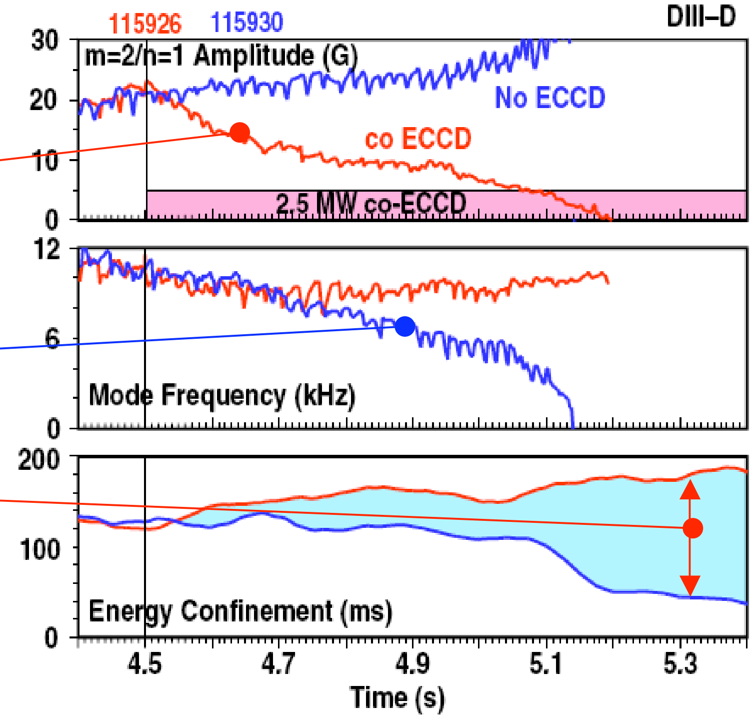
Sawtooth control on JET with Minority Current Drive on JET



- ICRF minority current drive can either increase or decrease period and amplitude
- Likely stabilization/destabilization mechanism – RF modification of current profile
- Need accurate calculation of current profile including:
  - RF driven electron and ion currents
  - Electron response to non-Maxwellian ions

It has been demonstrated experimentally that suppression of NTM by RF leads to improvement in confinement

- Electron cyclotron current drive drives down mode amplitude
- keeps mode rotating (no drop in frequency)
- improves energy confinement



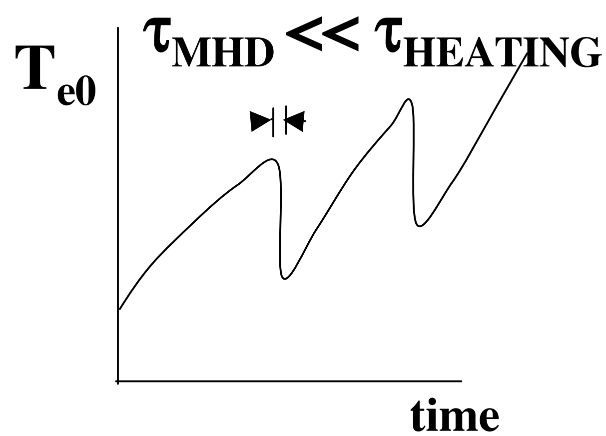
R. Prater
APS 2003

- **Empirical scaling of NTM pressure limits in ITER leave no margin in performance**
- **"Understanding the physics of neoclassical island modes and finding means for their avoidance or for limiting their impact on plasma performance are therefore important issues for reactor tokamaks and ITER" – ITER Physics Basis (1999)**

SWIM has two sets of physics goals distinguished by the time scale of unstable MHD motion

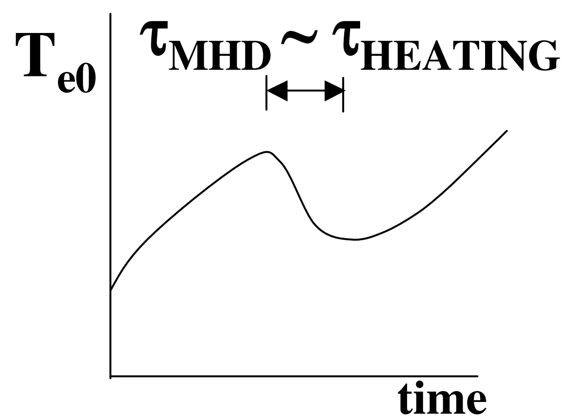
Fast MHD phenomena – separation of time scales

- Response of plasma to RF much slower than fast MHD motion
- RF drives slow plasma evolution, sets initial conditions for fast MHD event
- Example: sawtooth crash



Slow MHD phenomena – no separation of time scales

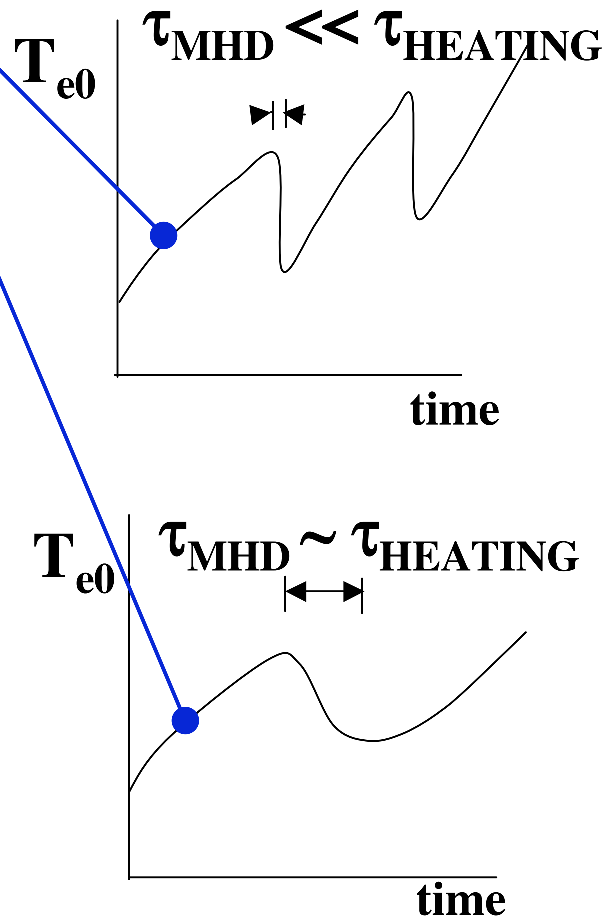
- RF affects dynamics of MHD events ⇔ MHD modifications affect RF drive plasma evolution
- Deals with multi-scale issue of parallel kinetic closure including RF – a new, cutting edge field of research
- Example: Neoclassical Tearing Mode



We are approaching these regimes in two *campaigns* of architecture development and physics analysis and validation

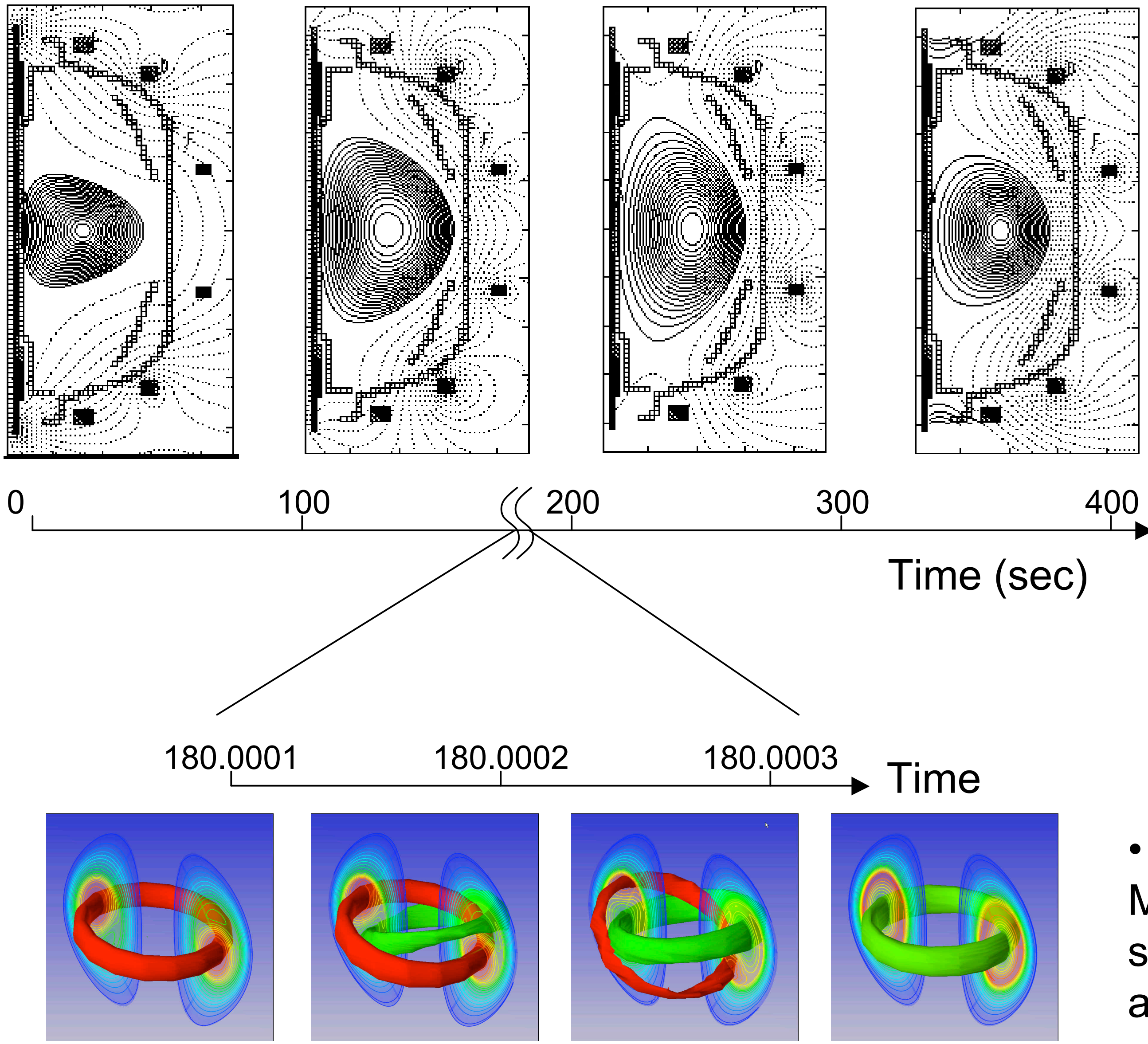
Simulation of plasma evolution requires complete model – Integrated Plasma Simulator (IPS)

- Heating and current drive sources
- Particle sources
- Transport
- Magnetic field evolution



Integrated Plasma Simulator will allow coupling of virtually any fusion fusion code, not just RF and MHD, and should provide the framework for a full fusion simulation

Simulation of plasma evolution requires complete model – Integrated Plasma Simulator (IPS)



- Plasma evolves through a series of 2D axisymmetric equilibrium states
  - Heating and current drive sources
  - Particle sources
  - Transport
  - Magnetic field evolution
- Instabilities occur as instantaneous events

- 3D Extended MHD simulation starts and ends in axisymmetric state

These two regimes are related

- Fast sawtooth crash can provide seed island for NTM growth
- Slow growth of NTM island can lead to fast disruption events
- Calculation of slow ramp of sawtooth, with incomplete reconnection or persistent islands, may actually require the same capabilities as NTM evolution

We calculate the plasma response, $f(\mathrm{x}, \mathrm{v}, t)$, from the Boltzmann equation

$$\frac{\partial f}{\partial t} + \mathrm{v} \cdot \nabla f + \tfrac{q}{m}\left[E + \mathrm{v} \times B\right] \cdot \nabla_{\mathrm{v}} f = C(f)$$

Nonlinear– *E* and *B* driven by current and charge described by *f*

There are two very helpful approximations we can make for externally injected RF waves

- **Separation of time scales - wave period 1/ω << time of equilibrium variation, τ_E**
- **The waves are stable (actually damped), so we can safely linearize the fast time equation:**

$$f = f_0 + f_1 e^{-i\omega t} + f_2 \qquad B(x,t) = B_0(x) + B_1(x) e^{-i\omega t}, \quad E(x,t) = E_1(x) e^{-i\omega t}$$

$$\frac{\partial}{\partial t} f_1 + \mathrm{v} \cdot \nabla f_1 + \tfrac{q}{m}\left[\mathrm{v} \times B_0\right] \cdot \nabla_{\mathrm{v}} f_1 = -\tfrac{q}{m}\left[E_1 + \mathrm{v} \times B_1\right] \cdot \nabla_{\mathrm{v}} f_0$$

Gives fast time scale variation – wave current

$$\frac{\partial f_2}{\partial t} + \mathrm{v} \cdot \nabla f_2 + \tfrac{q}{m}\left[\mathrm{v} \times B_0\right] \cdot \nabla_{\mathrm{v}} f_2 = -\tfrac{q}{m}\left[E_1 + \mathrm{v} \times B_1\right] \cdot \nabla_{\mathrm{v}} f_1 + C(f_2)$$

Contains Fokker Planck equation

Gives slow time scale variation of f_0 – power deposition, equilibrium evolution

Basic equations of wave propagation and absorption

$$\nabla \times \nabla \times \mathbf{E} + \frac{\omega^2}{c^2}\mathbf{E} = \mathbf{J}_P \circ \mathbf{E} + \mathbf{J}_{ant} \quad : \quad + \text{boundary conditions}$$

plasma wave current: an integral operator on E

- **Time harmonic ↔ real ω, coherent waves, spatial damping**
- **$\mathbf{J}_{ant}$ = antenna source current**
- **Boundary conditions: bounded domain – conducting or inhomogeneous source region**

- **Weakly non-linear, time average distribution function f_0(v, t) evolves slowly:**

$$f(\mathbf{x},\mathbf{v},t) = f_0(\mathbf{x},\mathbf{v},t) + f_1(\mathbf{x},\mathbf{v})e^{-i\omega t}$$

slow, quasilinear time scale ~ τ_E

Fast, RF time scale

- **$\mathbf{J}_p$ = fluctuating plasma current due to wave – non-local, integral operator on E**

$$\mathbf{J}_P(\mathbf{x},t) = e\int d^3\mathrm{v}\,\mathbf{v} f_1(\mathbf{x},\mathbf{v},t) \qquad f_1(\mathbf{x},\mathbf{v},t) = -\frac{e}{m}\int_{-\infty}^{t} dt'\,\mathbf{E}_1\left(\mathbf{x}'(\mathbf{x},\mathbf{v},t'),t'\right)\cdot\frac{\partial f_0}{\partial \mathbf{v}'}$$

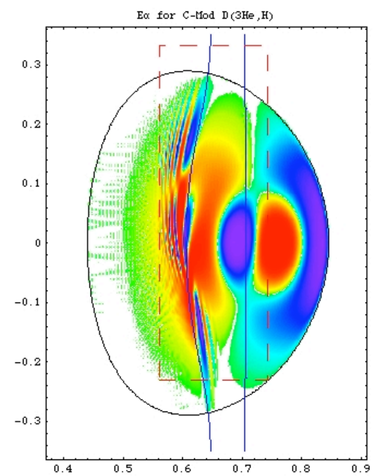
- **Approximate operator locally by integrating along guiding center orbits**
- **Effectively uniform plasma conductivity (Stix) →** $\sigma(k_{\parallel},k_{\perp},\omega) \Rightarrow I_{\ell}(k_{\perp}\rho),\ Z\left(\frac{\omega-\ell\Omega_c}{k_{\parallel}v_{th}}\right)$

Several ICRF full-wave solvers have been advanced in recent years

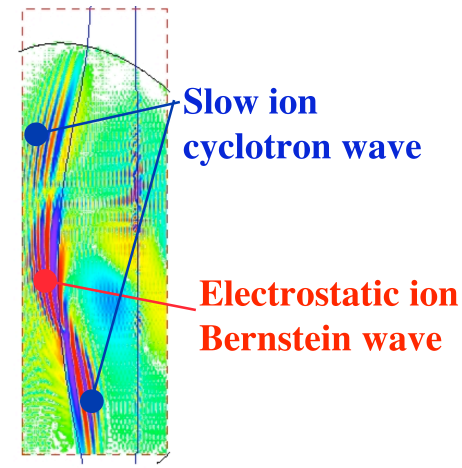
- **All Orders Spectral Algorithm (AORSA) – 1D, 2D & 3D (Jaeger)**
  - **Spectral in all 3 dimensions**
  - **Cartesian/toroidal coordinates** (x, y, ϕ)

$$\mathbf{E}(\mathbf{x}) = \sum_{n,m,l} \mathbf{E}_{\mathbf{n,m,l}} e^{i(nx+my+l\phi)}, \quad \sigma \to \sigma(x,y,\phi),$$

  - **Includes all cyclotron harmonics**
  - **No approximation of small particle gyro radius ρ compared to wavelength λ**
  - **Produces huge, dense, non-symmetric, indefinite, complex matrices**



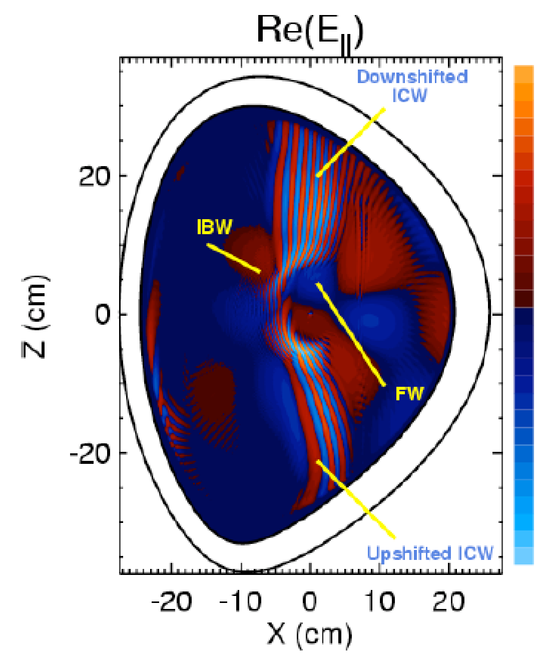
Blowup region



- **TORIC – 2D (Brambilla/Bonoli/Wright)**
  - **Mixed spectral (toroidal, poloidal), finite element (radial)**
  - **Flux coordinates** (ρ, θ, ϕ)

$$\mathbf{E}(\rho,\theta,\phi) = \sum \mathbf{E}_{\mathbf{m,l}}(\rho) e^{i(m\theta+l\phi)}, \quad \sigma \to \sigma(\rho,\theta)$$

  - **Up 2nd cyclotron harmonic**
  - **Expanded to 2nd order in ρ/λ**
  - **Sparse banded matrices**



CQL3D: Bounce-averaged Fokker-Planck Code

Solves for bounce-averaged distribution at torus equatorial plane ($\theta_P = 0$), $f_0(\rho, v_{\parallel}, v_{\perp}, t)$

$$\frac{\partial(\lambda f)}{\partial t} = \frac{\partial}{\partial \mathbf{v}} \cdot [\Gamma_E + \Gamma_{RF} + \Gamma_{coll}] + R(f) + S_{NB} + L$$

ρ=generalized radial coordinate labeling (non-circular) flux surface

λ = field line connection length

Γ_E = velocity space flux due to toroidal electric field (Ohmic)

$\Gamma_{RF} = \overleftrightarrow{\mathbf{D}}_{QL} \cdot \frac{\partial f}{\partial \mathbf{v}}$ = velocity space flux due to full, bounce average, RF Quasi-linear operator (all harmonics, Bessel functions, all wave modes)

Γ_{coll} = full, nonlinear, 2D, relativistic collisional operator

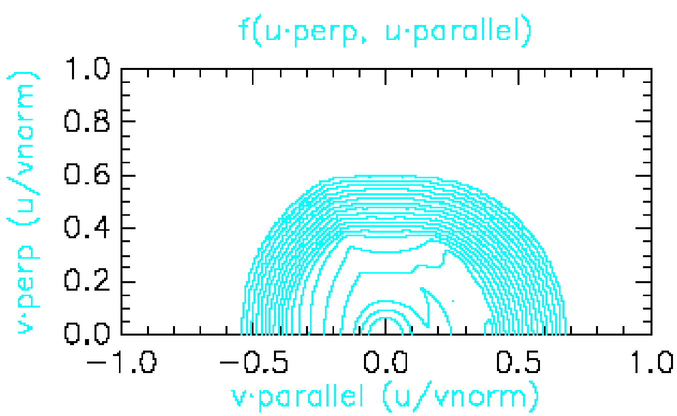
R(f) = Radial diffusion and pinch operator with v dependent coefficients

S_{NB} = Monte Carlo neutral beam source (NFREYA)

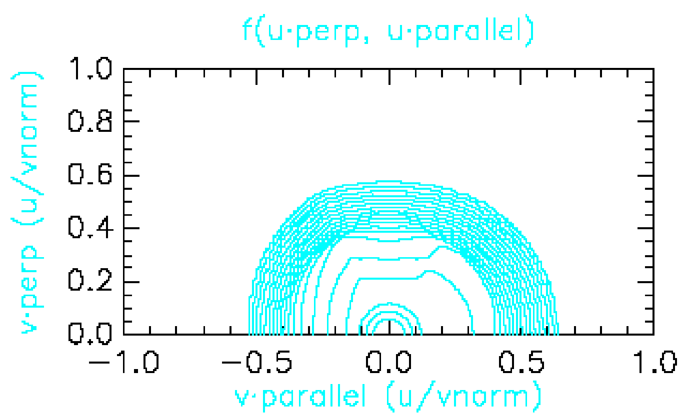
L(v) = velocity dependent prompt loss term

CQL3D distribution for NBI on NSTX (no RF)

r/a=0.10

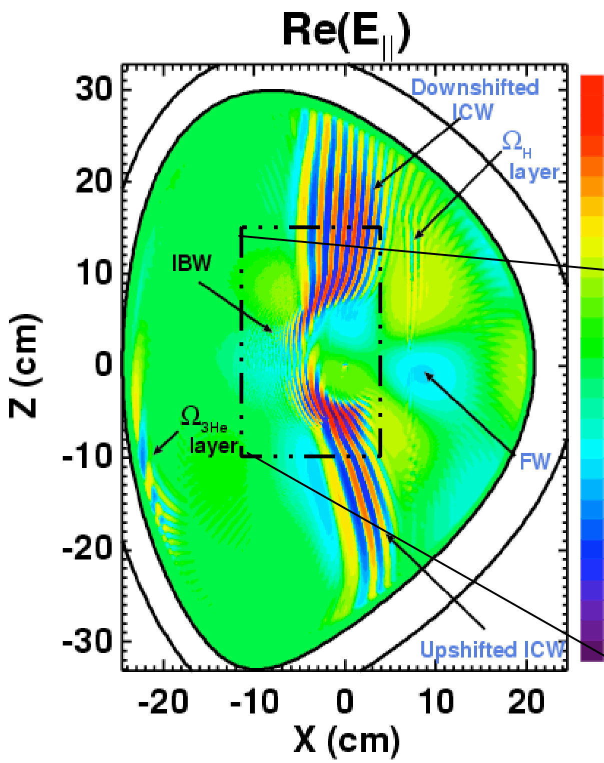


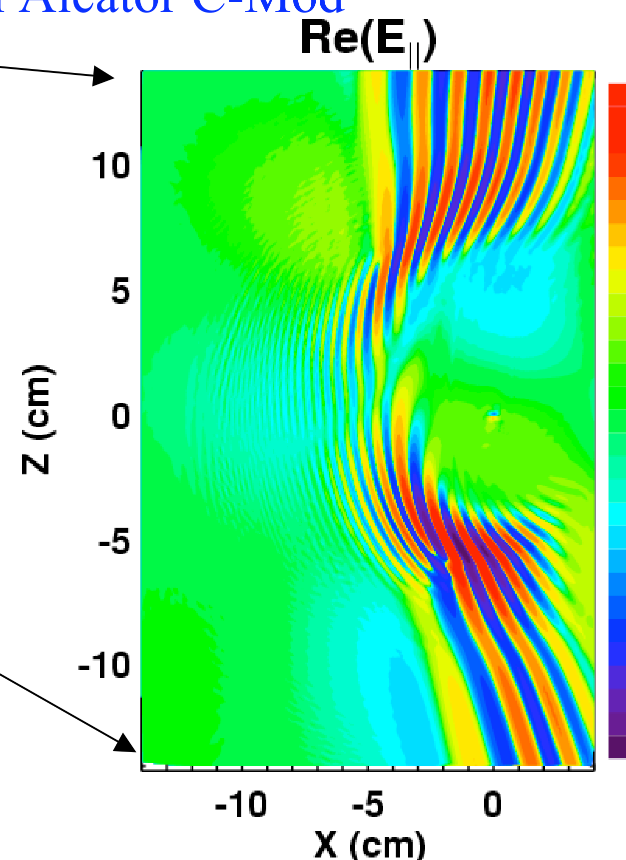
r/a=0.15



Understanding 2D ICRF mode conversion physics has been a triumph of ICRF full-wave modeling

- Three waves – IBW, ICW, FW – are resolved at the MC layer.
- The 2-3 wavelengths of IBW seen in simulations have been verified by PCI measurements on Alcator C-Mod





- The intermediate wavelength ICW travel towards the low field side above and below the midplane.
- Poloidal field allows coupling to ICW (Perkins 1977)

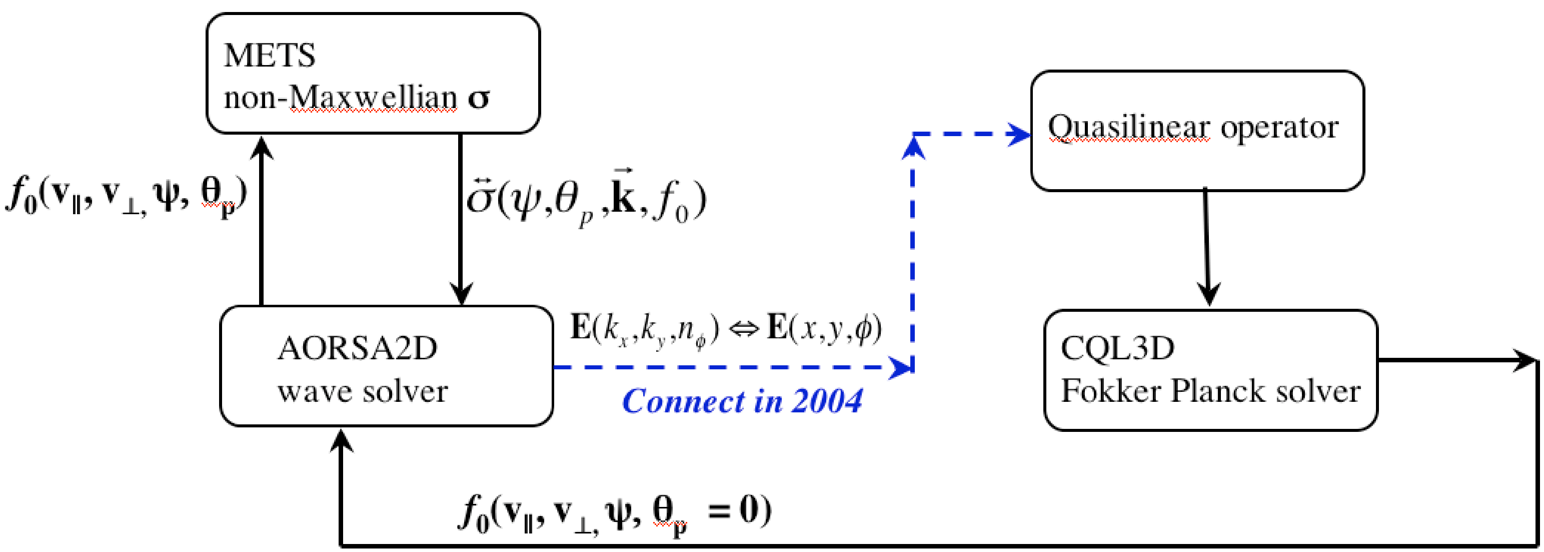
Self-consistency: All the results shown so far assume that f_0 is Maxwellian

- **High power waves can drive the distribution far from Maxwellian**
- **Significant non-Maxwellian components can be produced by neutral injection or fusion alpha particles**

Non-Maxwellian distributions can:

- **Affect local damping rate and wavelength**
- **Modify heating and current drive profiles**
- **Change partition of power deposition among plasma species**
- **Affect mode-conversion**

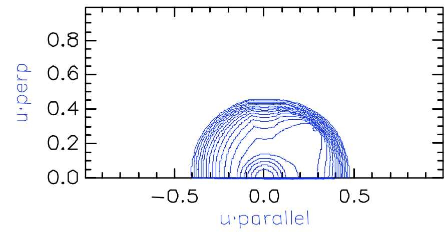
Calculation of wave fields self-consistently with the plasma distribution requires closed loop coupling of four significant physics models

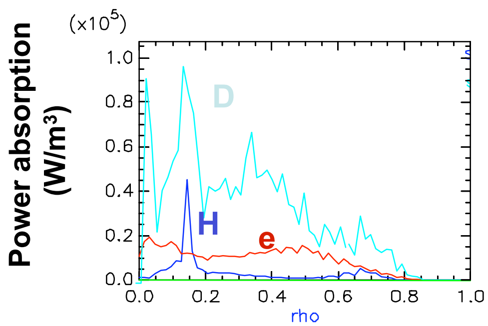


AORSA and CQL3D have been iterated to solve for the wave fields and distribution function self-consistently

NSTX shot 108251 with beams and HHFW (A. Rosenberg, 2003)

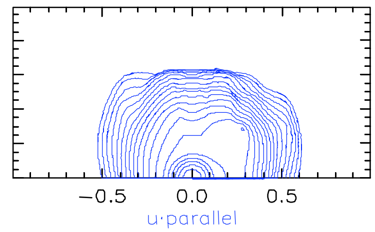
0th iteration
$P_{RF} = 0$

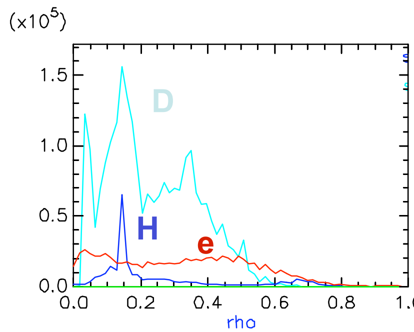




P(D) = .183 MW
= 70.6 %

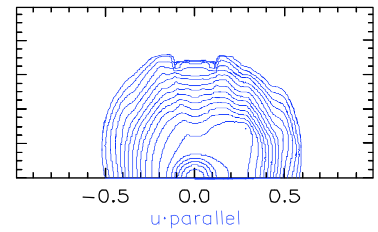
1st iteration
P_{RF} = .24 MW

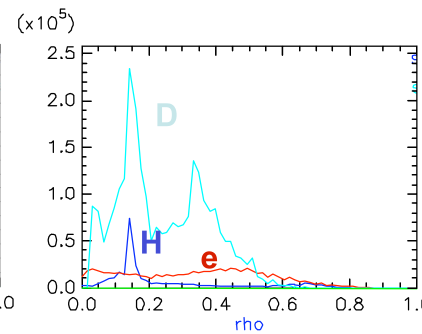




P(D) = .149MW
= 59.8 %

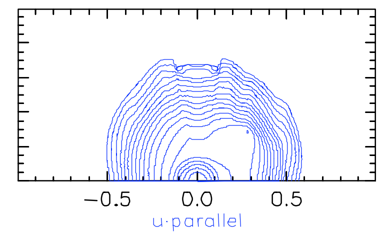
2nd iteration
P_{RF} = .24 MW

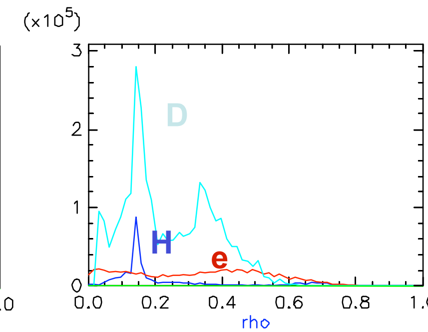




P(D) = .176 MW
= 64.4 %

3rd iteration
P_{RF} = .24 MW

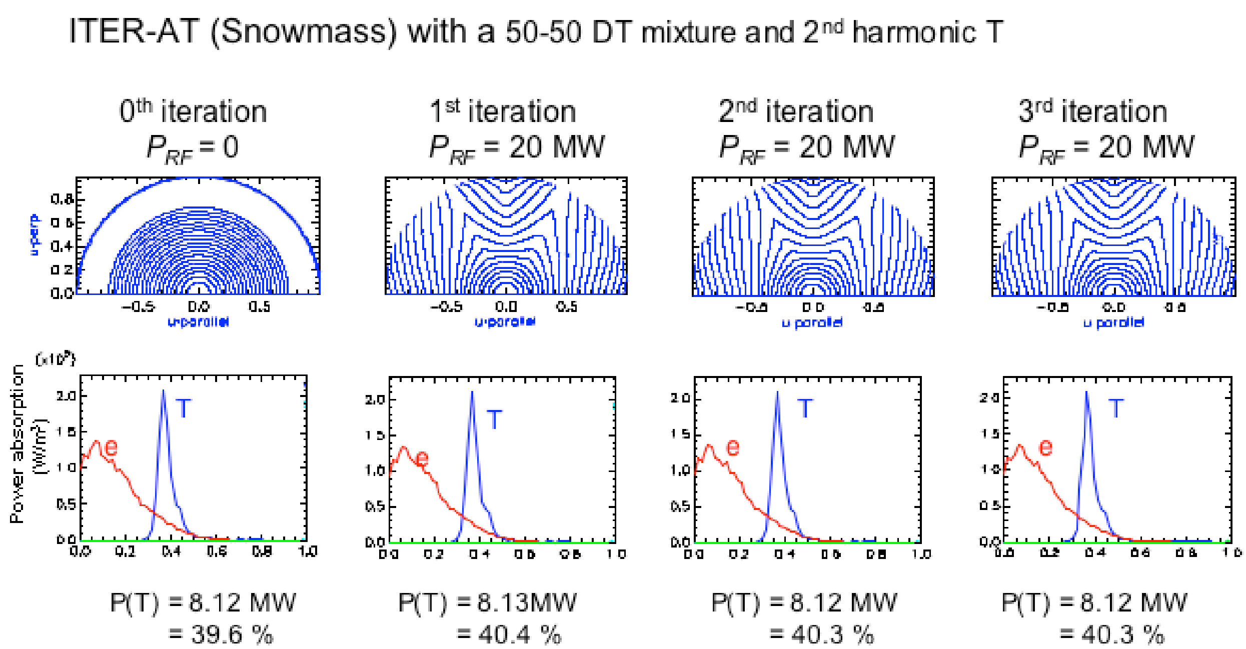




P(D) = .186 MW
= 65.1 %

For higher powers the tail continues to grow without reaching steady-state. Either the plasma itself doesn't reach steady state, or a radial diffusion term is needed in the Fokker Planck solver (CQL3D).

Parasitic absorption by tritium or alphas in ITER could reduce direct electron heating and the associated current drive



- **For 50-50 DT the tritium tail is weak and has little effect on the electron absorption, but could affect the neutron rate.**
- **For alphas the effect can be stronger.**
- **The main point here is that there is a converged, steady-state solution at full power with AORSA2D + CQL3D for ICRH in ITER.**

Monte-Carlo Orbit Solver (ORBIT-RF) and Full Wave Solver (TORIC)

- **ORBIT-RF solves Hamiltonian guiding center drift orbit equations**
- **Interaction of resonant ion with wave is modeled as stochastic RF resonant kicks**

$$\Delta\mu_{rf} = \overline{\Delta\mu_{rf}} + 2\sqrt{3}(R_s - \frac{1}{2})\sqrt{\left\langle \overline{\overline{\Delta\mu_{rf}}}^{2} \right\rangle}$$

$$\overline{\Delta\mu_{rf}} = \frac{\pi q^2 l^2 \Omega^2}{m\omega^2 B}\left|E_+\right|^2 \left[\left| J_{l-1}(k_\perp \rho_i) + e^{2i\vartheta_k}\frac{E_-}{E_+}J_{l+1} \right|^2 + \mu\left\{ 2\left(J_{l-1} + e^{2i\vartheta_k}\frac{E_-}{E_+}J_{l+1} \right)\left(\frac{\partial J_{l-1}}{\partial \mu} + e^{2i\vartheta_k}\frac{E_-}{E_+}\frac{\partial J_{l+1}}{\partial \mu} \right) \right\} \right] \frac{K}{\left|\dot{w}_l\right|}$$

$$\left\langle \overline{\overline{\Delta\mu_{rf}}}^{2} \right\rangle = 2\mu\left[\left| J_{n-1} + e^{2i\vartheta_k}\frac{E_-}{E_+}J_{n+1} \right|^2 \Big/ \left\{ 2\left(J_{n-1} + e^{2i\vartheta_k}\frac{E_-}{E_+}J_{n+1} \right)\left(\frac{\partial J_{n-1}}{\partial \mu} + e^{2i\vartheta_k}\frac{E_-}{E_+}\frac{\partial J_{n+1}}{\partial \mu} \right) \right\} \right]\overline{\Delta\mu_{rf}}$$

- **TORIC4 provides radial profiles for $|E_+|$, $e^{2i\vartheta_k}(E_-|/E_+|)$, $k_\perp$ and $k_{//}$ as a function of (R,Z)**
- **Presently, FW is represented by a single dominant toroidal and poloidal Fourier component.**

Fast MHD phenomena $\tau_{MHD} \ll \tau_{QL}, \tau_{Transp} \Rightarrow$ Sawtooth crash, (Disruptions?, 1/1 precursor, sawtooth trigger?)

- **RF and transport drive initial conditions for MHD – assumes closed nested flux surfaces**
  - **pressure profile**
  - **current profile**
  - **energetic particle populations**
  - **plasma flows**
- **Gaps in present theory and existing codes that must be filled:**
  - **Consistent separation of moments evolution(on transport timescale) and distribution function evolution**
  - **Distribution function evolution that includes both RF and transport e.g.**
    - **Neoclassical bootstrap current calculations do not include velocity space scattering by RF**
    - **RF calculations of f(Ψ, v) do not consistently include cross-flux-surface transport**
  - **Adequate model of plasma rotation/flows**
  - **The magnetic geometry during slow growth phase of sawtooth may contain magnetic islands and other essential 3D effects.**

Slow MHD phenomena $\tau_{MHD} \sim \tau_{QL}, \tau_{Transp} \Rightarrow$ NTM, (Sawtooth precursor?)

- **RF and transport do affect MHD perturbed currents and distributions on MHD mode time-scale $\Rightarrow$ RF and transport effects appear in terms of MHD equations**
- **Perturbed MHD mode fields (e.g. inductive E, magnetic islands) affect RF, transport, and evolution of distribution function $\Rightarrow$ MHD effects appear in terms of RF and transport equations**
- **MHD closure with RF**
  - **Sources: RF (and beams) – heating, current drive, flows – local, but become non-local on NTM time scale**
  - **RF scattering in velocity space – force and heat frictions like collisions**
- **Coupling 3D structures coming out of MHD back into source codes and transport**
  - **Detecting and describing islands – 3D RF calculations, 3D plasma state**
  - **Dealing with turbulent transport – active on this time scale**

Outstanding questions for CEMM related to sawteeth

- What will be the period and inversion radius of the sawtooth oscillation in an ITER-class burning plasma, as a function of plasma current and pressure?
- What physics underlies complete and incomplete reconnection during the sawtooth?
- Under what conditions will the sawtooth instability trigger the onset of a metastable island (neoclassical tearing mode) or lead to a disruption?
- How does this picture change in the presence of energetic particles?

Extensions related to SWIM:

- To what extent can RF waves be used to modify plasma current, pressure and energetic particle populations and affect the period and inversion radius of the sawtooth oscillation in ITER-class burning plasma
- What physics underlies the control that RF waves are observed to exert on sawtooth oscillations

Outstanding questions for CEMM related to NTMs

- Which extended-MHD models give adequate agreement with existing experiments for the formation, growth, and rotation velocity of the observed islands?
- What type of disturbance can cause the neoclassical tearing mode to form in an ITER-class tokamak?
- Under what conditions do "spontaneous NTMs" form?
- What will be the saturated island size as a function of plasma current and beta?
- What level of external current drive power is required to fully stabilize the NTM?

Extensions related to SWIM:

- Which extended-MHD + RF models give adequate agreement with existing experiments for the formation, growth, and rotation velocity of the observed islands?
- What will be the saturated island size as a function of plasma current and beta and launched RF power and spectrum?
- What level of external current drive power is required to fully stabilize the NTM?

Maybe we can take advantage of separations of scale, or of regions where physics may be simpler

- Different processes are dominant in different regions
- Perhaps we need detailed 3D (or 5D) kinetic model only in region 3
- Perhaps processes in regions 1,2,4 affect region 3, but effect of region 3 on 1,2, and 4 can be treated in flux surface average way
- Perhaps outer MHD solution (regions 1,2,4) can be calculated as quasi-static equilibrium
- RF field solutions may not care about islands (2D), but Fokker Planck solutions (power, CD, energetic) particles do

Region 1:

1/1 island from sawtooth, slow growth:

3D MHD, RF heating, CD, transport, kinetics

Region 2:

closed flux surfaces:

RF heating, CD and 1.5D transport

Region 3:

NTM magnetic island chain: 3D MHD, RF heating CD, transport

Region 4:

magnetic separatrix to island region