Parallel neoclassical closures for plasma fluid simulations.

- Need forms for parallel closures that include:
  1. rigorous treatment of linearized collision operator,
  2. interesting magnetic geometry,
  3. time dependence, and
- allow for an efficient numerical implementation in plasma fluid codes.

Close fluid equations with kinetically derived $\vec{q}$ and $\mathbf{\Pi}$.

- Species evolution equations and closure moments for five moment model:

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot n\vec{u} = 0 \quad \rightarrow \text{density}$$

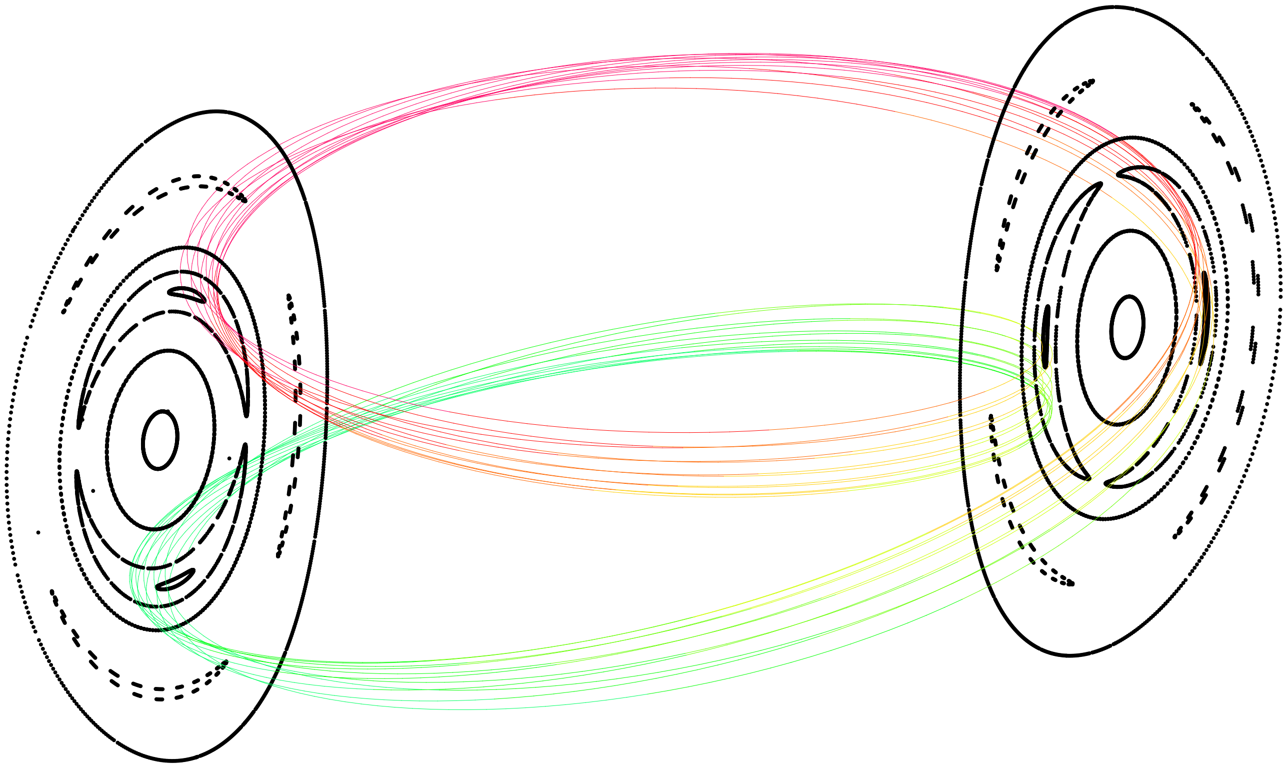
$$mn\left(\frac{\partial}{\partial t} + \vec{u} \cdot \vec{\nabla}\right)\vec{u} = en(\vec{E} + \frac{1}{c}\vec{u} \times \vec{B}) - \vec{\nabla}p - \underline{\vec{\nabla} \cdot \mathbf{\Pi}} + \vec{R} \quad \rightarrow \text{flow}$$

$$\frac{3}{2}n\left(\frac{\partial}{\partial t} + \vec{u} \cdot \vec{\nabla}\right)T = -p\vec{\nabla} \cdot \vec{u} - \underline{\mathbf{\Pi}:\vec{\nabla}\vec{u}} - \underline{\vec{\nabla} \cdot \vec{q}} + Q \quad \rightarrow \text{temperature}$$

$$\underbrace{\vec{q} \equiv \int d^3v' \frac{1}{2}mv'^2\vec{v}' f,}_{\text{heat flow}} \qquad \underbrace{\mathbf{\Pi} \equiv \int d^3v' m[\vec{v}'\vec{v}' - \frac{v'^2}{3}\mathbf{I}]f}_{\text{stress tensor}}.$$

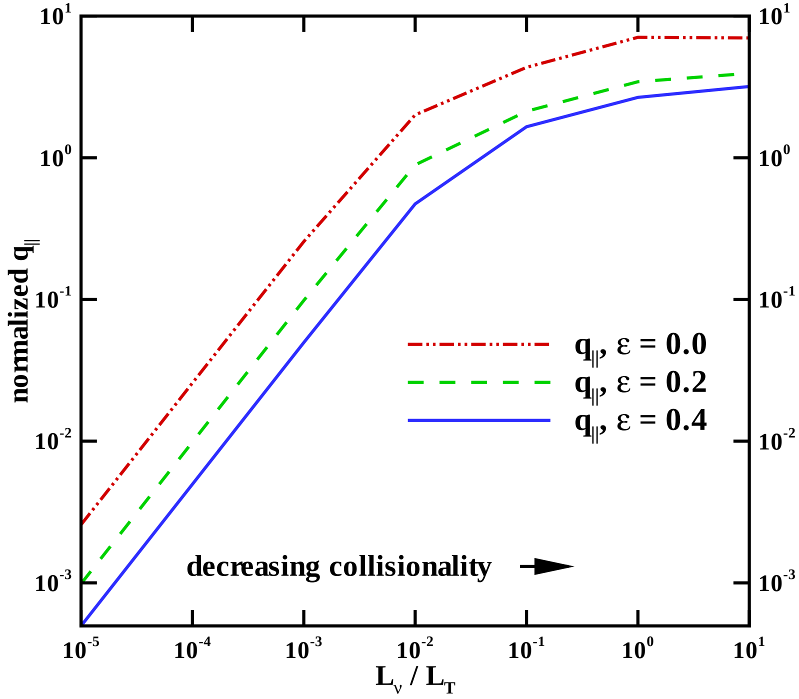
Changing magnetic topology results in large $q_{\|}$.

- Particles see T perturbations of scale length, L_T, which is comparable to the collision length, L_ν.



Particle trapping significantly reduces $q_{\|}$.

- $q_{\|}$ for homogeneous and inhomogeneous $|B|$ shows effect of trapped particles as collisionality varies.



Previous $q_{||}$ derivation lacking.

- Simple, “drift" kinetic equation:

$$\sigma\left[\vec{v}_{||}\cdot\vec{\nabla}_L\left(F^0+f^0\right)\right]=\left[C(F^0+f^0)\right].$$

- Solve separately for Cordey eigenfunctions:

$$\frac{\partial}{\partial\xi}\frac{1-\xi^2}{\xi}\langle\frac{v_{||}B_0}{vB}\rangle\frac{\partial C_n}{\partial\xi}+\lambda_n\langle\frac{v\xi B}{v_{||}B_0}\rangle C_n=0.$$

- Expand F^0 and solve system of ODEs:

$$\mathbf{I}\vec{F}+\frac{v}{\bar{\nu}}\mathbf{A}\frac{\partial\vec{F}}{\partial L}=-\frac{v}{\bar{\nu}}\vec{G}\frac{\partial f}{\partial L}.$$

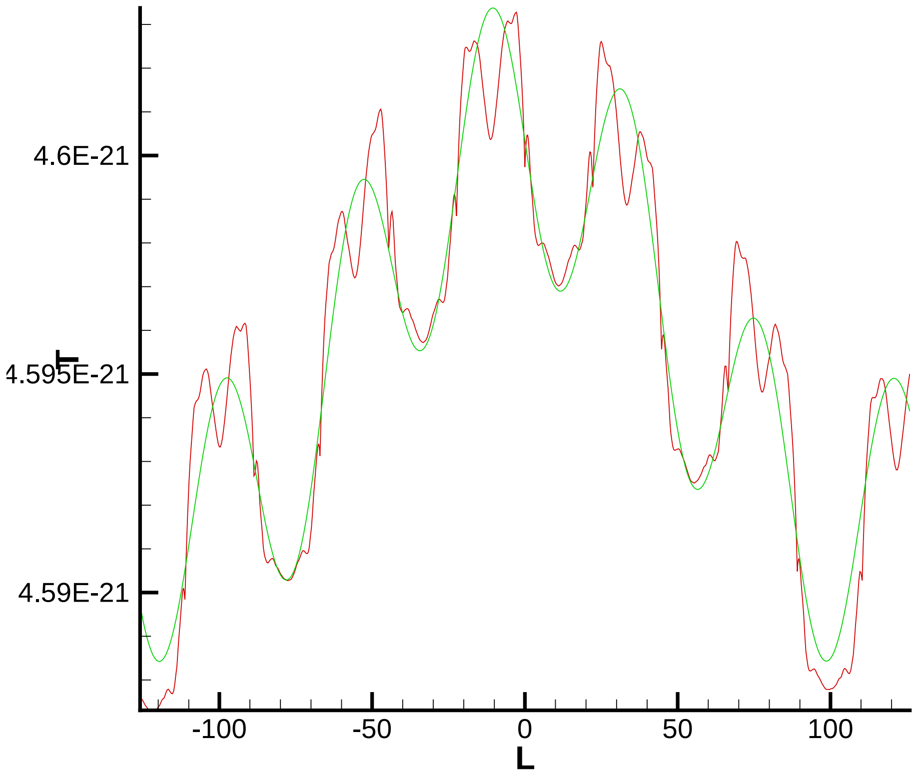
- Write $q_{||}$ in integral form:

$$q_{||}=\frac{n^{eq}v_{th}}{\pi^{3/2}}\int_0^\infty dL\left[T(-L)-T(+L)\right]K(L).$$

Numerical implementation for $q_\parallel$ in NIMROD in place.

- Determine spectral content of T using periodogram and linearly fit to

$$T_0 + \sum_i (T_i^c \cos k_{\parallel i} L + T_i^s \sin k_{\parallel i} L).$$



Provide for more complete closure scheme.

- Employ CEL approach writing:

$$f = f_M(n(\vec{x},t), T(\vec{x},t)) \left[1 + \frac{2}{v_{th}^2}\vec{v}\cdot\vec{u}\right] + F$$

- Derive first-level recursive equation for gyrophase independent $\bar{F}$:

$$\left[\frac{\partial}{\partial t} + \vec{v}_{\|}\cdot\vec{\nabla} + q\vec{v}_{\|}\cdot\vec{E}\frac{\partial}{\partial \epsilon}\right]\bar{F} - C(f_M + \bar{F}) =$$
$$-\frac{m}{T}(v_{\|}^2 - \frac{v_{\perp}^2}{2})(\hat{\mathbf{b}}\hat{\mathbf{b}} - \frac{\mathbf{I}}{3}):\vec{\nabla}\vec{u} f_M + \vec{v}_{\|}\cdot\left(\vec{\nabla}\cdot\mathbf{\Pi} - \vec{R}\right)\frac{f_M}{p}$$
$$-L_1^{1/2}(\mathbf{\Pi}:\vec{\nabla}\vec{u} + \vec{\nabla}\cdot\vec{q} - \tilde{Q})f_M + L_1^{3/2}\vec{v}_{\|}\cdot\vec{\nabla}T\frac{f_M}{T}.$$

Provide for more complete closure scheme.

- Employ CEL approach writing:

$$f = f_M(n(\vec{x},t), T(\vec{x},t)) \left[1 + \frac{2}{v_{th}^2}\vec{v}\cdot\vec{u}\right] + F$$

- Can use T equation to eliminate $\vec{q}$ and $\mathbf{\Pi}{:}\vec{\nabla}\vec{u}$ and ignore acceleration as first cut:

$$\left[\frac{\partial}{\partial t} + \vec{v}_{\|}\cdot\vec{\nabla}\right]\bar{F} - C(f_M + \bar{F}) =$$
$$-\frac{m}{T}(v_{\|}^2 - \frac{v_{\perp}^2}{2})(\hat{\mathbf{b}}\hat{\mathbf{b}} - \frac{\mathbf{I}}{3}){:}\vec{\nabla}\vec{u} f_M + \vec{v}_{\|}\cdot\left(\vec{\nabla}\cdot\mathbf{\Pi} - \vec{R}\right)\frac{f_M}{p}$$
$$+L_1^{1/2}\left[\vec{\nabla}\cdot\vec{u} + \frac{3}{2}\frac{\partial \ln T}{\partial t}\right] f_M + L_1^{3/2}\vec{v}_{\|}\cdot\vec{\nabla}T\frac{f_M}{T}.$$

Consider linearized C_{ss}.

- Keep full test particle (pitch-angle scattering, speed drag and diffusion) and field terms.
- Use limited expansion for $\bar{F} = \sum_{kl} M^{kl} v^l L_k^{l+1/2}(v) P_l(v_\parallel / v)$ to treat linearized collision operator introducing closures as collisional drives.
- Simple test problem to calculate collisional transport coefficients:

moments $\rightarrow$ coefficient $\downarrow$	2	3	4	Braginskii
$r_\parallel / p$	0.65	0.65	0.67	**0.71**
$\chi_{\parallel i}/(v_{thi}^2 \tau_i)$	4.60	2.36	2.76	**2.76**
$\chi_{\parallel e}/(v_{the}^2 \tau_e)$	2.73	1.49	1.64	**1.6**

In general, invert $\frac{\partial}{\partial t} + \vec{v}_{\parallel} \cdot \vec{\nabla}$ approximately.

- Expand $F = \sum_{n=0}^{N} F_n P_n(v_{\parallel}/v)$ to form system of hyperbolic equations for $\vec{F} = (F_0, F_1, ..., F_N)$:

$$\mathbf{I}\frac{\partial \vec{F}}{\partial t} + (\mathbf{A}v\frac{\partial}{\partial L} + \mathbf{B}(\hat{\mathbf{b}} \cdot \vec{\nabla} \ln B))\vec{F} = \vec{g}(\nabla_{\parallel} T, \vec{\nabla}\vec{u}, \vec{q}, \mathbf{\Pi}, ...).$$

  where $\mathbf{B}(\hat{\mathbf{b}} \cdot \vec{\nabla} \ln B)$ matrix arises from spatial dependence of $v_{\parallel}/v = \pm\sqrt{1 - \mu B(\vec{x})/(mv^2/2)}$.

- Further work needed to:
  1. determine roles of passing and trapped distributions.
  2. approximately invert algebraic, PDE operator.
  3. treat time dependent characteristics which complicate numerical implementation.