TWO-FLUID MODELS: WHAT ORDER ?

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TOPICS

- What are the most important parts of the 'two-fluid' model?
- Two-fluid plasma model extends MHD
  - Independent electron and ion motion.
  - Breaks MHD geometrical symmetries $\rightarrow$ important in nonaxisymmetric devices/processes.
- M3D: large amount of work with MHD and 2F+ models shows that even lower order moments are not well understood, esp. NL
  - MHD (simplest: $\partial n/\partial t \equiv 0$, no thermal conductivity)
  - Simplest 2F is drift ordered drift motions + MHD
  - Anisotropic pressures/temperatures
- Important processes — parallel thermal equilibration and $\partial n/\partial t$, even in MHD
- Particle effects (orbits, Landau Damping, etc)
- For physics, how high an order should "two-fluid" equations be?
  Lower order equations (up to heat flux q, with anisotropic temperatures) + particle closures?

Large amount of work done with M3D "two-fluid" models: The view from simulation — Don't need all terms in complete 'two-fluid' expansion to get the important physics for fusion plasmas. Other processes may be more important.

- **'Two-fluid' models break the MHD single fluid constraint.**

- **For toroidal plasmas, this also breaks the MHD geometrical symmetries. Thus even a few 2F terms can have big effects.**

  The MHD equations in a toroidal plasma with

  (i) density n uniform and fixed in time;

  (ii) up-down symmetry;

  (iii) no equilibrium rotation;

  allow two possible parities in (θ, ϕ), $f(r, \theta, \phi) = \pm f(r, -\theta, -\phi)$, where the sign gives the parity. If one MHD variable has only a single parity ($+$ or $-$), then the parities of all other MHD variables are determined. An axisymmetric equilibrium has a single parity, eg, p is $+$. Many common MHD instabilities tend to remain in same parity space (eg, tearing and ballooning modes). Also, in many cases, breaking of the conditions above adds only small corrections. The two-fluid additional terms mix the parities, as do other kinetic, etc terms.

  (In stellarator, where equilibrium has both parities of equal size, the 2F steady states are very different from MHD.)

- **Keep M3D multilevel approach, limits are simpler physical or geometrical models**

  - **Drift-ordered (but keeps full MHD velocity) (reference velocity = $\mathrm{v}_{mhd} = \mathrm{v}_{GC}$ for fluid**

(ion) guiding center), to get diamagnetic effects.
Mostly scalar pressure, some anisotropy trials.

- **Full 2F (ref. velocity = v_i ion fluid velocity). Has numerical problem with whistler wave, but in practice there should be a scale-length cutoff $L \simeq \rho_i$ at ion gyro-radius.**
- **Ion GK particles + electron fluid.**

MHD model

- Solves MHD equations.

$$\begin{cases} \rho\partial \mathbf{v}/\partial t + \rho\mathbf{v}\cdot\nabla\mathbf{v} = -\nabla p + \mathbf{J}\times\mathbf{B} + \mu\nabla^2\mathbf{v} \\ \partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E}, \quad \mathbf{E} = (-\mathbf{v}\times\mathbf{B} + \eta\mathbf{J}), \quad \mathbf{J} = \nabla\times\mathbf{B} \\ \partial\rho/\partial t + \nabla\cdot(\rho\mathbf{v}) = 0 \\ \partial p/\partial t + \mathbf{v}\cdot\nabla p = -\gamma p\nabla\cdot\mathbf{v} + \rho\nabla\cdot\kappa\nabla(p/\rho) \end{cases}$$

The fast parallel equilibration of T is modeled using wave equations;

$$\begin{cases} \partial T/\partial t = s\,\mathbf{B}/\rho\cdot\nabla u \\ \partial u/\partial t = s\,\mathbf{B}\cdot\nabla\mathbf{T} + \upsilon\nabla^2 u \end{cases}$$

s = wave speed/ v_A

Two-fluid MH3D-T

- Solves the two fluid equations with gyro-viscousity and neoclassical parallel viscousity terms in a torus.

- **Equations**

$$\begin{cases} \mathbf{v} \equiv \mathbf{v}_i - \mathbf{v}_i^* = \mathbf{v}_e - \mathbf{v}_e^* + \mathbf{J}_{\parallel}/en, \\ \mathbf{v}_e^* \equiv -\mathbf{B}\times\nabla p_e\ /(enB^2), \quad \mathbf{v}_i^* \equiv \mathbf{v}_e^* + \mathbf{J}_{\perp}/en, \end{cases}$$

$$\rho\partial\mathbf{v}/\partial t + \rho\mathbf{v}\cdot\nabla\mathbf{v} + \rho(\mathbf{v}_i^*\cdot\nabla)\mathbf{v}_{\perp} = -\nabla p + \mathbf{J}\times\mathbf{B} - \mathbf{b}\cdot\nabla\cdot\Pi i,$$

$$\partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E}, \quad \mathbf{E} = (-\mathbf{v}\times\mathbf{B} + \eta\mathbf{J}) - \nabla_{\parallel} p_e/en - \mathbf{b}\cdot\nabla\cdot\Pi e,$$
$$\mathbf{J} = \nabla\times\mathbf{B},$$

$$\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{v}_i) = 0,$$

$$\begin{aligned} \partial p/\partial t + \mathbf{v}\cdot\nabla p = {} & -\gamma p\nabla\cdot\mathbf{v} + \rho\nabla\cdot\kappa_{\parallel}\nabla_{\parallel}(p/\rho) \\ & - \mathbf{v}_i^*\cdot\nabla p + (1/en)\mathbf{J}\cdot\nabla p_e \\ & - \gamma p\nabla\cdot\mathbf{v}_i^* + \gamma p_e\mathbf{J}\cdot\nabla(1/en) \end{aligned}$$

$$\begin{aligned} \partial p_e/\partial t + \mathbf{v}\cdot\nabla p_e = {} & -\gamma p_e\nabla\cdot\mathbf{v} + \rho\nabla\cdot\kappa_{\parallel}\nabla_{\parallel}(p_e/\rho) \\ & + (1/en)\mathbf{J}_{\parallel}\cdot\nabla p_e - \gamma p_e\nabla\cdot(\mathbf{v}_e^* - \mathbf{J}_{\parallel}/en) \end{aligned}$$

GK Hot Particle /MHD Hybrid MH3D-K

- **Fluid equations**

$\rho\partial\mathbf{v}/\partial t + \rho\mathbf{v}\cdot\nabla\mathbf{v} = -\nabla p - (\nabla\cdot\mathbf{P}_h)_\perp + \mathbf{J}\times\mathbf{B}$ (Pressure coupling)

or

$\rho\partial\mathbf{v}/\partial t + \rho\mathbf{v}\cdot\nabla\mathbf{v} = -\nabla p + (\nabla\times\mathbf{B} - \mathbf{J}_h)\times\mathbf{B} + q_h\mathbf{V}\times\mathbf{B}$

(Current coupling)

$\partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E}$, $\quad \mathbf{E} = \mathbf{v}\times\mathbf{B} - \eta(\mathbf{J} - \mathbf{J}_h)$, $\quad \mathbf{J} = \nabla\times\mathbf{B}$

$\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{v}) = 0$

$\partial p/\partial t + \mathbf{v}\cdot\nabla p = -\gamma p\nabla\cdot\mathbf{v} + \rho\nabla\cdot\kappa\cdot\nabla(p/\rho)$

- **Gyrokinetic equations for energetic particles**

$d\mathbf{R}/dt = u[\,\mathbf{b} + (u/\Omega)\mathbf{b}\times(\mathbf{b}\cdot\nabla\mathbf{b})\,] + (1/\Omega)\mathbf{b}\times(\mu\nabla\mathbf{B} - q\mathbf{E}/m)$,

$du/dt = -[\,\mathbf{b} + (u/\Omega)\mathbf{b}\times(\mathbf{b}\cdot\nabla\mathbf{b})\,]\cdot(\mu\nabla\mathbf{B} - q\mathbf{E}/m)$.

GK Particle Ion / Fluid Electron Hybrid

- **Pressure coupling**

$\rho\partial\mathbf{v}/\partial t + \rho\mathbf{v}\cdot\nabla\mathbf{v} = -\nabla\cdot\mathbf{P}i - \nabla Pe + \mathbf{J}\times\mathbf{B}$

$= -\nabla\cdot\mathbf{P}i^{CGL} - \nabla\cdot\Pi i - \nabla Pe + \mathbf{J}\times\mathbf{B}$

$\nabla\cdot Pi^{CGL}$: from particles following GK eqns.

$\nabla\cdot\Pi i$: fluid picture as 2 fluid eqns,

or from particles.

- **Fluid electrons**

$\mathbf{E} = -\mathbf{V}e\times\mathbf{B} + \eta\mathbf{J} + \nabla\cdot\mathbf{P}e/ne$

$= -\mathbf{V}e\times\mathbf{B} + \eta\mathbf{J} + \nabla Pe/ne + \mathbf{bb}\cdot\nabla\cdot\Pi e/ne$

$\partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E}$, $\quad \mathbf{J} = \nabla\times\mathbf{B}$

Pe eqn currently, but $P_{\parallel}$ and $P_\perp$ eqns are planned.

Relative sizes of two-fluid terms

- **Anisotropic pressures (temperatures)**
  In tokamak, find that $\delta T_j/T_j$ is small (few $\times 10^{-4}$), and δT_j is noisy, although flux-surface averages are smoother.

- Two other fluid-based processes have strong effects, even in MHD.
  - Parallel heat flux, expressed as a parallel thermal equilibration (along magnetic field lines).
  - Plasma density evolution $\partial n/\partial t$ from continuity equation have strong effects. ($\partial n/\partial t$ with nonuniform $n_o(\mathrm{x})$ breaks MHD symmetries, although effect is mostly small in axisymmetric plasmas.)
  - Little explored, especially in nonlinear MHD.

- What should parallel heat flux be? Want Landau damping and FLR, anomalous turbulent effects.

- Easier to make and test physical models of heat fluxes than next order moment r's.
  Also, harder to maintain convergence of higher moment equations.

- In many other cases, non-fluid effects are strong: Need particles.
  - NSTX (ie, tight aspect ratio plasma) with strong toroidal rotation
  - Edge pedestals in H-mode (ion orbit losses)
  - Sawtooth, fishbones with hot particles
  - Landau damping in coll'less and semi-collisional tearing

Anisotropic temperature evolution equations

(P.B. Snyder et al, Phys. Plasmas 4 3974 (1997))

$$
\begin{aligned}
\left(\frac{\partial}{\partial t} + \mathrm{v}\cdot\nabla\right) T + \frac{2}{3} T\,\nabla\cdot\mathrm{v} &= -\frac{2}{3}\Pi : \nabla\mathrm{v} - \frac{2}{3}\frac{1}{n}\nabla\cdot\left(q\hat{\mathrm{b}}\right) \\
\left(\frac{\partial}{\partial t} + \mathrm{v}\cdot\nabla\right) \delta T + \frac{2}{3}\delta T\,\nabla\cdot\mathrm{v} &= -\Pi : \nabla\mathrm{v} - 3T\left[\hat{\mathrm{b}}\cdot\left(\hat{\mathrm{b}}\cdot\nabla\right)\mathrm{v} + \hat{\mathrm{b}}\cdot(\mathrm{v}\cdot\nabla)\,\hat{\mathrm{b}}\right] + T\nabla\cdot\mathrm{v} \\
&\quad -(\nu_s\delta T) - \frac{1}{n}\nabla\cdot(\delta q\hat{\mathrm{b}}) + \frac{1}{n}3q_\perp(\nabla\cdot\hat{\mathrm{b}}) \\
\Pi : \nabla\mathrm{v} &= \delta T\left(\frac{1}{3}\nabla\cdot\mathrm{v} + \hat{\mathrm{b}}\cdot\nabla\mathrm{v}\cdot\hat{\mathrm{b}} + \hat{\mathrm{b}}\cdot(\mathrm{v}\cdot\nabla)\hat{\mathrm{b}}\right)
\end{aligned}
$$

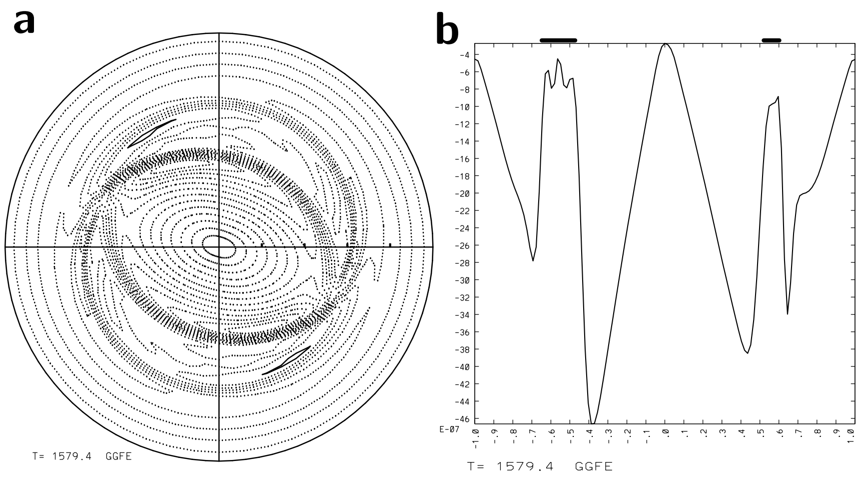
- $(\nu_s\delta T)$ represents the collision term (simplest form shown, using the BGK collision operator).
- The heat fluxes $q_\parallel$, $q_\perp$ from standard M3D models.

Momentum equations: stress tensor term $\nabla\cdot\Pi_\parallel$

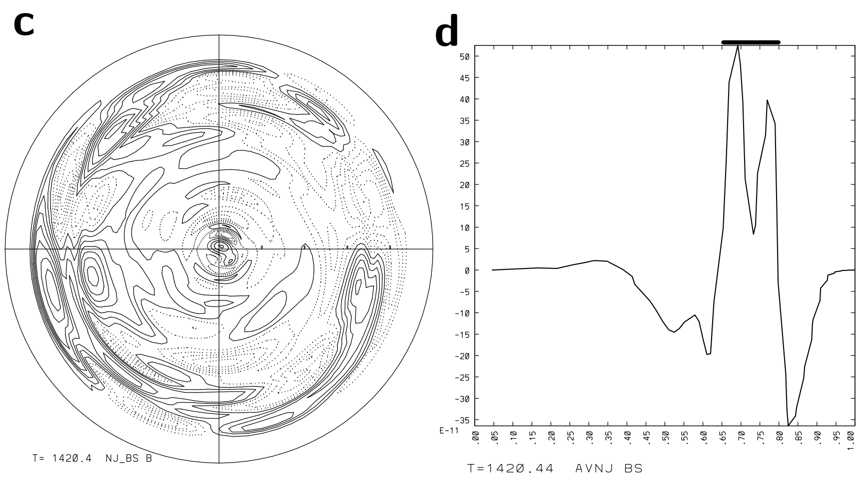
$$
\begin{aligned}
\nabla\cdot\Pi_\parallel &= \nabla p_\perp + \frac{1}{2}f\,\nabla B^2 + f\mathrm{J}\times\mathrm{B} + \mathrm{B}\,(\mathrm{B}\cdot\nabla)\,f \\
f &= \left(p_\parallel - p_\perp\right)/B^2
\end{aligned}
$$

- The neoclassical parallel viscous stress and the perpendicular components appear naturally.

Parallel component of the electron collisional parallel viscous stress, $X = (1/(enB^2))\mathrm{B}\cdot\nabla\cdot\Pi_{e\|}$, for large (2,1) islands.



Island calculated using the flux-surface-averaged neoclassical expressions for $\hat{\mathrm{b}}\cdot\nabla\cdot\Pi_{j\|}$ for the electrons and ions, $X \simeq (1/3enB)(nm_e/\tau_e)\mu_{0e}U_{\theta e}$ where μ_{0e} is the neoclassical coefficient for the flow $U_{\theta e}$, with equilibrium $\nabla T_j = 0$. a) Contours and b) profile as a function of minor radius, left side along $\theta = \pi/2$ in (a) (upper vertical axis) and right side along $\theta = 0$ (right horizontal axis). Helical symmetry.

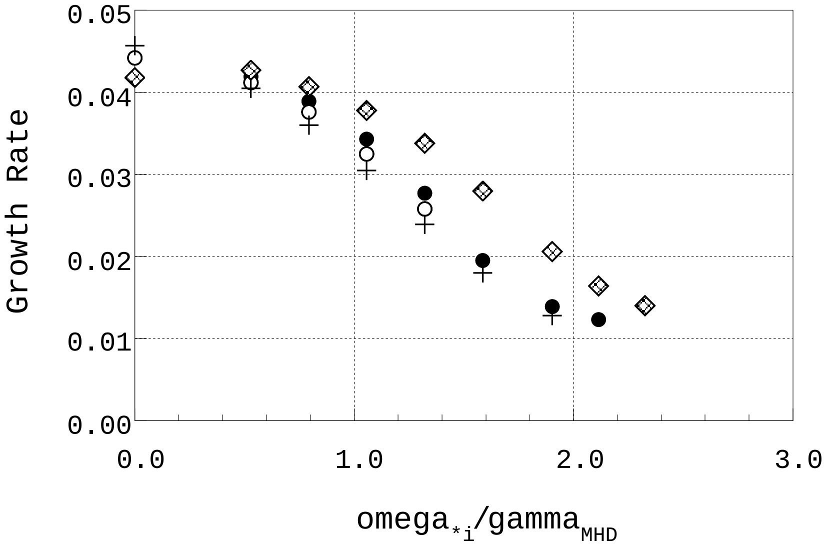


Island calculated by evolving δT_j with MHD, with $\nabla T_j \neq 0$, starting from initial MHD island in torus. c) Contours and d) flux surface average, based on the equilibrium field, as function of minor radius [0,a]. Axis and edge values are zero. Top bars show location of islands.

- **CYLINDRICAL 1/1 INTERNAL KINK shows the ω_{*i}-stabilizing effect is related to internal rotation of the mode. The kink inside $q < 1$ rotates poloidally relative to the magnetic X-point at $q \simeq 1$, most out of phase at maximum stabilization $\omega_{*i}/\gamma_{MHD} \simeq 2$.**

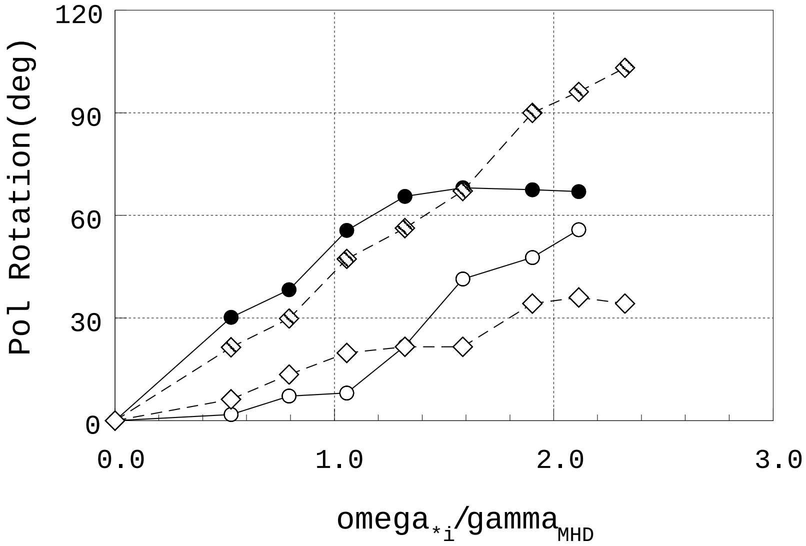
Growth rate

o $p_e = p_i$, $\diamond$ $p_e \equiv 0$

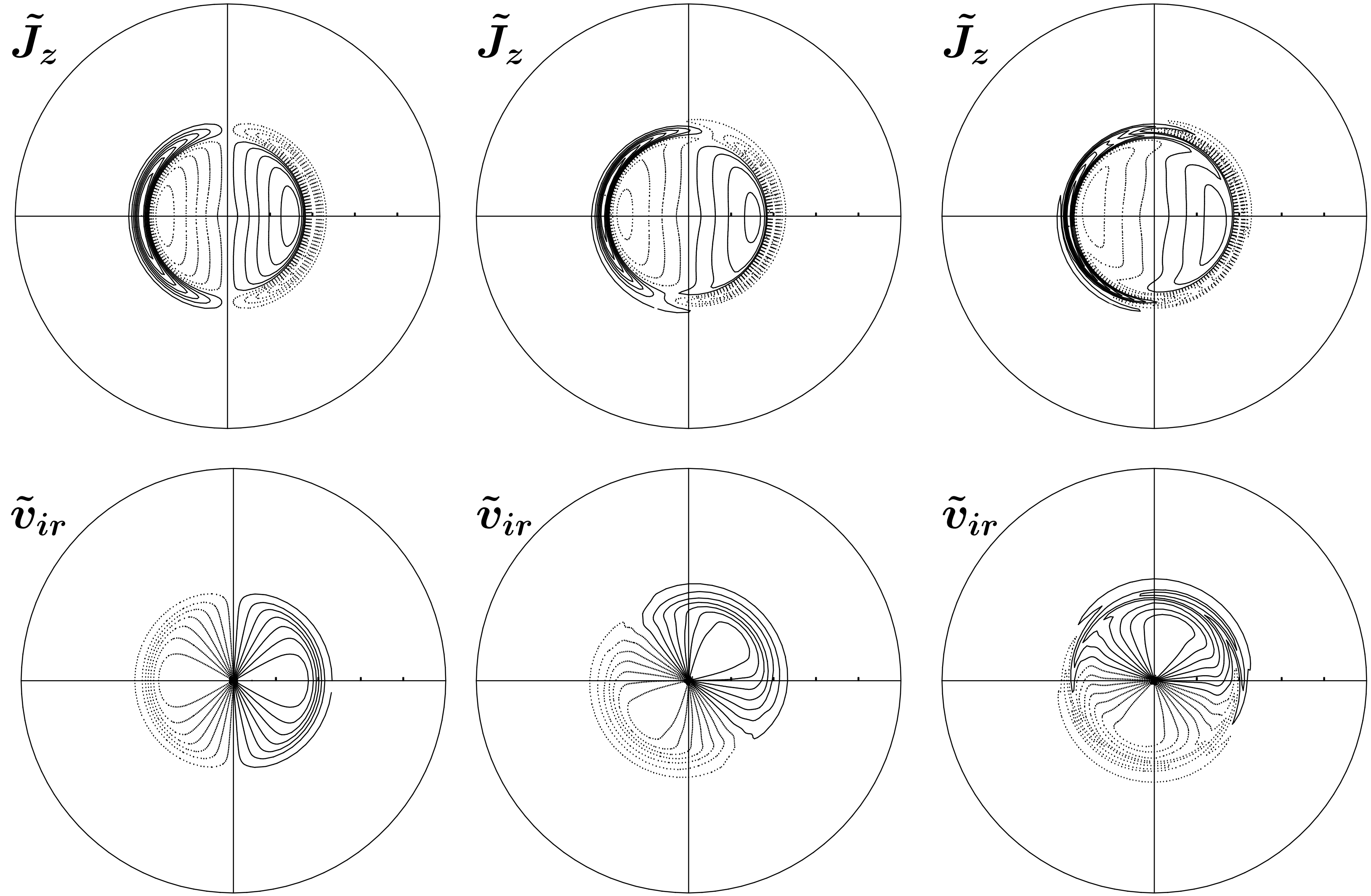


Internal poloidal shift

solid v_{ir}, open $v_{r,GC}$



- **Radial perturbed diamagnetic flow $\tilde{v}_{*ir} = (\mathrm{B} \times \widetilde{\nabla p_i / enB^2})_r \simeq -B_{\phi o}(im\tilde{p}_i/r)/(enB_o^2)$ is always $\pi/2$ out of phase with $\tilde{p}$.**
  Total ion flow $\tilde{v}_{ir} = \tilde{v}_{*ir} + \tilde{v}_{r,GC}$ (GC is guiding center flow perpendicular to B).
  As ω_{*i}/γ_{MHD} increases, $\tilde{v}_{*ir} > \tilde{v}_{r,GC}$ and the kink flow rotates poloidally (counterclockwise). $\tilde{v}_{r,GC}$ also rotates in the same direction.



$\omega_{*i}/\gamma_T = 0$
MHD

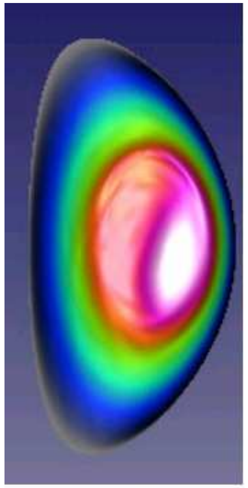
$\omega_{*i}/\gamma_T = 1.06$

$\omega_{*i}/\gamma_T = 1.9$

Two-fluid NSTX sawtooth with toroidal rotation

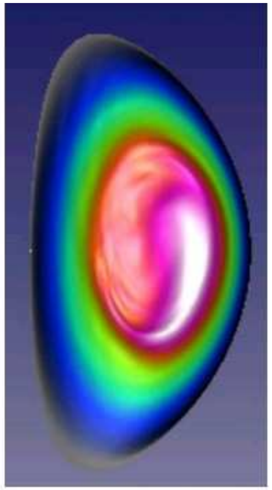
Two-fluid effect depends on direction of rotation, unlike MHD.

(Temperature shown)



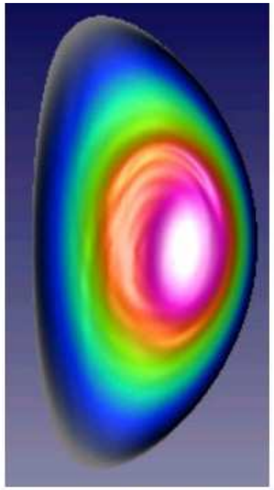
MHD

$M_A = \pm 0.3$



Ctr-rot 2Fluids

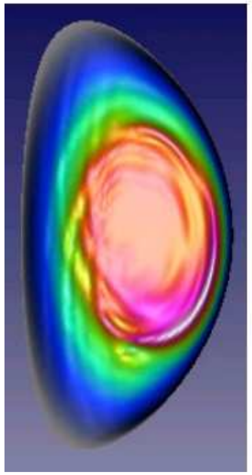
$M_A = -0.3$

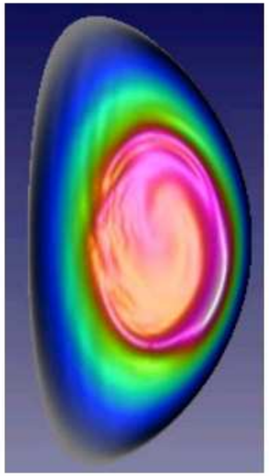


Co-rot 2Fluids

$M_A = +0.3$

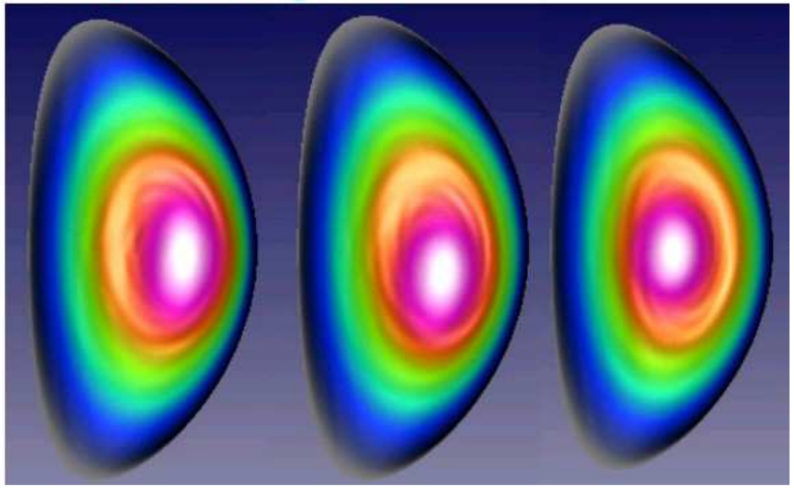
(expt)





Crash faster than MHD

Saturates: hot spot pulls away from X-point



$\nabla_{\|} p_e$ destabilization of tearing

- In a tokamak, $\nabla_{\|} p_e$ in Ohm's law increases nonlinear magnetic reconnection rates and saturated island sizes. Turning on the term from MHD increases the nonlinear reconnection rate and island size. Turning off the term reduces the island to the original size.

- Time evolution of 2/1 island width in a TFTR supershot-like tokamak $p_e/p = 0.5$ with MH3D-T. (Steep pressure profiles $\longrightarrow$ strong effect.)

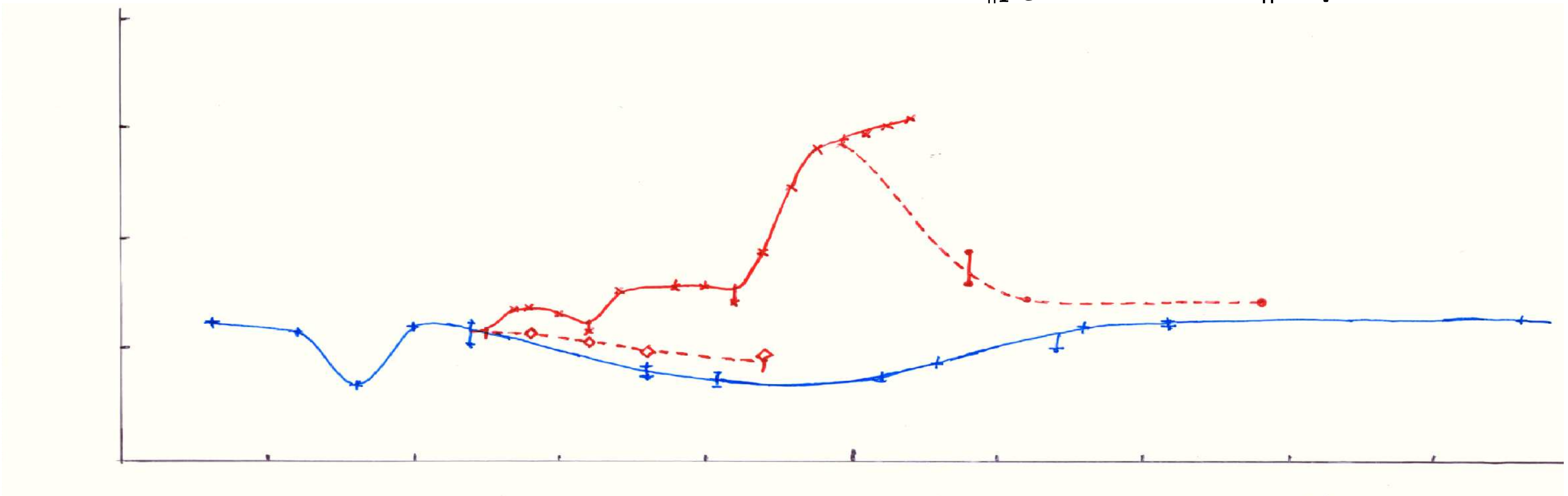
blue MHD island (saturates)

red x $\nabla_{\|} p_e$ on at $t = 440$

red ● $\nabla_{\|} p_e$ turned off at $t = 980$

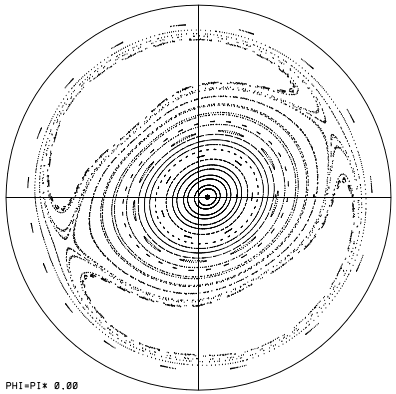
red ◇ $\nabla_{\|} p_e$ on, but fast $\|$ equilibration.

Island Width

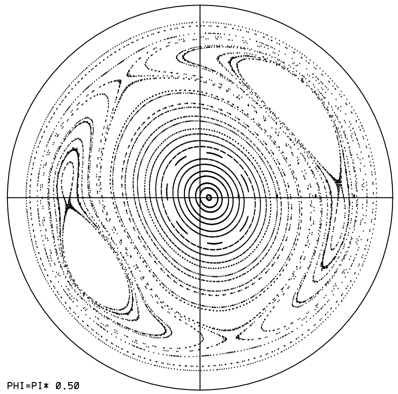


0 1000 2000

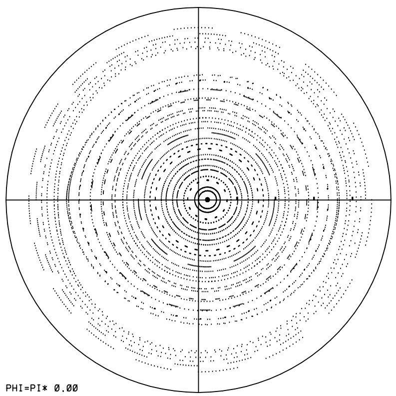
time (τ_A)



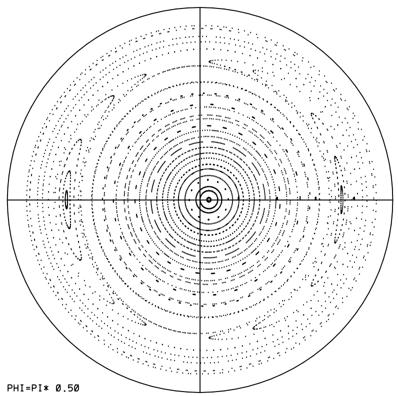
a) ∇p_e in Ohm's law, $\phi = 0$



b) $\phi = \pi/2$



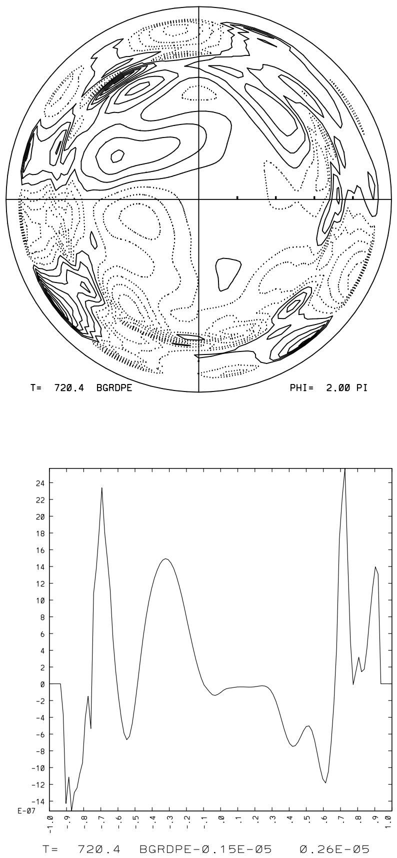
c) MHD, $\phi = 0$



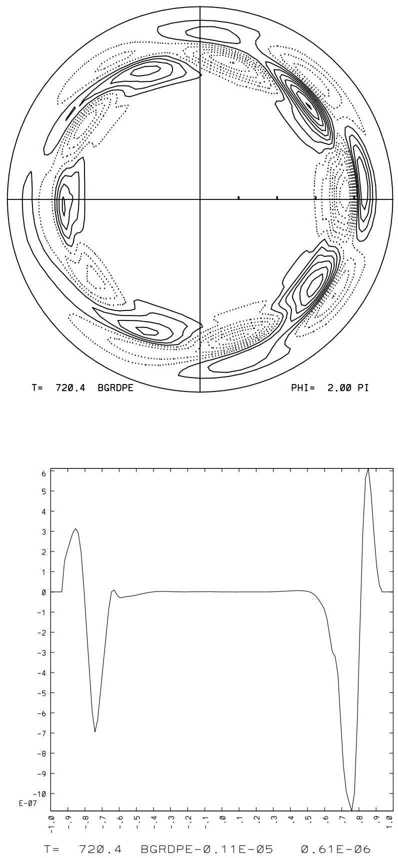
d) $\phi = \pi/2$

$\nabla_{\|} p_e$ in Ohm's law causes fast nonlinear growth of the resistive island over MHD, when the density evolves.

Two-fluid resembles MHD, except more $m = 4$ component.



a) $\mathrm{B}\cdot\boldsymbol{\nabla}\mathbf{T_e}\rightarrow\mathbf{0}$ **accel.**



b) $\mathrm{B}\cdot\boldsymbol{\nabla}\mathbf{p_e}\rightarrow\mathbf{0}$ **accel, at fixed n.**

Parallel gradient $\mathrm{B}\cdot\boldsymbol{\nabla}p_e$ in Ohm's law drives island growth.

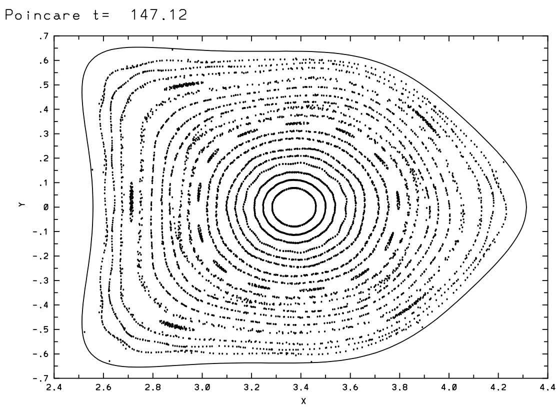
Acceleration of $\mathrm{B}\cdot\boldsymbol{\nabla}T_e\rightarrow 0$ leaves density contribution to $\mathrm{B}\cdot\boldsymbol{\nabla}p_e$.

Acceleration of $\mathrm{B}\cdot\boldsymbol{\nabla}p_e\rightarrow 0$ reduces island growth to $\boldsymbol{\nabla}p_e\simeq 0$ level.

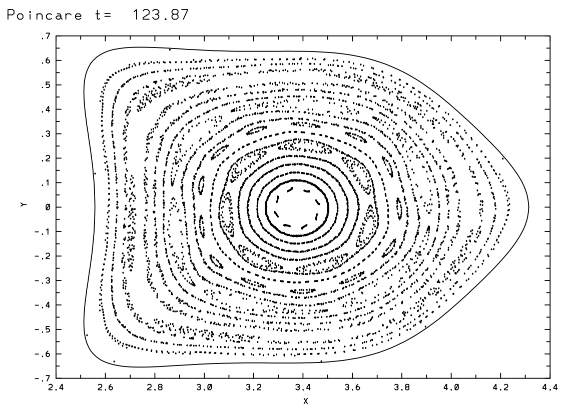
Two-fluid magnetic reconnection may set the intrinsic limit on beta in stellarators — a soft beta limit.

- **Nonlinear magnetic reconnection rates at low-order rational surfaces are enhanced by increasing electron pressure, due to $\nabla_{\|} p_e / en$ in Ohm's law.**

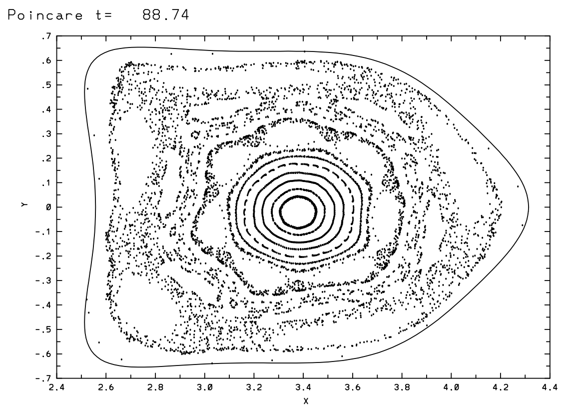
$$p_e/p = 0.05$$



$$p_e/p = 0.5$$

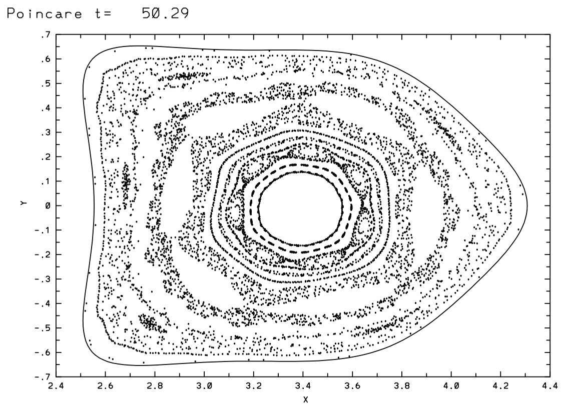


$$p_e/p = 0.95$$



- **Two-fluid reconnection rates increase strongly with beta at high beta.**

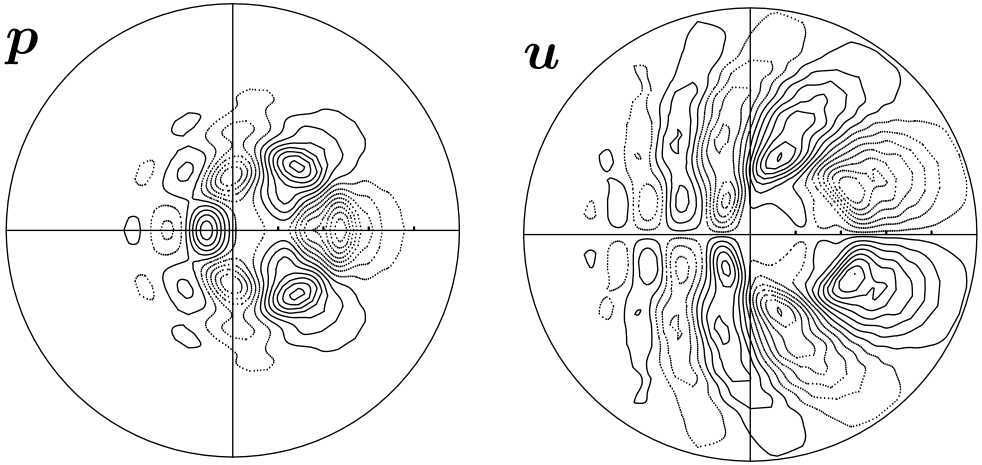
$$\beta = 8\%$$
$$p_e/p = 0.5$$
$$t = 50.3$$



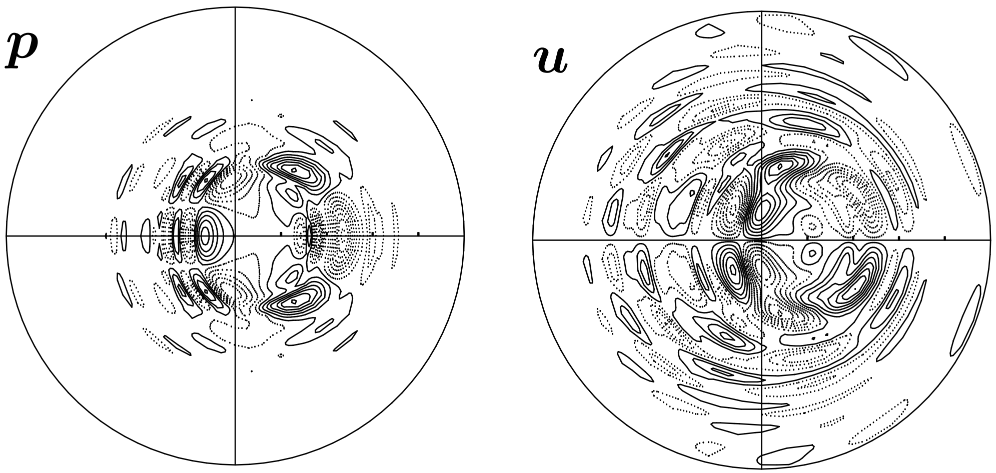
Parallel thermal equilibration and density evolution

- M3D parallel heat flux due to thermal equilibration, not diffusive temperature gradient $\kappa_{\|}\nabla_{\|}T$.
  - Modeled by wave propagation ('artificial sound' method) at given velocity (typical $v_a = v_{th}$).
  - For T, preserves flux-tube-integrated pressure.
  - Time-dependent, not instantaneous.
- Parallel thermal equilibration has very strong effect on typical tokamak macro-instabilities.
  - Physically, decouples cross-field dependence in favor of along field line.
  - But, to equilibrate temperature completely over a large island takes a very long time, multiple 10^5 toroidal transits.
- Density evolution, $\partial n/\partial t$ by continuity equation, has opposing effect.
  - Parallel thermal equilibration operates only on T_j, not n, and n is not well equilibrated except for slow processes.
  - Compressibility across field lines introduced, sim to non-reduced vs reduced MHD.
- Density evolution in MHD couples different-n modes (islands) more closely (NL).
- In 2F, parallel thermal equilibration effect reduced still further, since T_e has fast (v_{the}) equilibration, while T_i slower.
  Find that T_e is often enough to simulate MHD.

Parallel thermal equilibration and density evolution have strong opposing effects on linear MHD resistive ballooning mode with $m > 1$ in a torus (m/n=3/2).



Parallel thermal equilibration on T and $\partial n/\partial t$, $\gamma = 0.029$.
Eigenfunctions similar to case $\kappa_{\|} \equiv 0$, $\tilde{n} \equiv 0$, $\gamma = 0.032$.

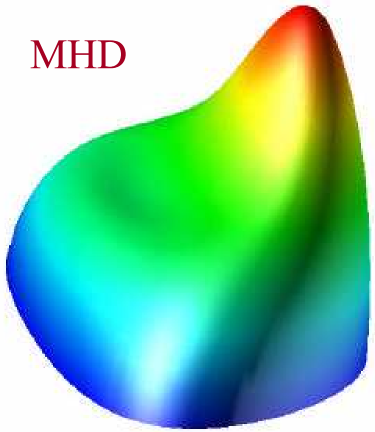


Parallel thermal equilibration (on p) with $\tilde{n} \equiv 0$ stabilizes mode ($\gamma < 0$).
Radial structure breaks up.

Density profile dependence on Physics model

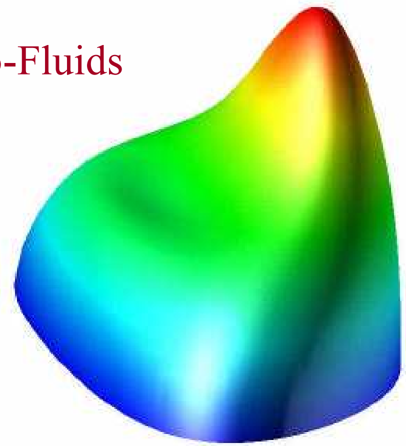
NSTX ε=1.3 q_o=0.8 q_b=5

MHD



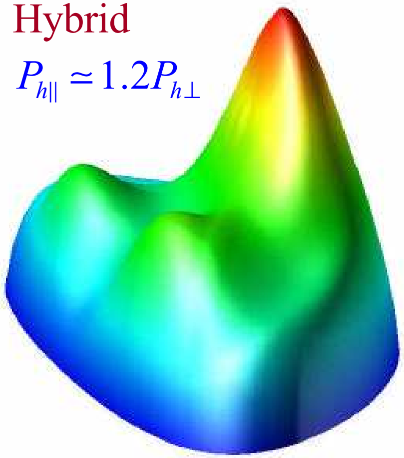
M_A=0.2
Sh=0.3
ρ_{max}=1.1
ρ_{min}=0.5
RelSh=1

Two-Fluids



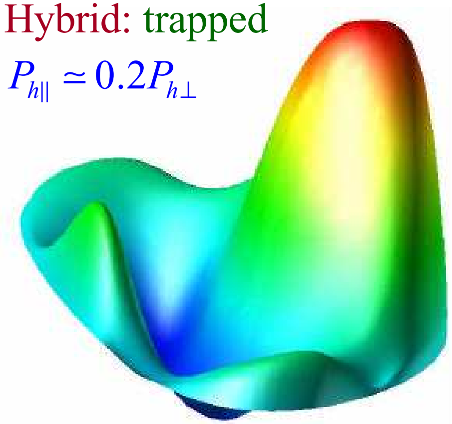
M_A=0.2
Sh=0.3
ρ_{max}=1.1
ρ_{min}=0.5
RelSh=1

Hybrid
$P_{h\parallel} \simeq 1.2 P_{h\perp}$



M_A=0.2
Sh=0.3
ρ_{max}=1.2
ρ_{min}=0.5
RelSh=0.8

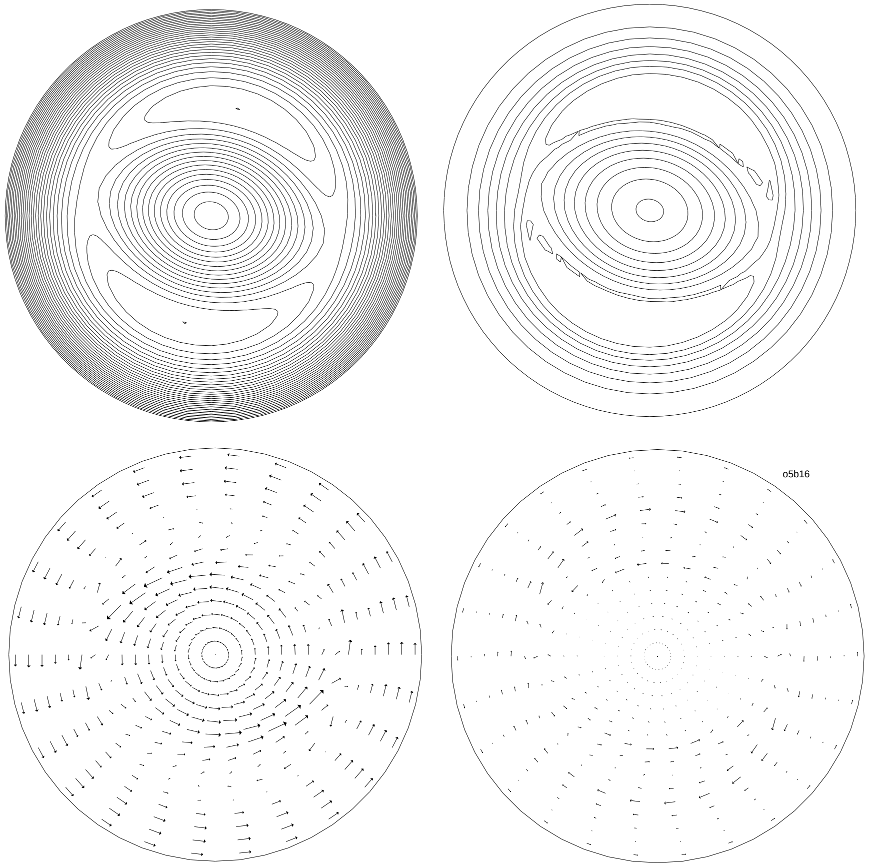
Hybrid: trapped
$P_{h\parallel} \simeq 0.2 P_{h\perp}$



M_A=0.2
Sh=0.3
ρ_{max}=1.8
ρ_{min}=0.15
RelSh=1.9

Neoclassical Parallel Viscous Stress

- **Major parts of neoclassical parallel viscous stress effect can be modeled very simply. Magnetic island driven by neoclassical MHD for $\nabla T_j = 0$, equilibrium $T_i = T_e$.**



The $m = 2$, $n = 1$ island is stable with resistive MHD, but becomes unstable when the ($\|$ part of) neoclassical MHD parallel viscous forces are included. The saturated island, shown here as contours of the helical magnetic flux (top left) and the electron pressure (top right), rotates with $\omega \simeq 0.7\omega_{*i}$ in the guiding center frame ($\mathrm{v}_\perp$ shown at bottom left) and with $\omega \simeq 0.3\omega_{*e}$ in the lab (plasma mass) frame ($\mathrm{v}_{i\perp}$, bottom right). Without the neoclassical terms, a saturated island would have $\omega = 0$. The ion poloidal momentum damping due to $\mathrm{B}\cdot\nabla\cdot\Pi_{i\|}$, also makes $\mathrm{v}_{i\perp} \simeq 0$ outside the island (bottom right).

SUMMARY

- **Two-fluid M3D code — extensive work with MHD and simple 2F and kinetic models shows that important extended MHD terms are not just 2F.**

- **Two-fluid plasma processes are important — axisymetric MHD symmetry broken. 2F diamagnetic terms important.**

- **Parallel thermal equilibration (conduction/Landau damping) with density evolution has strong effects, related to 2F terms.**
  Need good model that can be related to physics. Better than going to next order.

- **Particle effects important for fusion plasmas, not in fluid closures. Combine fluid and particle models.**