For the Phase-II comparisons, we are taking the Hall-MHD subset of the Extended MHD equations, with a fixed ion-to-electron pressure ratio. They may be written, in MKS units, as follows:

Dimensional Equations:

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} \qquad \mu_0 \vec{J} = \nabla \times \vec{B}$$

$$\vec{E} = -\vec{V} \times \vec{B} + \eta \vec{J} + \frac{1}{ne}[\vec{J} \times \vec{B} - \nabla p_e]$$

$$nM_i(\frac{\partial \vec{V}}{\partial t} + \vec{V} \bullet \nabla \vec{V}) + \nabla p = \vec{J} \times \vec{B} + \nu \nabla \bullet \left[\nabla \vec{V} + \nabla \vec{V}^{\dagger}\right]$$

$$\frac{\partial n}{\partial t} + \nabla \bullet (n\vec{V}) = 0$$

$$\frac{3}{2}\frac{\partial p}{\partial t} + \nabla \bullet \left(\frac{3}{2} p\vec{V}\right) = -p\nabla \bullet \vec{V} + \nu\left[\nabla \vec{V} + \nabla \vec{V}^{\dagger}\right] : \nabla V + \nabla \bullet \kappa \nabla \bullet (p/n) + \eta \vec{J}^2$$

$$p_e = \frac{1}{6} p$$

These equations have the energy integral:

$$\frac{\partial}{\partial t}\left(\frac{1}{2\mu_0} B^2 + \tfrac{1}{2} nM_i V^2 + \tfrac{3}{2} p\right) = -\nabla \bullet \left(\frac{5}{2} p\vec{V} + \tfrac{1}{2} nM_i V^2 \vec{V} + \frac{1}{\mu_0} \vec{E} \times \vec{B} - \nu\left(\nabla \vec{V} + \nabla \vec{V}^{\dagger}\right) \cdot \vec{V} - \kappa \nabla\left(p/n\right)\right)$$

UNITS:

$$\vec{B} - \text{tesla} \; = \; \frac{\text{m}}{\text{s} \bullet \text{coulomb}}$$

$$\vec{V} - \frac{\text{m}}{\text{s}}$$

$$\eta - \text{Ohm-m} = \frac{(\text{kg}) \bullet \text{m}^3}{\text{s} \bullet (\text{coulomb})^2}$$

$$\nu - \frac{\text{kg}}{\text{m} \bullet \text{s}}$$

$$\kappa - \frac{1}{\text{m} \bullet \text{s}}$$

$$\mu_0 - \frac{(\text{kg}) \bullet \text{m}}{\left(\text{coulomb}\right)^2}$$

Dimensionless equations: (Here, $\vec{B}, n, \nabla, \vec{V}, t, p$ are dimensionless variables.)

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left[\vec{V} \times \vec{B} - \hat{\eta} \nabla \times \vec{B} - \frac{1}{n} \left(J \times B - \tfrac{1}{6} \nabla p \right) \right]$$

$$n\left(\frac{\partial \vec{V}}{\partial t} + \vec{V} \bullet \nabla \vec{V}\right) + \nabla p = \left(\nabla \times \vec{B}\right) \times \vec{B} + \hat{\nu} \nabla \bullet \left[\nabla \vec{V} + \nabla \vec{V}^{\dagger} \right]$$

$$\frac{\partial n}{\partial t} + \nabla \bullet (n\vec{V}) = 0$$

$$\frac{3}{2}\frac{\partial p}{\partial t} + \nabla \bullet \left(\frac{3}{2} p \vec{V} \right) = -p \nabla \bullet \vec{V} + \hat{\nu} \left[\nabla \vec{V} + \nabla \vec{V}^{\dagger} \right] : \nabla \vec{V} + \nabla \bullet \hat{\kappa} \nabla \bullet (p/n) + \hat{\eta} \left(\nabla \times \vec{B} \right)^2$$

To convert between MKS units and dimensionless units, we now need to specify only 2 dimensional quantities since it is implied that the unit of length is given by:

$\ell_0 = \frac{c}{\omega_{pi}} \equiv \left(\frac{M_i}{n_0 e^2 \mu_0} \right)^{1/2}$. To be specific, let: $B_0 = 1$ tesla, $n_0 = 10^{20} m^{-3}$..

Then, the conversion from the dimensionless to dimensional quantities are:

Velocity:	$V_0 = V_A \equiv \left(\frac{B_0^2}{\mu_0 n_0 M_i} \right)^{1/2} = 2.1812 \times 10^6 \ m/s$
time:	$t_0 = \frac{\ell_0}{V_A} = \ell_0 4.58 \times 10^{-7} \ s$
pressure:	$p_0 = \frac{B_0^2}{\mu_0} = 7.957 \times 10^5$ pascals
resistivity:	$\eta = \mu_0 \ell_0 V_A \times \hat{\eta} = \ell_0 2.739 \times \hat{\eta}$
viscosity:	$\nu = \ell_0 n_0 M_i V_A \times \hat{\nu} = \ell_0 0.364 \times \hat{\nu}$
thermal conductivity:	$\kappa = \ell_0 V_A n_0 \times \hat{\kappa} = \ell_0 2.186 \times 10^{26} \times \hat{\kappa}$
kinetic energy:	$M_i n_0 V_0^2 = 7.957 \times 10^5 \ J/m^3$

Problem Definition:

Dimensionless/Dimensional problem specification is as follows:

	Dimensionless	*Dimensional*
$L_x/2 < x < L_x/2$	$L_x=25.6$	$L_x=25.6 \ \ell_0$
$L_y/2 < y < L_y/2$	$L_y=12.8$	$L_y=12.8 \ \ell_0$
$\psi(x,y) = \psi_0 \left[\begin{array}{l} \frac{1}{2}\ln\left(\cosh 2y\right) \\ +0.1\cos\left(\frac{2\pi x}{L_x}\right)\cos\left(\frac{\pi y}{L_y}\right) \end{array} \right]$	$\psi_0=1$	$\psi_0 = 1$ Tesla/m^2
$p(x,y) = \frac{p_0}{2}\left(\text{sech}^2(2y) + 0.2\right)$	$p_0 = 1$	$p_0 = 7.957 \times 10^5$ pascals
$n(x,y) = n_0 \left(\text{sech}^2(2y) + 0.2\right)$	$n_0 = 1$	$n_0 = 1 \times 10^{20}$ m^{-3}

resistivity	$\hat{\eta} = 0.005$	η = 0.0137 Ohm-m
viscosity	$\hat{\nu} = 0.050$	ν = 0.0182 kg/m-s
thermal conductivity	$\hat{\kappa} = 0.02$	$\kappa = 4.37 \times 10^{24}$ m^{-1}-s^{-1}

To compare the kinetic energies, we need to multiply the dimensionless one by the conversion factor 7.957×10^5 J/m^3