

RMP Effects On Pedestal Plasma Transport

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Issue To Be Addressed:

Is RMP-flutter-induced¹ pedestal plasma transport important?²

Theses:

- At pedestal top² RMPs reduce $|\vec{\nabla}T_e|$ more than $|\vec{\nabla}n_e|$, with changes $\propto I_{\text{coil}}^2$; they also change plasma toroidal rotation Ω_t .
- Flow-screening averts stochasticity-induced³ but not flutter¹ xport.
- RMP-flutter-induced transport¹ might cause observed transport.²
- Model implications are different at low⁴ & high⁵ collisionality.

¹J.D. Callen, “Drift-Wave Turbulence Effects on Magnetic Structure and Plasma Transport in Tokamaks,” Phys. Rev. Lett. **39**, 1540 (1977).

²J.D. Callen, A.J. Cole, C.C. Hegna, S. Mordijck, R.A. Moyer, “RMP effects on pedestal structure and ELMs,” UW-CPTC 11-13R (to be pub. in NF).

³A.B. Rechester and M.N. Rosenbluth, “Electron heat transport in a tokamak with destroyed magnetic surfaces,” Phys. Rev. Lett. **40**, 38 (1978).

⁴T.E. Evans et al., “RMP ELM suppression in DIII-D plasmas with ITER similar shapes and collisionalities,” Nucl. Fusion **48**, 024002 (2008).

⁵W. Suttrop et al., “Studies of edge localized mode mitigation with new active in-vessel saddle coils in ASDEX Upgrade,” PPCF **53**, 124014 (2011).

RMPs Reduce Pressure Gradient At Pedestal Top

- RMP-induced reductions in $|\vec{\nabla}P|$ are:
 - small in core,
 - largest at the **pedestal top**, and
 - small (increase!?) at the edge.

- Key transport issue for ELM suppression is:

How do RMPs reduce $|\vec{\nabla}P|$ at the **pedestal top**?

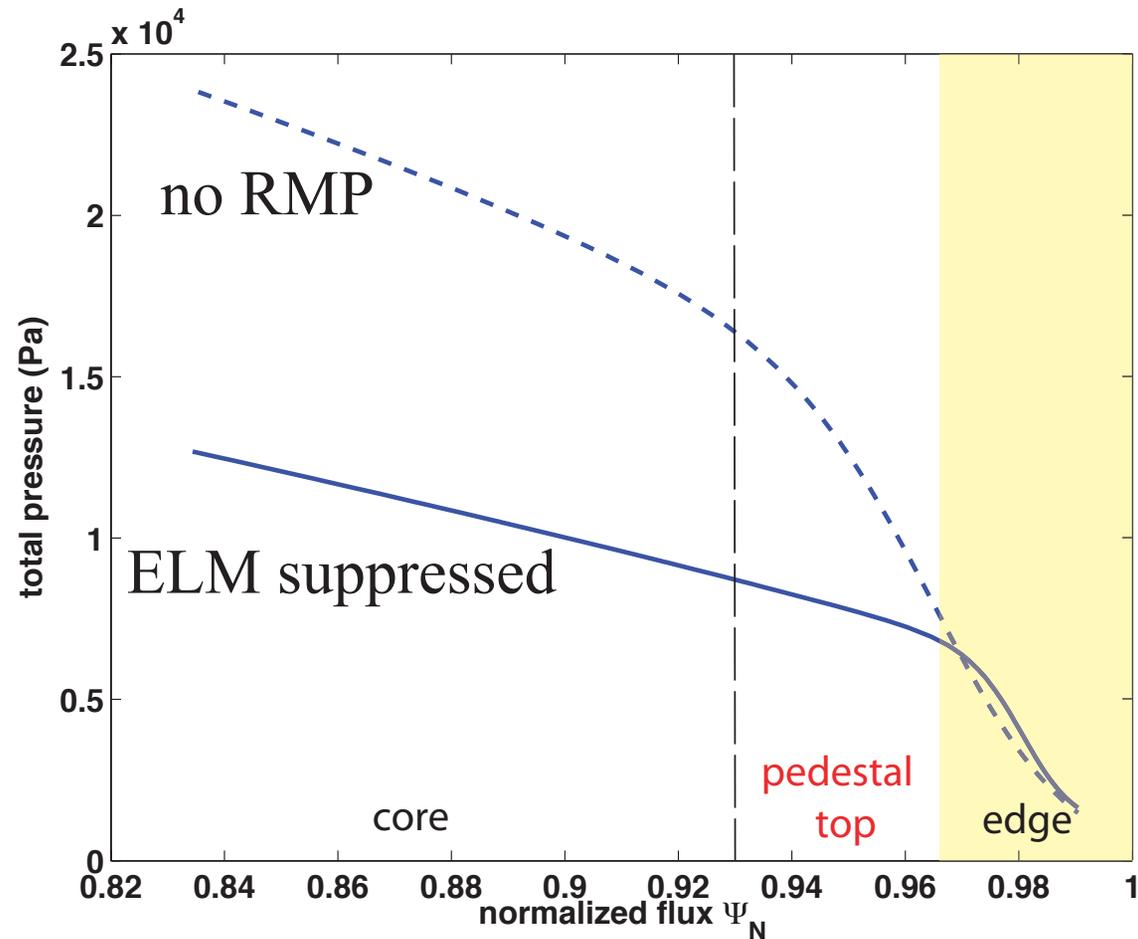


Figure 1: Edge pressure profile without/with RMP ELM suppression. Courtesy of O. Schmitz, R. Nazikian, 2011.

RMPs Increase T_e , n_e Gradient Lengths At Pedestal Top

- T_e , n_e gradient length ratios with RMPs to wo (sym) $\propto \chi_e$, D :

$$\frac{[L_{Te}]_{\text{RMP}}}{[L_{Te}]_{\text{sym}}} \simeq \frac{\chi_e^{\text{RMP}} + \chi_e^{\text{sym}}}{\chi_e^{\text{sym}}};$$

plus a similar formula for L_n , D^{RMP} .

- Apparently RMPs increase:

$$\chi_e \text{ by } \lesssim \times 6,$$

$$D \text{ by } \lesssim \times 2.$$

- Changes peak at **pedestal top**:

$$0.93 \lesssim \Psi_N \lesssim 0.97,$$

near 11/3 surface.

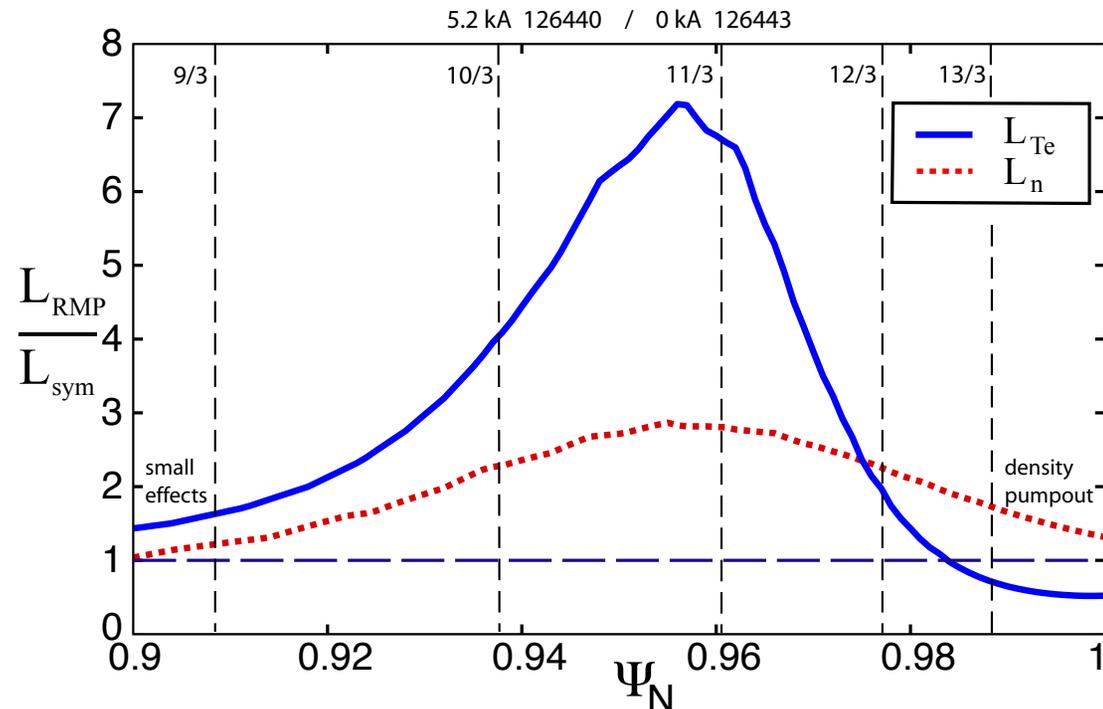


Figure 2: Ratio of T_e and n_e gradient scale lengths with RMPs to without (sym) versus radius.²

Peak Of RMP-induced Extra Transport $\propto I_{\text{coil}}^2$

- Peak T_e , n_e gradient length ratios scale approximately with I_{coil}^2 .
- Peak L_{Te} ratio increases $\sim 3\times$ more than peak L_n ratio does, which indicates $D^{\text{RMP}}/\chi_e^{\text{RMP}} \sim 1/3$.

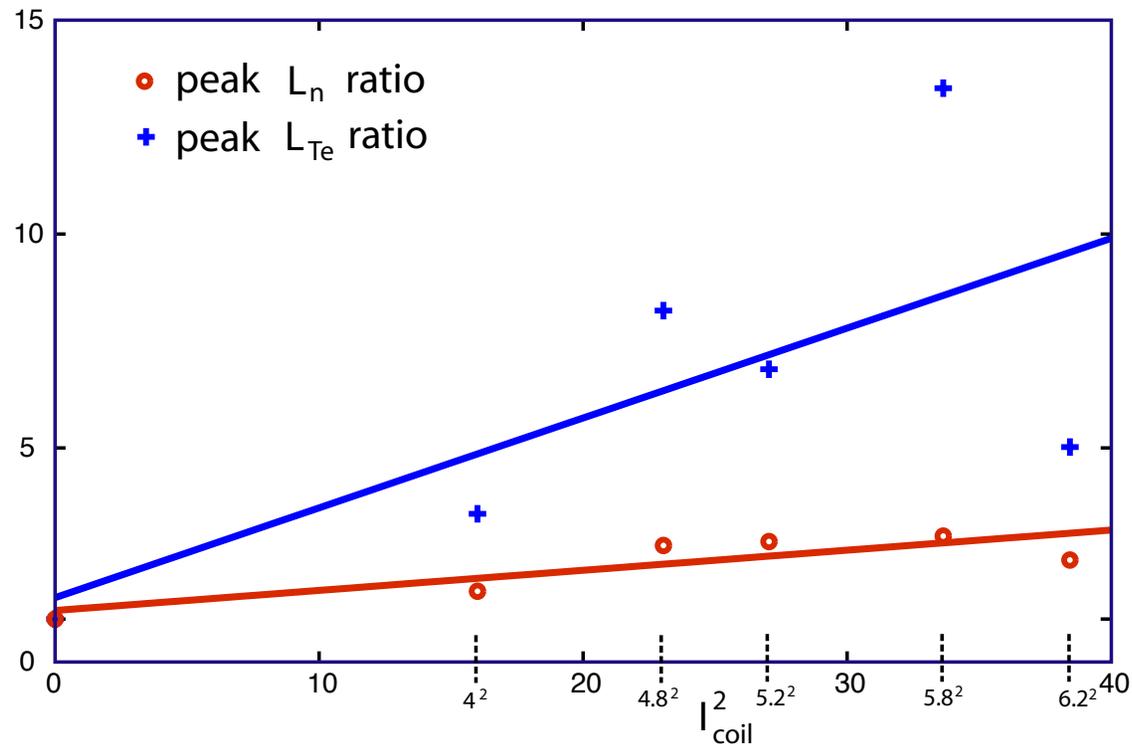


Figure 3: Peak ratios of L_{Te} and L_n with to wo RMPs vs. square of I-coil current.²

Toroidal Rotation Of Carbon Increases With I_{coil}

- When ELMs are suppressed with $I_{\text{coil}} > 4.0$ kA in DIII-D ISS discharges,⁴ carbon toroidal rotation at **pedestal top** “jumps up” to a toroidal rotation frequency of $\Omega_t \simeq V_{\text{tor}}/R \sim 10^4 \text{ s}^{-1}$.

- Increases in V_{tor} induced by RMPs

do not systematically increase with I_{coil} ,

but are always large when ELMs are suppressed ($I_{\text{coil}} \geq 4.7$).

- Changes are largest at the **pedestal top**:

near $\Psi_N \sim 0.96$.

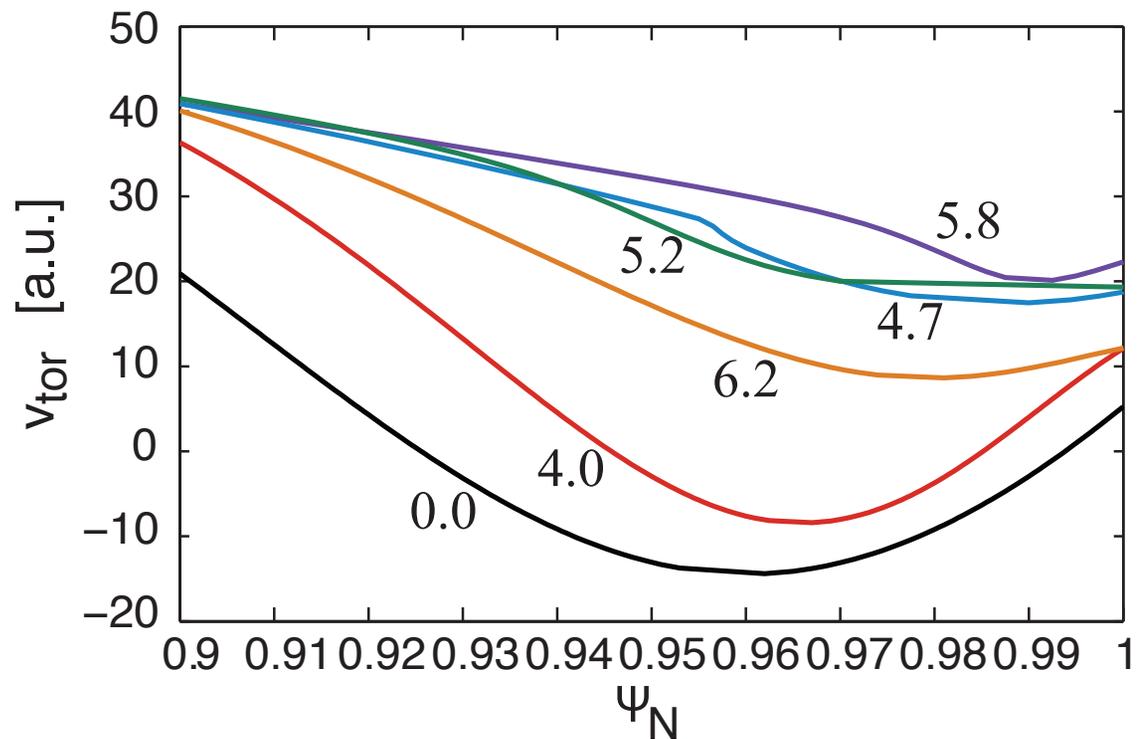


Figure 4: Carbon toroidal rotation in edge as function of I_{coil} . Fig. 4.19b in S. Mordijck thesis, UCSD 2011.

Expt. Summary:² There Are Some Key RMP-induced Plasma Transport Effects That Need To Be Explained

- RMPs increase gradient scale lengths and electron diffusivities:
 $L_{Te}, L_n \sim I_{\text{coil}}^2$ implies $\chi_e^{\text{RMP}}, D^{\text{RMP}} \sim I_{\text{coil}}^2$ ($\chi_e^{\text{RMP}} \sim 4 \text{ m}^2/\text{s}$ at $I_{\text{coil}} = 5.2 \text{ kA}$),
 $L_{Te}/L_n \sim 1/3$ implies $D^{\text{RMP}} \sim \chi_e^{\text{RMP}}/3$,
both are larger than values without RMPs ($\chi_e^{\text{sym}} \sim 0.6 \text{ m}^2/\text{s}$), and
their increases are localized to **pedestal top** region ($0.93 \lesssim \Psi_N \lesssim 0.97$).
- In DIII-D low collisionality ISS discharges⁴ plasma toroidal rotation Ω_t increases at **pedestal top** when there is ELM suppression.
- RMP effects apparently depend on collisionality:
low⁴ (DIII-D) — ELMs are suppressed in narrow q_{95} “resonance windows,”
high⁵ (ASDEX-U) — ELMs mitigated at high density, no resonance effects.

Transport Effects Of RMPs: Flutter Or Stochasticity?

- RMP-induced radial (ρ) magnetic perturbations δB_ρ :
 - mostly just non-resonantly spatially “flutter” the field lines, flux surfaces, but can induce stochasticity if islands are created and overlap (Chirikov).
- Transport can be induced by magnetic flutter and stochasticity:
 - flutter causes¹ $\chi_e^{\delta B} \sim v_{Te} \lambda_e (\delta B_\rho / B_0)^2$ via finite collisional (\sim Braginskii) parallel electron heat conduction, which along with perpendicular electron heat transport and multiple RMP fields produces the needed irreversibility,
 - stochasticity causes $(RR^3) \chi_e^{\delta B_{st}} \sim v_{Te} L_{||c} (\delta B_{st} / B_0)^2$ via \parallel motion along \vec{B}_{st} .
- But flow screening of RMP fields inhibits reconnection, island formation & overlap, and hence stochasticity—next 2 viewgraphs.
- Nonetheless, $\delta B_\rho \neq 0$ off rational surfaces induces flutter xport.¹

Flow Screening Reduces δB_ρ At Rational Surfaces, But RMPs Induce Many $\delta B_{\rho m/n} \neq 0$ At Pedestal Top

- Resonant $\delta B_{\rho 10/3}$ “screened” ($\sim \times 30$) at **10/3** surface \implies no island.
- But, other $\delta B_{\rho m/n}/B_0 \sim 3.3 \times 10^{-4}$ MPs are nonzero there.

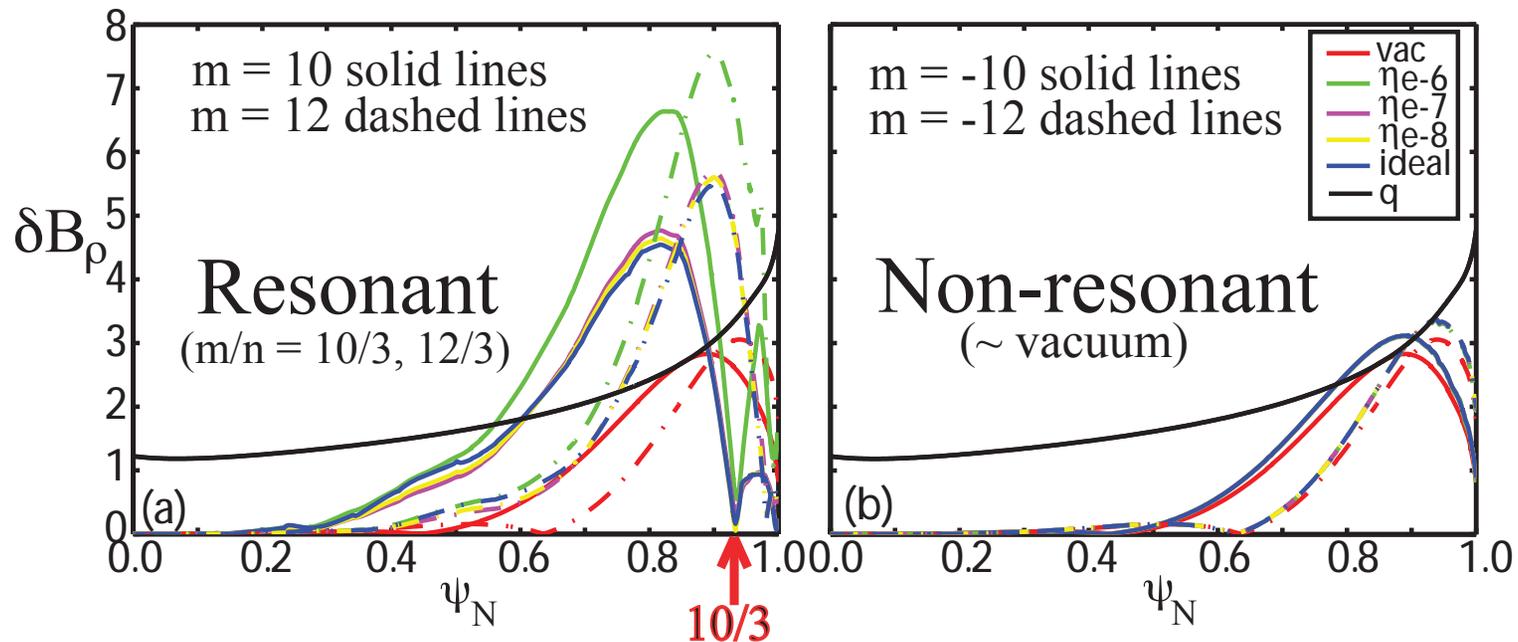


Figure 5: Ideal/resistive MHD RMP-induced radial $\delta B_{\rho m/n}$ profiles: flow-screened (left), \sim vacuum (right). Fig. 3 in M.S. Chu et al., Nucl. Fus. 51, 073036 (2011).

Two-fluid Modeling⁶ Yields Less Flow-Screening

- M3D-C1 visco-resistive 2-fluid modeling⁶ is more realistic. In Fig. 6 solid lines are with flow and the dashed lines are in vacuum.

⁶N.M. Ferraro, "Calculations of two-fluid linear response to non-axisymmetric fields in tokamaks," 2011 APS-DPP invited paper JI2.02, sub. to PoP.

- Features of say 11/3 (purple) RMP field in rotating 126440 plasma:

it is screened by $\sim \times 5$
near 11/3 surface,

its amplitude grows
linearly away from min,

it extends over many
 m/n rational surfaces.

- 8/3, 9/3 RMP fields are not flow screened since $\vec{V}_{e\perp} \simeq 0$ at $\Psi \simeq 0.86$.

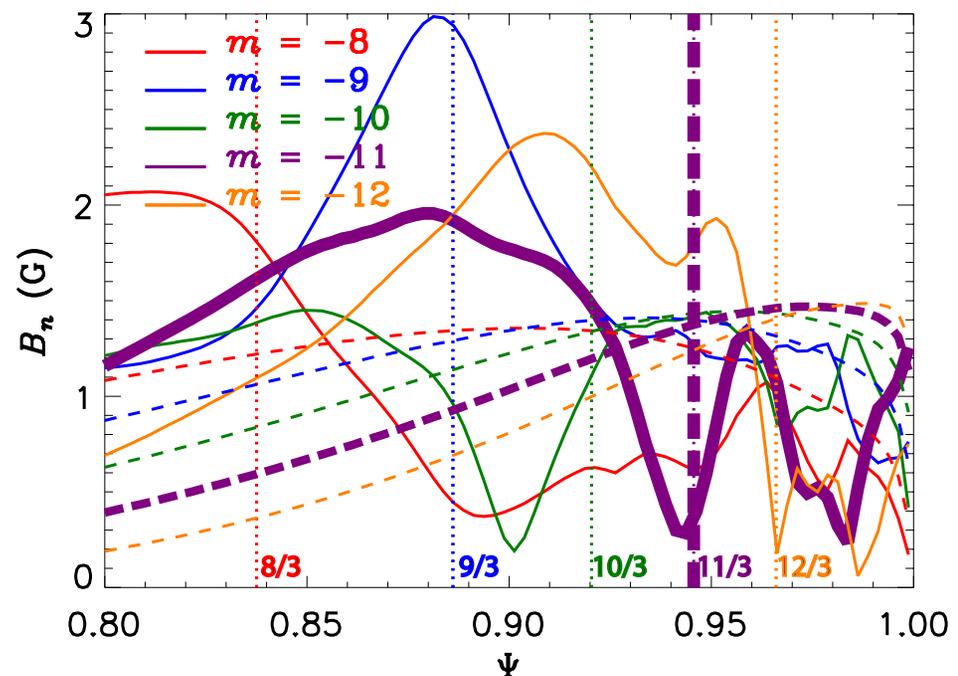


Figure 6: Spatial variation of $\delta B_{\rho m/3}$ (G/kA) for 126440. Courtesy of N. Ferraro, March 2012.

A Model Field Can Be Used For Flow-Screened δB_ρ

- Some key characteristics of flow-screened RMP m/n fields are:
 - on rational surfaces their magnitudes are reduced from their vacuum values,
 - they increase linearly with distance $|x| = |\rho - \rho_{m/n}|$ off rational surfaces,
 - ideal MHD responses outside dissipative layer ($\delta_{\mu\eta} \sim 0.5$ cm) grow linearly until they reach vacuum values at cylindrical analytic estimate of $|x| \sim 1/k_\theta$.

- Define a flow screening factor as the ratio of vacuum $\delta B_{\rho m/n}$ to its flow-screened value on the rational flux surface $q(\rho_{m/n}) \equiv m/n$:

$$f_{\text{scr}} \equiv \left[\frac{\delta B_{\rho m/n}^{\text{vac}}}{\delta B_{\rho m/n}^{\text{plasma}}} \right]_{\rho_{m/n}}$$

flow screening factor² in the plasma (≥ 1),
 ∞ (ideal MHD), ~ 30 (res. MHD), ~ 4 (2-fluid⁶).

- Cyl. model of flow-screened m/n RMP in plasma is² ($|x| < 1/k_\theta$)

$$\frac{\delta B_{\rho m/n}^{\text{plasma}}(x)}{\delta B_{\rho m/n}^{\text{vac}}(\rho_{m/n})} = \sqrt{1/f_{\text{scr}}^2 + k_\theta^2 x^2} \simeq \begin{cases} 1/f_{\text{scr}}, & |x| \ll 1/k_\theta f_{\text{scr}}, \\ k_\theta |x|, & 1/k_\theta f_{\text{scr}} \ll |x| \leq 1/k_\theta. \end{cases}$$

What Physics Causes Magnetic Flutter Transport?^{1,2}

- Consider parallel heat conduction in a perturbed \vec{B} field:

Represent magnetic field by axisymmetric \vec{B}_0 plus RMP field: $\vec{B} = \vec{B}_0 + \delta\vec{B}$.

Radial perturbation $\delta\hat{B}_{\rho m/n}(x) \cos(m\theta - n\zeta)$ induces \parallel heat flow along \vec{B} near m/n surface for $T_e = T_e(\rho)$ and neglecting magnetic shear [$k_{\parallel}(x)v_{Te} < \nu_e$]:

$$\delta\vec{q}_{\parallel m/n} \equiv -n\chi_{\parallel} \left(\frac{\vec{B}\vec{B}}{B^2} \cdot \vec{\nabla}T \right) \simeq -n\chi_{\parallel} \vec{B} \frac{\delta\hat{B}_{\rho m/n}}{B_0^2} \cos(m\theta - n\zeta) \frac{dT}{d\rho} \equiv \frac{\vec{B}_0 + \delta\vec{B}}{B_0} \delta q_{\parallel m/n}.$$

Average radial ($\hat{e}_{\rho} \cdot$) heat flow induced by collisional \parallel heat flow along \vec{B} is

$$\langle \hat{e}_{\rho} \cdot \vec{q} \rangle \equiv \langle \hat{e}_{\rho} \cdot \frac{\vec{B}_0 + \delta\vec{B}}{B_0} \delta q_{\parallel m/n} \rangle = \langle \delta\hat{B}_{\rho m/n} \cos(m\theta - n\zeta) \delta q_{\parallel m/n} \rangle.$$

This results in a radial electron thermal diffusivity of^{1,2}

$$\chi_e^{m/n} = \frac{1}{2} \left(\frac{\delta\hat{B}_{\rho m/n}}{B_0} \right)^2 \chi_{e\parallel}, \quad \text{which is to be summed over all } mn \text{ magnetic perturbations.}$$

- Very small RMP-induced fields can induce significant $\chi_e^{m/n}$ values:

$$\text{For DIII-D pedestal}^2 \chi_{e\parallel} \sim 10^{10} \frac{\text{m}^2}{\text{s}}, \text{ so } \frac{\delta\hat{B}_{\rho m/n}^{\text{vac}}/B_0}{f_{\text{scr}}} \gtrsim 10^{-5} \text{ yields } \chi_e^{m/n} \gtrsim 1 \frac{\text{m}^2}{\text{s}}.$$

- But magnetic shear reduces effective $\chi_{e\parallel}$ for $|x| > \delta_{\parallel} \sim 0.2$ cm.

Various Spatial Scale Lengths Can Be Important

- Radial distances from $11/3$ rational surface in terms of the radial coordinate $\rho \equiv \sqrt{\psi_t/\pi B_{t0}}$ for DIII-D pedestal top parameters² (distances on outboard mid-plane are about half these numbers):

ion sound gyroradius $\rho_S \sim 0.2$ cm

width for small magnetic shear effects on $\chi_{e\parallel}$ $\delta_{\parallel} \sim 0.2$ cm

resistive MHD layer width $\delta_{\eta} \sim 0.2$ cm

visco-resistive (2-fluid⁶) MHD layer width $\delta_{\mu\eta} \sim 0.5$ cm

$\chi_{e\perp}$ causes T_e to not follow island topology⁷ $W_c^{Te}/2 \sim 0.5$ cm

magnetic island half-width (for $f_{scr} = 4$) $W/2 \sim 0.7$ cm

distance between rational surfaces $1/nq' \sim 2.8$ cm

radial extent of $\delta B_{\rho m/n}^{\text{plasma}}(x)$ increasing with $|x|$ $1/k_{\theta} \sim 6.7$ cm

⁷R. Fitzpatrick, "Helical temperature perturbations associated with tearing modes in tokamak plasmas," Phys. Plasmas **2**, 825 (1995).

Flutter Transport Model Employs Some Assumptions

- 1) RMP-induced perturbations are gyroradius small: $\delta B_\rho/B_0 \sim \rho_s/R_0 \lesssim 10^{-3}$. Effects are linearly independent, quasilinear-type.
- 2) Use flow-screened RMPs from linear extended MHD codes — explore self-consistency with nonlinear RMP-induced torque later.
- 3) If magnetic islands are present, they are thin, isolated ones that do not overlap. (Model okay outside islands that do occur.)
- 4) Significant perpendicular electron heat transport and multiple flow-screened RMP responses occur at each radius ρ , which causes T_e and n_e to be \sim constant on axisymmetric flux surfaces.
- 5) Significant flow-screened RMP-flutter responses extend radially over a number of rational surfaces: $\Delta m \simeq 2nq'/k_\theta$ (~ 5).

RMP-flutter Induces Radial Electron Heat Transport²

- A cylindrical screw pinch model of the radial plasma transport induced by RMP-induced flutter has been developed;² it yields

$$\chi_e^{\text{RMP}} \equiv \sum_{mn} \chi_e^{m/n}, \quad D_e^{\text{RMP}} \lesssim \chi_e^{\text{RMP}}/2.5,$$

$$\chi_e^{m/n} \equiv \chi_{m/n}^{\text{ref}} F_{m/n}(x), \quad \text{reference } \chi_{m/n}^{\text{ref}} \text{ times spatial factor } F_{m/n},$$

$$\chi_{m/n}^{\text{ref}} \equiv \frac{1}{2} \left[\frac{\delta \hat{B}_{\rho m/n}^{\text{vac}}}{B_0} \right]^2 \chi_{\parallel c}^{\text{ref}} [k_\theta \delta_{\parallel c}]^2_{\rho_{m/n}} \simeq 0.086 \text{m}^2/\text{s at DIII-D pedestal top},^2$$

$$\chi_{\parallel c}^{\text{ref}} \simeq \frac{5}{4} \left[\frac{\langle n_{ut} \rangle}{n_0} \right]^3 \nu_e \lambda_e^2 \sim 1.45 \times 10^9 \text{ m}^2/\text{s}, \sim \text{Braginskii with } ut \text{ particles},$$

$$\delta_{\parallel c} \simeq \left[\frac{\langle n_{ut} \rangle}{n_0} \right]^{-2} \frac{\sqrt{2} L_S}{k_\theta \lambda_e} \sim 0.22 \text{ cm}, \quad \lambda_e \equiv \frac{v_{Te}}{\nu_e}, \text{ electron collision length},$$

$$F_{m/n}(x) \equiv \frac{1}{[k_\theta \delta_{\parallel c}]^2} \left[\frac{\delta \hat{B}_{\rho m/n}(x)^{\text{plasma}}}{\delta \hat{B}_{\rho m/n}(\rho_{m/n})^{\text{vac}}} \right]^2 \underbrace{\frac{\chi_{e\parallel}^{\text{eff}}(x)}{\chi_{\parallel c}^{\text{ref}}}}_{1/(1+x^2/\delta_{\parallel c}^2)} \simeq \frac{1/k_\theta^2 f_{\text{scr}}^2 + x^2}{\delta_{\parallel c}^2 + x^2},$$

in which $x \equiv \rho - \rho_{m/n}$ is radial distance off the rational surface.

Spatial Factor $F_{m/n}(x)$ Varies Significantly & Is Important

- $F_{m/n}(x)$ has different scalings in various radial regions:

$$F_{m/n}(x) \equiv \frac{1/f_{\text{scr}}^2 k_{\theta}^2 + x^2}{\delta_{\parallel c}^2 + x^2} \simeq \begin{cases} 1/(\delta_{\parallel c}^2 f_{\text{scr}}^2 k_{\theta}^2), & |x| \ll \delta_{\parallel c}, \\ 1/(x^2 f_{\text{scr}}^2 k_{\theta}^2), & \delta_{\parallel c} \ll |x| \ll 1/f_{\text{scr}} k_{\theta}, \\ 1, & 1/f_{\text{scr}} k_{\theta} \ll |x| \ll 1/k_{\theta}. \end{cases}$$

- This causes different behavior for various screening models:

Resistive MHD with $f_{\text{scr}} \sim 30$, $\delta_{\parallel c} \sim 1/f_{\text{scr}} k_{\theta} \implies F_{m/n} \simeq 1 = \text{constant}$.

Visco-res. 2-fluid⁶ MHD with $f_{\text{scr}} \sim 4 \implies$ highly peaked, $F_{m/n}(0) \simeq 60$.

Collisional pedestal⁵ with $\delta_{\parallel c} \sim 1/f_{\text{scr}} k_{\theta} \sim 2 \text{ cm} \implies$ also $F_{m/n} \sim 1$.

- This spatial factor has a significant effect on the T_e gradient:²

$$\boxed{-\frac{dT_e}{d\rho}} \simeq \frac{P_e/V' \langle |\vec{\nabla} \rho|^2 \rangle}{n_e \chi_e^{\text{RMP}}} = \frac{P_e/V' \langle |\vec{\nabla} \rho|^2 \rangle}{n_e \sum_{mn} \chi_{m/n}^{\text{ref}} F_{m/n}(x)} \sim \boxed{\frac{1}{\sum_{mn} F_{m/n}(x)}}.$$

Spatial Profile Of $F_{m/n}$ Modifies Predicted T_e Profile²

- Obtain T_e profile by integrating $dT_e/d\rho$ inward from 11/3 surface:

$$T_e(\rho) = T_e(\rho_{11/3}) - \int_{\rho_{11/3}}^{\rho} d\rho \frac{P_e / (n_e \chi_{m/n}^{\text{ref}} V' \langle |\vec{\nabla} \rho|^2 \rangle)}{\sum_{mn} F_{m/n}(x)}, \quad \text{average } \chi_e \sim \frac{\dots}{-\Delta T_e / \Delta \rho},$$

which yields² an average $\bar{\chi}_e^{\text{RMP}}$ of 0.5 m²/s for $f_{\text{scr}} = 30$ or 1.2 m²/s for $f_{\text{scr}} = 4$.

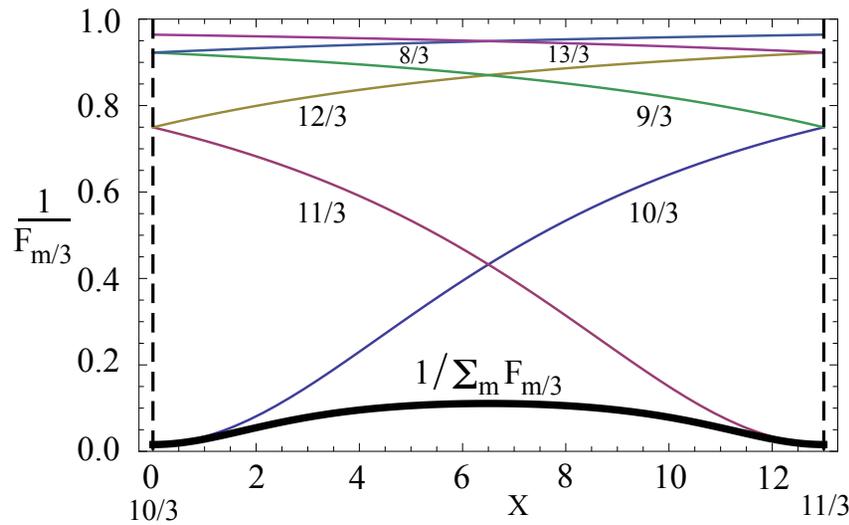


Figure 7: Radial variation of $1/F_{m/n}$ between 10/3 and 11/3 surfaces and the inverse of their sum ($X \equiv x/\delta_{||c}$).

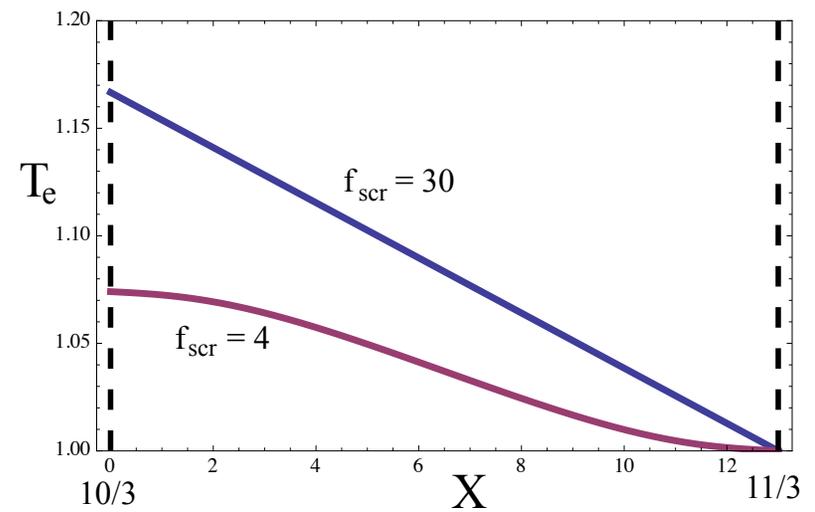


Figure 8: T_e profiles between 10/3 & 11/3 surfaces for constant $F_{m/n}$ ($f_{\text{scr}} = 30$) & due to $1/\sum_{mn} F_{m/n}(x)$ for $f_{\text{scr}} = 4$.

RMP Flutter Induces Toroidal Torques On Edge Plasma

- Plasma toroidal momentum balance⁸ for $L_t \equiv m_i n_i \langle R^2 \rangle \Omega_t$ is:

$$\frac{\partial L_t}{\partial t} = \sum T_\zeta, \text{ torques } T_\zeta \equiv -q_s \langle \vec{\Gamma}_s \cdot \vec{\nabla} \psi_p \rangle \text{ caused by non-ambipolar fluxes } \vec{\Gamma}_s.$$

- Ion & electron 3D density fluxes cause oppositely directed torques:

$\vec{\Gamma}_i$ (NTV, ripple) create⁸ counter-current torques because $q_i = +e$ ($J_\rho < 0$), whereas electron density fluxes create co-current torques because $q_e = -e$.

- RMP-flutter induces a non-ambipolar radial electron density flux and hence a co-current torque (for offset frequency $\Omega_{fl} > \Omega_t$):

$$T_\zeta^{\text{RMP}} \equiv e \langle \vec{\Gamma}_e^{\text{RMP}} \cdot \vec{\nabla} \psi_p \rangle \simeq n_e m_i \langle R^2 \rangle \nu_e \frac{L_S^2 B_p^2}{6\rho_S^2 B_0^2} \left(\sum_{mn} \frac{\delta \hat{B}_{\rho m/n}^{\text{vac} 2}}{B_0^2} F_{m/n} \right) (\Omega_{fl} - \Omega_t),$$

$$\text{flutter offset freq. } \Omega_{fl} \simeq -\frac{1}{n_e e} \frac{dP}{d\psi_p} \sim 10^4 \text{ s}^{-1} (> 0 \implies \vec{V}_{e\perp}^t \simeq -\frac{cT_t}{eB_0} \frac{dT_e}{d\rho}).$$

- Low ν_e RMP-induced torque is localized near rational surfaces.

⁸See Eqs. (3)–(7) in J.D. Callen, “Effects of 3D magnetic perturbations on toroidal plasmas,” Nucl. Fusion **51**, 094026 (2011).

Cylindrical Model Has Been Compared² To DIII-D Results

- Flutter predictions in qualitative agreement with experiment are:
 - scaling of χ_e^{RMP} and D^{RMP} with $\delta B_\rho^2 \sim I_{\text{coil}}^2$,
 - ratio of $D^{\text{RMP}}/\chi_e^{\text{RMP}} \lesssim 1/3$, and
 - RMP-induced change in toroidal rotation (E_ρ) to $\Omega_t \sim 10^4 \text{ s}^{-1}$.
- But cylindrical χ_e^{RMP} is smaller than experiment at pedestal top:
 - in DIII-D $\chi_{e \text{ exp}}^{\text{RMP}} \sim 4 \text{ m}^2/\text{s}$ (#126440, 5.2 kA) while $\chi_{e \text{ exp}}^{\text{sym}} \sim 0.6 \text{ m}^2/\text{s}$ (#126443),
 - versus $\chi_e^{\text{RMP}} \equiv \sum_{m,n} \chi_{m/n}^{\text{ref}} F_{mn}(\boldsymbol{x}) \sim (5-12)(10^{-7})(10^6) \lesssim 0.5-1.2 \text{ m}^2/\text{s}$;
 - but should use and sum flow-screened $\delta \hat{B}_{\rho m/n}^{\text{plasma}}(\boldsymbol{x}) +$ all possible mn fields.
- Additional routes to improved agreement with experiment:
 - toroidal model could increase magnitude of χ_e^{RMP} with
 - $q_{95} \sim 3.5$ resonance if 10/3, 11/3, 12/3 RMP responses are modified.

Toroidal Model Yields || Flows & Radial Transport Fluxes

- RMP-induced parallel electron flow and conductive heat flux are⁹ ($y \equiv v^2/v_{Te}^2$)

$$\begin{bmatrix} n_e \delta V_{e\parallel} \\ \delta q_{e\parallel} \end{bmatrix} = \int_u d^3v v_{\parallel} \begin{bmatrix} L_0^{(3/2)} \\ -L_1^{(3/2)} T_e \end{bmatrix} \delta f_e = -2\pi \frac{B_0}{B_{\max}} \int_0^{\infty} v^3 dv \begin{bmatrix} 1 \\ T_e(y - \frac{5}{2}) \end{bmatrix} \int_0^1 d\lambda |\mathcal{R}e\{\delta h_u\}|.$$

- These RMP-induced || flows cause radial electron density and heat fluxes:

$$\begin{aligned} \begin{bmatrix} \delta \Gamma_{ep}^{m/n} \\ \delta \Upsilon_{ep}^{m/n} \end{bmatrix} &\equiv \begin{bmatrix} \langle \delta \vec{\Gamma}_e \cdot \vec{\nabla} \rho \rangle \\ \langle \delta \vec{q}_e \cdot \vec{\nabla} \rho \rangle \end{bmatrix} = \left\langle \int_u d^3v \delta f_e \begin{bmatrix} 1 \\ T_e(y - \frac{5}{2}) \end{bmatrix} \vec{v}_g \cdot \vec{\nabla} \rho \right\rangle = \left\langle \int_u d^3v \delta h_u v_{\parallel} \begin{bmatrix} 1 \\ T_e(y - \frac{5}{2}) \end{bmatrix} \frac{\delta \vec{B} \cdot \vec{\nabla} \rho}{B_0} \right\rangle \\ &= \left\langle \begin{bmatrix} n_e \overline{\delta V_{e\parallel}} \\ \overline{\delta q_{e\parallel}} \end{bmatrix} \frac{\delta \vec{B} \cdot \vec{\nabla} \rho}{B_{t0}} \right\rangle = -n_e \begin{bmatrix} D_e^{m/n} & D_T^{m/n} \\ \chi_n^{m/n} & \chi_e^{m/n} \end{bmatrix} \cdot \begin{bmatrix} d \ln p_e / d\rho - (e/T_e) d\Phi_0 / d\rho \\ dT_e / d\rho \end{bmatrix}. \end{aligned}$$

- Transport coefficients here are defined by kinetic coefficients matrix K_{ij} :

$$\boxed{\begin{bmatrix} D_e^{m/n} & D_T^{m/n} \\ \chi_n^{m/n} & \chi_e^{m/n} \end{bmatrix} = \frac{v_{Te}^2}{\nu_e} \frac{1}{2} \left(\frac{\langle \delta B_{\rho n} \rangle}{B_{t0}} \right)^2 \begin{bmatrix} K_{00} & K_{01} \\ K_{10} & K_{11} \end{bmatrix},}$$

$$\begin{bmatrix} K_{00} & K_{01} \\ K_{10} & K_{11} \end{bmatrix} = \frac{B_{t0}}{B_{\max}} \int_0^{\infty} \frac{dy y^3 e^{-y}}{2\sqrt{\pi}} \int_0^1 d\lambda |\mathcal{R}e\{\Lambda\}| \begin{bmatrix} 1 & y - \frac{5}{2} \\ y - \frac{5}{2} & (y - \frac{5}{2})^2 \end{bmatrix}, \text{ in which } y \equiv v^2/v_{Te}^2.$$

⁹J.D. Callen, A.J. Cole and C.C. Hegna, "Magnetic-perturbation-induced plasma transport in H-mode pedestals," UW-CPTC 11-15, to be at <http://www.cptc.wisc.edu> "soon."

Kinetic Matrix Coefficients K_{ij} Result From Λ Solutions

- Resultant kinetic coefficients and their regimes of applicability are

$$|k_{\parallel}| \simeq 0: \quad \begin{bmatrix} K_{00} & K_{01} \\ K_{10} & K_{11} \end{bmatrix}^{k_{\parallel}=0} = \frac{2}{3\sqrt{\pi}} f_c \frac{B_{\max}}{B_{t0}} \begin{bmatrix} 6 & 9 \\ 9 & 75/2 \end{bmatrix}, \quad \text{for } y < \frac{1}{|X_t|^{1/2}} \text{ in which } X_t \equiv \frac{x}{\delta_{\parallel t}},$$

$$k_{\parallel} \neq 0: \quad \begin{bmatrix} K_{00} & K_{01} \\ K_{10} & K_{11} \end{bmatrix}^{k_{\parallel} \neq 0} = \frac{1}{|X_t|^{3/2}} \frac{B_{t0}/B_{\max}}{8\sqrt{\pi} \langle v_{\parallel} |_{\lambda=1/v} \rangle} \begin{bmatrix} 1 & -3/2 \\ -3/2 & 13/4 \end{bmatrix},$$

$$\text{for } y > y_{\min} \equiv \max \left\{ \frac{1}{|X_t|^{1/2}}, \frac{1}{X_{\text{crit}}^{1/2}} \right\}, \quad X_{\text{crit}} \equiv \frac{B_{t0}}{2B_{\max}} \frac{\lambda_e}{\bar{R}q} \simeq 22 \text{ (so } \nu_{\text{eff}}^{k_{\parallel}} < \omega_{ut}).$$

- Normalizing the kinetic coefficients to $(13/4)K_{11}^{k_{\parallel}}$ and using energy smoothing¹⁰ to develop a Padé approximation yields for the kinetic coefficient matrix

$$\begin{bmatrix} K_{00}^t & K_{01}^t \\ K_{10}^t & K_{11}^t \end{bmatrix} = \frac{13}{32\sqrt{\pi}} \frac{B_{t0}/B_{\max}}{\langle v_{\parallel} |_{\lambda=1/v} \rangle} \begin{bmatrix} G_{00} & G_{01} \\ G_{10} & G_{11} \end{bmatrix}, \quad \text{in which coefficient } \sim 0.38 \text{ and matrix is}$$

$$\begin{bmatrix} G_{00} & G_{01} \\ G_{10} & G_{11} \end{bmatrix} = \frac{4}{13 |X_t|^{3/2}} \left(\frac{|X_t|^{3/2}}{c_{\parallel t}} \int_0^{1/|X_t|^{1/2}} dy y^3 e^{-y} + \int_{y_{\min}}^{\infty} dy e^{-y} \right) \begin{bmatrix} 1 & y - \frac{5}{2} \\ y - \frac{5}{2} & (y - \frac{5}{2})^2 \end{bmatrix}, \quad c_{\parallel t} \simeq 0.94.$$

- Properties of the geometric coefficient matrix that determine diffusivities are

$$\lim_{|X_t| \rightarrow 0} G_{11} = \frac{150}{13 c_{\parallel t}} \implies \chi_{\parallel t}^{\text{ref}} \sim 3.3 \times 10^{10}, \quad \text{for } |X_t| \rightarrow \infty, \quad \frac{G_{00}}{G_{11}} \simeq \frac{4}{13} \implies \frac{D_e^{m/n}}{\chi_e^{m/n}} \simeq \frac{1}{3.25}.$$

¹⁰K.T. Tsang and J.D. Callen, "Smooth transition of neoclassical diffusion from the banana to Pfirsch-Schlüter regime," Phys. Fluids 19, 667 (1976).

Toroidal Flutter Results⁹ Are Similar To Cylindrical Model

- Parallel electron thermal diffusivity on a rational surface:
toroidal, Lorentz collision value is larger — $\chi_{\parallel t}^{\text{ref}} \simeq 3.3 \times 10^{10} \text{ m}^2/\text{s}$, while $\chi_{\parallel c}^{\text{ref}} \simeq 1.5 \times 10^9 \text{ m}^2/\text{s}$.
- Layer width beyond which magnetic shear effects dominate:
toroidal width is smaller — $\delta_{\parallel t} \simeq 0.11 \text{ cm}$, while $\delta_{\parallel c} \simeq 0.22$.
- Spatial decay of effective parallel diffusivities away from rational surface:
toroidal decays slower with $|x| \gg \delta_{\parallel}$ — toroidal $G \sim |x|^{-3/2}$, while cylindrical $\chi_{e\parallel}^{\text{eff}} \sim |x|^{-2}$.
- Magnitude of a single $\chi_e^{m/n}$ midway between rational surfaces:
using $\delta B_{\rho m/n}^{\text{plasma}}(x)$ on p 10, they are about equal — $\chi_{et}^{m/n} \sim 0.23 \text{ m}^2/\text{s}$, while $\chi_{ec}^{m/n} \sim 0.21 \text{ m}^2/\text{s}$;
but using $\delta B_{\rho 11/3}^{\text{plasma}}(x)$ profile from Fig. 6, both estimates give $\chi_e^{11/3} \sim 2 \text{ m}^2/\text{s}$ at $\Psi_N \sim 0.955$.
- Ratio of RMP-induced electron density to electron thermal diffusivity:
results are similar — $D_{et}^{m/n}/\chi_{et}^{m/n} \simeq 1/3.25$, while $D_{ec}^{m/n}/\chi_{ec}^{m/n} \simeq 1/2.5$.
- Radial RMP-flutter-induced electron transport fluxes:
toroidal fluxes have off-diagonal K_{ij} matrix elements, while cylindrical model fluxes don't.

Some Predictions Are Different At High Collisionality

- ASDEX-U⁵ electron collision frequency ν_e is $\gtrsim \times 10$ greater which
 - 1) increases shear-effects width parameter by a factor $\sim \times 10$ to $\delta_{\parallel} \sim 2$ cm,
 - 2) increases reconnection layer width by a factor ~ 2 to $\delta_{\mu\nu} \sim 1$ cm,
 - 3) causes most “smoothing” processes to exceed half of the distance between rational surfaces and hence overlaps the effects around various m/n surfaces — this causes q_{95} resonance effects and magnetic islands to be less likely.
- Model predictions for approximate ASDEX-U conditions are:

$$1) \chi_e^{\text{RMP}} \simeq \frac{\nu_e L_S^2}{2} \sum_{mn} \left[\frac{\delta \hat{B}_{\rho m/n}^{\text{vac}}}{B_0} \right]^2 \gtrsim 1 \text{ m}^2/\text{s}, \quad L_S \equiv \frac{R_0 q}{s} \text{ magnetic shear length,}$$

- 2) which reduces gradients throughout the pedestal if it exceeds a typical level of $D_{\eta} \sim \nu_e \delta_e^2$ transport there and yields an ELM mitigation criterion:

$$\delta_e^2 \equiv \frac{c^2}{\omega_{pe}^2} \simeq \frac{3 \times 10^{19}}{n_e} 10^{-6} \lesssim \frac{L_S^2}{2} \sum_{mn} \left[\frac{\delta \hat{B}_{\rho m/n}^{\text{vac}}}{B_0} \right]^2 \implies n_e \gtrsim 5 \times 10^{19} \text{ m}^{-3}?$$

Summary

- Experimental effects of ELM-suppressing RMPs on pedestals are:²
reduced $|\vec{\nabla}P|$ at top via increasing L_{Te} ($\lesssim \times 6$) & L_n ($\lesssim \times 2$) $\propto I_{\text{coil}}^2$, Ω_t change.
- Flow screening inhibits RMPs in forming islands, stochasticity.
- But RMP-flutter induces^{1,2} radial transport at pedestal top:
both cylindrical² screw pinch and toroidal⁹ models have been developed
— results are qualitatively similar and differ quantitatively mostly by $\lesssim \times 2$,
in high collisionality pedestals q_{95} resonances and islands are less likely.
- Comparison between flutter model predictions and DIII-D data:²
 $\chi_e^{\text{RMP}} \propto I_{\text{coil}}^2$, $D^{\text{RMP}}/\chi_e^{\text{RMP}} \lesssim 1/3$, Ω_t changes agree qualitatively, but
need increase of cyl. χ_e^{RMP} by $\gtrsim \times 3$ and $q_{95} \sim 3.5$ resonance via modified
10/3, 11/3, 12/3 RMP responses (\sim Fig. 6 $\delta B_{\rho m/n}^{\text{plasma}}$ profiles might do both).

What Are Next Steps For Model Testing, Validation?

- 1) Use M3D-C1 flow-screened $\delta B_{\rho m/n}(\rho)$ to predict RMP-flutter-induced transport and compare to ONETWO interpretive transport modeling of electron thermal and density diffusivities,
to see if model magnitude, scaling are correct & it can capture q_{95} resonances.
- 2) Explore low versus high collisionality RMP cases with M3D-C1 RMP modeling plus ONETWO interpretive transport modeling,
to understand differences between RMP effects at low, high collisionality.
- 3) Employ quasilinear flutter model toroidal torque in M3D-C1 to explore its effects on toroidal rotation at the pedestal top,
to determine if flutter-induced torque plays a significant role in RMP effects.
- 4) Ultimately, treat toroidal rotation (radial electric field) on par with n , T transport by coupling M3D-C1 to predictive ONETWO,
to develop a predictive capability for RMP effects in ITER plasmas.

Toroidal:⁹ RMP-Induced Flutter Modifies e Distribution

- Electron drift kinetic equation for $\vec{v}_g \equiv v_{\parallel} \vec{B}/B + \vec{v}_d$ with $\vec{B} \rightarrow \vec{B}_0 + \delta\vec{B}$ is^{2,9}

$$\frac{\partial f_e}{\partial t} + \left[\frac{v_{\parallel}}{B} (\vec{B}_0 + \delta\vec{B}) + \vec{v}_d \right] \cdot \vec{\nabla} f_e + \frac{d\varepsilon}{dt} \frac{\partial f_e}{\partial \varepsilon} = \mathcal{C}\{f_e\}, \quad \varepsilon \equiv \frac{m_e v^2}{2} - e\Phi = \frac{m_e v_{\parallel}^2}{2} + \mu B - e\Phi.$$

- Lowest order solution is a Maxwellian constant along \vec{B}_0 : $f_e = f_{Me}(\rho, \varepsilon)$.
- For $\delta B_{\rho}(\vec{x}, t) \equiv \sum_n \delta B_{\rho n}(\rho, \theta) \mathcal{R}e\{e^{-i(n\alpha - \omega t + \varphi_n)}\}$, in helical coordinates $\rho, \theta, \alpha \equiv \zeta - (m/n)\theta$, equation for $\delta h \equiv \delta f_e - (e/T_e) \delta\Phi f_{Me}$ induced by $\delta B_{\rho n}$ is⁹

$$\frac{v_{\parallel}}{B_0} (\vec{B}_0 \cdot \vec{\nabla} \theta) \left[\frac{\partial \delta h_n}{\partial \theta} + \underbrace{i(m - nq)}_{(d\alpha/d\theta)(\partial/\partial\zeta)} \delta h_n \right] - i(\omega - \omega_d) \delta h_n - \mathcal{C}\{\delta h_n\} = - \frac{v_{\parallel}}{B_0} \delta\vec{B}_n \cdot \vec{\nabla} f_{Me}.$$

- Trapped particle solution vanishes — bounce average yields no **drive** because trapped particles don't carry any parallel flow over $\lambda_e \gg 2\pi R_0 q$.
- Magnitudes of frequencies indicate ω and ω_d can be neglected ($\ll \nu_{\text{eff}}$):

$$v_{Te}/R_0 q \sim 3 \times 10^6, \quad k_{\parallel} v_{Te} \sim (0-8) \times 10^6, \quad \omega \sim 10^4, \quad \omega_d \sim 10^4, \quad \nu_{\text{eff}} \sim 2 \times 10^5.$$

Bounce-averaging Yields⁹ Non-adiabatic Response Eq.

- Neglecting ω , ω_d & operating on δh_n equation with $\int_{-\pi}^{\pi} d\theta B_0 / [v_{\parallel} \vec{B}_0 \cdot \vec{\nabla} \theta]$ yields

$$\boxed{ik_{\parallel} v \delta \bar{h}_u - 2\nu_{\perp} \frac{B_{\max}}{B_{t0}} \frac{\partial}{\partial \lambda} \left(\lambda \frac{\langle v_{\parallel} \rangle}{v} \frac{\partial \delta \bar{h}_u}{\partial \lambda} \right) = -v \frac{\langle \delta B_{\rho n} \rangle}{B_{t0}} \frac{\partial f_{Me}}{\partial \rho}}, \quad \text{in which } \langle \dots \rangle \text{ is FSA,}$$

$$k_{\parallel}(x) \equiv -\frac{n[q(\rho) - m/n]}{\bar{R}q} \simeq k'_{\parallel} x, \quad \lambda \equiv \frac{\mu B_{\max}}{\varepsilon}, \quad \delta \bar{h}_u = \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} \delta h_u, \quad \langle \delta B_{\rho n} \rangle \equiv \langle \delta \vec{B} \cdot \vec{\nabla} \rho e^{in\alpha} \rangle.$$

- Since $\delta \bar{h}_u$ is a mostly separable function, it is useful to write it as:

$$\delta \bar{h}_u \equiv -D(\rho) V(v, \rho) \Lambda(\lambda, x, v), \quad \text{in which } D \equiv \frac{\langle \delta B_{\rho n} \rangle}{B_{t0}}, \quad V = \frac{v}{\nu_{\perp}(v)} \frac{d f_{Me}}{d \rho} = \frac{v^4}{v_{Te}^4} \frac{v_{Te}}{\nu_e} \frac{d f_{Me}}{d \rho}.$$

- Then, equation above reduces to an equation for the pitch-angle function Λ :

$$\boxed{\frac{\partial}{\partial \lambda} \left(\lambda \frac{\langle v_{\parallel} \rangle}{v} \frac{\partial \Lambda}{\partial \lambda} \right) - i \left(\frac{k_{\parallel} v}{2\nu_{\perp}(v)} \frac{B_{t0}}{B_{\max}} \right) \Lambda = -1}, \quad \text{with B.C.: 1) } \Lambda(\lambda=1) = 0, \text{ 2) } \Lambda(0) \text{ finite.}$$

- This equation will be solved for $\Lambda(\lambda)$ in two limits:

$k_{\parallel} = 0$ — on $q = m/n$ rational surface, which will yield Braginskii-like result for $\chi_{e\parallel}$, and

$k_{\parallel} \neq 0$ — for $|x| \equiv |\rho - \rho_{m/n}| \gg \delta_{\parallel}$ where magnetic shear effects become dominant.

Pitch-angle Function Λ Can Be Solved For⁹ In Limits

- On a rational surface $k_{\parallel} = 0$ and Λ equation can be integrated directly:

$$\Lambda^{k_{\parallel}=0}(\lambda) \equiv \int_{\lambda}^1 \frac{d\lambda'}{\langle v_{\parallel}(\lambda', \theta) \rangle / v} \implies \int_0^1 d\lambda \Lambda^{k_{\parallel}=0} = \int_0^1 \frac{\lambda d\lambda}{\langle \sqrt{1 - \lambda B/B_{\max}} \rangle} \equiv \frac{4}{3} \frac{B_{\max}^2}{\langle B_0^2 \rangle} f_c.$$

- For large k_{\parallel} the solution will be localized in λ near the untrapped-trapped particle boundary where $\lambda \lesssim 1$. Thus, defining $\tilde{\lambda} \equiv 1 - \lambda$, equation becomes

$$\frac{\partial^2 \Lambda^{k_{\parallel} \neq 0}}{\partial \tilde{\lambda}^2} - i 2 k_{\lambda}^2 \frac{k_{\parallel}}{|k_{\parallel}|} \Lambda^{k_{\parallel} \neq 0} = - \frac{1}{\langle v_{\parallel} |_{\lambda=1/v} \rangle}, \quad \text{in which} \quad k_{\lambda}(x, v) \equiv \left[\frac{|k_{\parallel}(x)| v (B_{t0}/B_{\max})}{4 \nu_{\perp} |\langle v_{\parallel} |_{\lambda=1/v} \rangle|} \right]^{1/2}.$$

- Complementary solutions of this equation are of form $e^{\pm \sqrt{\pm 2i} k_{\lambda} \tilde{\lambda}} = e^{\pm (1 \pm i) k_{\lambda} \tilde{\lambda}}$. Boundary-layer-type particular solution that satisfies boundary conditions is

$$\Lambda^{k_{\parallel} \neq 0} = - \frac{(i/2 k_{\lambda}^2)(k_{\parallel}/|k_{\parallel}|)}{\langle v_{\parallel} |_{\lambda=1/v} \rangle} \left[1 - e^{-k_{\lambda} \tilde{\lambda}} (\cos k_{\lambda} \tilde{\lambda} - i \frac{k_{\parallel}}{|k_{\parallel}|} \sin k_{\lambda} \tilde{\lambda}) \right].$$

- Key parameters of this solution can be written as

$$k_{\lambda}(x, v) = |X_t|^{1/2} \frac{v^2}{v_{Te}^2}, \quad X_t \equiv \frac{x}{\delta_{\parallel t}}, \quad \delta_{\parallel t} \equiv \frac{c_t}{|k'_{\parallel}| \lambda_e} = \frac{c_t L_S}{k_{\theta} \lambda_e} \simeq 0.11 \text{ cm}, \quad c_t \equiv 4 |\langle v_{\parallel} |_{\lambda=1/v} \rangle| \frac{B_{\max}}{B_{t0}}.$$

- Pitch-angle integral of this solution for $k_{\lambda} \tilde{\lambda} \gg 1$ ($v^2/v_{Te}^2 \gg 1/|X_t|^{1/2}$) is

$$\int_0^1 d\lambda \mathcal{R}e\{\Lambda^{k_{\parallel} \neq 0}\} = \frac{1}{2 k_{\lambda}^2 |\langle v_{\parallel} |_{\lambda=1/v} \rangle|} \int_0^1 d\lambda e^{-k_{\lambda} \tilde{\lambda}} \sin k_{\lambda} \tilde{\lambda} \simeq \frac{1}{4 k_{\lambda}^3 |\langle v_{\parallel} |_{\lambda=1/v} \rangle|}.$$