



# ECCD-induced tearing mode stabilization in coupled IPS/NIMROD/GENRAY simulations



Center for Simulation of  
RF Wave Interactions with  
Magnetohydrodynamics

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**SciDAC**

Scientific Discovery through Advanced Computing

CEMM/NIMROD Meeting  
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# Problem: modeling mitigation/control of tearing modes (magnetic islands) by electron cyclotron current drive (ECCD)



- Neoclassical tearing modes generate magnetic islands in tokamaks; pressure profile flattening → altered plasma bootstrap current profile → self-reinforcing altered confinement.
- Islands grow to macroscopic scales before nonlinearly saturating, causing degraded confinement and the possibility of disruption.

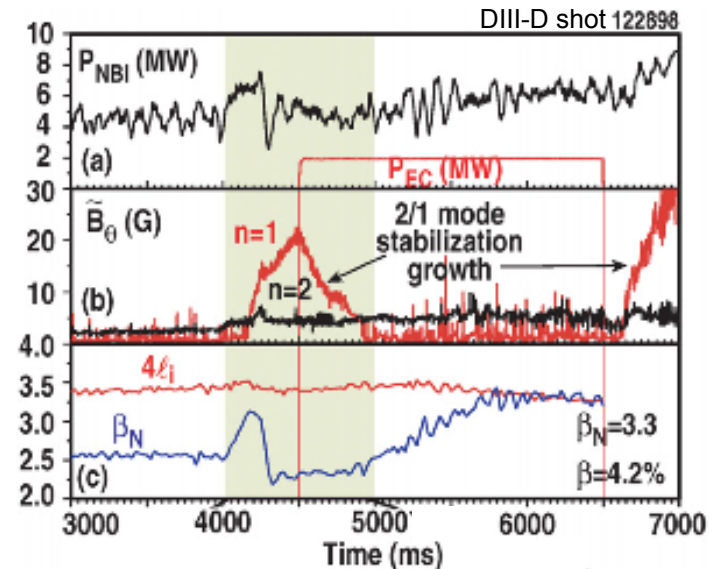
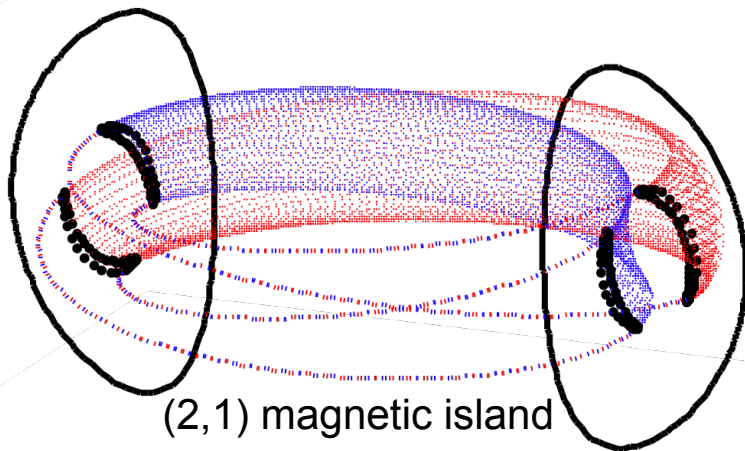


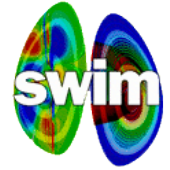
Figure from Prater *et al.*, Nucl. Fusion **47**, 371 (2007).



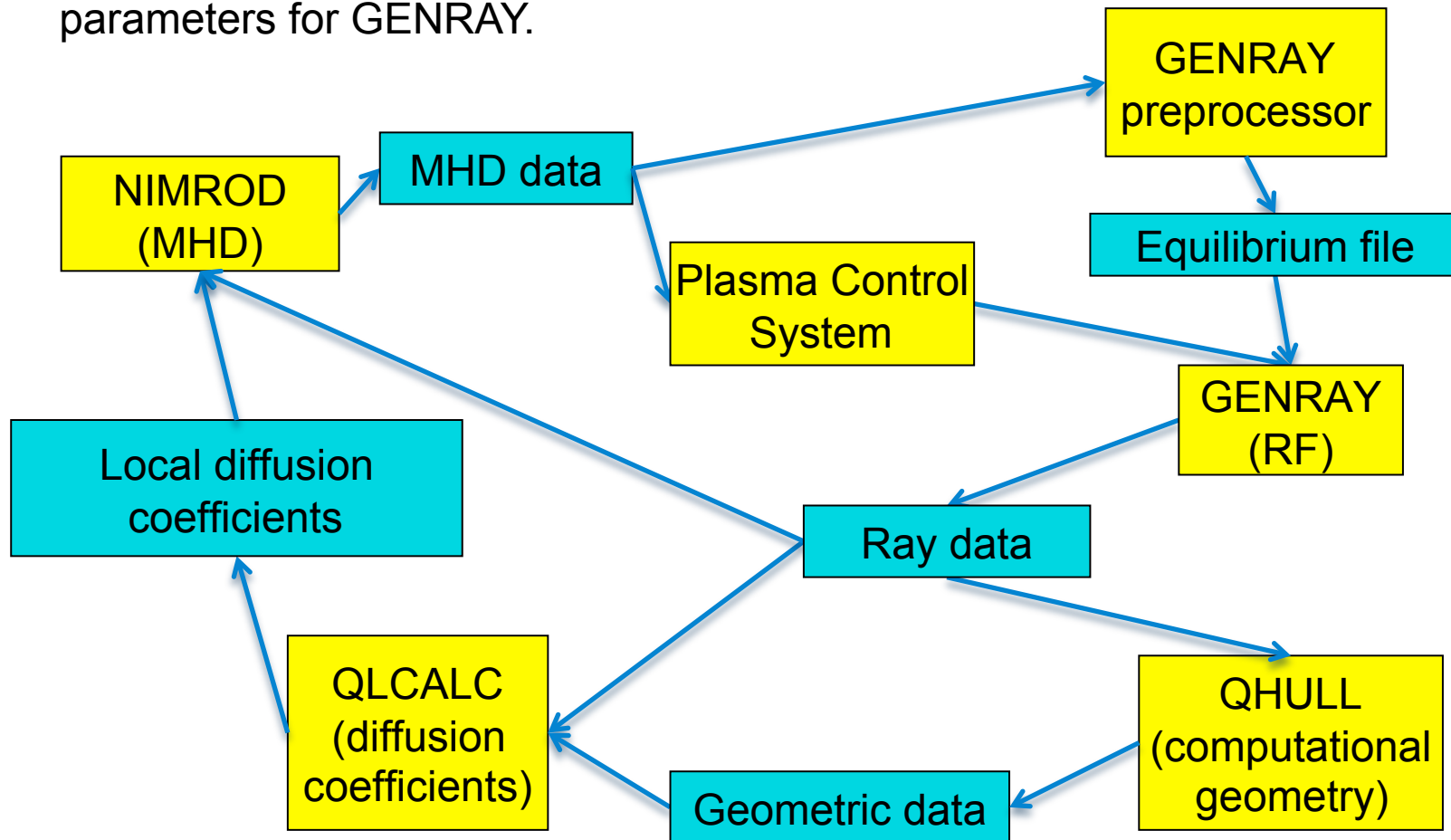
- Experimentally, RF waves resonant with electron cyclotron motion can drive currents that alter or suppress the island structures.
- Numerically, we want to detect the modes and suppress them in the same way that experimentalists do; however, also want to find ways to optimize control systems.



# Plasma Control System monitors MHD physics and adjusts RF inputs accordingly

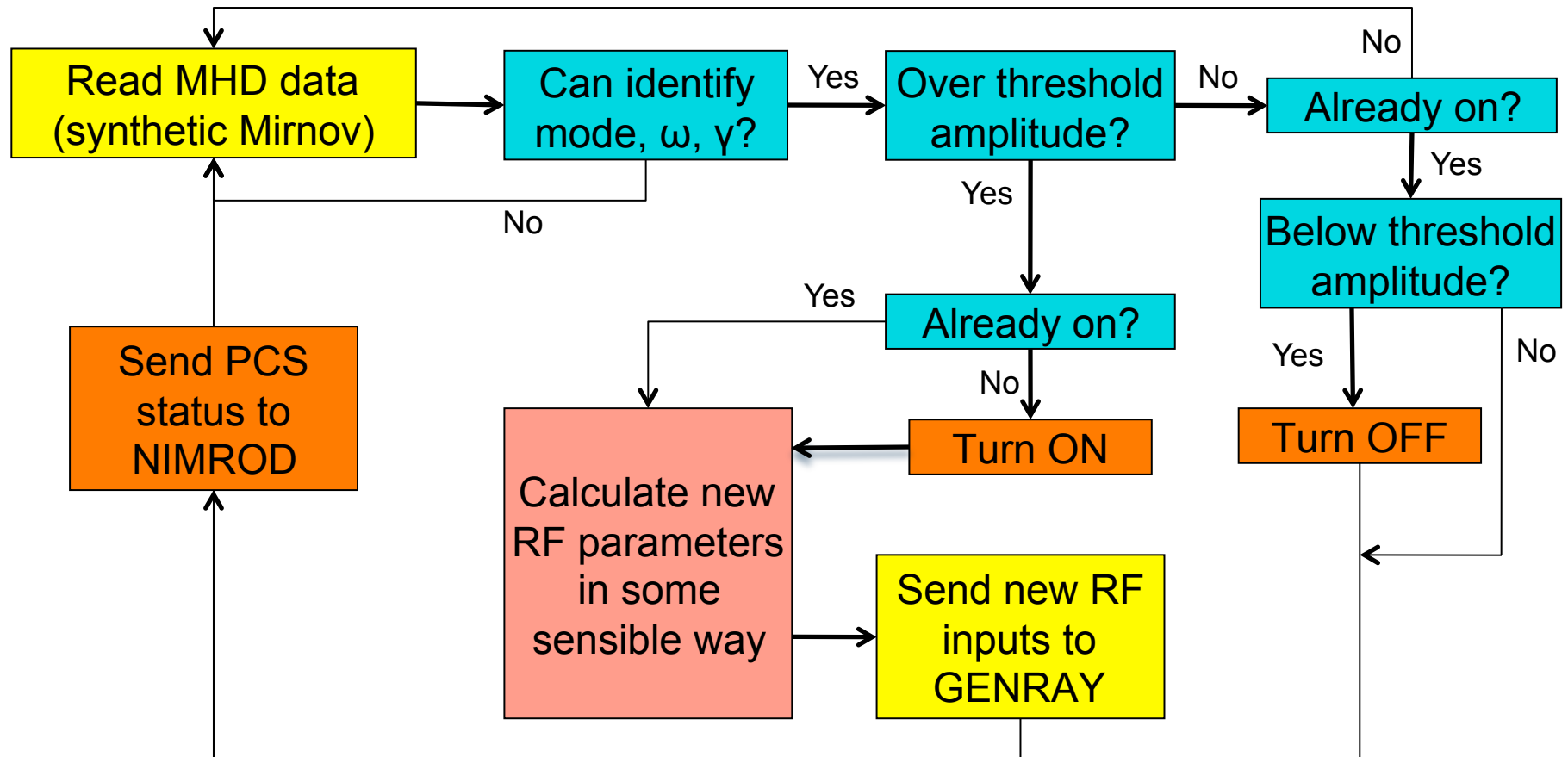


- Synthetic diagnostics in NIMROD provide data to the control system and enable calculation (or ad hoc adjustment, as in experiments) of RF input parameters for GENRAY.





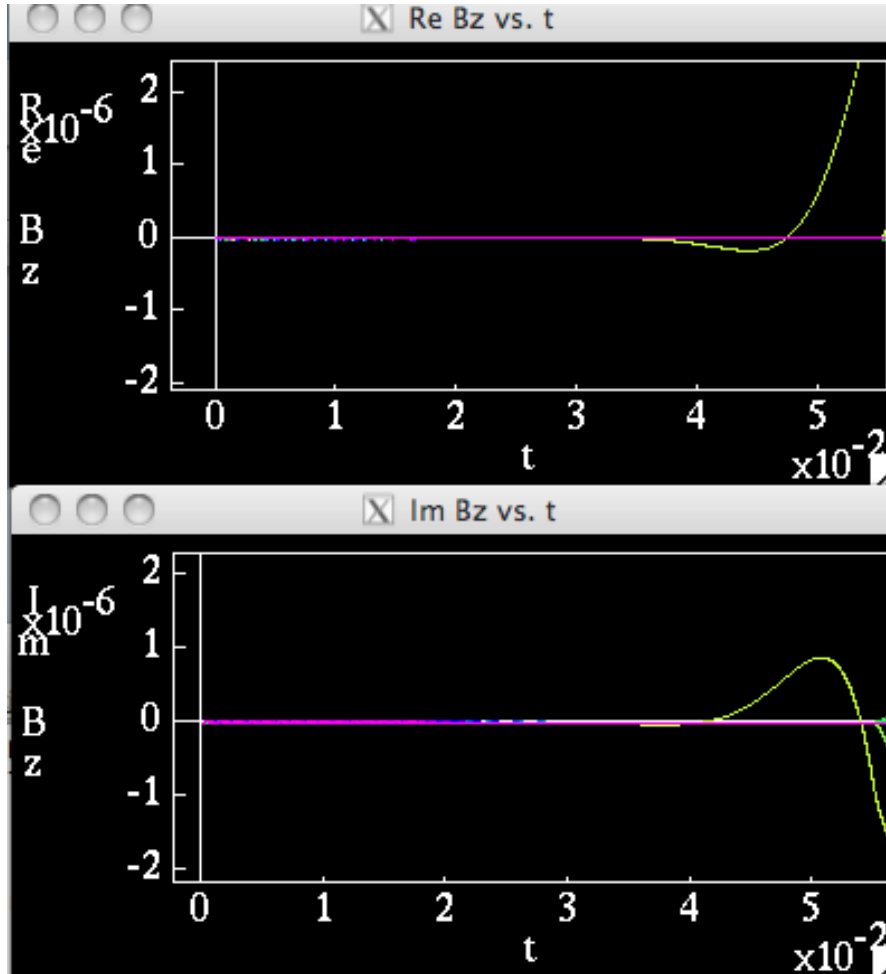
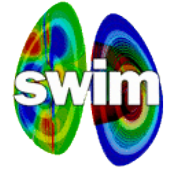
# A basic numerical Plasma Control System for the RF/MHD problem



- Time modulated RF vs. continuous RF
- Relative size of island and RF deposition
- “Perfect” vs. “pragmatic” control system; optimization



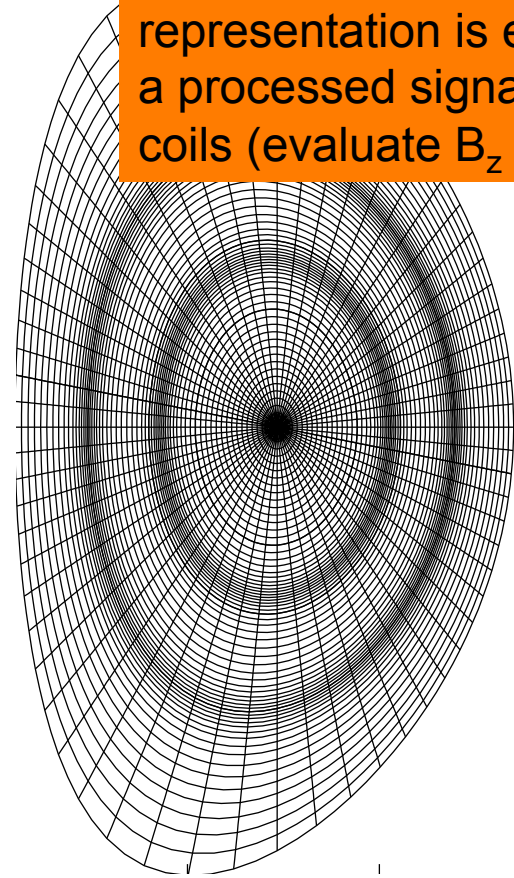
# Mode detection is carried out using synthetic Mirnov coil diagnostics in NIMROD



•n=1 tearing mode

$$\sim Ae^{i\omega t + \gamma t + \varphi}$$

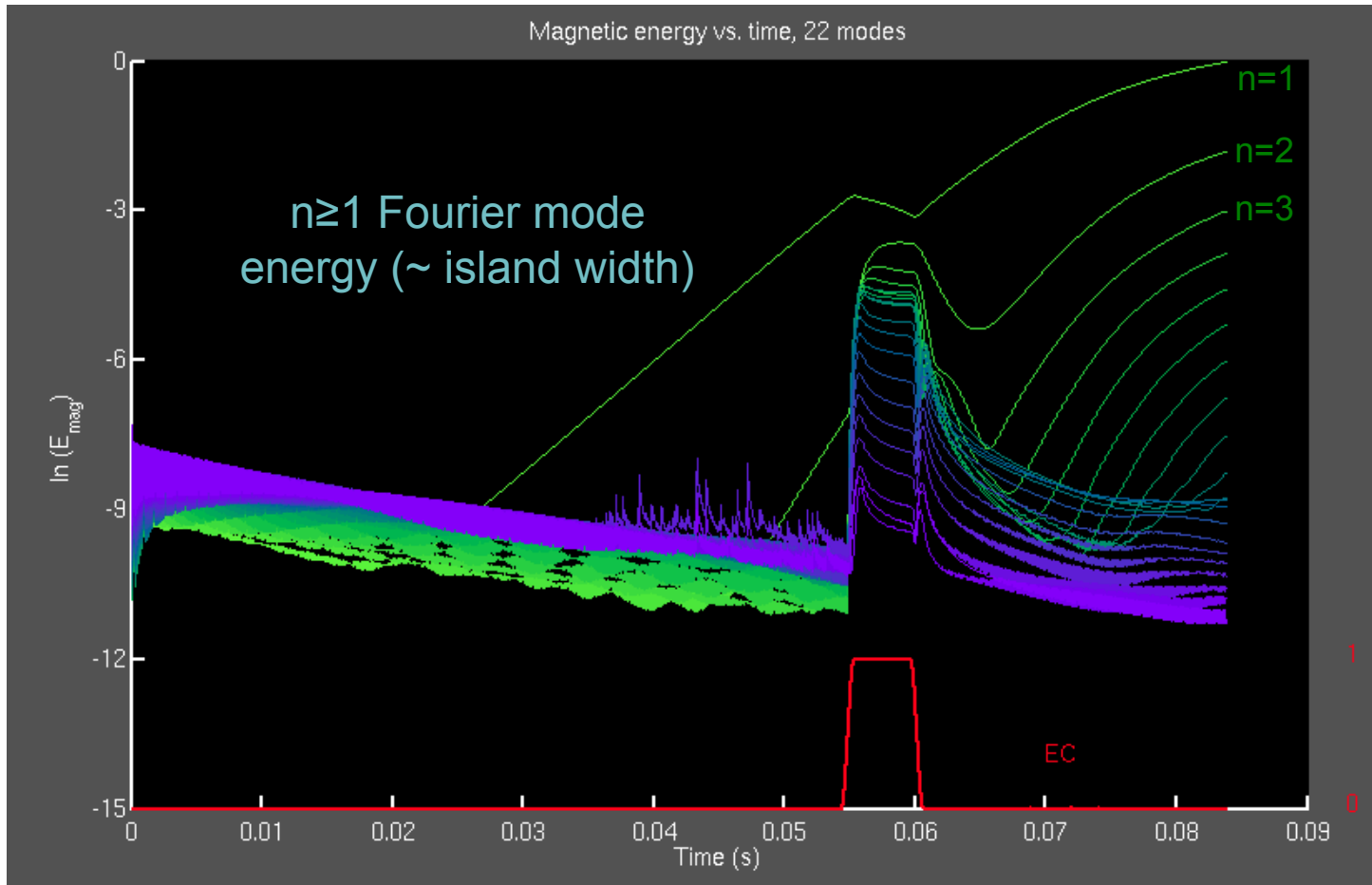
•NIMROD's toroidal Fourier representation is essentially a processed signal from Mirnov coils (evaluate  $B_z$  at the wall)



•Extract NIMROD data with python; fit data; extract amplitude, growth rate, and toroidal rotation frequency.



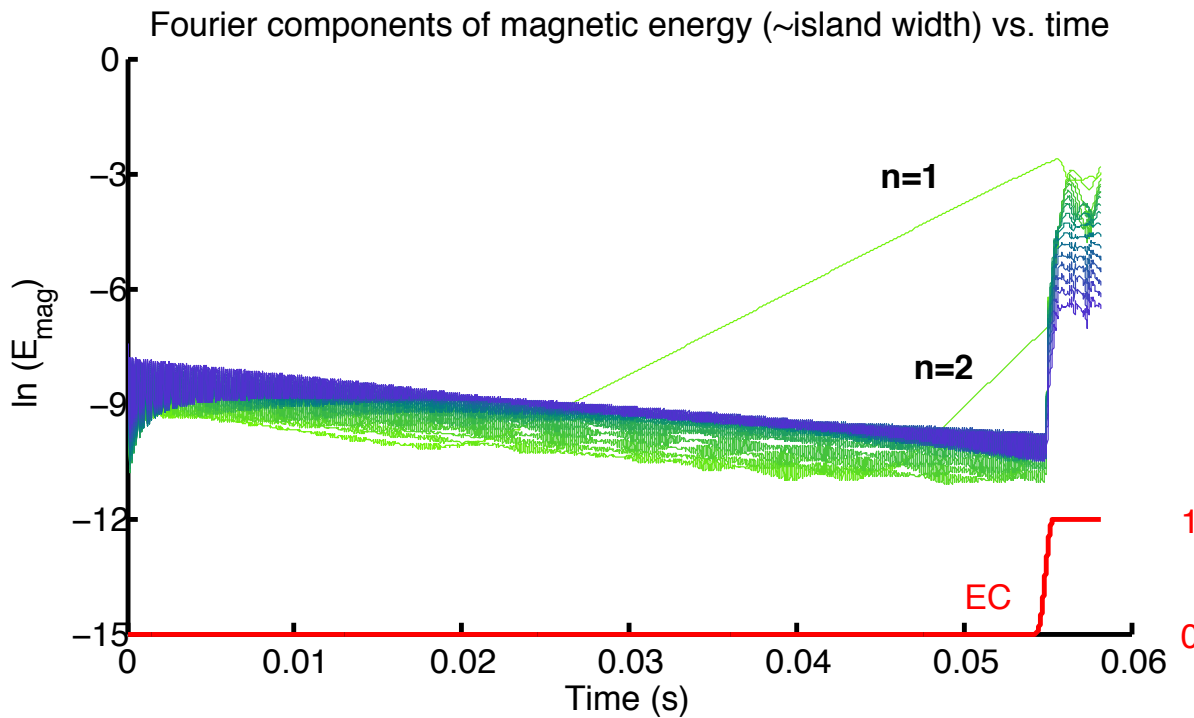
## Initial control system tests seem promising



- When mode amplitude exceeds threshold, PCS injects ECCD at island O-point; island shrinks.
- Mode growth resumes when PCS is shut off.



# Further work on the PCS algorithms is needed to optimize the mode stabilization

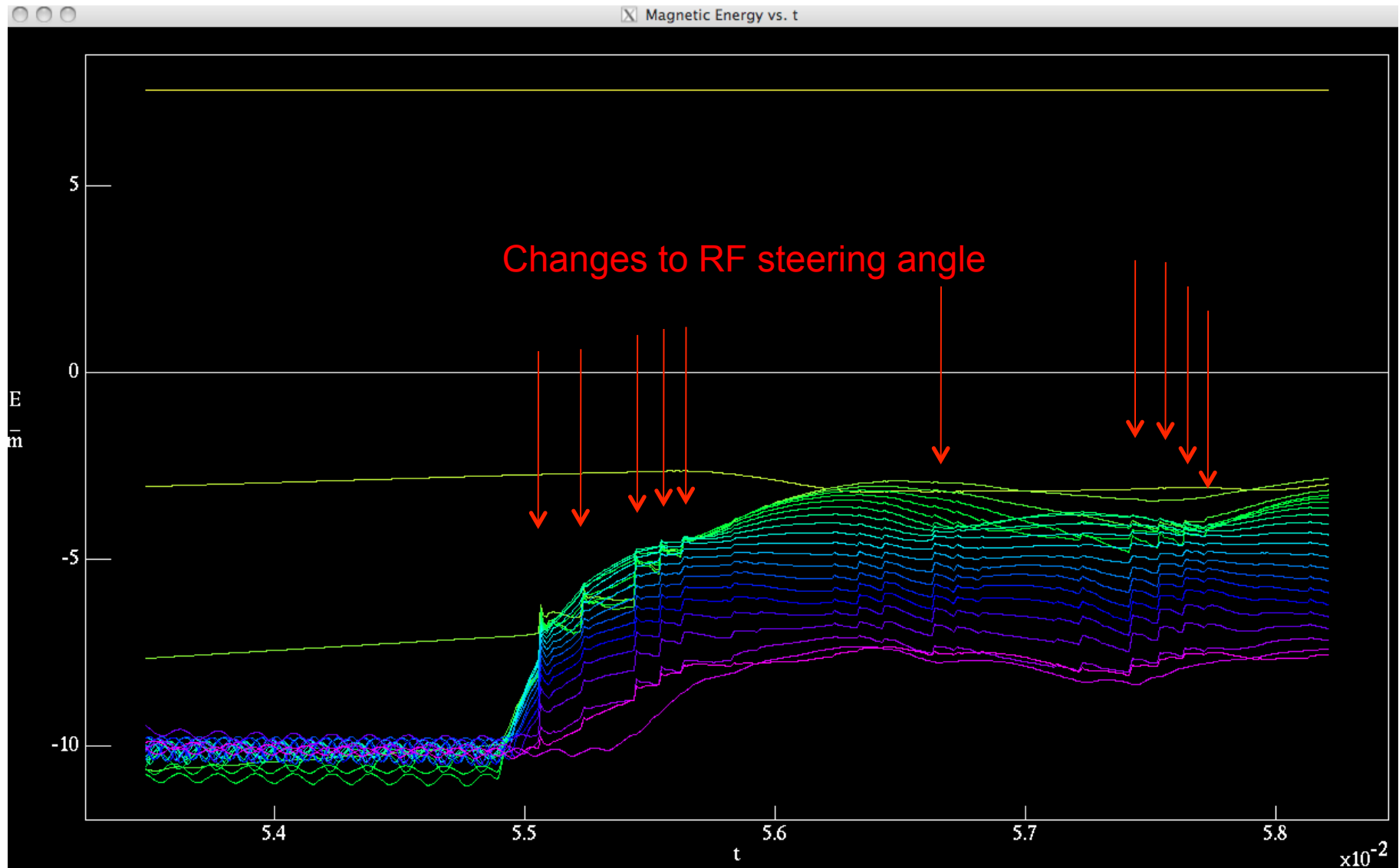


- RF deposition slightly outside the rational surface gives a more rapid initial stabilizing effect (consistent with observations from 2010 PoP paper).

- Very sensitive to RF alignment (La Haye NF 2009)



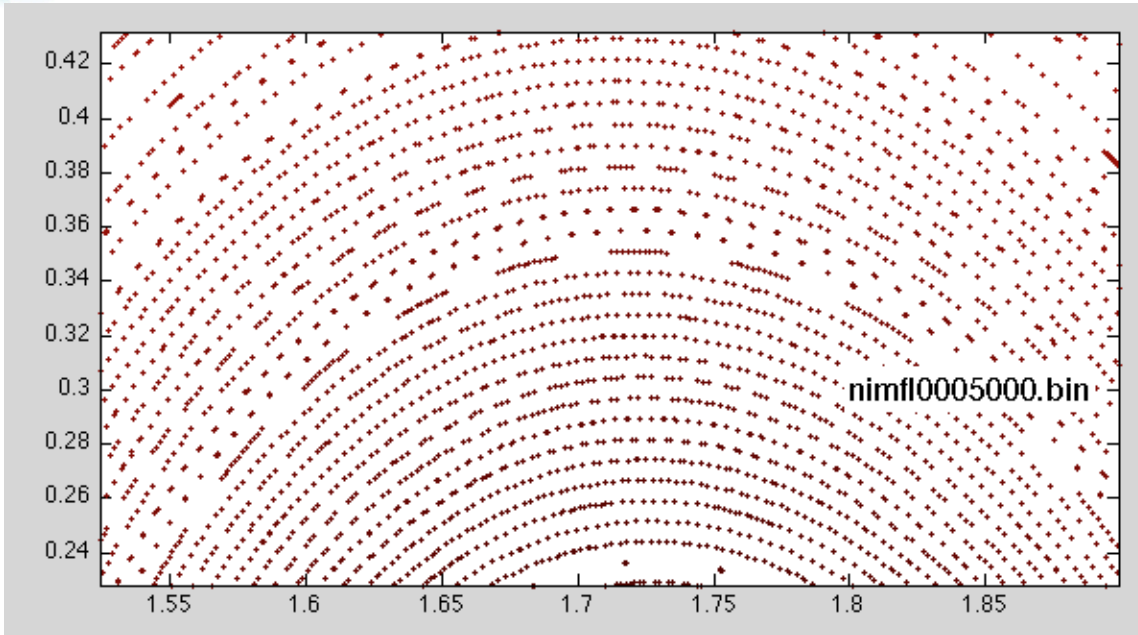
# Rapid changes in the steering angle of the RF may lead to numerical error



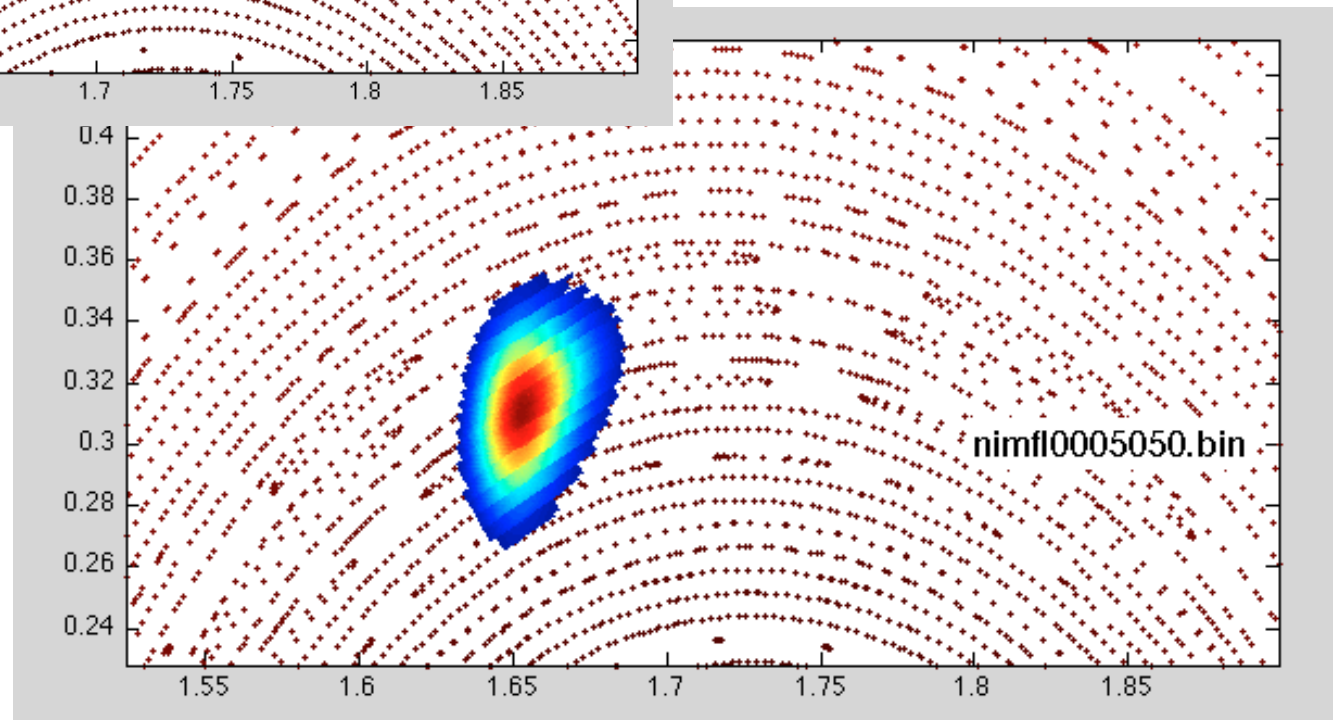




# Can study RF effects with Poincare maps

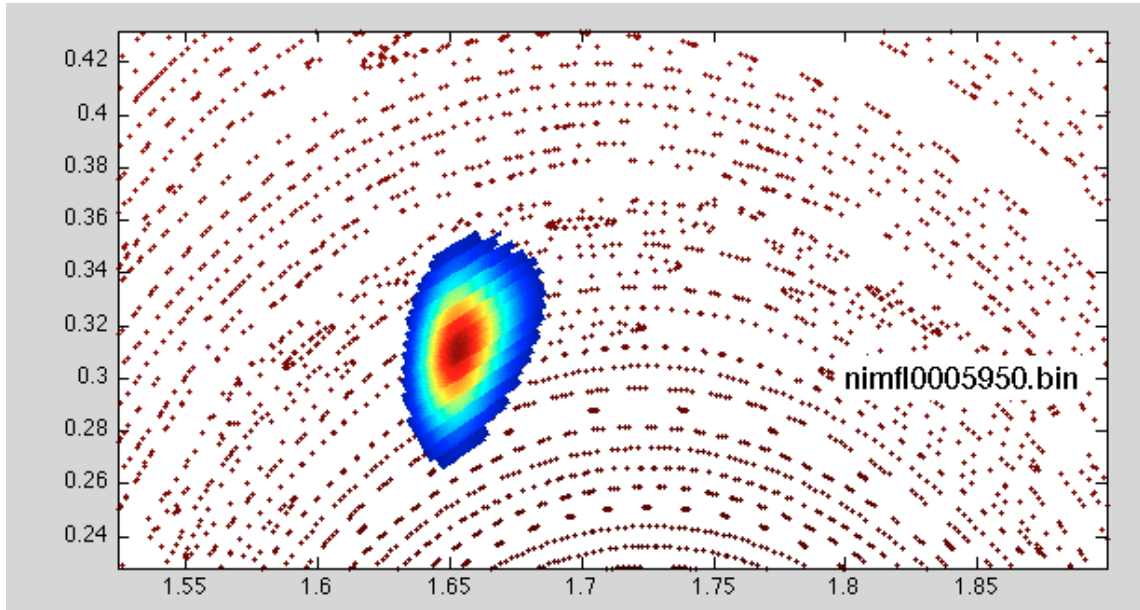


RF introduces  
filamentary current  
structures, as  
anticipated

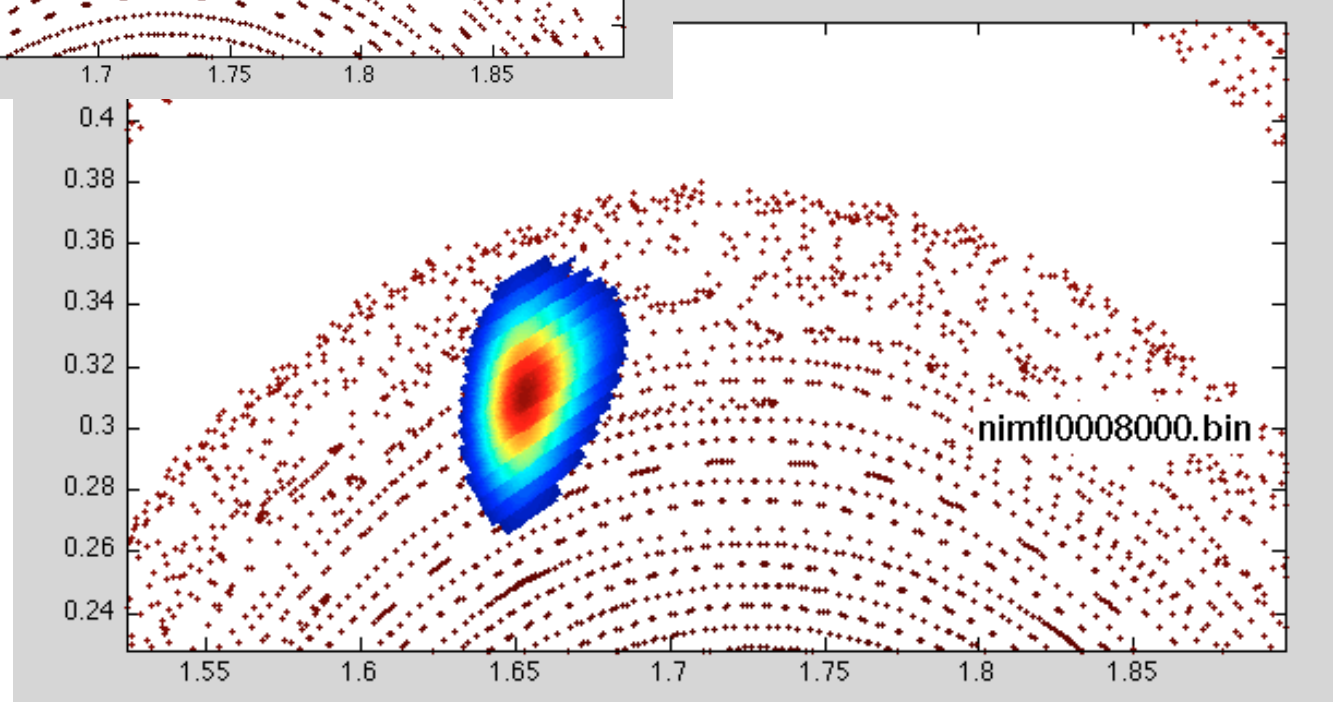




# Can study RF effects with Poincare maps



Here, misalignment is severe enough that the stabilization effects are entirely lost.





# How is RF power transferred to the plasma?



$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \nabla f_\alpha + \frac{q_\alpha}{m_\alpha} [\mathbf{E} + \mathbf{v} \times \mathbf{B}] \cdot \frac{\partial f_\alpha}{\partial \mathbf{v}} = \sum_\beta C(f_\alpha, f_\beta)$$

Kinetic +  
Maxwell:

$$\nabla \cdot \mathbf{B} = 0 \quad \nabla \cdot \mathbf{E} = \sum_\alpha \frac{q_\alpha}{\epsilon_0} \int f_\alpha d^3\mathbf{v}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \nabla \times \mathbf{B} = \mu_0 \sum_\alpha q_\alpha \int f_\alpha \mathbf{v} d^3\mathbf{v} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$$

Use vector identities to get

$$\nabla \cdot \left( \frac{\mathbf{E} \times \mathbf{B}}{2\mu_0} \right) = -\frac{\partial}{\partial t} \left( \frac{B^2}{2\mu_0} + \frac{\epsilon_0 E^2}{2} \right) - \frac{\mathbf{E} \cdot \mathbf{J}}{2}$$

Poynting vector  
(power/area)

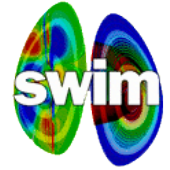
Wave energy density

Wave-plasma  
interaction

All power-related terms are quadratic in the field amplitudes...



# Decompose fields and distribution functions to account for RF and MHD spatiotemporal scales



Local Maxwellian

Kinetic distortion  
(for closure)

Distribution function perturbations  
(induced by RF, evolving on MHD  
time and spatial scales, complex)

$$f_\alpha = f_{M\alpha s}(\mathbf{x}, \mathbf{v}, t) + \epsilon F_\alpha(\mathbf{x}, \mathbf{v}, t) + (\epsilon/2) f_{\alpha s}^{RF}(\mathbf{x}, \mathbf{v}, t) e^{i\psi(\mathbf{x}, t)} + (\epsilon/2) f_{\alpha s}^{RF*}(\mathbf{x}, \mathbf{v}, t) e^{-i\psi^*(\mathbf{x}, t)}$$

$$\mathbf{E} = \mathbf{E}_{0s}(\mathbf{x}, t) + (\epsilon/2) \mathbf{E}_s(\mathbf{x}, t) e^{i\psi(\mathbf{x}, t)} + (\epsilon/2) \mathbf{E}_s^*(\mathbf{x}, t) e^{-i\psi^*(\mathbf{x}, t)},$$

$$\mathbf{B} = \mathbf{B}_{0s}(\mathbf{x}, t) + (\epsilon/2) \mathbf{B}_s(\mathbf{x}, t) e^{i\psi(\mathbf{x}, t)} + (\epsilon/2) \mathbf{B}_s^*(\mathbf{x}, t) e^{-i\psi^*(\mathbf{x}, t)},$$

MHD fields

RF field amplitudes (evolving on MHD  
time and spatial scales, complex)

Rapidly varying phase of  
RF fields/d.f. perturbations  
(complex)

$$\text{RF terms are real, } \sim \epsilon \text{Re} \left[ f_{\alpha s}^{RF}(\mathbf{x}, \mathbf{v}, t) e^{i\psi(\mathbf{x}, t)} \right]$$

Phase satisfies

$$k_s(x, t) = \nabla \psi(x, t) \quad \omega_s(x, t) = -\frac{\partial \psi(x, t)}{\partial t}$$

where  $k_s(x, t)$  and  $\omega_s(x, t)$  variation is on the MHD spatiotemporal scales;

$$\psi \approx k_s \cdot x - \omega_s t \quad \text{on RF scales}$$



# Averaging over the RF period gives a separable system



Several kinds of terms appear in the kinetic equation:

No phase information – MHD equations & closures

$e^{i\psi(x,t)}$  – RF equations

$e^{i[\psi(x,t)-\psi^*(x,t)]}$  – quasilinear terms

$e^{2i\psi(x,t)}$  – nonlinear terms

Define an average over the RF period;

$$\langle \xi(\mathbf{x}, t) \rangle \equiv \frac{1}{T} \int_{t-T/2}^{t+T/2} \xi(\mathbf{x}, t) dt$$

MHD and QL pieces do not vanish in the averaging;

$$\frac{\partial n_{\alpha s}}{\partial t} + \nabla \cdot (n_{\alpha s} \mathbf{V}_{\alpha s}) = 0$$

$$m_{\alpha} n_{\alpha s} \left( \frac{\partial \mathbf{V}_{\alpha s}}{\partial t} + (\mathbf{V}_{\alpha s} \cdot \nabla) \mathbf{V}_{\alpha s} \right) = -\nabla (n_{\alpha s} T_{\alpha s}) - \epsilon \nabla \cdot \mathbf{\Pi}_{\alpha} + q_{\alpha} n_{\alpha s} [\mathbf{E}_{0s} + \mathbf{V}_{\alpha s} \times \mathbf{B}_{0s}] + \mathbf{R}_{\alpha}$$

$$+ \frac{\epsilon^2 \langle e^{i(\psi - \psi^*)} \rangle}{4} \delta \mathbf{R}_{\alpha} + \frac{\epsilon^2 \langle e^{i(\psi - \psi^*)} \rangle}{4} q_{\alpha} \int f_{\alpha s}^{RF} [\mathbf{E}_s^* + \mathbf{v} \times \mathbf{B}_s^*] d^3 \mathbf{v} + \text{c. c.}$$

$$\frac{3}{2} n_{\alpha s} \left( \frac{\partial T_{\alpha s}}{\partial t} + (\mathbf{V}_{\alpha s} \cdot \nabla) T_{\alpha s} \right) + n_{\alpha s} T_{\alpha s} \nabla \cdot \mathbf{V}_{\alpha s} = -\epsilon \nabla \cdot \mathbf{q}_{\alpha} - \epsilon \mathbf{\Pi}_{\alpha} : \nabla \mathbf{V}_{\alpha s} + Q_{\alpha}$$

$$+ \frac{\epsilon^2 \langle e^{i(\psi - \psi^*)} \rangle}{4} \delta Q_{\alpha} + \frac{\epsilon^2 \langle e^{i(\psi - \psi^*)} \rangle}{4} q_{\alpha} \left[ (\mathbf{E}_s^* + \mathbf{V}_{\alpha s} \times \mathbf{B}_s^*) \cdot \int f_{\alpha s}^{RF} (\mathbf{v} - \mathbf{V}_{\alpha s}) d^3 \mathbf{v} \right] + \text{c. c.}$$

along with Maxwell's equations (separable) and closure equation (complicated)



# RF wave propagation – linearized fluctuations about the averaged (MHD) physics



RF terms  $\sim e^{i\psi(x,t)}$ ; neglect nonlinearity, collisions, MHD spatiotemporal variation, terms small in MHD...

$$\begin{aligned}
 & \frac{\epsilon e^{i\psi}}{2} \left[ \frac{\partial f_{\alpha s}^{RF}}{\partial t} + \mathbf{v} \cdot \nabla f_{\alpha s}^{RF} + i \left( \frac{\partial \psi}{\partial t} + \mathbf{v} \cdot \nabla \psi \right) f_{\alpha s}^{RF} + \frac{q_\alpha}{m_\alpha} (\mathbf{E}_{0s} + \mathbf{v} \times \mathbf{B}_{0s}) \cdot \frac{\partial f_{\alpha s}^{RF}}{\partial \mathbf{v}} \right. \\
 & \left. + \frac{q_\alpha}{m_\alpha} (\mathbf{E}_s + \mathbf{v} \times \mathbf{B}_s) \cdot \frac{\partial}{\partial \mathbf{v}} (f_{M\alpha s} + \epsilon F_\alpha) - \sum_\beta [C(f_{M\alpha s} + \epsilon F_\alpha, f_{\beta s}^{RF}) + C(f_{\alpha s}^{RF}, f_{M\beta s} + \epsilon F_\beta)] \right] + \text{c. c.} \\
 & + e^{2i\psi} \frac{\epsilon^2}{4} \left( \frac{q_\alpha}{m_\alpha} (\mathbf{E}_s + \mathbf{v} \times \mathbf{B}_s) \cdot \frac{\partial f_{\alpha s}^{RF}}{\partial \mathbf{v}} - \sum_\beta C(f_{\alpha s}^{RF}, f_{\beta s}^{RF}) \right) + \text{c. c.} \\
 & + \frac{\epsilon^2}{4} [e^{i(\psi-\psi^*)} - \langle e^{i(\psi-\psi^*)} \rangle] \left( \frac{q_\alpha}{m_\alpha} (\mathbf{E}_s^* + \mathbf{v} \times \mathbf{B}_s^*) \cdot \frac{\partial f_{\alpha s}^{RF}}{\partial \mathbf{v}} - \sum_\beta C(f_{\alpha s}^{RF}, f_{\beta s}^{RF*}) \right) + \text{c. c.} = 0
 \end{aligned}$$

Solution (generalization of Kennel/Engelmann [Phys. Fluids **9**, 2377 (1966)] result):

$$\begin{aligned}
 f_{\alpha s}^{RF} = & e^{iz \sin \phi} \sum_{n=-\infty}^{\infty} \frac{ie^{-in\phi} q_\alpha n_{\alpha s} e^{-v_\perp^2/2v_{t\alpha s}^2} e^{-(v_\parallel - V_{\parallel\alpha s})^2/2v_{t\alpha s}^2}}{\omega m_\alpha v_{t\alpha s}^5 (2\pi)^{3/2}} \left[ -J_n(z) V_{\parallel\alpha s} \hat{z} \right. \\
 & \left. + \frac{(\omega - k_\parallel V_{\parallel\alpha s})}{(\omega - k_\parallel v_\parallel - n\Omega_{\alpha s})} \left( \frac{n\Omega_{\alpha s}}{k_\perp} J_n(z) \hat{x} + iv_\perp J'_n(z) \hat{y} + v_\parallel J_n(z) \hat{z} \right) \right] \cdot \mathbf{E}_s
 \end{aligned}$$



With the solution for  $f_{\alpha}^{RF}$ , we can get the dispersion relation from Maxwell's equations



$$\mathbf{k}_s \times \mathbf{E}_s = \omega \mathbf{B}_s ;$$

$$c^2[\mathbf{k}_s \times \mathbf{B}_s] = -\omega \mathbf{E}_s - \sum_{\alpha} \frac{iq_{\alpha}}{\epsilon_0} \int f_{\alpha s}^{RF} \mathbf{v} d^3\mathbf{v}$$

➔  $\mathbf{N}_s \times (\mathbf{N}_s \times \mathbf{E}_s) + \mathbf{E}_s + \sum_{\alpha} \chi_{\alpha} \cdot \mathbf{E}_s = 0$  or  $\vec{D} \cdot \mathbf{E}_s = 0$  for tensor  $D$ .

and an equation for the local RF power balance;

$$k_s \cdot \left( \frac{\mathbf{E}_s \times \mathbf{B}_s^*}{2\mu_0} + \frac{\mathbf{E}_s^* \times \mathbf{B}_s}{2\mu_0} \right) = \omega \left( \frac{B_s^2}{2\mu_0} + \frac{\epsilon_0 \mathbf{E}_s^* \cdot (\mathbf{I} + \chi^h) \cdot \mathbf{E}_s}{2} \right) + \frac{i\epsilon_0 \omega \mathbf{E}_s^* \cdot \chi^a \cdot \mathbf{E}_s}{2}$$

Poynting vector (power/area)
Wave/plasma energy density (nondissipative)
Dissipative wave-plasma interaction

real part:

$$k_{sr} \cdot S = \omega E$$

along surfaces of constant phase:

$$\frac{d\psi[x(t),t]}{dt} = \frac{\partial\psi[x(t),t]}{\partial t} + \frac{\partial\psi[x(t),t]}{\partial x} \cdot \frac{dx(t)}{dt} = -\omega + k_s \cdot v_g = 0$$

so that  $S = E v_g$  (power flux) = (energy density) x (velocity)



Can also calculate local power deposition along the ray



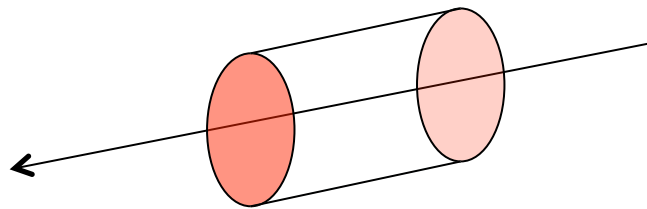
$$k_s \cdot \left( \frac{E_s \times B_s^*}{2\mu_0} + \frac{E_s^* \times B_s}{2\mu_0} \right) = \omega \left( \frac{B_s^2}{2\mu_0} + \frac{\epsilon_0 E_s^* \cdot (I + \chi^h) \cdot E_s}{2} \right) + \frac{i\epsilon_0 \omega E_s^* \cdot \chi^a \cdot E_s}{2}$$

Poynting vector (power/area)
Wave/plasma energy density (nondissipative)
Dissipative wave-plasma interaction

imaginary part:

$$2k_{si} \cdot S e^{i[\psi - \psi^*]} \Rightarrow \nabla \cdot [S e^{i[\psi - \psi^*]}] = \omega \epsilon_0 E_s^* \cdot \chi^a \cdot E_s e^{i[\psi - \psi^*]}$$

Integrate over a volume element along a ray trajectory:



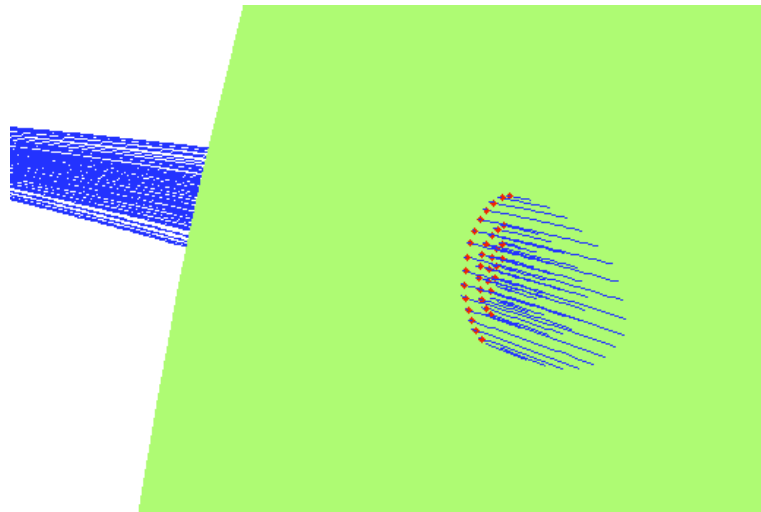
$$S \sim \text{Power} / \text{Area}$$

Net decrease in power flux = volumetric dissipation

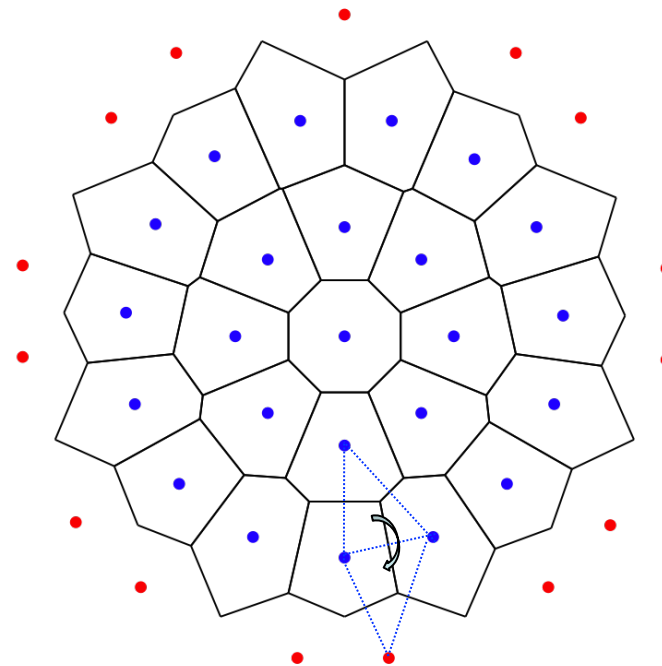




## Need to calculate necessary areas

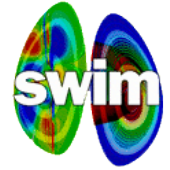


- Use Voronoi tessellations to calculate area belonging to each ray





Ray tracing equations for a given mode can be obtained



For a given solution  $\omega = \omega_0[x, t, k(x, t)]$

$$\nabla \omega = \nabla \omega_0 + \frac{\partial \omega_0}{\partial k} \cdot (\nabla k)$$

$$\nabla \omega = -\frac{\partial k}{\partial t} \quad ; \quad \frac{\partial \omega_0}{\partial k} = v_g \quad ; \quad v_g \cdot (\nabla k) = (v_g \cdot \nabla)k$$

$$\frac{\partial k}{\partial t} + (v_g \cdot \nabla)k = \frac{dk}{dt} = -\nabla \omega_0 \quad ; \quad \frac{dx}{dt} = \frac{\partial \omega_0}{\partial k}$$

Variation in local solution  $\omega_0$  alters local wavenumber and trajectory path.

$$\mathbf{N}_s \times (\mathbf{N}_s \times \mathbf{E}_s) + \mathbf{E}_s + \sum_{\alpha} \chi_{\alpha} \cdot \mathbf{E}_s = 0$$

tells us about all the modes; ray tracing codes only tell us about one at a time.



# New terms in the MHD equations match the linear RF terms



$$\begin{aligned} \mathbf{k}_s \times \mathbf{E}_s &= \omega \mathbf{B}_s; & \mathbf{k}_s \cdot \mathbf{B}_s &= 0; \\ c^2 [\mathbf{k}_s \times \mathbf{B}_s] &= -\omega \mathbf{E}_s - \sum_{\alpha} \frac{iq_{\alpha}}{\epsilon_0} \int f_{\alpha s}^{RF} \mathbf{v} d^3\mathbf{v} & \mathbf{k}_s \cdot \mathbf{E}_s &= -\sum_{\alpha} \frac{iq_{\alpha}}{\epsilon_0} \int f_{\alpha s}^{RF} d^3\mathbf{v}; \end{aligned}$$

$$m_{\alpha} n_{\alpha s} \left( \frac{\partial \mathbf{V}_{\alpha s}}{\partial t} + (\mathbf{V}_{\alpha s} \cdot \nabla) \mathbf{V}_{\alpha s} \right) = -\nabla (n_{\alpha s} T_{\alpha s}) - \epsilon \nabla \cdot \mathbf{\Pi}_{\alpha} + q_{\alpha} n_{\alpha s} [\mathbf{E}_{0s} + \mathbf{V}_{\alpha s} \times \mathbf{B}_{0s}] + \mathbf{R}_{\alpha}$$

$$+ \frac{\epsilon^2 \langle e^{i(\psi - \psi^*)} \rangle}{4} \delta \mathbf{R}_{\alpha} + \frac{\epsilon^2 \langle e^{i(\psi - \psi^*)} \rangle}{4} q_{\alpha} \int f_{\alpha s}^{RF} [\mathbf{E}_s^* + \mathbf{v} \times \mathbf{B}_s^*] d^3\mathbf{v} + \text{c. c.}$$

$$= \frac{\epsilon^2 \langle e^{i(\psi - \psi^*)} \rangle \epsilon_0 k_s}{2} E_s^* \cdot \chi^a \cdot E_s$$

$$\frac{3}{2} n_{\alpha s} \left( \frac{\partial T_{\alpha s}}{\partial t} + (\mathbf{V}_{\alpha s} \cdot \nabla) T_{\alpha s} \right) + n_{\alpha s} T_{\alpha s} \nabla \cdot \mathbf{V}_{\alpha s} = -\epsilon \nabla \cdot \mathbf{q}_{\alpha} - \epsilon \mathbf{\Pi}_{\alpha} : \nabla \mathbf{V}_{\alpha s} + Q_{\alpha}$$

$$+ \frac{\epsilon^2 \langle e^{i(\psi - \psi^*)} \rangle}{4} \delta Q_{\alpha} + \frac{\epsilon^2 \langle e^{i(\psi - \psi^*)} \rangle}{4} q_{\alpha} \left[ (\mathbf{E}_s^* + \mathbf{V}_{\alpha s} \times \mathbf{B}_s^*) \cdot \int f_{\alpha s}^{RF} (\mathbf{v} - \mathbf{V}_{\alpha s}) d^3\mathbf{v} \right] + \text{c. c.}$$

$$= \frac{\epsilon^2 \langle e^{i(\psi - \psi^*)} \rangle \epsilon_0}{2} (\omega - k_{\parallel} V_{\parallel \alpha}) E_s^* \cdot \chi^a \cdot E_s$$



## Analytic form for new terms

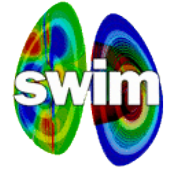


$$2E_s^* \cdot \chi^a \cdot E_s = \sum_{n=-\infty}^{\infty} \frac{\omega_{p\alpha s}^2}{\omega^2} e^{-\lambda_{\alpha s} \xi_{0s}} \sqrt{\pi} e^{-\xi_{ns}^2} \times$$
$$\left[ [I_n(\lambda_{\alpha s}) - I_{n+1}(\lambda_{\alpha s})] \left( 2\lambda_{\alpha s} |E_{ys}|^2 + n \left| E_{xs} - iE_{ys} + \frac{\sqrt{\lambda_{\alpha s}}(\omega - n\Omega_{\alpha s})E_{zs}}{nk_{\parallel}v_{t\alpha s}} \right|^2 \right) \right.$$
$$\left. + [I_n(\lambda_{\alpha s}) - I_{n-1}(\lambda_{\alpha s})] \left( 2\lambda_{\alpha s} |E_{ys}|^2 - n \left| E_{xs} + iE_{ys} + \frac{\sqrt{\lambda_{\alpha s}}(\omega - n\Omega_{\alpha s})E_{zs}}{nk_{\parallel}v_{t\alpha s}} \right|^2 \right) \right]$$

Highly localized at cyclotron resonance;  $\xi_n = \frac{\omega - k_{\parallel}V_{\parallel\alpha} - n\Omega_{\alpha}}{\sqrt{2}k_{\parallel}v_{t\alpha}}$



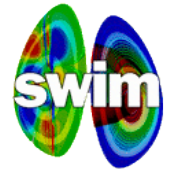
Closures will add additional, significant physics  
as we move toward self-consistent NTM simulations



$$\begin{aligned}
 & \epsilon \left( \frac{\partial F_\alpha}{\partial t} + \mathbf{v} \cdot \nabla F_\alpha + \frac{q_\alpha}{m_\alpha} (\mathbf{E}_{0s} + \mathbf{v} \times \mathbf{B}_{0s}) \cdot \frac{\partial F_\alpha}{\partial \mathbf{v}} \right) + \frac{\epsilon^2 q_\alpha}{4m_\alpha} \langle e^{i(\psi - \psi^*)} \rangle \frac{\partial}{\partial \mathbf{v}} \cdot [(\mathbf{E}_s^* + \mathbf{v} \times \mathbf{B}_s^*) f_{\alpha s}^{RF}] + \text{c. c.} \\
 & + f_{M\alpha s} \left\{ \frac{m_\alpha}{T_{\alpha s}} \left( \mathbf{v}' \mathbf{v}' - \frac{v'^2 \mathbf{I}}{3} \right) : \nabla \mathbf{V}_{\alpha s} + \left( \frac{m_\alpha v'^2}{2T_{\alpha s}} - \frac{5}{2} \right) \frac{\mathbf{v}' \cdot \nabla T_{\alpha s}}{T_{\alpha s}} \right. \\
 & \quad \left. + \frac{\mathbf{v}'}{n_{\alpha s} T_{\alpha s}} \cdot \left( \mathbf{R}_\alpha + \frac{\epsilon^2 \langle e^{i(\psi - \psi^*)} \rangle}{4} \delta \mathbf{R}_\alpha - \epsilon \nabla \cdot \mathbf{\Pi}_\alpha \right) \right. \\
 & \quad \left. + \left( \frac{m_\alpha v'^2}{3T_{\alpha s}} - 1 \right) \frac{1}{n_{\alpha s} T_{\alpha s}} \left( Q_\alpha + \frac{\epsilon^2 \langle e^{i(\psi - \psi^*)} \rangle}{4} \delta Q_\alpha - \epsilon \nabla \cdot \mathbf{q}_\alpha - \epsilon \mathbf{\Pi}_\alpha : \nabla \mathbf{V}_\alpha \right) \right. \\
 & \quad \left. + \frac{\epsilon^2 \langle e^{i(\psi - \psi^*)} \rangle q_\alpha \mathbf{v}'}{4n_{\alpha s} T_{\alpha s}} \cdot \left( \int f_{\alpha s}^{RF} [\mathbf{E}_s^* + \mathbf{v} \times \mathbf{B}_s^*] d^3 \mathbf{v} \right) + \text{c. c.} \right. \\
 & \quad \left. + \frac{\epsilon^2 \langle e^{i(\psi - \psi^*)} \rangle q_\alpha}{4n_{\alpha s} T_{\alpha s}} \left( \frac{m_\alpha v'^2}{3T_{\alpha s}} - 1 \right) \left[ (\mathbf{E}_s^* + \mathbf{V}_{\alpha s} \times \mathbf{B}_s^*) \cdot \left( \int f_{\alpha s}^{RF} (\mathbf{v} - \mathbf{V}_{\alpha s}) d^3 \mathbf{v} \right) \right] + \text{c. c.} \right\} \\
 & = \sum_\beta C(f_{M\alpha s} + \epsilon F_\alpha, f_{M\beta s} + \epsilon F_\beta) + \frac{\epsilon^2 \langle e^{i(\psi - \psi^*)} \rangle}{4} \sum_\beta [C(f_{\alpha s}^{RF}, f_{\beta s}^{RF*}) + C(f_{\alpha s}^{RF*}, f_{\beta s}^{RF})];
 \end{aligned}$$



# Fokker-Planck physics effects we'll need to capture



$$m_\alpha n_\alpha \left[ \frac{\partial \vec{V}_\alpha}{\partial t} + (\vec{V}_\alpha \cdot \vec{\nabla}) \vec{V}_\alpha \right] = n_\alpha q_\alpha (\vec{E} + \vec{V}_\alpha \times \vec{B}) - \vec{\nabla} p_\alpha - \vec{\nabla} \cdot \vec{\pi}_\alpha + \eta \left[ \vec{J} + \frac{3e\vec{q}}{5T_e} \right] + \vec{F}_\alpha^{rf}$$

- RF interaction increases electron  $v_\perp$
- Lower collisionality ( $\sim 1/v^3$ )
- Net momentum transfer between ions and electrons; current

- RF interaction moves particles across trapped-passing boundary
- Symmetric detrapping, asymmetric trapping; current (opposite direction)

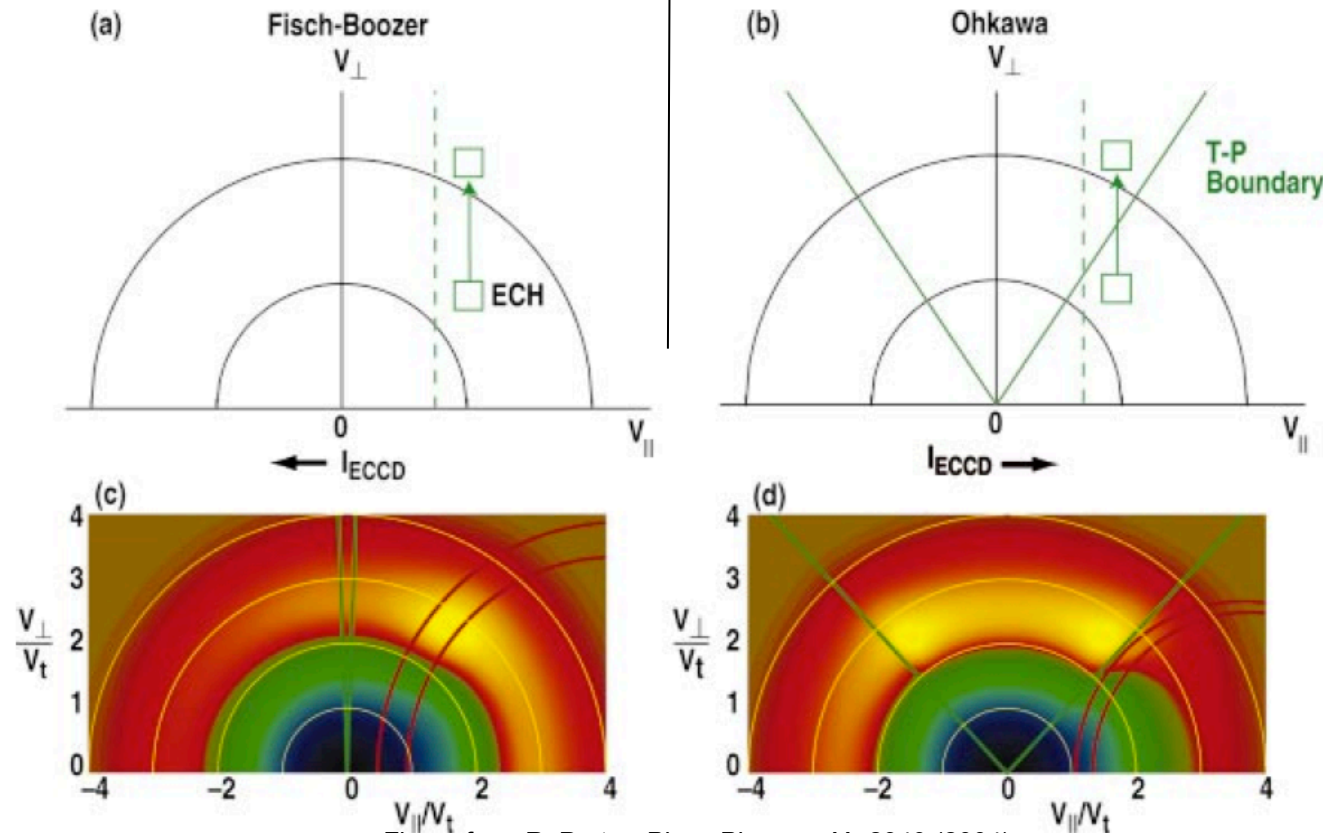
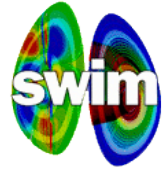


Figure from R. Prater, *Phys. Plasmas* 11, 2349 (2004).



## Present status and plans



- Developments to the control system are ongoing
- Writing several papers:
  - Form of new terms in the fluid equations and kinetic equation for closure
  - Computational methods, and how to translate between the various physics objects in this problem (MHD fluids, individual RF rays, and the RF ray bundle).
  - Physics results
- QL operator to be implemented in Eric's closure scheme (S. Kruger's talk yesterday)