Verification of tearing-mode drift effects with extended MHD.

Jacob King, Scott Kruger

Tech-X Corporation

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Motivation

- Large-guide-field tearing impacts the performance of state-of-the-art and planned tokamaks.
- FLR effects impact tearing dynamics through 'drift' effects.
- These drift effects are manifest in the model through advective terms, gyroviscosity and the cross heat flux.
- For a physically descriptive model, one must include drift contributions from all sources.

The diamagnetic drift velocity, ω_*, is significant in experimental configurations.

	DIII-D core	DIII-D edge	ITER core	ITER edge
m	1	3	2	3
n	1	1	1	1
β	0.0849	0.0158	0.0376	0.0195
$k\rho_i$	0.0356	0.0150	0.0089	0.0080
$\omega_{*i}\ \tau_a$	0.0594	0.1097	0.0032	0.0035
$\omega_{*e}\ \tau_a$	0.0585	0.1099	0.0033	0.0035
S	1.03E+07	3.85E+05	1.63E+09	4.41E+08

Estimates local to the outboard mid-plane

Experimental conditions have moderate β, d_i, and ω_*.

- Previous analytic drift-tearing works typically make one or multiple simplifying assumptions:
  - Low β
  - Complete gyroviscous cancellation
  - No cross heat flux
  - Reduced models
- We need to verify our unreduced extended-MHD models on this problem.
- This exercise also allows us to understand the relation between our model and previous works.

Analytic work has described tearing with a two-fluid model (no FLR effects).

- Analytic calculation [1] with an extended-MHD model has shown that the diffusion of $\tilde{B}_{||}$ can produce two-fluid tearing which is modified from the traditional semi-collisional result [2].
- The effect is relevant both to experiment and computational modeling.
- The analytic work [3] which bridges the dispersion relation between the single-fluid, the $\tilde{B}_{||}$-diffusion, and the semi-collisional regime has been used to verify the NIMROD code.
- We are working on adding FLR 'drift' effects to these analytics.

[1] Mirnov et al., Phys. Plasmas 11(9), 4468 (2004).

[2] Drake and Lee, Phys. Fluids 20, 1341 (1977).

[3] Ahedo and Ramos, PPCF 51, 055018 (2009).

Our analytics begin with an <u>unreduced</u> extended-MHD model.

$$\frac{\partial n}{\partial t} = -\nabla \cdot n\mathbf{v} + D_n \nabla^2 n \ ,$$

$$m_i n \frac{d\mathbf{v}}{dt} = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \mathbf{\Pi}_i - \nabla \cdot \nu m_i n \mathbf{W} \ ,$$

$$\frac{n}{\Gamma - 1} \frac{d^\alpha T_\alpha}{dt} = -p_\alpha \nabla \cdot \mathbf{v}_\alpha - \nabla \cdot \mathbf{q}_\alpha + \nabla \cdot \chi n \nabla T_\alpha \ ,$$

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B} + \frac{\mathbf{J} \times \mathbf{B}}{ne} - \frac{\nabla p_e}{ne} + \eta \mathbf{J} - \frac{m_e}{e} \frac{d^e \mathbf{v}_e}{dt} \ ,$$

$$p_\alpha = n T_\alpha \ , \quad \mathbf{J} = \nabla \times \mathbf{B} \ , \quad \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \ , \quad \mathbf{v}_e = \mathbf{v}_i - \frac{\mathbf{J}}{ne} \ ,$$

$$\mathbf{W}_\alpha = \nabla \mathbf{v}_\alpha + \nabla \mathbf{v}_\alpha^T - (2/3) \mathbf{I} \nabla \cdot \mathbf{v}_\alpha \ ,$$

The Braginskii closure includes first-order FLR effects.

$$\mathbf{\Pi}_{gv\alpha} = \frac{m_\alpha p_\alpha}{4eB}\left[\hat{\mathbf{b}} \times \mathbf{W}_\alpha \cdot \left(\mathbf{I} + 3\hat{\mathbf{b}}\hat{\mathbf{b}}\right) - \left(\mathbf{I} + 3\hat{\mathbf{b}}\hat{\mathbf{b}}\right) \cdot \mathbf{W}_\alpha \times \hat{\mathbf{b}}\right] ,$$

$$\mathbf{q} = \frac{5p_\alpha}{2eB_0}\hat{\mathbf{b}} \times \nabla T_\alpha ,$$

- Our study does not include heat flux contributions to the gyroviscous tensor.
- We also neglect electron gyroviscosity (more on this later).

These equations result after linearization.

$$(\hat{\gamma}_i - i\hat{\omega}_*)\,\hat{B}_r = i\hat{k}_\parallel \hat{\gamma}\hat{\xi} + \hat{k}_\parallel \hat{d}_i \hat{Q} + S_g^{-1}\hat{B}_r''$$

$$\hat{\gamma}_e \hat{B}_\parallel = -\hat{\nabla}_\perp \cdot \hat{\mathbf{v}} + \hat{\omega}_* \frac{\hat{\gamma}\hat{\xi}}{\hat{d}_i} - i\hat{\omega}_*\hat{Q} + \hat{k}_\parallel \hat{d}_i \left[\hat{B}_r'' + i\hat{k}_\parallel \hat{B}_\parallel' - \hat{B}_r\right] + \hat{\omega}_{*n}\frac{\Gamma}{\hat{c}_s^2}\hat{\lambda}_0 \hat{B}_r - \left(i\hat{\omega}_{*i}\hat{n} + i\hat{\omega}_{*n}\frac{\Gamma}{\hat{c}_s^2}\hat{p}_e\right) + S_g^{-1}\hat{B}_\parallel''$$

$$\hat{\gamma}_i \hat{\gamma}\hat{\xi} = \frac{\hat{\omega}_*}{\hat{d}_i}\hat{B}_\parallel + i\hat{k}_\parallel \hat{B}_r - \hat{B}_\parallel' - \hat{p}' - \left(\hat{\nabla}\cdot\hat{\mathbf{\Pi}}\right)_r \quad , \quad \hat{\gamma}_i \hat{v}_\perp = -i\hat{Q} - i\hat{p} - \left(\hat{\nabla}\cdot\hat{\mathbf{\Pi}}\right)_\perp$$

$$\hat{\gamma}_i \hat{v}_\parallel = -\frac{\hat{\omega}_*}{\hat{d}_i}\hat{B}_r - i\hat{k}_\parallel \hat{p} - \left(\hat{\nabla}\cdot\hat{\mathbf{\Pi}}\right)_\parallel \quad , \quad \hat{\gamma}_i \hat{n} = -\hat{\omega}_{*n}\frac{\Gamma}{\hat{c}_s^2}\frac{\hat{\gamma}\hat{\xi}}{\hat{d}_i} - \hat{\nabla}\cdot\hat{\mathbf{v}}$$

$$\hat{p} = -E_t \frac{\hat{\gamma}\hat{\xi}}{\hat{d}_i} - \frac{\hat{c}_{sp}^2}{\hat{\gamma}_i}\hat{\nabla}\cdot\hat{\mathbf{v}} + \left(C_{pe} + \hat{c}_{sq}^2\right)\hat{Q} - \left(C_{pe} + 4\hat{c}_{sq}^2\right) i\hat{\lambda}_0 \hat{B}_r + 2\hat{c}_{sq}^2 i\hat{k}_\parallel \hat{B}_r'$$

$$\hat{p}_e = -E_e \frac{\hat{\gamma}\hat{\xi}}{\hat{d}_i} - \frac{\hat{c}_{spe}^2}{\hat{\gamma}_i}\hat{\nabla}\cdot\hat{\mathbf{v}} + \left(C_{pe} + \hat{c}_{spe}^2\right)\hat{Q} - \left(C_{pe} + 4\hat{c}_{sqe}^2\right) i\hat{\lambda}_0 \hat{B}_r + 2\hat{c}_{sqe}^2 i\hat{k}_\parallel \hat{B}_r'$$

The hats indicate normalization by Alfven time/velocity, characteristic n/B/T or wavenumber

$$\gamma\tilde{\xi} = \tilde{v}_r$$

$$\hat{Q} = \hat{B}_\parallel - i\hat{k}_\parallel \hat{B}_r' + i\hat{\lambda}_0 \hat{B}_r \qquad \hat{\gamma}_\alpha = \hat{\gamma} + i\hat{\mathbf{k}}\cdot\hat{\mathbf{v}}_\alpha$$

Heat flux contributions become c_{sq} terms in the energy eqn.

$$\hat{p} = -E_t \frac{\hat{\gamma}\hat{\xi}}{\hat{d}_i} - \frac{\hat{c}_{sp}^2}{\hat{\gamma}_i} \hat{\nabla} \cdot \hat{\mathbf{v}} + \left(C_{pe} + \hat{c}_{sq}^2\right) \hat{Q} - \left(C_{pe} + 4\hat{c}_{sq}^2\right) i\hat{\lambda}_0 \hat{B}_r + 2\hat{c}_{sq}^2 i\hat{k}_{\|} \hat{B}_r'$$

$$\hat{c}_{sq}^2 = C_{qe}\hat{c}_{se}^2 + C_{qi}\hat{c}_{si}^2$$

$$C_{q\alpha} = \sigma_{q\alpha}\left(i\hat{\omega}_{*\alpha} - f_{T\alpha} i\hat{\omega}_{*n}\right) / \left(\hat{\gamma}_\alpha + i\hat{\omega}_{*q\alpha}\right)$$

$$i\hat{\omega}_{*n\alpha} = \sigma_{q\alpha}\left(\Gamma i\hat{\omega}_{*\alpha} - \hat{c}_{s\alpha}^2 i\hat{\omega}_*\right)$$

$$i\hat{\omega}_{*q\alpha} = \sigma_{q\alpha} f_{T\alpha}\left(\Gamma i\hat{\omega}_{*n} - \hat{c}_{s\alpha}^2 i\hat{\omega}_{*\alpha}\right)$$

$$C_{pe} = \frac{\sigma_{pe}}{\hat{\gamma}_{pe} + i\hat{\omega}_{*qe}}\left(i\hat{\omega}_{*e} - \Gamma f_{Te} i\hat{\omega}_{*n}\right)$$

Note all terms ~ to ω_*.

* Blue terms from advection of p_e by v_e.

We make standard large-guide-field tearing assumptions.

- Small tearing layer width
  - allows expansion of $k_{||} \rightarrow k'_{||} x$
  - Ignore flow shear
- Subsonic growth: $\hat{\gamma}_i^2 << \hat{c}_{sp}^2$
- 'Constant-Psi': $\hat{B}_r = \hat{B}_r(0)$
- Cartesian or 'slab' geometry
- Large guide field
- These assumptions are captured with the following ordering:

$$\hat{\gamma} \sim \hat{\omega}_* \sim \epsilon^{3/2} \quad , \quad \hat{x} \sim \epsilon$$

$$\hat{\lambda}_0 \sim \hat{k}'_{||} \sim \epsilon^{3/4} \quad , \quad \epsilon << 1$$

$$\hat{d}_i \sim \left[\epsilon^{3/2} - 1\right] \quad , \quad \hat{c}_s^2 \sim \beta \sim \left[\epsilon^{3/2} - 1\right]$$

We reduce this set to a system of three eqns., as in Ref. [3].

$$\left(\hat{\gamma}_i - i\hat{\omega}_*\right)\hat{B}_r = i\hat{k}_\parallel \hat{\gamma}\hat{\xi} + \hat{k}_\parallel \hat{d}_i \hat{Q} + S_g^{-1}\hat{B}_r'' \qquad (1)$$

$$\hat{\tau}_Q \hat{Q} = \hat{k}_\parallel \hat{d}_i \hat{B}_r'' + S_g^{-1}\hat{Q}'' - \frac{\hat{k}_\parallel^2}{\hat{\gamma}_{gvi}}\hat{Q} + \left(\hat{\tau}_B - \frac{\hat{k}_\parallel \hat{\omega}_*}{\hat{\gamma}_{gvi}\hat{d}_i\hat{\lambda}_0}\right) i\hat{\lambda}_0 \hat{B}_r + \hat{\tau}_\xi \frac{\hat{\gamma}\hat{\xi}}{\hat{d}_i}$$
$$- \sigma_{gv} i\hat{\lambda}_0 \frac{\hat{c}_{si}^2}{\Gamma}\hat{d}_i \frac{\hat{k}_\parallel}{\hat{\gamma}_{gvi}}\hat{\gamma}\hat{\xi}' - \sigma_{gv}\left(i\hat{\gamma}_i \frac{A-1}{\hat{c}_{sp}^2} - \frac{i\hat{k}_\parallel^2}{\hat{\gamma}_{gvi}}\right)\frac{\hat{c}_{si}^2}{\Gamma}\frac{\hat{d}_i}{2}\hat{\gamma}\hat{\xi}'' \qquad (2)$$

$$\hat{\tau}_Q = \hat{\gamma}_i + i\hat{\omega}_{*n}\frac{\Gamma}{\hat{c}_s^2}\left(C_{pe} + \hat{c}_{sqe}^2\right) - \frac{\hat{\gamma}_i}{\hat{c}_{sp}^2}\left(1 + C_{pe} + \hat{c}_{sq}^2\right)(A-1)$$

$$\hat{\tau}_B = i\hat{\omega}_{*n}\frac{\Gamma}{\hat{c}_s^2}\left(1 - C_{pe} - 4\hat{c}_{sqe}^2\right) + \hat{\gamma}_i\left(C_{pe} + 4\hat{c}_{sq}^2\right)\frac{(A-1)}{\hat{c}_{sp}^2}$$

$$\hat{\tau}_\xi = \hat{\omega}_* + i\hat{\omega}_{*n}\frac{\Gamma}{\hat{c}_s^2}E_n - \frac{(A-1)}{\hat{c}_{sp}^2}\hat{\gamma}_i E_t$$

$$\hat{\gamma}_{gvi}\hat{\gamma}\hat{\xi}'' = i\hat{k}_\parallel \hat{B}_r'' - 2\frac{\hat{\omega}_*}{\hat{d}_i}i\hat{\lambda}_0\hat{B}_r + \sigma_{gv}\frac{i\hat{\omega}_*\hat{\omega}_{*i}}{\hat{d}_i}\hat{\nabla}\cdot\hat{\mathbf{v}} - i\sigma_{gv}\frac{\hat{c}_{si}^2}{\Gamma}\hat{d}_i\left(\left(\hat{\nabla}\cdot\hat{\mathbf{v}}\right)'' + i\hat{k}_\parallel \hat{v}_\parallel''\right) \qquad (3)$$

Gyroviscous terms enter both the parallel induction and vorticity eqns.

$$\hat{\tau}_Q \hat{Q} = \hat{k}_\parallel \hat{d}_i \hat{B}_r'' + S_g^{-1} \hat{Q}'' - \frac{\hat{k}_\parallel^2}{\hat{\gamma}_{gvi}} \hat{Q} + \left(\hat{\tau}_B - \frac{\hat{k}_\parallel \hat{\omega}_*}{\hat{\gamma}_{gvi} \hat{d}_i \hat{\lambda}_0} \right) i \hat{\lambda}_0 \hat{B}_r + \hat{\tau}_\xi \frac{\hat{\gamma} \hat{\xi}}{\hat{d}_i}$$
$$\underline{- \sigma_{gv} i \hat{\lambda}_0 \frac{\hat{c}_{si}^2}{\Gamma} \hat{d}_i \frac{\hat{k}_\parallel}{\hat{\gamma}_{gvi}} \hat{\gamma} \hat{\xi}' - \sigma_{gv} \left(i \hat{\gamma}_i \frac{A-1}{\hat{c}_{sp}^2} - \frac{i \hat{k}_\parallel^2}{\hat{\gamma}_{gvi}} \right) \frac{\hat{c}_{si}^2}{\Gamma} \frac{\hat{d}_i}{2} \hat{\gamma} \hat{\xi}''} \quad (2)$$

$$\hat{\gamma}_{gvi} \hat{\gamma} \hat{\xi}'' = i \hat{k}_\parallel \hat{B}_r'' - 2 \frac{\hat{\omega}_*}{\hat{d}_i} i \hat{\lambda}_0 \hat{B}_r + \sigma_{gv} \frac{i \hat{\omega}_* \hat{\omega}_{*i}}{\hat{d}_i} \underline{\hat{\nabla} \cdot \hat{\mathbf{v}} - i \sigma_{gv} \frac{\hat{c}_{si}^2}{\Gamma} \hat{d}_i \left(\left(\hat{\nabla} \cdot \hat{\mathbf{v}} \right)'' + i \hat{k}_\parallel \hat{v}_\parallel'' \right)} \quad (3)$$

- The modified gyroviscous growth rate is defined as

$$\hat{\gamma}_{gvi} = \hat{\gamma}_i - \sigma_{pe} \left(i \hat{\omega}_{*i} + i \hat{\omega}_* \frac{\hat{c}_{si}^2}{\Gamma} \right)$$

- It appears in the parallel vorticity and parallel momentum eqns.
- The last term appears a result of the gradient of the equilibrium magnetic field in the coefficient of the gyroviscous tensor.

Treating the gyroviscous contributions requires add't eqns.

$$\hat{\gamma}_{gvi}\hat{\gamma}\hat{\xi}'' = i\hat{k}_{\parallel}\hat{B}_r'' - 2\frac{\hat{\omega}_*}{\hat{d}_i}i\hat{\lambda}_0\hat{B}_r + \sigma_{gv}\frac{i\hat{\omega}_*\hat{\omega}_{*i}}{\hat{d}_i}\hat{\nabla}\cdot\hat{\mathbf{v}} - i\sigma_{gv}\frac{\hat{c}_{si}^2}{\Gamma}\hat{d}_i\left(\left(\hat{\nabla}\cdot\hat{\mathbf{v}}\right)'' + i\hat{k}_{\parallel}\hat{v}_{\parallel}''\right) \tag{3}$$

- Still a focus of current work.
- Need to use equations for the divergence and parallel flows.
- This increases the number of equations in the system.
- We will proceed without gyroviscosity.

We eliminate B$_r$ and normalize the equations as in Ref. [3].

$$\bar{D}_R \frac{\partial^2 \bar{\xi}}{\partial \bar{x}^2} = \bar{x}^2 \left(\bar{\xi} + \bar{Q} \right) - \bar{x} \tag{4}$$

$$\frac{\partial^2 \bar{Q}}{\partial \bar{x}^2} = \left(\bar{D}_R^{-1} \bar{x}^2 + \bar{\tau}_Q \right) \bar{Q} + \bar{D}_R \bar{\sigma}^2 \frac{\partial^2 \bar{\xi}}{\partial \bar{x}^2} + \bar{\tau}_\xi \bar{\xi} - \bar{\tau}_B + \bar{\Lambda} \bar{x} \tag{5}$$

$$\bar{x} = \frac{\hat{x}}{\hat{d}_0} , \quad \bar{\xi} = \frac{i \hat{k}_{||}' \hat{d}_0 \hat{\gamma} \hat{\xi}}{\hat{B}_r(0) \left(\hat{\gamma}_i - i \hat{\omega}_* \right)} , \quad \bar{Q} = \frac{\hat{k}_{||}' \hat{d}_0 \hat{d}_i \hat{Q}}{\hat{B}_r(0) \left(\hat{\gamma}_i - i \hat{\omega}_* \right)} , \quad \hat{d}_0 = \left(\frac{\hat{\gamma}}{S_g} \right)^{1/4} \left(\hat{k}_{||}' \right)^{-1/2} ,$$

$$\bar{\sigma}^2 = \frac{\hat{\gamma}^2 \hat{d}_i^2}{\hat{k}_{||}'^{\,2} \hat{d}_0^4} = \hat{\gamma} \hat{d}_i^2 S , \quad \bar{D}_R = \frac{\hat{\gamma}_{gvi}}{\hat{\gamma}} , \quad \bar{\Lambda} = \frac{i \hat{\omega}_*}{\left(\hat{\gamma}_i - i \hat{\omega}_* \right)} \frac{\hat{\gamma}}{\hat{\gamma}_{gvi}} ,$$

$$\bar{\tau}_Q = \frac{\hat{\gamma}}{\left(\hat{k}_{||}' \hat{d}_0 \right)^2} \hat{\tau}_Q , \quad \bar{\tau}_\xi = \frac{i \hat{\gamma}}{\left(\hat{k}_{||}' \hat{d}_0 \right)^2} \hat{\tau}_\xi , \quad \bar{\tau}_B = \frac{i \hat{\gamma} \hat{d}_i}{\left(\hat{\gamma}_i - i \hat{\omega}_* \right) \hat{d}_0} \left(\hat{\tau}_B + 2 i \hat{\omega}_* \right) .$$

The dispersion relation is found by integrating the radial induction eqn. after solving for ξ and Q.

$$\frac{\left(\hat{\gamma}_i - i\hat{\omega}_*\right)\hat{\gamma}^{1/4} S_g^{3/4}}{\hat{k_\parallel'}^{1/2}\hat{\Delta}'}\int_{-\infty}^{\infty} d\bar{x}\left(1 - \bar{x}\bar{\xi} - \bar{x}\bar{Q}\right) = 1$$

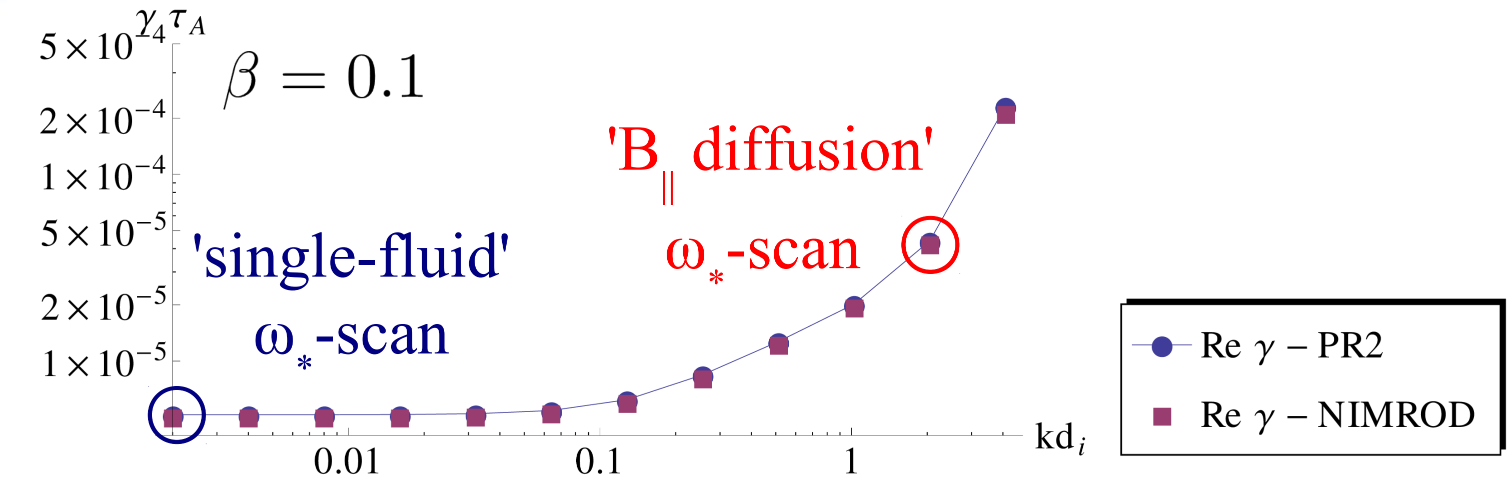
- Some results without drifts (see [3]):

$$\hat{\gamma}_{MHD} = S_g^{-3/5}\left(\frac{\hat{\Delta}'}{\sqrt{2}\Gamma\left(3/4\right)^2}\right)^{4/5}\hat{k_\parallel'}^{2/5} \qquad \bar{\tau} >> \bar{\sigma}^2,\ or\ \bar{\sigma} << 1$$

$$\hat{\gamma}_{SC} = S_g^{-1/3}\left(\hat{\rho}_s\frac{\hat{\Delta}'\hat{k_\parallel'}}{\pi}\right)^{2/3} \qquad \bar{\sigma} >> 1,\ and\ \bar{\sigma} << \bar{\tau} << \bar{\sigma}^2$$

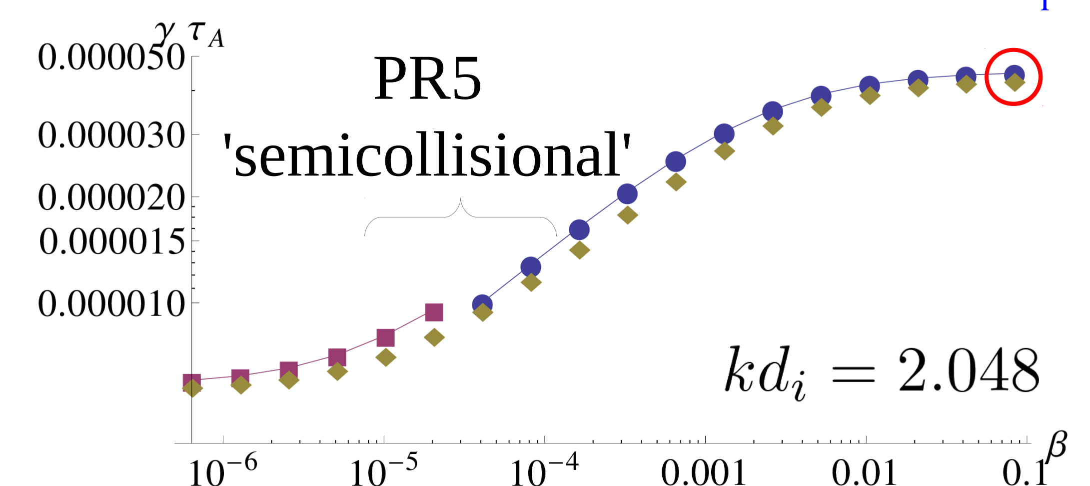
$$\hat{\gamma}_{B\parallel D} = S_g^{-1/2}\left(\hat{d}_i\hat{k_\parallel'}\right)^{1/2}\frac{\hat{\Delta}'}{\sqrt{2}\Gamma\left(3/4\right)^2} \qquad \bar{\sigma} >> 1,\ and\ \bar{\tau} << \bar{\sigma}$$

Two scans without drifts orient us in parameter space.

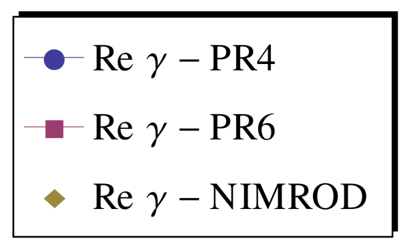


To PR3, 'B$_{\|}$ diffusion'; large β and d$_i$ →

← To PR1, 'collisional'; small β or d$_i$



PR5/6 drift scans difficult as $\omega_* \sim k p_0' \beta d_i$



Common parameters:

$kL_B = 0.15$

$S = 3.5 \times 10^7$

$m_e/m_i = 2.72 \times 10^{-4}$

$k^{-1}\Delta' = 1.46$

$T_{i0} = T_{e0}$

$\mathbf{v}_0 = 0$

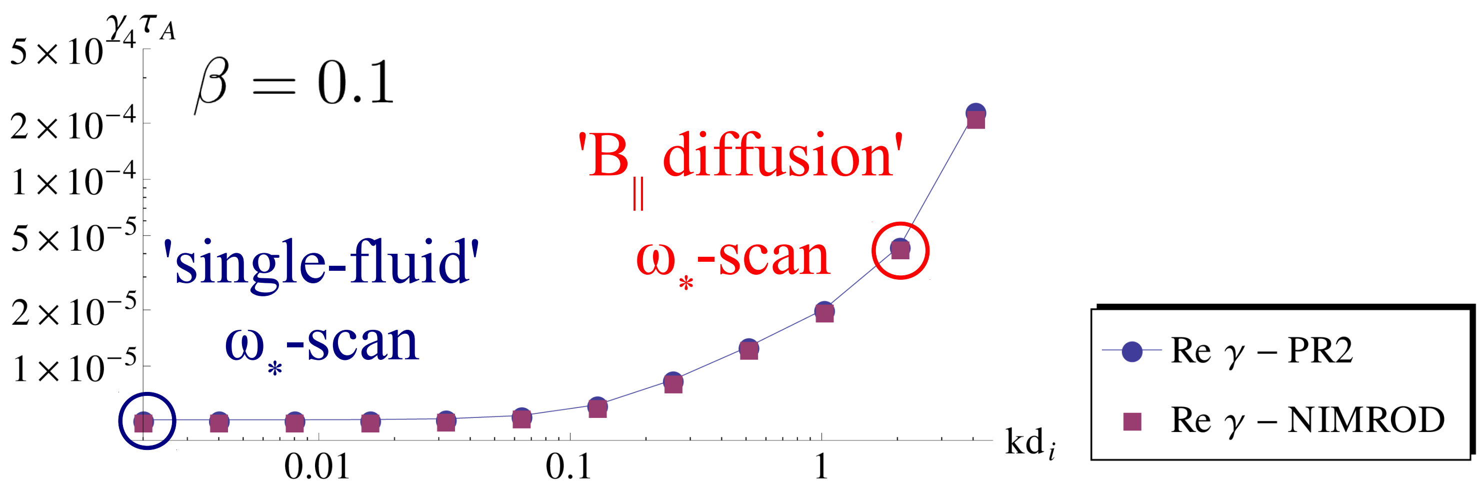
Drift tearing modes are modeled with a hyperbolic tangent pressure profile.

- The equilibria are the sheared slab equilibria of Ref. [3] with the addition of a pressure (density) gradient:

$$n_0(x) = n_0\left(1 + \frac{n_1}{n_0}\tanh\left[\frac{x}{L_B}\right]\right)$$

- We choose a flat temperature profile to avoid ITG-like modes.
- Cases are run with a resistive-MHD model to verify the outer solution is not significantly modified by the pressure profile.

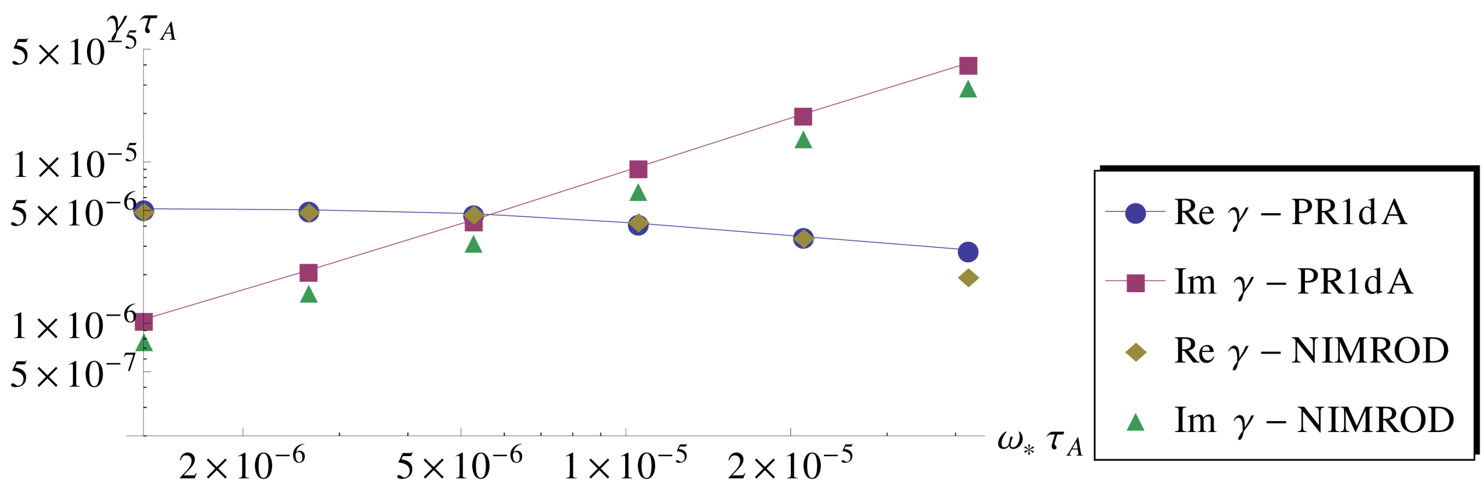
'Single-fluid' ω_* scan



To PR3, 'B$_\|$ diffusion'; large d_i →

Our scan at small d_i has contributions from new terms.

PR1dB Λ significant   PR1dC Λ, τ_Q and τ_ξ significant



- PR1dA (no Λ, τ_Q and τ_ξ contributions) is difficult to model as $\Lambda \sim \omega_* d_i^{-1} \ll 1$ is not easily satisfied at small d_i.
- However, Re γ is approximated by the dispersion relation of PR1dA.

$$\frac{\partial^2 \bar{Q}}{\partial \bar{x}^2} = \left(\bar{D}_R^{-1}\bar{x}^2 + \bar{\tau}_Q\right)\bar{Q} + \bar{D}_R\bar{\sigma}^2\frac{\partial^2\bar{\xi}}{\partial\bar{x}^2} + \bar{\tau}_\xi\bar{\xi} - \bar{\tau}_B + \bar{\Lambda}\bar{x}$$

The single-fluid drift regime at moderate β is subdivided into drift regimes. (1)

PR1dA

$$\bar{\Lambda} \sim \bar{\tau}_Q \sim \bar{\tau}_\xi \sim \bar{\tau}_B \sim \bar{\sigma}^2 << 1$$

$$\bar{D}_R \bar{\xi}'' = \bar{x}^2 \bar{\xi} - \bar{x}$$

$$\bar{\xi} = \frac{\bar{x}}{2\bar{D}_R} \int_0^{\sqrt{\bar{D}_R}} d\mu \left(1 - \frac{\mu^2}{\bar{D}_R}\right)^{-1/4} \exp\left[-\frac{\mu \bar{x}^2}{2\bar{D}_R}\right]$$

$$\hat{\gamma}_{MHD}^{5/4} = \left(\hat{\gamma}_i - i\hat{\omega}_*\right) \hat{\gamma}_i^{1/4}$$

All regimes: $\bar{x} \sim \bar{\xi} \sim 1$

The single-fluid drift regime at moderate β is subdivided into drift regimes. (2)

PR1dB

$$\bar{\tau}_Q \sim \bar{\tau}_\xi \sim \bar{\tau}_B \sim \bar{\sigma}^2 << 1 \sim \bar{\Lambda}$$

$$\bar{D}_R \bar{Q}'' = \bar{x}^2 \bar{Q} + \bar{D}_R \bar{\Lambda} \bar{x}$$

$$\bar{D}_R \bar{\xi}'' = \bar{x}^2 \left(\bar{\xi} + \bar{Q}\right) - \bar{x}$$

- Not yet solved.
- Q has a soln. as a parabolic cylinder ftn.
- The ξ eqn. is then an inhomogenous parabolic cylinder eqn., where the inhomogenous term is a parabolic cylinder ftn. minus x.

All regimes: $\bar{x} \sim \bar{\xi} \sim 1$

The single-fluid drift regime at moderate β is subdivided into drift regimes. (3)

PR1dC

$$\bar{\tau}_B \sim \bar{\sigma}^2 << 1 \sim \bar{\Lambda} \sim \bar{\tau}_Q \sim \bar{\tau}_\xi$$

$$\bar{Q}'' = \left(\bar{D}_R^{-1}\bar{x}^2 + \bar{\tau}\right)\bar{Q} + \bar{\tau}_\xi\bar{\xi} + \bar{\Lambda}\bar{x}$$

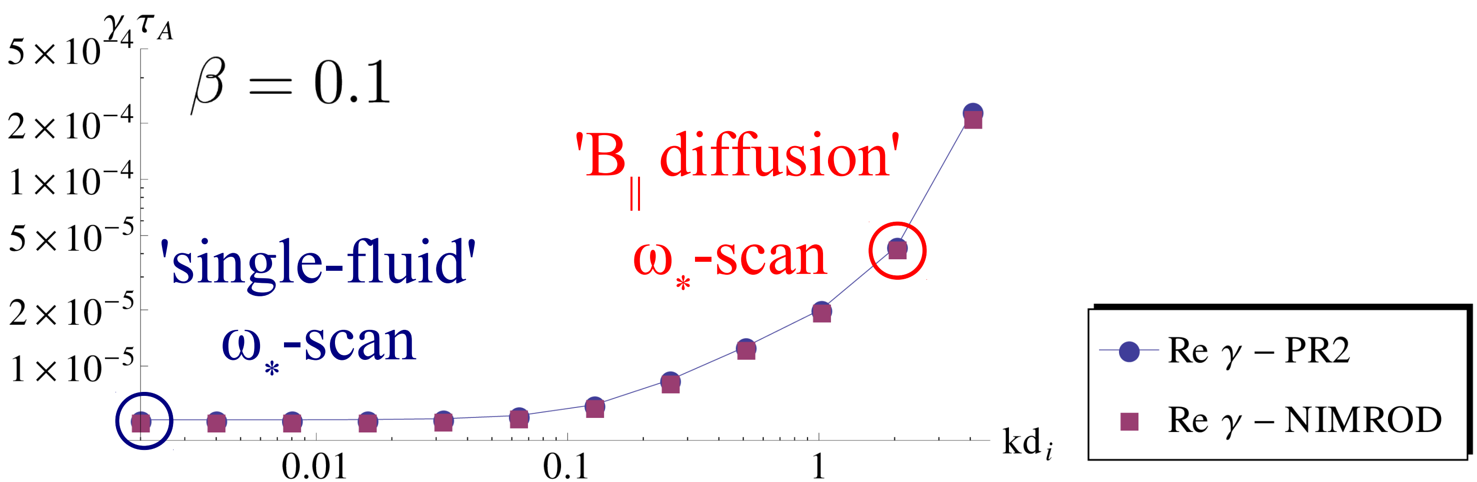
$$\bar{D}\bar{\xi}'' = \bar{x}^2\left(\bar{\xi} + \bar{Q}\right) - \bar{x}$$

Not solved.

All regimes: $\bar{x} \sim \bar{\xi} \sim 1$

'Large d_i' ω_* scan

Phyiscally relevant regime



To PR3, 'B$_{||}$ diffusion'; large d_i →

We can solve for the dispersion relation in the two-fluid drift regime, PR4d.

PR4d

$$\bar{x}^{-1} \sim \bar{\sigma}^{1/2} \sim \bar{Q}\,, \quad \bar{\xi} \sim \bar{\sigma}^{-3/2}\,, \quad \bar{\sigma} \sim \bar{\tau} >> 1$$

$$\bar{\tau}_\xi << \bar{\sigma}^3\,, \quad \bar{\tau}_B << \bar{\sigma}^{3/2}\,, \quad \bar{\Lambda} << \bar{\sigma}^2$$

$$\bar{Q}'' = \bar{\tau}\bar{Q} + \bar{D}_R \bar{\sigma}^2 \bar{\xi}''$$

$$\bar{D}_R \bar{\sigma}^2 \bar{\xi}'' = \bar{\sigma}^2 \bar{x}^2 \bar{Q} - \bar{\sigma}^2 \bar{x}$$

$$\left(\hat{\gamma}_i - i\hat{\omega}_*\right) \frac{\Gamma\left[\left(3 + \bar{\tau}_Q/\bar{\sigma}\right)/4\right]}{\Gamma\left[\left(1 + \bar{\tau}_Q/\bar{\sigma}\right)/4\right]} = S_g^{-1/2} \sqrt{\hat{k}_\parallel' \hat{d}_i}\, \frac{\hat{\Delta}'}{2\pi}$$

- Solution given in Ref. [1].
- Next we examine two limits of τ_Q with respect to σ (PR3d or '$B_\parallel$ diffusion' and PR5d or 'semi-collisional').

As a surprising result, in the $B_{||}$ diffusion regime there is rotation but no stabilization.

PR3d

$$\bar{\tau}_Q << \bar{\sigma}$$

$$(\hat{\gamma}_i - i\hat{\omega}_*) = \hat{\gamma}_{B||D}$$

- The real part of the growth rate is unchanged.
- There is rotation at the electron diamagnetic frequency.
- Can be found as both a limit of the PR4d dispersion relation, and by taking a limit of the tearing layer equations and solving.

The large-τ_Q (small-β) limit resembles the drift tearing of Ref. [2].

PR5d

$$\bar{\tau}_Q >> \bar{\sigma}$$

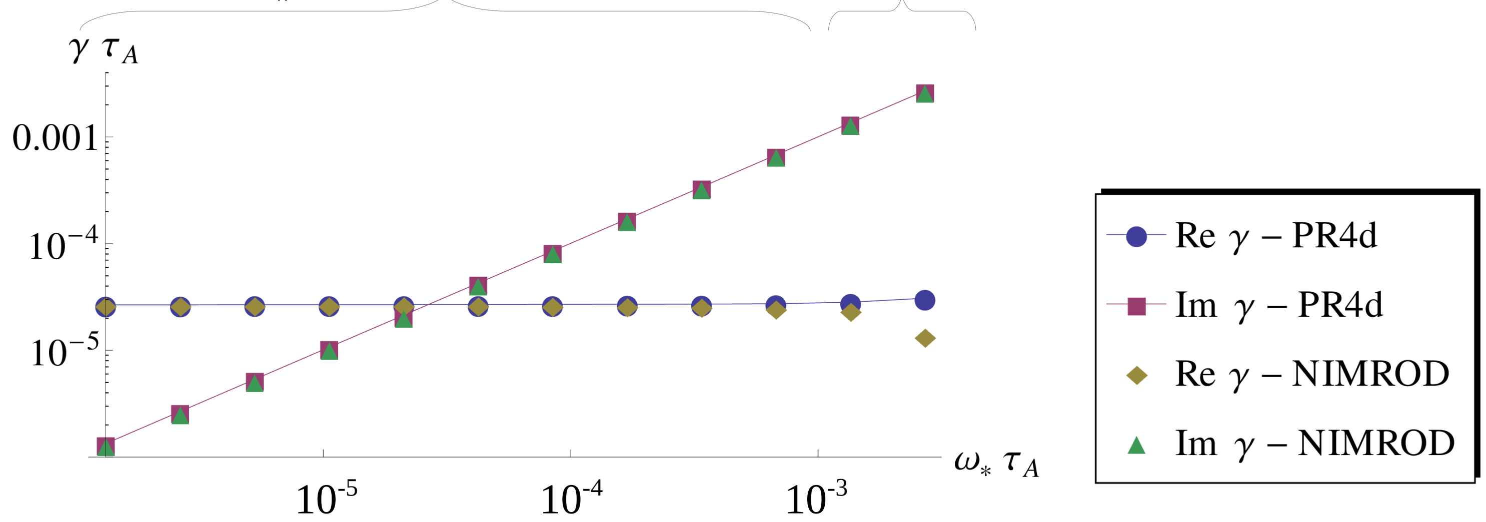
$$\frac{(\hat{\gamma}_i - i\hat{\omega}_*)\,\hat{\gamma}^{1/4} S_g^{3/4}}{\hat{k}_\parallel'^{\,1/2}\,\hat{\Delta}'}\pi\frac{\sqrt{\bar{\tau}_Q}}{\bar{\sigma}} = 1$$

$$(\hat{\gamma}_i - i\hat{\omega}_*)\left[\hat{\gamma}_i + i\hat{\omega}_{*n}\frac{\Gamma}{\hat{c}_s^2}\left(C_{pe}+\hat{c}_{sqe}^2\right) - \frac{\hat{\gamma}_i}{\hat{c}_{sp}^2}\left(1+C_{pe}+\hat{c}_{sq}^2\right)(A-1)\right]^{1/2} = S^{-1/2}\frac{\hat{k}_\parallel'\hat{\Delta}'}{\pi}\hat{d}_i = \hat{c}_s^{-1}\hat{\gamma}_{SC}^{3/2}$$

- Can be found as both a limit of the PR4d dispersion relation, and by taking a limit of the tearing layer equations and solving.

Theory and computation in PR3d are in good agreement until large ω_* at small m_e.

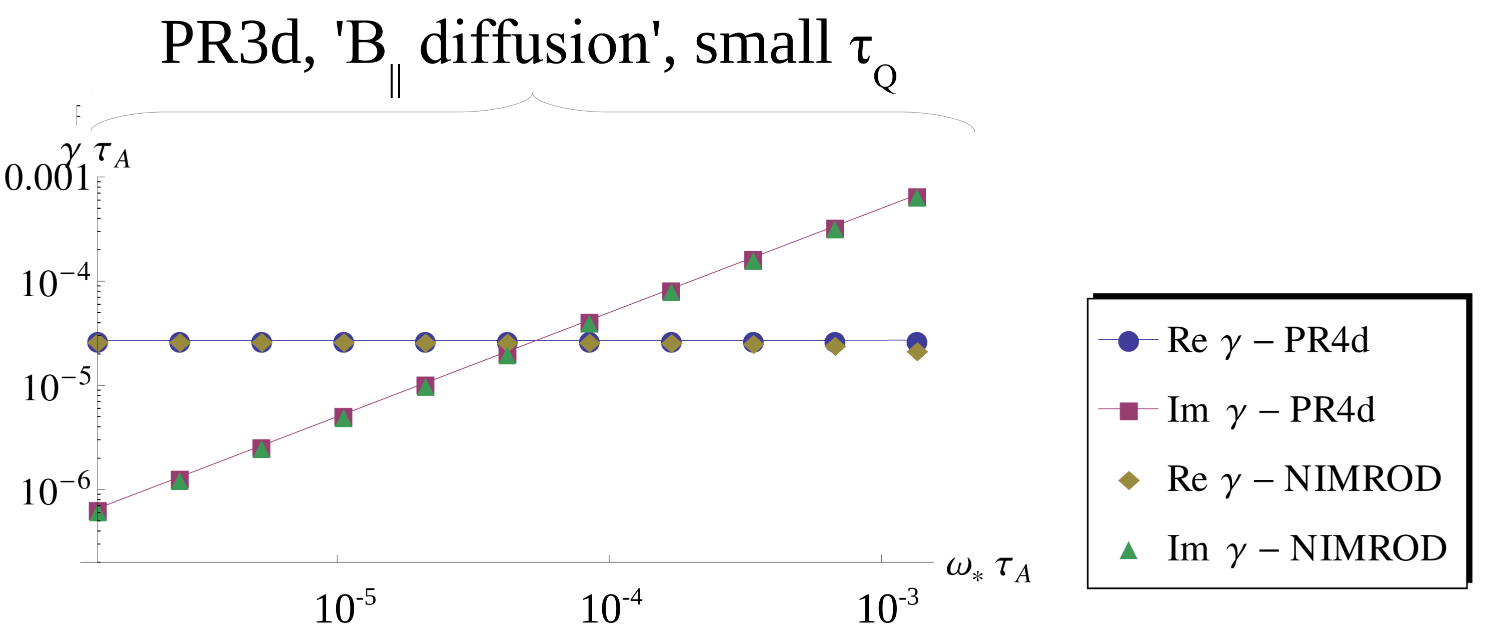
PR3d, 'B$_{\|}$ diffusion', small τ_Q PR4d $\tau_Q \sim \sigma$



$$\mathbf{v}_0 = 0$$
$$m_e/m_i = 2.72 \times 10^{-6}$$

No $\mathbf{v}_e \cdot \nabla \mathbf{v}_e$
in electron inertia.

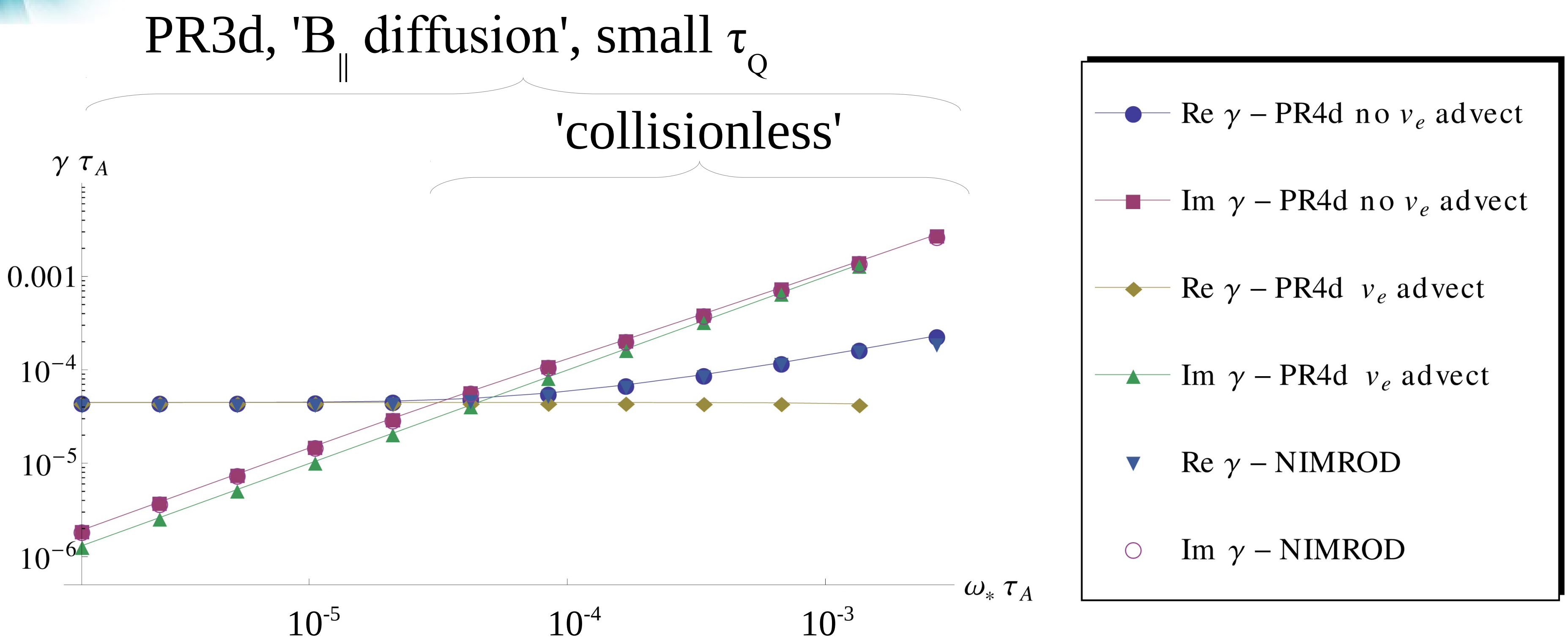
Good agreement is seen with an equilibrium diamagnetic flow.



$$\mathbf{v}_0 = \mathbf{v}_{*i}$$
$$m_e/m_i = 2.72 \times 10^{-6}$$

No $\mathbf{v}_e \cdot \nabla \mathbf{v}_e$ in electron inertia.

Cases with moderate m_e show the importance of electron advection.



$$\mathbf{v}_0 = 0$$
$$m_e/m_i = 2.72 \times 10^{-4}$$

No $\mathbf{v}_e \cdot \nabla \mathbf{v}_e$ in electron inertia in the computations.

Examining the PR3d growth rate with and without advection in electron inertia provides insight into its importance.

In the collisionless regime the generalized Lundquist number becomes

$$S_g^{-1} \simeq \hat{d}_e^2 \hat{\gamma}_e = \hat{d}_e^2 \left(\hat{\gamma}_i - i\hat{\omega}_*\right) \text{ with electron inertia,}$$

$$S_g^{-1} \simeq \hat{d}_e^2 \hat{\gamma} \text{ without electron inertia.}$$

Thus the growth rate $\left(\hat{\gamma}_i - i\hat{\omega}_*\right) = S_g^{-1/2} \left(\hat{d}_i \hat{k}_\parallel'\right)^{1/2} \dfrac{\hat{\Delta}'}{\sqrt{2}\Gamma\left(3/4\right)^2}$

may be approximated as (in the collisionless regime)

$$\left(\hat{\gamma}_i - i\hat{\omega}_*\right) = \hat{d}_e^2 \hat{d}_i \hat{k}_\parallel' \left(\frac{\hat{\Delta}'}{\sqrt{2}\Gamma\left(3/4\right)^2}\right)^2 \quad \text{with electron inertia,}$$

$$\left(\hat{\gamma}_i - i\hat{\omega}_*\right)^2 \hat{\gamma}^{-1} = \hat{d}_e^2 \hat{d}_i \hat{k}_\parallel' \left(\frac{\hat{\Delta}'}{\sqrt{2}\Gamma\left(3/4\right)^2}\right)^2 \text{without electron inertia.}$$

The PR5d regime is similar to the semicollisional regime of Ref. [2]

We expect better agreement when gyroviscosity is included. Numerical results in PR5 are difficult as $\omega_* \sim k p_0' \beta d_i$, and β is typically small.

Assuming small β and pressure/density advection by v_e, or

$$\beta << 1 \, , \quad \sigma_{pe} = 1 \, , \quad \hat{\gamma}_{pe} = \hat{\gamma}_e \ .$$

We find

$$\hat{\gamma}_i^{1/3} \left(\hat{\gamma} - \Gamma f_{Te} i \hat{\omega}_{*n}\right)^{1/3} \left(\hat{\gamma}_e - i\hat{\omega}_{*i}\right)^{1/3} \left(\frac{\hat{c}_s}{\hat{c}_{sp}}\right)^{2/3} = \hat{\gamma}_{SC} \ .$$

In the limit of small T_i this becomes

$$\hat{\gamma}_e^{2/3} \left(\hat{\gamma} - \Gamma f_{Te} i \hat{\omega}_{*n}\right)^{1/3} = \hat{\gamma}_{SC} \ .$$

The single-fluid limit of very small β with drifts is similar to the result of Ref. [4].

Assuming the ordering:

$$\bar{\Lambda} \sim \bar{\tau}_B \sim \bar{\sigma}^2 << \bar{\tau}_Q \sim \bar{\tau}_\xi \ , \quad 1 << \bar{\tau}_Q \sim \bar{\tau}_\xi$$

The layer equations become

$$\bar{D}\bar{\xi}'' = \bar{x}^2 \left(1 - \frac{\bar{\tau}_\xi}{\bar{\tau}_Q}\right) \bar{\xi} - \bar{x} \ , \quad \bar{\tau}_Q \bar{Q} = -\bar{\tau}_\xi \bar{\xi}$$

The dispersion relation is

$$\hat{\gamma}_{MHD}^{5/4} = \frac{(\hat{\gamma}_i - i\hat{\omega}_*)}{\left(1 - \frac{\bar{\tau}_\xi}{\bar{\tau}_Q}\right)^{1/4}} \hat{\gamma}_i^{1/4}$$

With pressure/density advection by v_e and in the limit where the electron temperature gradient dominates this becomes similar to Ref. [5]:

$$\hat{\gamma}_{MHD}^{5/4} = \hat{\gamma}_e^{3/4} \hat{\gamma}_i^{1/4} \hat{\gamma}^{1/4}$$

[4] Coppi, Phys. Fluids 7, 1501 (1964).

Verification of the physics of separate pressures and heat flux is on going.

- Initial attempts have been unsuccessful and exhibit numerical modes.
- A concurrent advance of pressure and magnetic field is a potential solution.
  - Electron-pressure-advection and heat-flux terms with current contributions would be treated fully implicitly.

Summary

- Progress with two-fluid drift tearing to date:
  - Tearing-ordered equations with diamagnetic drifts, gyroviscosity and heat flows.
  - Solutions with diamagnetic drifts and heat flows.
  - Verification of diamagnetic drift effects.
- The moderate-β single-fluid regime (small d_i) is weakly stabilized by diamagnetic drifts when compared to the dispersion relation of Coppi (1964) [5].
- Drift effects in the $B_{||}$-diffusion regime (PR3d) do not produce stabilization as in the semicollisional (PR5d) and the small-β single-fluid regimes.
- As could be expected, advection from electron inertia is important in the collisionless regime.
- Future work will include verification with diamagnetic heat flows and finding solutions with ion gyroviscous terms.