Continuum kinetics in the NIMROD Code

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Outline

- Kinetic equations which have been implemented into NIMROD:
  - Fokker-Planck problems (2D FE/1D Fourier)
    - What are the properties of the collision operator that is important to get right?
  - Hazeltine form
    - axisymmetric plasmas for neoclassical benchmarks
      - Can we benchmark with other DKE codes with NIMROD?
    - Hot particle form for energetic particle modes
      - Can we do energetic particle modes implicitly?
  - CEL form of Ramos
    - Can we do long-time scale simulations with accurate closures?

Hazeltine's form of DKE has been implemented into NIMROD

- Summary of form (pitch angle, speed):

$$\partial_t f + (\mathbf{v}_{||} + v_D)\cdot\nabla f - (\mathbf{v}_{||} + v_D)\cdot\left[\frac{1-\xi^2}{2\xi}\nabla \ln B\partial_\xi + s\nabla \ln v_0\partial_s\right] f + \left(\frac{e}{2e_0 s^2}(\mathbf{v}_{||} + \mathbf{v}_D)\cdot\mathbf{E}\right)\left(s\partial_s f + 2g(\xi)\partial_\xi f\right) = C$$

with drift defined as:

$$v_D = \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{e_0 s^2}{eB}\left[\mathbf{b}\times\left(\left(1-\xi^2\right)\nabla \ln B + 2\xi^2\kappa - \frac{v_0 s\xi}{e_0 B}\nabla\times\mathbf{E}\right) + (1-\xi^2)\frac{\mu_0\mathbf{J}_{||}}{B}\right]$$

- Criticisms of form exist (*) but useful for benchmarks because it is used in NEO, similar to EP form, ...

* A. N. Simakov and P.J. Catto. Phys. Plasmas 12 (2005), 012105.

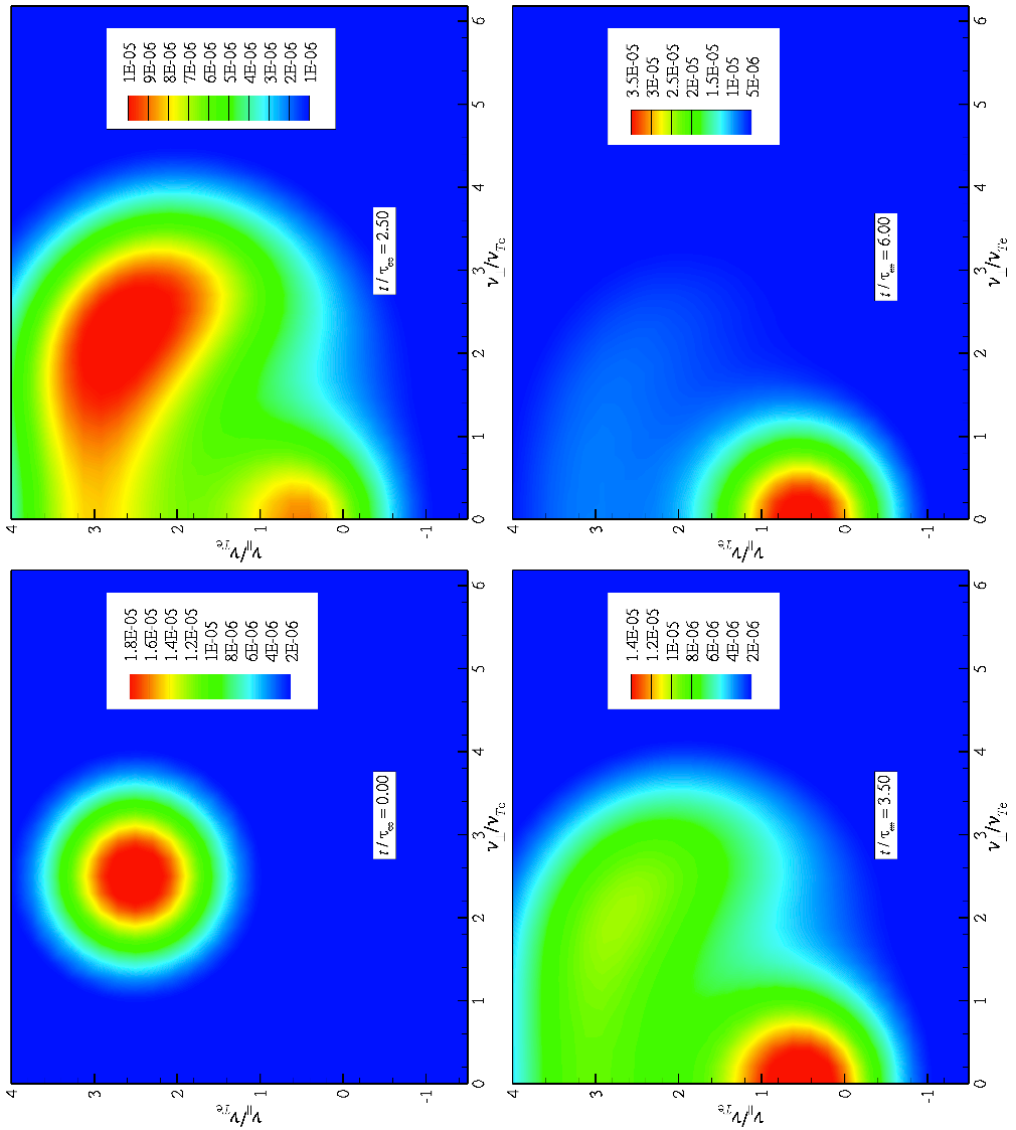
FP problems used to study advanced schemes for collision operator

- Use NIMROD discretization scheme to consider velocity space dynamics of uniform plasma
  - Map vpar, vperp, gamma -> R,Z,phi
- Long history of problems to study for velocity space dynamics to study basic numerical properties

FP problems used to study advanced schemes for collision operator

- Test problems studied
  - Resistivity of unmagnetized plasma
    - Balance of electric field and collision operator (Spitzer)
  - Thermalization of test particles
    - B.A. Trubnikov, Rev. Plasma Phys. 1 (1965).
    - Z. Xiong, R. Cohen, et.al., Journal of Comp. Phys. 227 (2008).
  - Tenuous beam on background electron and ions
  - …
  - See: A. Spencer, J.Y. Ji, E. Held
    coming soon to a JCP near you

Thermalization of tenuous electron beam demonstrates basic capabilities



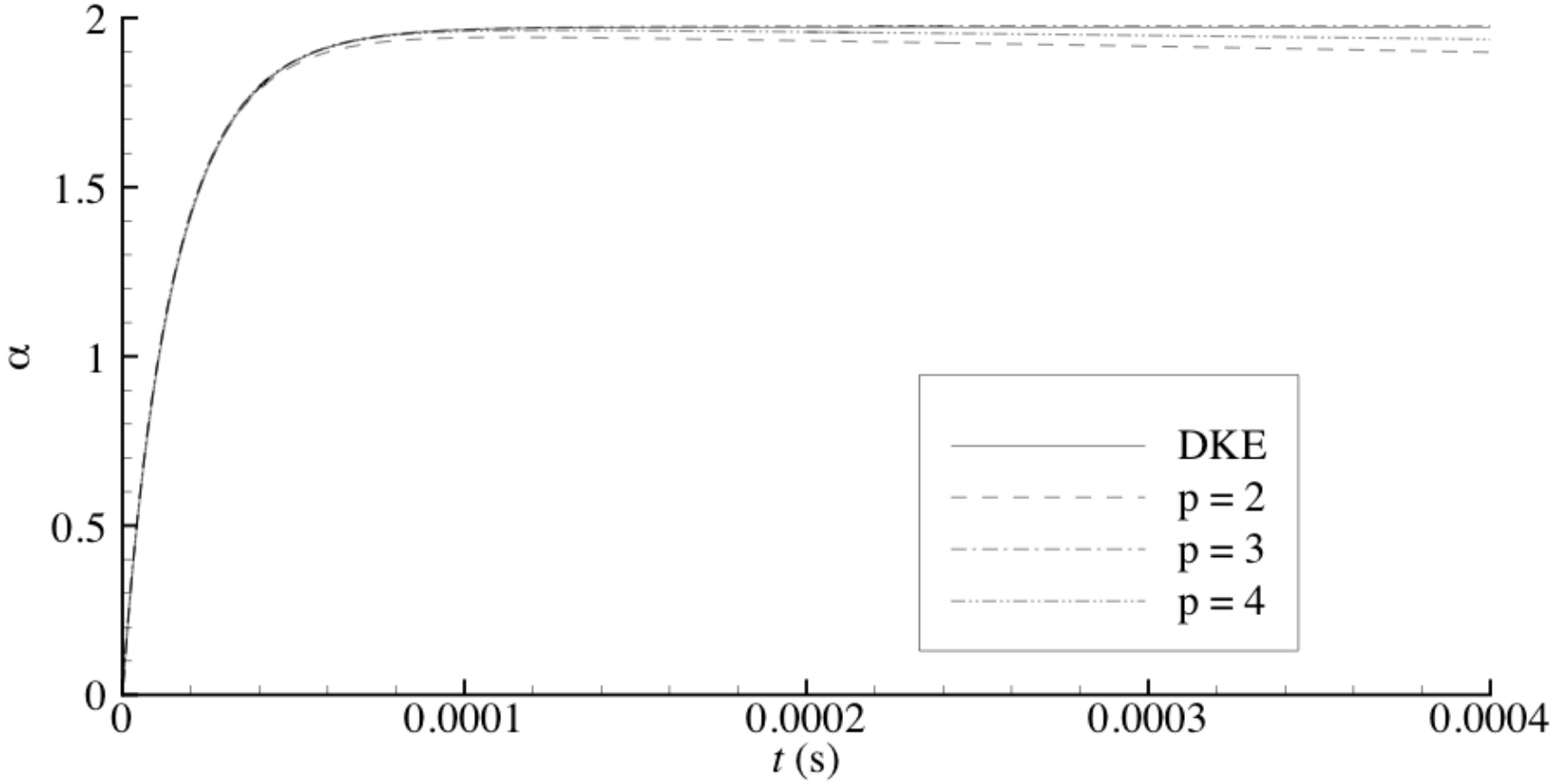
- Big changes in velocity distribution function as it evolves

Thermalization of test electrons demonstrates efficacy of method

- Xiong (COGENT code):
  - Finite volume with E, μ coordinates
- NIMROD
  - 4^{th} order polynomials in each cell
- Comparable results of 5x5 FE grid with 60x60 FV grid
- 441 degrees of freedom versus 54,000 dof
- HOE FE considerably more efficient

Momentum transfer over long time scales is an issue

Spitzer problem requiring field terms for both ions and electrons



Packing was required to achieve results this good.

Long time scales require lots of resolution

Summary of finite-element/FFT investigations

- High-order finite elements offers an attractive method of discretizing velocity space
  - packing especially for electron-ion collisions
  - spectral convergence
- Conservation properties (density, momentum) requires careful consideration of convergence, especially for long time scales

Hazeltine DKE in NIMROD

Implementation for closures is using polynomial basis functions

- Collision operator from Ji and Held 2006:

$$
\begin{aligned}
C^{ab} &= C(f_{1a}, f_{0b}) + C(f_{0a}, f_{1b}) \\
&= \sum_{lk} \frac{f_a^{(0)}}{\sigma_k^l} P_l(v_{||}/v) \left(\nu_{ab}^{lk,0} M_{||a}^{lk}(\mathbf{r}, t) + \nu_{ab}^{0,lk} M_{||b}^{lk}(\mathbf{r}, t) \right)
\end{aligned}
$$

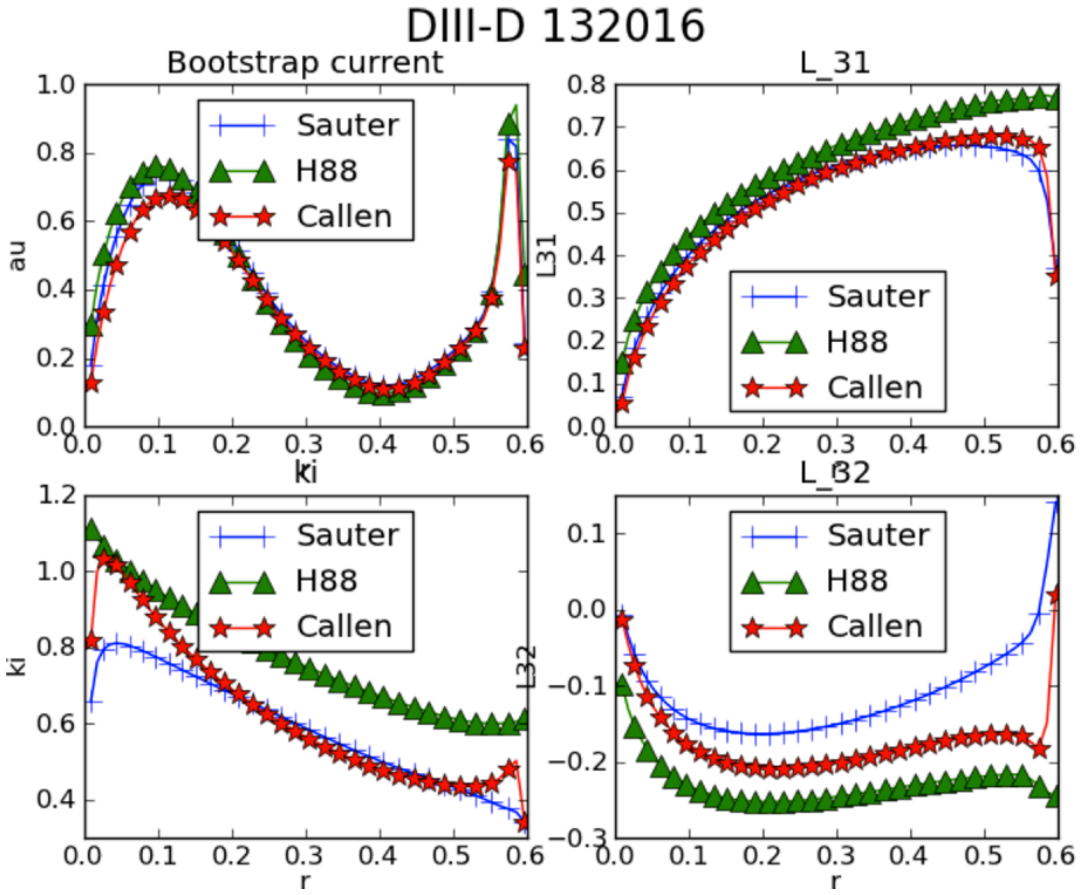
with

$$
n_a M_{||a}^{lk} = \frac{l!}{(2l-1)!!} v_{Ta}^{l+2k} \int d\mathbf{v} L_k^{l+1/2}(s^2) s^l P_l(v_{||}/v) F_a
$$

- Pitch angle uses finite-elements motivated from earlier work:
  - Modal built from Legendre functions
  - Nodal (Lagrange, Gauss-Legendre-Lobatto)
- Speed variable decouples from pitch angle
  - Good for parallelization
  - Use evaluation with weighted quadrature to handle

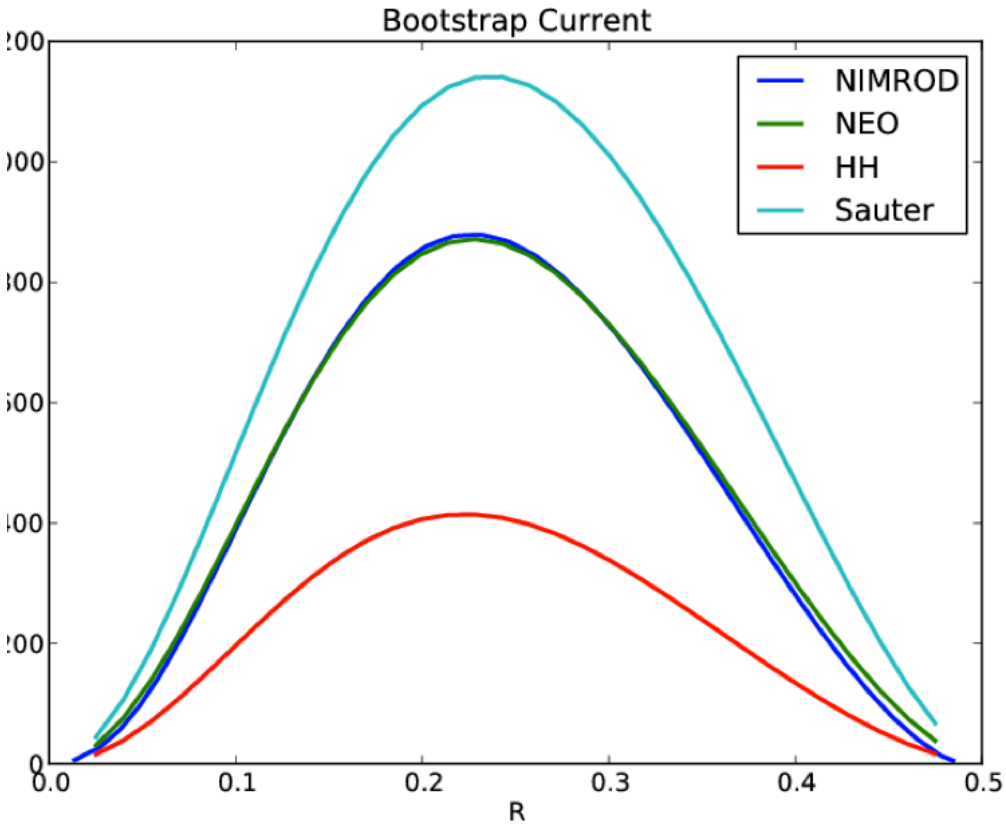
Library of cases have been developed to verify cases

- Motivated by need to benchmark implementation in NIMROD, NEO cases were developed
- Interest in analytic formulas (Callen, C.S. Chang) have broadened benchmark effort
- Currently including NEO, NIMROD, Wang, Ernst and analytic formulas



Benchmark demonstrates NIMROD has continuum capabilities

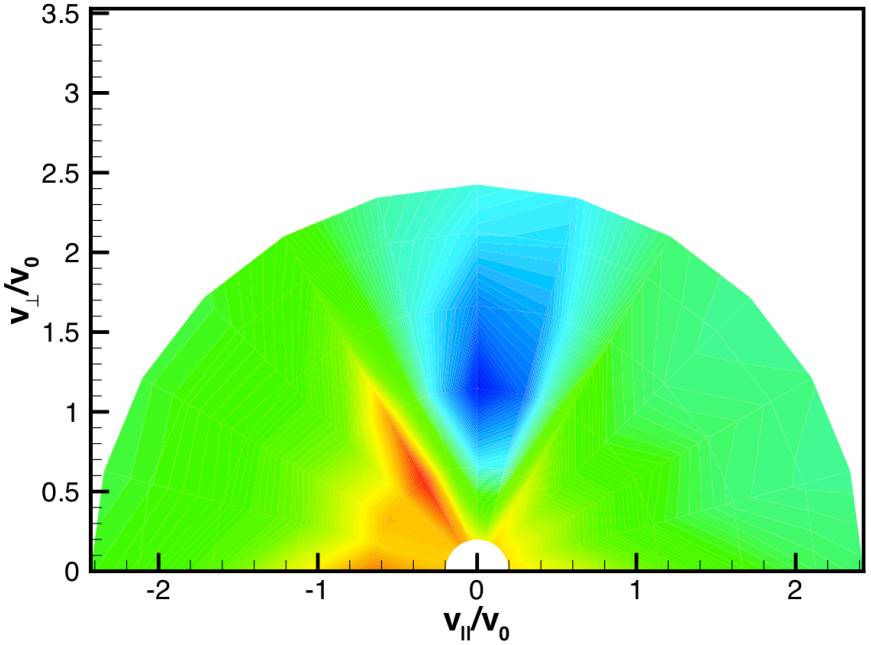
- Quantitative benchmark
- Moderate collisionality
- Required resolving electron-ion velocity space simultaneously
- Done in full NIMROD geometry (3D even though BM is 2D)
- Since APS:
  - Implemented ability to do annular regions
  - Near collisionless regimes working
  - Merged with trunk



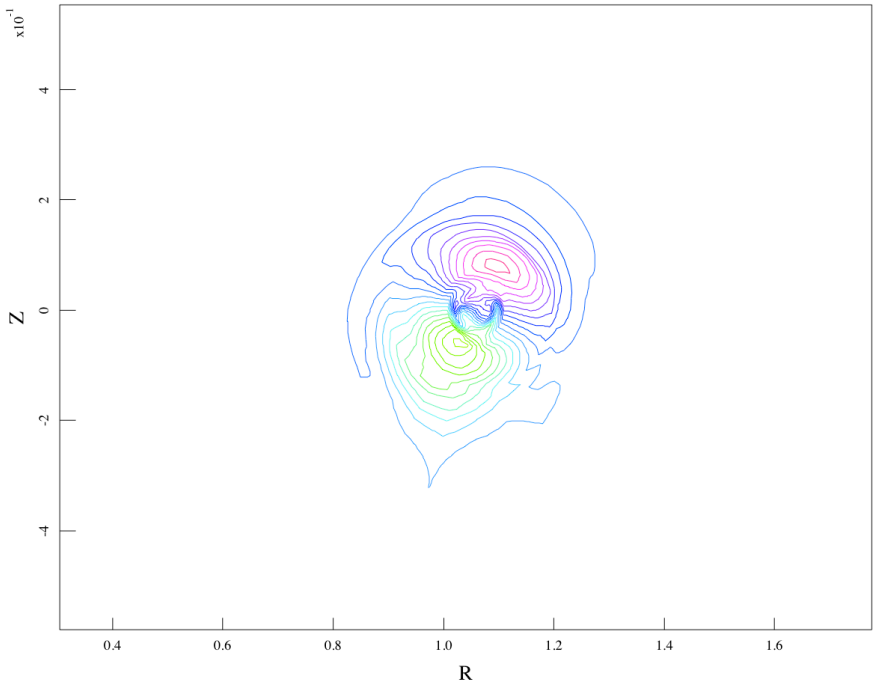
Continuum velocity space methods

- Using PIC methods for the energetic particle "species" is well-developed
- Often uses drift-kinetic equation
  - Kim PHYSICS OF PLASMAS **15, 072507 2008**
  - Fu PHYSICS OF PLASMAS **13, 052517 2006**
- Can continuum formalism be used instead of PIC?
  - No noise
  - Enable implicit methods which is important as maximum velocity considered increases
  - Giant sawtooth problem: 800 keV
  - ITER with alphas: 4 MeV

Initial results using continuum methods is promising



Major difficulty for quantitative benchmark seems to be in implementing the right DKE



Moving to long time scale simulations

- Hazeltine equation is not the best closure scheme for extended MHD simulations
  - Not of sufficient accuracy (Simakov and Catto, Ramos)
  - Typically not formulated for evolving equilibrium/ Maxwellian
- CEL form is better for long time scale simulations
  - No contributions to low order moments, …
  - Ramos form has all of the orders necessary to get bootstrap current, conservation, …
- Ramos form is implemented into NIMROD
  - Benchmarking has not started yet

Time advance of distribution functions introduces challenges to split scheme

- Normal NIMROD advance:
  - V^n => $\mathbf{B}^{n+1/2}$, $T_i^{n+1/2}$, $T_e^{n+1/2}$, $n^{n+1/2}$
  - With predictor-correct on B and T used for strongly coupled cases
- For CEL advance:
  - V^n=> $\mathbf{B}^{n+1/2}$, $T_i^{n+1/2}/F_i^n$, $T_e^{n+1/2}/F_e^{n+1/2}$, $n^{n+1/2}$

  Or
  - V^n/F_i^n => $\mathbf{B}^{n+1/2}/F_e^{n+1/2}$, $T_i^{n+1/2}$, $T_e^{n+1/2}$, $n^{n+1/2}$
  - Solving simultaneously greatly improves capabilities
  - Solving simultaneously greatly stresses the solvers

Brief comment on ITG problem

- Normally we consider just the parallel stress tensor and parallel heat flux from DKE closures
  - From calculation of F, we can calculate off-diagonal stress tensors and perpendicular heat fluxes
  - Coupling with the full equations is a bit more complicated
  - How much further in k_perp rho_i can we go and get good agreement with the full gyrokinetic model?

Summary

- Many efforts currently in the community for continuum discretization methods for kinetic equations
  - GYRO, NEO, NIMROD, M3D-C1, Ernst, COGENT, GKEYLL, …
- High-order FE elements offer attractive properties for discretizing velocity space
  - Trading off on HP convergence
- NIMROD now has:
  - 1D FE elements for pitch angle
  - collocation/weighted quadrature scheme for the speed variable

  ⇒Many opportunities for using accelerators/multi-threading as route to parallelization

⇒Extended MHD codes are becoming 5D codes for evolving tokamak plasmas on long time scales

Future work

- Finish NEO and EP benchmarks. Publish JCP
- Start on production cases this summer
- Implement QL operator over summer