12/29/04 Dylan Brennan

Report on completion of first quarter elm simulation milestone

1st Q (end of December 04) Quantify the scaling of unstable modes with resistivity and thermal conduction, and compare to linear codes

The study of the nonlinear evolution of edge localized modes in theoretical and computational plasma physics has only recently begun. A new initiative was formed to study the extended MHD evolution of these modes with NIMROD, and compare with linear results of codes like ELITE and GATO, and the nonlinear results from recent simulations with BOUT, to understand the basic physics of the nonlinear evolution from a number of perspectives. The first quarterly milestone of the year, stated above, is the subject of this report. Only linear computations and results are considered in the first quarter milestone.

A simple equilibrium configuration was constructed to be unstable only to a robust edge instability with low shear and shaping, with both ballooning and pealing characteristics, and yet to reside in the zero dimensional parameter space of typical DIII-D discharges. This equilibrium is shown in Figure 1. Using this equilibrium the linear results of ELITE and GATO were favorably compared to the NIMROD results, showing the same eigenfunctions and that the growth rates in a NIMROD simulation depend weakly on the resistivity and viscosity in the vicinity of the mode but are in qualitative agreement with GATO and ELITE, as seen in Figure 2.



Figure 1. The equilibrium profiles and shape used in this study.

These linear computations were the culmination of an aggressive campaign to understand and solve the challenges associated with simulating edge localized modes in tokamaks with the NIMROD code. In carrying out this study, the linear behavior of 22 toroidal mode numbers were each computed with 3 values of resistivity and 3 values of thermal conductivity, for a total of $22 \times 9 = 198$ cases. Each case required 16 processors on the NERSC SP3, for a total of 3168 processors. Each calculation required approximately 24 hours to complete, so this study represents the use of 76032 processor hours. Nonlinear simulations will take much longer.



Figure 2. An example of an n=21 eigenfunction (left) and a comparison between the n=7 eigenfunction from ELITE (center) and NIMROD (right), showing the overall agreement in mode structure. The exterior part of the ELITE eigenfunction is not plotted.

As mentioned, several computational parameters were varied in this campaign, and vacuum was found to have a small but significant effect on the growth rate of the mode, which implies that the mode has a resistive component, as shown in Figure 3. Here the Prandtl number (ratio of the kinetic viscosity to resistivity) is held fixed while both the core and vacuum resistivities are varied.



Figure 3. The effects of resistivity changes. Between the blue and black curves the resistivity is only changed in the vacuum region and slightly across the mode region, while viscosity is held fixed.

In NIMROD, the dvac parameter is the ratio of the vacuum resistivity to the core resistivity, while kinetic viscosity is constant everywhere. The profile of resistivity smoothly transitions at the separatrix between the two values, but since these modes are localized in that very region, both the width and magnitude of the transition to the vacuum resistivity effect the mode growth rate somewhat. The scaling with resistivity is very weak since the mode is predominantly ideal, and changes in core resistivity also change the viscosity at fixed Prandtl number. This is not so when dvac alone is changed. In Figure 3 the growth rates increase significantly with dvac. Changes in viscosity alone had little effect on the growth rates of the mode, but further studies are needed on viscous effects in future.

However, no systematic study of the effects of core resistivity and parallel thermal diffusivity had been completed. These physical effects will be important in the nonlinear phase of the evolution, but it is best to interpret nonlinear results in terms of linear results. As a first step we consider the linear stability of a two dimensional set of equilibria, with three values of core resistivity and three values of parallel thermal diffusivity, holding the vacuum resistivity fixed.

In Figure 4 are shown a subset of these results for three toroidal mode numbers n=1,6 and 21. The growth rates increase with n as expected without a hyperviscosity filter, but also increase with S. This is likely due, in part, to the reduction of the viscosity as the Prandtl number is held fixed, but previous results (above) have shown that the resistivity alone can change the mode growth rates. The effect of dissipation on the linear behavior of ideal modes can be either stabilizing or destabilizing, depending on whether damping or the breaking of the frozen field line constraint is dominant. In general the dependence of the growth rates on thermal diffusivity is weak in the linear phase. The ratio of

diffusivities should decrease near the edge and into the vacuum region, which can be important in the nonlinear evolution. But linearly the scaling is weak.



Figure 4. The growth rates vs. S for a series of three parallel thermal diffusivities. In the linear phase, growth rate is relatively insensitive to the parallel thermal diffusivity. Growth rates increase weakly with S, as the Prandtl number is held fixed and kinetic viscosity is reduced.



Figure 5. The growth rates as a function of n for three values of S and parallel conduction. The Prandtl number is held fixed. Note that the higher mode number growth rates begin to decrease with n at higher resistivity and viscosity.

In Figure 5 is shown the full set of growth rate results. The convergence of the S=5e8 cases is marginal, and error bars would be larger than 10%. Several intermediate to low n modes are not quite settled into a set eigenfunction and growth rate, and have very slowly decaying if not overstable oscillations in the growth rates as seen in Figure 6. Most are close enough to suffice, but by taking an average of the data over time we set the convergence condition to be when the average of the kinetic energy and magnetic energy growth rates differ by less than 0.1%. The persistent oscillations in growth rate could also be the signature of overstable modes, which are not ruled out when the system is dissipative.





These results suggest that the resistivity profile across the region of the separatrix is important, even in the linear calculations of edge modes. Although there is a weak scaling of the growth rates with the resistivity, the resistivity varies strongly across this region, and the vacuum resistivity setting can be important. In Figure 3 the resistivity everywhere is decreased by a factor of 10, along with viscosity, from S=1e6 to S=1e7, this decreases the growth rates. When the vacuum resistivity alone is increased the growth rates increase. In these latest results we see an increase of growth rates with decreasing core resistivity while the vacuum resistivity is held fixed. A theoretical explanation of this is pending.