Magnetic X-points, stochasticity, and Edge Localized Modes

Linda E. Sugiyama[1] **and the M3D Team**

[1]Laboratory for Nuclear Science
Massachusetts Institute of Technology, Cambridge MA 2139-4307 U.S.A.

E-mail: `sugiyama@mit.edu`

Abstract. The confinement and stability properties of the edge of a magnetically confined fusion plasma have long resisted theoretical explanation. Numerical simulation with the M3D extended MHD code shows that large periodic Edge Localized Modes (ELMs), associated with otherwise good confinement, represent a new type of nonlinear MHD plasma instability, in which a coherent plasma instability couples to the favorable part of a chaotic magnetic field. The magnetic surface bounding the plasma typically contains at least one X-point. Under small perturbations, such a Hamiltonian surface splits into two different asymptotic limits. A ballooning-type plasma instability can couple to the "unstable" field manifold that forms bulges around the original magnetic surface and then grow to penetrate deep into the plasma. The magnetic field becomes chaotic over the affected region, allowing plasma loss from the core to the outside. Eventually the instability saturates and the plasma relaxes back towards its original axisymmetric shape. The interaction between the plasma and the homoclinic field perturbation drives a multi-stage process that reproduces many experimental observations. This type of instability may help to explain the wide range of ELM and ELM-free behavior observed in fusion plasmas with X-points, as well as properties of plasma confinement.

1. Plasma boundaries with X-points

Magnetically confined toroidal plasmas routinely achieve high temperatures in a configuration with one or two magnetic X-points on the bounding magnetic surface, where the field becomes purely toroidal. X-points allow good control of the plasma shape against instabilities. They support a steep edge pressure gradient with good plasma edge confinement (H-mode), favorable for fusion reactivity. The outer legs of the X are intended to channel plasma losses along the field lines, to divertors designed to handle them. In practice, the steep edge pressure gradient and associated plasma current can drive large periodic instabilities (Edge Localized Modes or ELMs) that erode the edge and dump plasma and energy onto the surrounding walls. The losses can be severe in fusion burning plasmas and are one of the major constraints on the design of next generation experiments such as ITER.

Despite its long history[1], the H-mode plasma edge has defied solid theoretical explanation. The size and onset of ELMs are still difficult to predict. Plasma stability theory almost universally assumes that the magnetic field responds to small perturbations in a coherent, unitary fashion. This may be justified in the interior of toroidal plasmas, where the magnetic flux surfaces often behave as surprisingly persistent KAM surfaces of the Hamiltonian system that describes the magnetic field.

The magnetic boundary surface with an X-point behaves differently[2]. The X-point is equivalent to a Hamiltonian hyperbolic saddle point. Under small perturbations, an X-point surface splits into two, defined by different asymptotic limits, depending on the direction in which the field line is traversed (eg, [4]). The "stable" limiting surface or manifold, on which the field line moves away from the X-point, remains close to the original surface. The "unstable" surface forms loops or bulges transverse to the stable surface, as the field line approaches the X-point. The asymptotic intersection points, called homoclinic points, become more closely spaced as the X-point is approached. Because the Hamiltonian is area preserving in a 2D surface of section, successive loops of the unstable manifold have equal areas and their excursions around the stable surface become longer and longer, becoming infinite as the X-point is approached.

Homoclinic intersections cause the magnetic field to become chaotic[4]. While a number of theoretical properties can be proven, they are primarily asymptotic and qualitative. No criterion exists for the limiting perturbation size that allows the theoretically predicted behavior. The manifold shapes and intersection points are determined by the driving mechanism, which may be far from Hamiltonian. Since in physical systems, infinite excursions are impossible because of natural length scales and propagation delays, the plasma magnetic field differs from the theoretical homoclinic tangle, although it may try to approach it.

ELMs have recently become accessible to numerical simulation with extended MHD models at meaningful parameters. Previous, more approximate simulations of ELMs[5, 6] saw field mixing, but did not consider field splitting. The present results show that ELMs in fact constitute a new type of plasma instability that grows in the presence of a split, chaotic magnetic field. This paper addresses certain aspects of the field splitting. Detailed results on ELMs are reported elsewhere[7].

2. Numerical model

The extended MHD code M3D[8] simulates a free boundary plasma surrounded by a "vacuum" region with open field lines that is bounded by a rigid, ideally conducting wall. The code solves the compressible resistive MHD equations, including separate density and temperature, over the entire region. The resistive MHD vacuum is characterized by low density, zero temperature, and high resistivity.

Initial conditions were taken from a DIII-D discharge with large ELMs, the ITER-similar shot 119690 at $t = 2656$ ms[3], chosen because it was relatively collisional, $\nu_e = 0.7$, and had a very steep, narrow edge pressure gradient strongly unstable to MHD ballooning modes. The plasma boundary had one lower X-point. A second, upper X-point existed a short distance above and inboard from the top of the plasma. The plasma had toroidal magnetic field $B_T = 1.60$ T at major radius $R_o = 1.76$ m, current $I_p = 1.042$ MA, and neutral beam heating power $P_{NB} = 4,8$ MW. The central density was $n_{eo} = 1.2 \times 10^{20}\mathrm{m}^{-3}$, temperature $T_{eo} \simeq T_{io} = 1.83$ keV, and normalized beta $\beta_N = 1.81$. The edge magnetic safety factor was $q_{95} = 3.75$ and central q_o just above unity. The simulations reported here neglect the equilibrium toroidal rotation.

The ELM is partly resistive. The model resistivity varied as $(T/T_o)^{-3/2}$ inside the plasma, up to the large, constant vacuum value. A relaxed-parameter case with large central plasma resistivity, Lundquist number $S = 10^6$, vacuum $S_{vac} = 10^3$, normalized kinematic ion viscosity $\mu_i/\rho = 10^{-5}$, and "vacuum" density $n_{vac}/n_o = 0.4$ clearly demonstrates the instability. Cases at more realistic parameters, including the actual resistivity $S = 3.3 \times 10^7$, with $\mu = 6 \times 10^6$ and $n_{vac}/n_o = 0.1$, show an analogous evolution, at reduced scale[7].

The parallel thermal conductivity, using the artificial sound wave method[8], was equivalent to a diffusion coefficient $\kappa_\| = 3.53(R_o/a_o)$. Perpendicular thermal and particle diffusivities were $\kappa_\perp = D_n = 10^{-5}$. The dissipation, other than resistivity, was uniform except very near the wall, where it became large, 10^{-2}. Length is normalized to $a_o = 100$ cm, not the plasma radius 55.7 cm. Time is normalized to the Alfvén time $\tau_A = R_o/v_A$, based on the plasma vacuum field and initial density at the magnetic axis R_o. The reference time is $\tau_A = 0.78$ μsec.

The grid in each poloidal plane was packed around the plasma edge, more tightly on the outboard side. The smaller S case used 66 radial by 400 poloidal points, for 12871 total points, for linear triangular finite elements. Other cases used 72 by 400 points, or 15769 total. FFTs were used in the toroidal direction. Typical toroidal mode numbers ranged over $|n| \leq 22$, or 72 poloidal planes with full dealiasing. Runs with $|n| \leq 48$ modes show that the primary nonlinear instability falls within the lower range.

3. Results

The initial configuration was taken from DIII-D experimental reconstructions that include the large bootstrap current in the plasma edge. The ELM was started with a small random perturbation of multiple toroidal harmonics at time $t = 0$.

The time evolution at large resistivity is shown in Figure 1. The top row shows temperature contours. Black represents the vacuum region and its border the vacuum vessel. The toroidal axis $R = 0$ lies to the left (inboard) side.

The early nonlinear instability at $t = 66\tau_A$ barely perturbs the outboard edge (large R) of the plasma, as seen in the small fuzzy region and uneven point spacing on the outermost magnetic surfaces. The perturbed magnetic region is a little wider at the bottom and top. By $t = 89$, the nonlinear instability is strongly developed. The midplane oscillations broaden poloidally and large plasma fingers grow above and below the midplane, around ± 60 degrees. In contrast, the standard ballooning eigenmode decreases away from the midplane. The growth rate also drops from initial levels.

As time progresses to $t = 133$, the low density fingers grow inward, penetrating deeply into the plasma. The protruding high density fingers disperse shortly outside the original magnetic boundary, where the plasma shears off in "blobs." Plasma spreads outside, over the outboard region and then into the outboard divertor, while the core density, and to a lesser extent the temperature, is reduced. (A central magnetic island grows as q_o drops below unity, then decays with little effect on the main instability.) At $t = 178$, smoothing of the edge field is visible. In this case, the instability is unphysically strong and the loss of interior plasma reduces the driving force before the later ELM stages can develop.

At more realistic parameters[7], including the actual resistivity, the growth rate and instability magnitude are reduced. The finger variation is smaller and outgoing fingers shorter, but the field still mixes over much of the plasma. Relatively early in the ELM, some of the outer magnetic field lines partially relocalize in the edge region, enough to support a new, partial edge density gradient. At a later stage, plasma moves into the inboard side and into the divertors at top and bottom. As the original instability saturates, the plasma relaxes towards the original equilibrium shape, with small changes in the profiles.

The chaotic field region mirrors the inward growth of the plasma fingers. A central core is mostly preserved, inside some limiting flux surface. The behavior agrees with the theoretical expectation[4] that the entire region containing homoclinic points becomes chaotic. This behavior differs from interior plasma instabilities, where relatively narrow magnetic island chains, interspersed with good, closely spaced KAM surfaces, often persist.

The magnetic boundary surface clearly splits. The $+\phi$-direction puncture plots clearly show lobes near the bottom X-point on the outboard side, as expected for the "unstable" manifold. The surfaces are relatively smooth on the inboard side of the lower X-point and the outboard side of the upper X-point, where this direction should coincide with the "stable" manifolds. Since the stable manifolds of two different X-points cannot intersect, the $+\phi$ points connected to the lower X-point cannot extend much above the top of the original plasma. The $-\phi$ points closely shadow the lobes of the "unstable" manifold of the upper X-point and form large lobes near the top of the plasma. They also show a smoother "stable" response on the outboard side

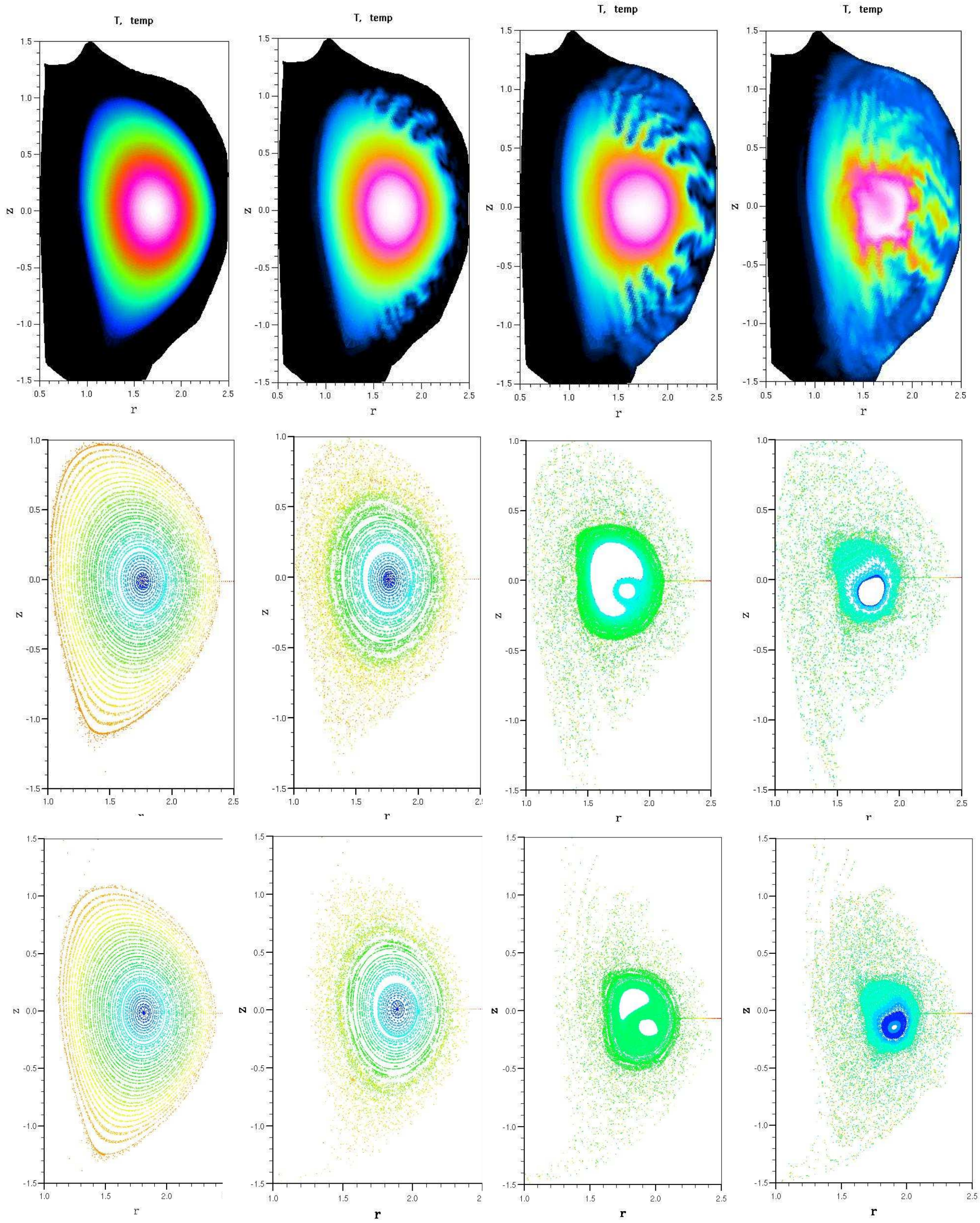

Figure 1. Time evolution of an ELM at large resistivity, shown at $t = 67$, 89, 133, and $178\tau_A$ for a perturbation started at $t = 0$. Temperature contours (top row), magnetic puncture plots traced in $+\phi$-direction (middle), and in the $-\phi$-direction (bottom). Axis labels are arbitary units.

of the lower X-point. The small wiggles on the inboard side are consistent with the "unstable" manifold of the lower X-point, which is less strongly unstable than on the outboard side.

In 3D, the field lines and the plasma perturbations have helical shapes that roughly follow the equilibrium field lines. Figure 3 shows a surface of constant temperature, colored according to values of the poloidal magnetic flux. A single nearby field line, traced in the $+\phi$-direction, shows how the overall "chaotic" field lines approximately coincide with the temperature contours for

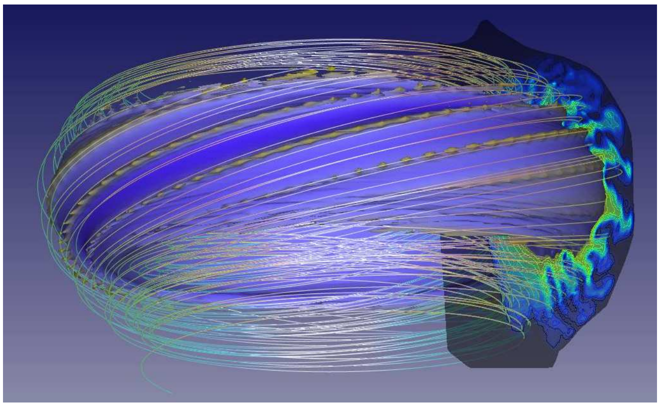

Figure 2. Helical structure at $t = 133$, showing one temperature contour, colored according to the poloidal magnetic flux, and one $+\phi$ field line, eventually lost to the bottom. Tilted upward to show the lower plasma fingers.

a significant number of toroidal transits, before (often) being lost at the X-point. The coupling of field and plasma is much tighter at the smaller resistivity of a real ELM[7], where the plasma slips through the field at a reduced rate.

The mismatch in shape between the idealized homoclinic tangle and the the plasma interchange-type instability, together with the fact that only part of the field participates constructively in the instability, reduces the growth rate over the case restricted field splitting[7].

4. Summary

Large scale MHD simulations show that Edge Localized Modes (ELMs) in high temperature, toroidal fusion plasmas constitute a new class of nonlinear plasma instability. A coherent plasma structure is able to couple dynamically to only part of a chaotic magnetic field, which then grows with the plasma instability. The magnetic X-point on the plasma boundary surface causes it to split into two asymptotic limits under perturbation. The field lines become chaotic, but retain their helical, torus-wrapping character. Instabilities driven by the strong edge pressure gradient, preferentially aligned along the equilibrium field, can proceed by a ballooning-like flux-tube interchange that couples nonlinearly to the "unstable" loops of the split field. Low density loops grow inwards by transferring plasma to the outward growing, higher density loops. The magnetic field approximates a near-Hamiltonian homoclinic tangle, modified by the plasma response. Over time, the plasma heals back to a nearly axisymmetric state. The resulting multi-stage ELM has many features similar to experiment.

The demonstration that a chaotic magnetic field with only asymptotically definable structure can couple effectively to the plasma motion may help to explain the large range of ELM and ELM-free behavior observed in H-mode fusion plasmas, as well as properties of plasma confinement.

Acknowledgments

Work supported by the U.S. Department of Energy OFES and SciDAC programs. Computation and visualization performed at NERSC. Thanks to T. Evans and L. Lao for DIII-D data.

References

[1] Wagner F, et al., 1982 *Phys. Rev. Lett.* **49** 1408

[2] Roeder R K W, Rapoport B I, and Evans T W, *Phys. Plasmas* 2003 **10** 2796-99; Evans T E, in *Chaos, Complexity and Transport: Theory and Applications* 2008, ed. Chandre C, Leoncini X, and Zaslavsky G (Singapore, World Scientific Publishing Co.) pp 147-176

[3] Evans T E, et al., *Nucl. Fusion* 2005 **45** 595-607; Evans T E, et al., 2004 *Phys. Rev. Lett* **92** 235003

[4] Lichtenberg A J and Lieberman M A 1992 *Regular and Chaotic Dynamics 2nd ed.*, (New York, Springer-Verlag); Most texts on Hamiltonian dynamics discuss these topics.

[5] Mizuguchi N, et al. 2007 *Nucl. Fusion* **47** 579-585; Huysmans G T A and Czarny O, 2007 *Nucl. Fusion* **47** 659-666; Snyder P B, Wilson H R, and Xu X Q, 2005 *Phys. Plasmas* **12** 056115

[6] Strauss H R, Sugiyama L, et al. 2009 *Nucl. Fusion* **49** 055025

[7] Sugiyama L E, and Strauss H R, in preparation.

[8] Park W, et al., 1999 *Phys. Plasmas* **6** 1796-1803; Sugiyama L E and W Park 2000 *Phys. Plasmas* **7** 4644-4658