AMR Simulations of Pellet Injection

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Outline

- Semi-implicit MHD code – Progress
- 3D AMR MHD code
- Pellet Injection - Progress
- Conclusion and future work

3D AMR MHD Code - Status

- Hyperbolic fluxes computed using an unsplit upwind method
- Implicit treatment of parabolic terms
- **r ¢ B**=0 by projection
- Inclusion of nonlinear coefficients in the elliptic solvers is under progress
  - *Reconnection (with Breslau's nonlinear h) will be the test case*

Single-fluid resistive MHD Equations

- Equations in conservation form

$$\frac{\partial U}{\partial t} + \boxed{\frac{\partial F_j(U)}{\partial x_j}} = \boxed{\frac{\partial \tilde{F}_j(U)}{\partial x_j}}$$

Parabolic

Hyperbolic

$$U = \{\rho, \rho u_i, B_i, e\}^T$$

$$F_j(U) = \left\{ \begin{array}{c} \rho u_j \\ \rho u_i u_j + p\delta_{ij} + \frac{1}{2} B_k B_k \delta_{ij} - B_i B_j \\ u_j B_i - B_j u_i \\ \left(e + p + \frac{1}{2} B_k B_k\right) u_j - B_i u_i B_j \end{array} \right\}$$

$$\tilde{F}_j(U) = \left\{ \begin{array}{c} 0 \\ Re^{-1} \tau_{ij} \\ S^{-1} \eta \left(\frac{\partial B_i}{\partial x_j} + \frac{\partial B_j}{\partial x_i} \right) \\ S^{-1} \eta \left(\frac{1}{2} \frac{\partial B_i B_i}{\partial x_j} - B_i \frac{\partial B_j}{\partial x_i} \right) + Re^{-1} \tau_{ij} u_i + Pe^{-1} \kappa \frac{\partial T}{\partial x_j} \end{array} \right\}$$

Reynolds no.

Lundquist no.

Peclet no.

$$\tau_{ij} = \rho \nu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right)$$

$$e = \frac{p}{\gamma - 1} + \frac{1}{2} \rho u_i u_i + \frac{1}{2} B_i B_i$$

Numerical Method

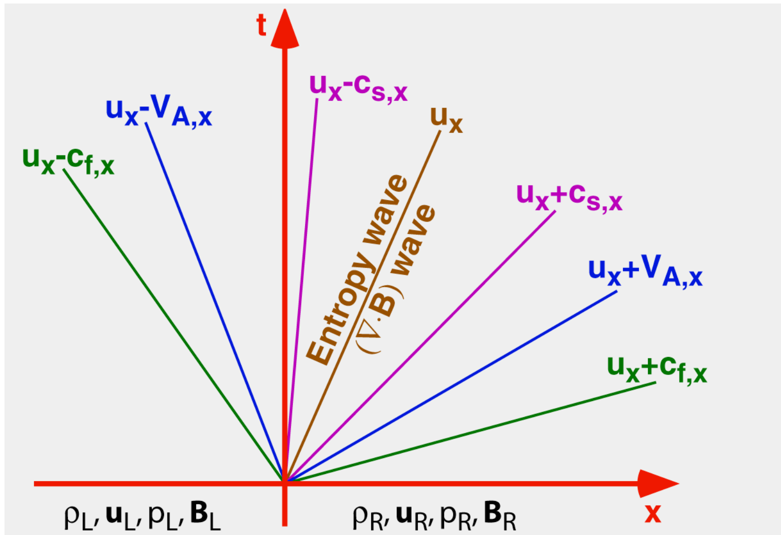
- MHD Equations written in symmetrizable near-conservative form (Godunov, Numerical Methods for Mechanics of Continuum Media, 1, 1972, Powell et al., J. Comput. Phys., vol 154, 1999).
  - *Deviation from total conservative form is of the order of $\nabla\cdot$**B** truncation errors*

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho \\ \rho\mathbf{u} \\ \frac{1}{2}\rho u^2+\frac{1}{\gamma-1}p+\frac{1}{2\mu_0}B^2 \\ \mathbf{B}\end{pmatrix}+\left\{\nabla\cdot\begin{pmatrix}\rho\mathbf{u} \\ \rho\mathbf{u}\mathbf{u}+\left(p+\frac{1}{2\mu_0}B^2\right)\mathbf{I}-\frac{1}{\mu_0}\mathbf{B}\mathbf{B} \\ \left[\frac{1}{2}\rho u^2+\frac{\gamma}{\gamma-1}p+\frac{1}{2\mu_0}B^2\right]\mathbf{u}-\frac{1}{\mu_0}(\mathbf{u}\cdot\mathbf{B})\mathbf{B} \\ \mathbf{u}\mathbf{B}-\mathbf{B}\mathbf{u}\end{pmatrix}^T\right\}^T=-(\nabla\cdot\mathbf{B})\begin{pmatrix}0 \\ \frac{1}{\mu_0}\mathbf{B} \\ \frac{1}{\mu_0}(\mathbf{u}\cdot\mathbf{B}) \\ \mathbf{u}\end{pmatrix}$$

- The symmetrizable MHD equations lead to the 8-wave method.
  - *The fluid velocity advects both the entropy and div(**B**)*
- Finite volume approach. Hyperbolic fluxes determined using the unsplit upwinding method (Colella, J. Comput. Phys., Vol 87, 1990)
  - *Predictor-corrector.*
  - *Fluxes obtained by solving Riemann problem*
  - *Good phase error properties due to corner coupling terms*

$$U_{\boldsymbol{i}}^{n+1}=U_{\boldsymbol{i}}^{n}-\frac{\Delta t}{h}\sum_{d=0}^{\mathbf{D}-1}\left(F_{\boldsymbol{i}+\frac{1}{2}\boldsymbol{e}^d}^{n+\frac{1}{2}}-F_{\boldsymbol{i}-\frac{1}{2}\boldsymbol{e}^d}^{n+\frac{1}{2}}\right)$$

$$F_{\boldsymbol{i}+\frac{1}{2}\boldsymbol{e}^d}^{n+\frac{1}{2}}=R\left(W_{\boldsymbol{i},+,d}^{n+\frac{1}{2}},W_{\boldsymbol{i}+\boldsymbol{e}^d,-,d}^{n+\frac{1}{2}},d\right)$$



r¢ B=0 by Projection

- Compute the estimates to the fluxes $F^{n+1/2}_{i+1/2,j}$ using the unsplit formulation
- Use face-centered values of B to compute **r¢** B.
  Solve the Poisson equation **r**$^2\phi$ = **r ¢** B
- Correct B at faces: B=B-**r**f
- Correct the fluxes $F^{n+1/2}_{i+1/2,j}$ with projected values of B
- Update conservative variables using the fluxes
  - *The non-conservative source term S(U) a* ***r¢*** *B has been algebraically removed*
- On uniform Cartesian grids, projection provides the smallest correction to remove the divergence of B. (Toth, JCP 2000)
- Does the nature of the equations change?
  - *Hyperbolicity implies finite signal speed*
  - *Do corrections to B via* ***r***^{2}*f=****r¢*** *B violate hyperbolicity?*
- Conservation implies that single isolated monopoles cannot occur. Numerical evidence suggests these occur in pairs which are spatially close.
  - *Corrections to B behave as a 1/r*^{2} *in 2D and 1/r*^{3} *in 3D*
- Projection does not alter the order of accuracy of the upwinding scheme and is consistent

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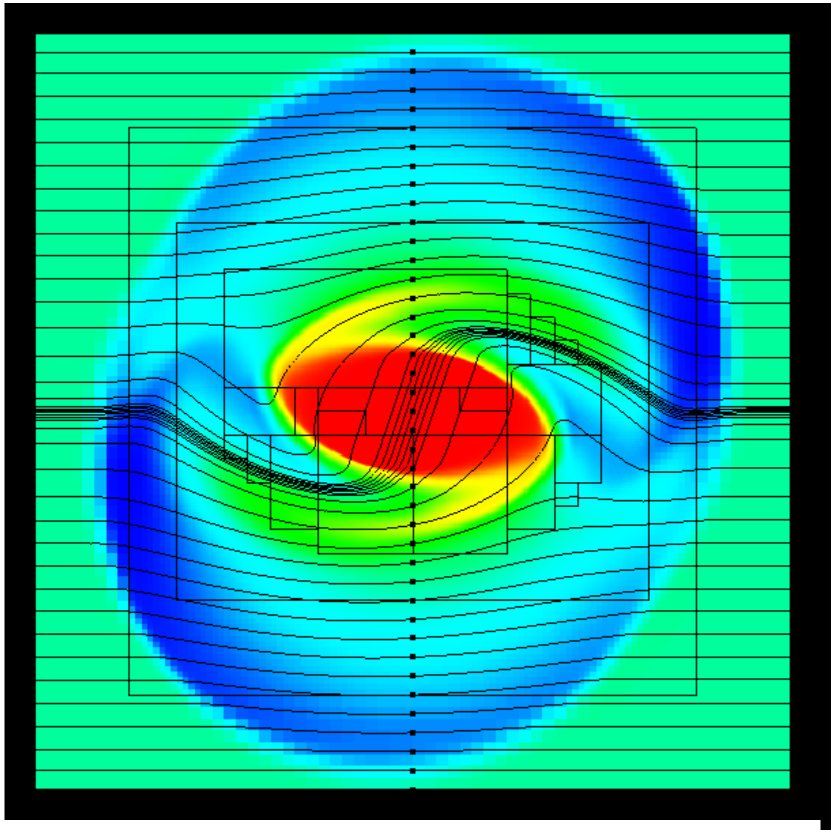
Unsplit + Projection AMR Implementation

- Implemented the Unsplit method using Chombo
- Solenoidal B is achieved via projection, solving the elliptic equation $\mathbf{r}^2\phi=\mathbf{r}$**¢** B
  - *Solved using Multgrid on each level (union of rectangular meshes)*
  - *Coarser level provides Dirichlet boundary condition for ƒ*
    - *Requires **O(h^3)** interpolation of coarser mesh ƒ on boundary of fine level*
  - *a “bottom smoother” (conjugate gradient solver) is invoked when mesh cannot be coarsened*
- Multigrid convergence is sensitive to block size
- Flux corrections at coarse-fine boundaries to maintain conservation
  - *A consequence of this step: **r¢** B=0 is violated on coarse meshes in cells adjacent to fine meshes.*

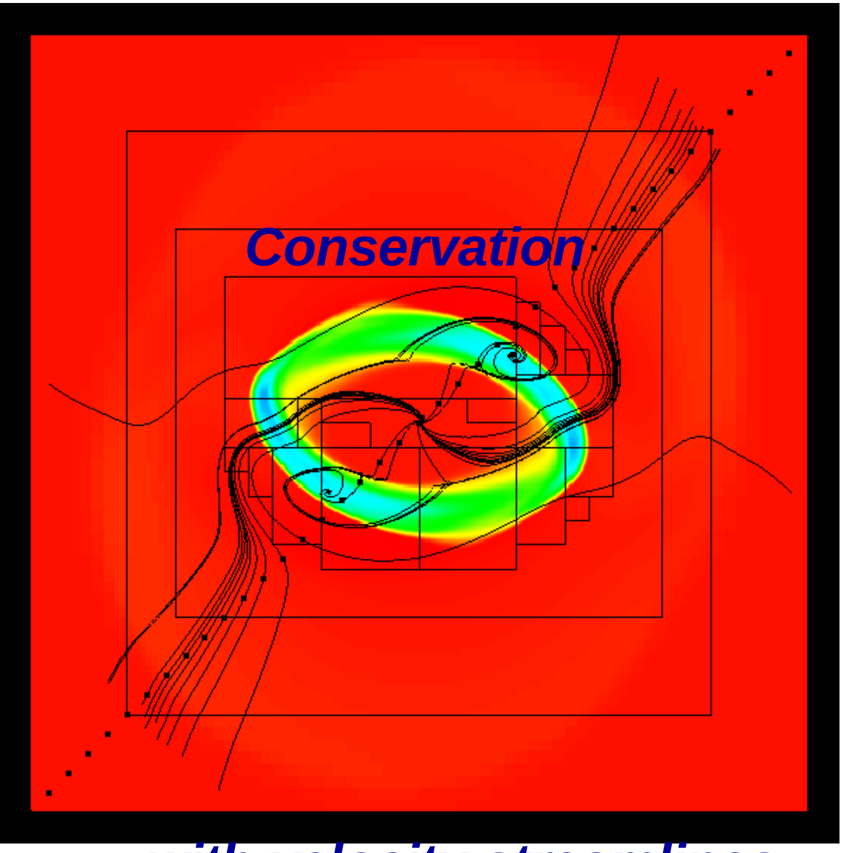
Implicit treatment of parabolic flux terms

- Implicit treatment requires the solution of elliptic equations (Helmholtz equation)
  - *Completed implicit treatment of viscous, heat conduction and resistive terms*
  - *Viscous and conduction terms require non-constant coefficient Helmholtz solvers - Completed*
  - *Favored approach: Implicit Runge Kutta, TGA Approach (Twizell, Gumel, Arigu, Advances in Comp. Math. 6(3):333-352, 1996)*
    - *Due to C++ abstractions, other solvers (Backward Euler, Crank-Nicholson) can also be used instead of TGA – choice can be made by the user.*
  - ***r¢ u is ignored in the shear stress tensor. If r¢ u is included, the resulting elliptic equations are coupled -such solvers are under development***
- Quadratic interpolation ($O(h^3)$) at coarse-fine boundaries
  - *Corner terms required and obtained by linear interpolation*
- Flux-refluxing step requires implicit solution on all levels synchronized at the current time step.
  - *Backward Euler used for this step*

Weak rotor – Resistive MHD



Pressure with B field lines



r *with velocity streamlines*

Pellet Injection: Objective and Motivation

- Objectives
  - *Identify the mechanisms for mass distribution during pellet injection in tokamaks*
  - *Quantify the differences between "inside launch" and "outside launch"*
- Motivation
  - *Fusion power depends upon efficient fueling*
  - *Gas puffing is limited in its ability to achieve core fueling*
  - *Injection of frozen hydrogen pellets is a viable method of fueling a tokamak (Bell et al., Nuclear fusion, 2000)*
    - *Pellet injection provides much deeper fueling*
  - *Pellet-plasma interactions:*
    - *Ablation: Considered well-understood*
    - *Mass deposition: Large scale MHD driven but poorly understood*

Background - Experimental

- Early pellet experiments showed improvement in energy confinement with pellet fueled plasmas (Greenwald, PRL, 1984)
- Pellet injection of frozen hydrogen is a viable method to fuel tokamaks (Bell et al., Nuclear Fusion 1992 – this TFTR experiment also exceeded empirical Greenwald density limit)
- Inside (HFS) vs. outside (LFS) launch
  - *HFS is more effective in fueling the center of the plasma* ***(Lang et al. PRL 1997, Baylor et al. Phys. Plasmas 2000)***
  - ***Example: DIIID fueling efficiency is 95% (HFS), 55% (LFS)***
- Pellets trigger formation of internal transport barrier with central heating
- Edge localized modes are triggered in H-mode by strong perturbations from pellets

Background – Simulation/Theory

- Earliest ablation model by Parks (Phys. Fluids 1978)
  - ***Accurate expression for pellet ablation once pellet is in contact with the high temperature plasma***
- Detailed multi-phase calculations in 2D of pellet ablation (MacAulay, PhD thesis, Princeton Univ 1993, Nuclear Fusion 1994)
  - *Agreement with Parks model of plasma ablation within a factor of 2*
- 3D Simulations by Park and Strauss (Phys. Plasmas, 1998)
  - *Solve an initial value problem . Initial condition consists of a prescribed MHD equilibrium and a large density "blob" to mimic a <u>fully ablated</u> pellet cloud with zero flux averaged pressure perturbation*
    - *Pellet cloud to device dimension was relatively large due to resolution restrictions*
  - *No motion of pellet modeled*
  - *Mass distribution along field lines*
  - *Scaling law for mass distribution established*
  - *Verified that MHD effects cause localized density perturbation to displace towards LFS.*

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Approach/ Model

- Detailed 3D AMR simulations of pellet injection – pellet treated as moving density source
  - *Ratio of pellet size to device size is ~O(10^{-3})*
- Phased approach
  - *Simple Cartesian geometry AMR simulation to understand the basic physics of mass redistribution with varying degrees of complexity*
    - *Ideal MHD with density and energy source terms*
    - *Resistive MHD with density source and anisotropic heat conduction*
  - *Force terms to mimic toroidal geometry*
- Physical assumptions for first phase of AMR simulations
  - *Pellet ablates with an analytic ablation model*
  - *Instantaneous heating of ablated mass by electrons to corresponding flux surface temperature*
  - *No drag coupling between pellet and ambient plasma*
  - *Single fluid/single phase*
- Mathematical model is Ideal MHD with source terms in density and energy equations

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Mathematical Model I

- Mass conservation

$$\frac{\partial n}{\partial t} + \nabla \bullet \left(n \vec{V} \right) = \boxed{\frac{dN}{dt} \delta\left(\vec{X} - \vec{X}_p(t) \right)}$$

Mass source active on pellet surface

- Momentum conservation

$$nM_i \left(\frac{\partial \vec{V}}{\partial t} + \vec{V} \bullet \nabla \vec{V} \right) = \vec{J} \times \vec{B} - \nabla p \boxed{+ \vec{T}_1}$$

- Maxwell equations

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}$$

$$\vec{E} + \vec{V} \times \vec{B} = \eta \vec{J} \boxed{+ \vec{T}_3}$$

$$\mu_0 \vec{J} = \nabla \times \vec{B}$$

Toroidal source terms zero in Cartesian geometry

- Energy conservation

$$\frac{\partial p}{\partial t} + \vec{V} \bullet \nabla p = -\tfrac{5}{3} p \nabla \bullet \vec{V} + \tfrac{2}{3} \left[\eta J^2 + \nabla \bullet \vec{q} \right] + \boxed{T_2}$$

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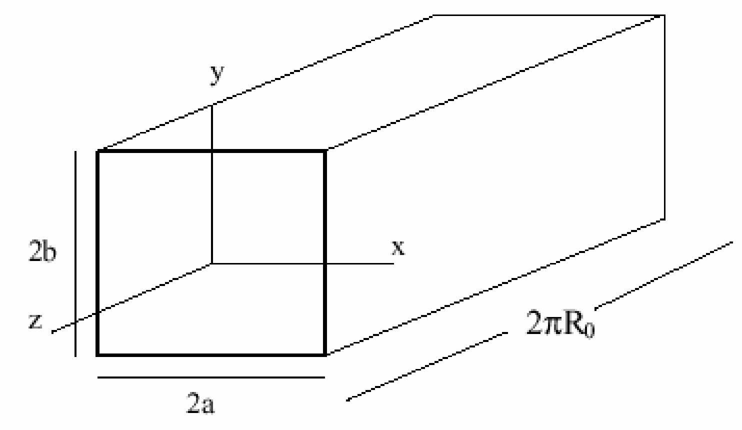
Mathematical Model II

- Mass source is given using the ablation model by Kuteev (Nuclear Fusion 1995)

$$\frac{dN}{dt} = -4\pi r_p^2 \frac{dr_p}{dt} 2n_m = 1.12\times10^{16} n_e^{0.333} T_e^{1.64} r_p^{1.33} M_i^{-0.333}$$

  - *Pellet shape is spherical for all t*
  - *Above equation uses cgs units*
- Delta function in source term approximated as a "top-hat" function of width ten times the pellet radius (motivated by Parks et al. Phys. Plasmas 2000)
- For numerical computations, equations re-written in strong conservation form
- Energy equation source term: dN/dt x 3T/2 where T is the temperature of the flux surface at the pellet center to model heating by electrons on the flux surface

AMR Simulation Parameters



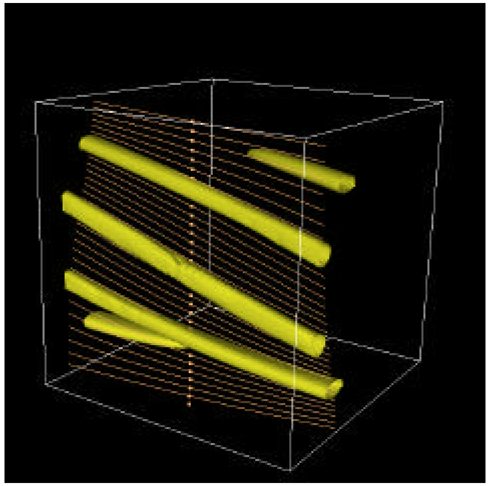
- Toroidal Magnetic Field
  B_T=0.2 T
- Device size a=0.2m
- Initial Pellet size r_p=0.1 cm
- Pellet velocity v_p=3000 m/s
- Plasma β=0.1
- Average plasma Temperature T=662 ev
- Initial average density n=1.5 x $10^{19}/m^3$
- Boundary conditions: $\mathbf{B_n}$ ¢ n=0, u¢ n =0 on sides
  Periodic in z-direction
- Initial condition: Ideal MHD equilibrium
  $\psi= \psi_0 \sin k_x x \cos k_y y$

  $p=p_0 + p_1 \psi^2$

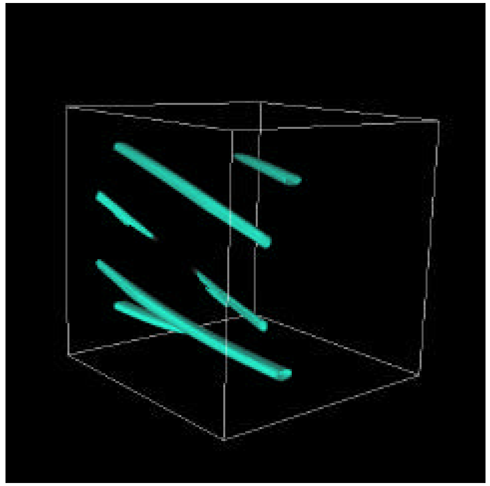
  $\mathbf{B}=\mathbf{r}\,\psi \times \mathbf{r}\,z + B_T\, k$

Results from AMR Simulations –Early time

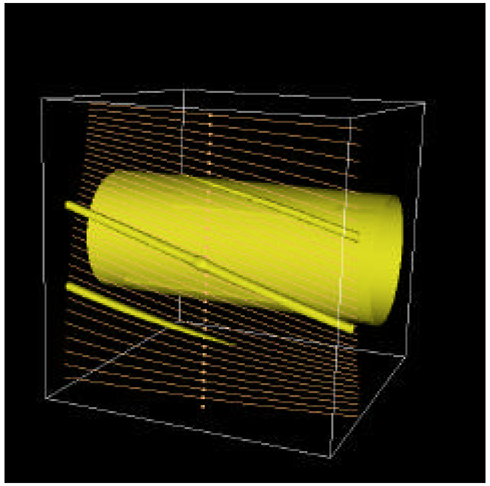
- Observations
  - *Even at early time, mass is rapidly distributed along field lines, and shows the appearance of striations (consistent with experimental observations)*



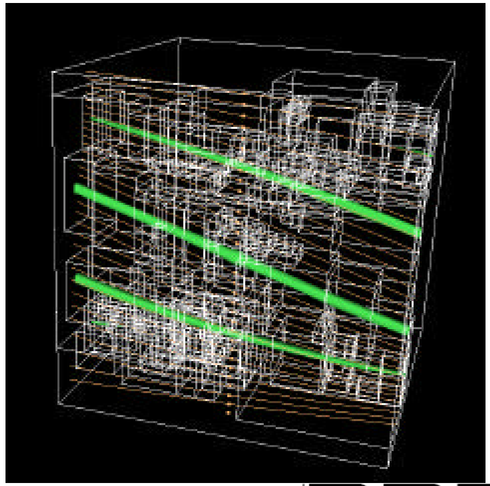
t=0.66. r



Mach No. (Peak 0.3)



Pressure



B_T

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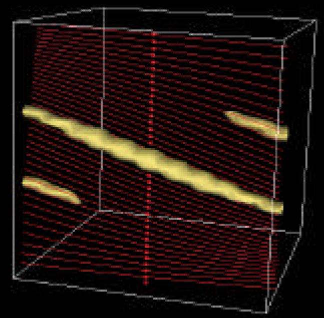
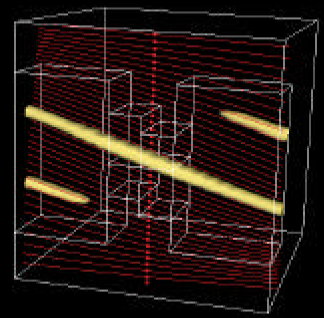
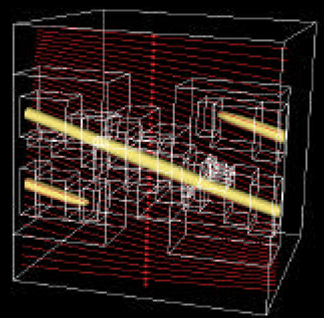
Results from AMR Simulations –Early time



Level 0

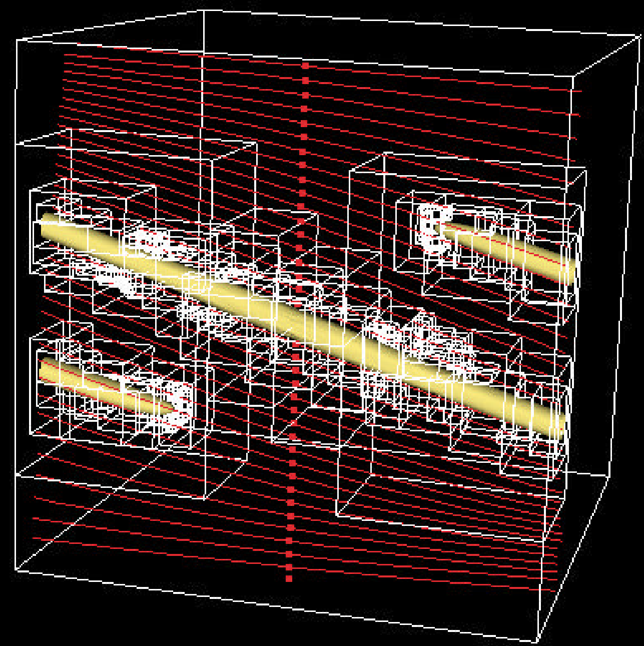


Level 0-1



Level 0-2

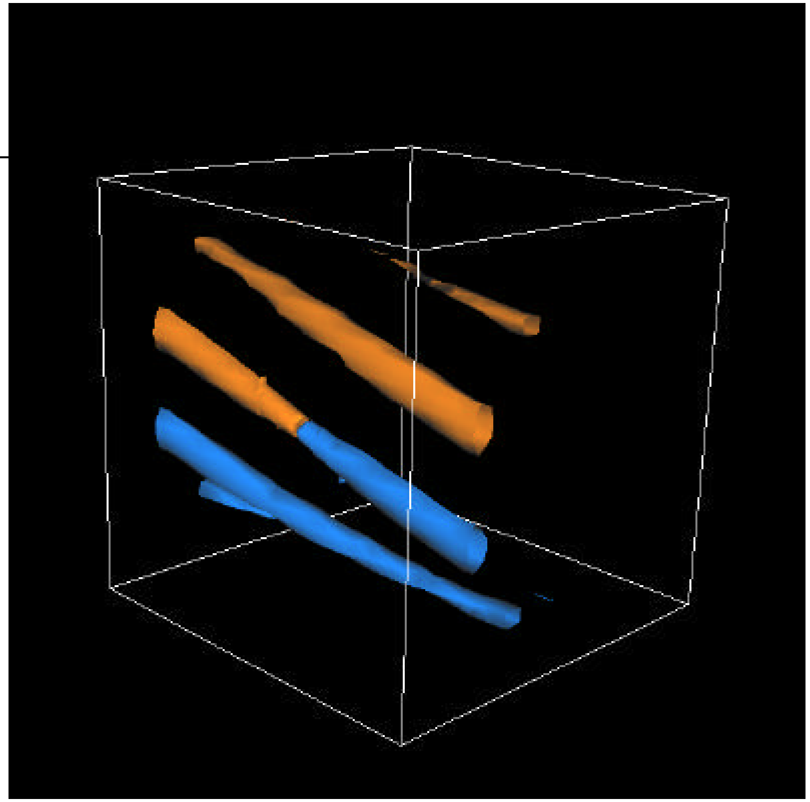
Density isosurfaces at t=0.45. Boxes indicate meshes at various AMR levels. Equivalent uniform mesh resolution: 512^3

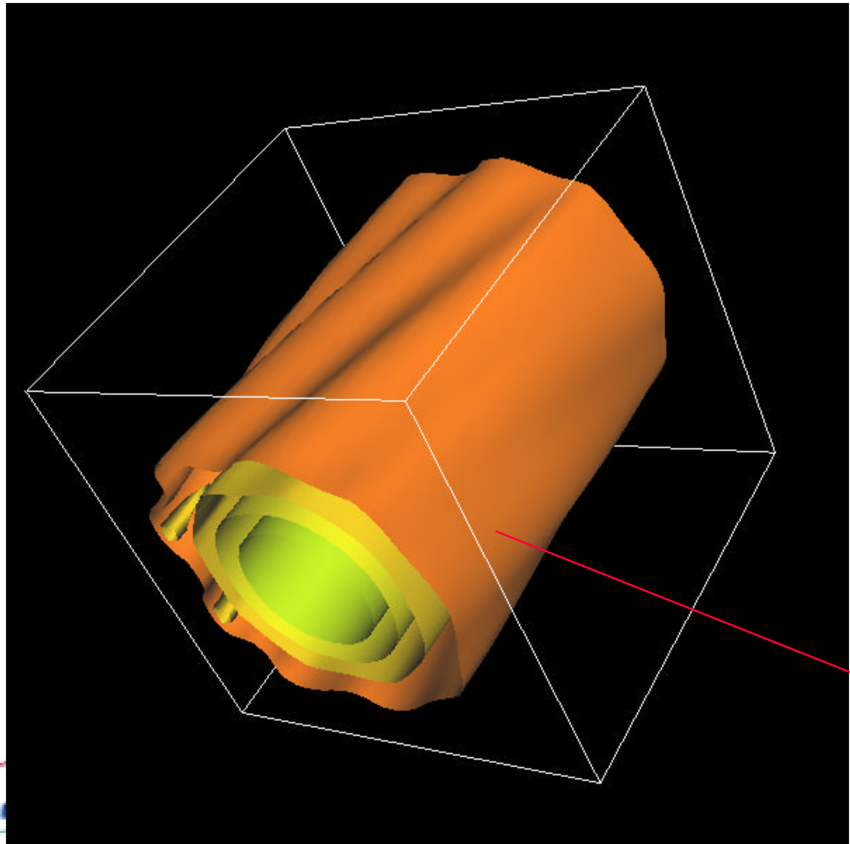


Level 0-3

Results from AMR Simulations

Parallel velocity isosurfaces at t=3.86





***t=3.86 Isosurfaces of plasma** b*

b_{max}***=0.51 at the pellet surface***

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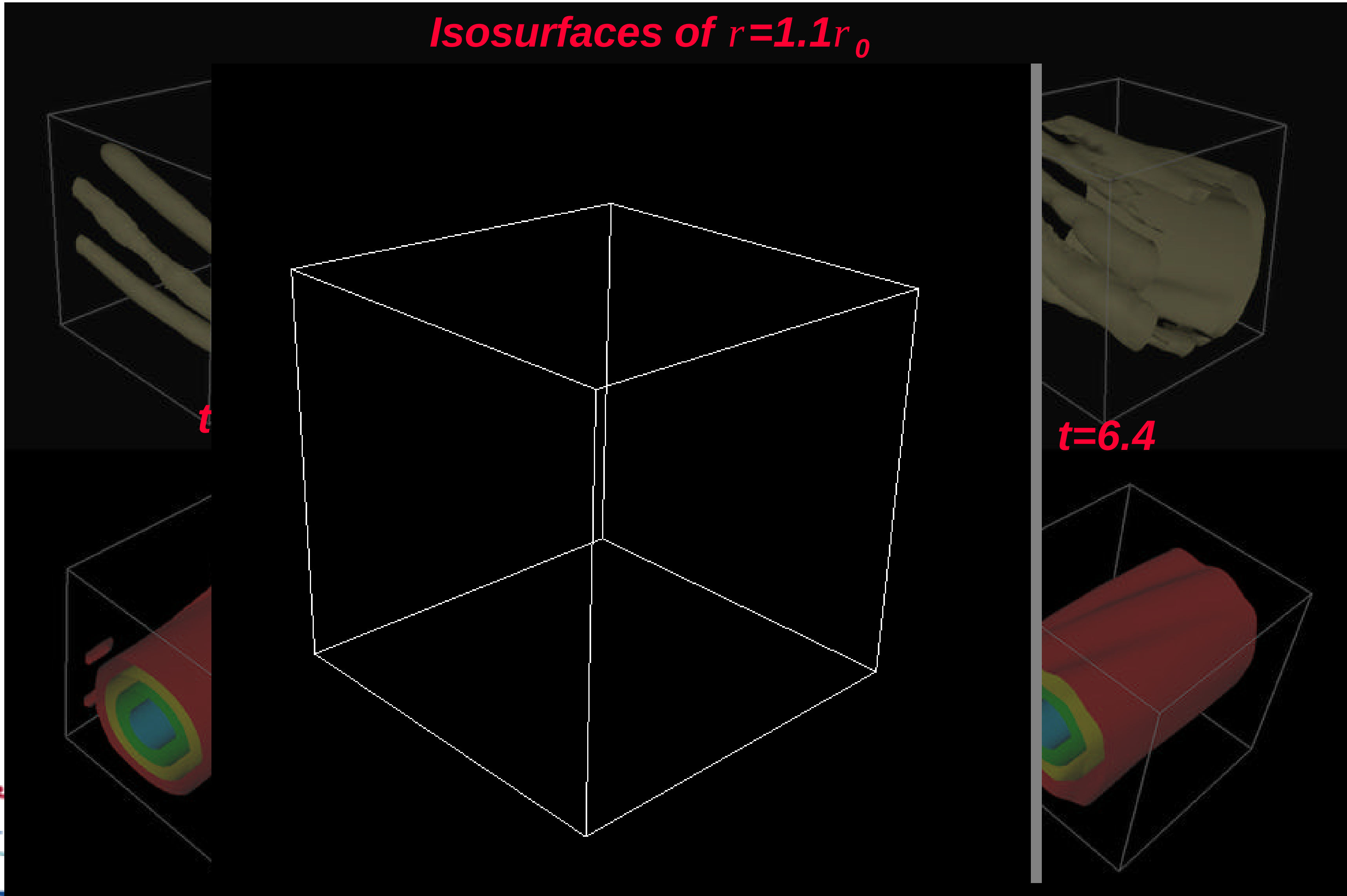
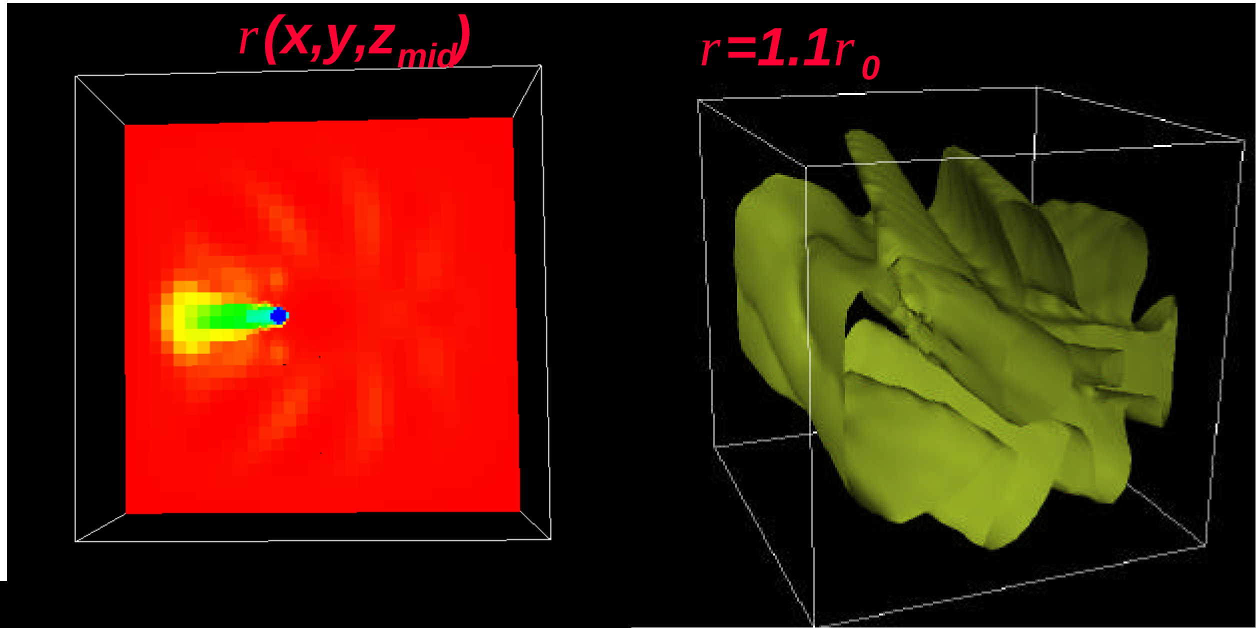
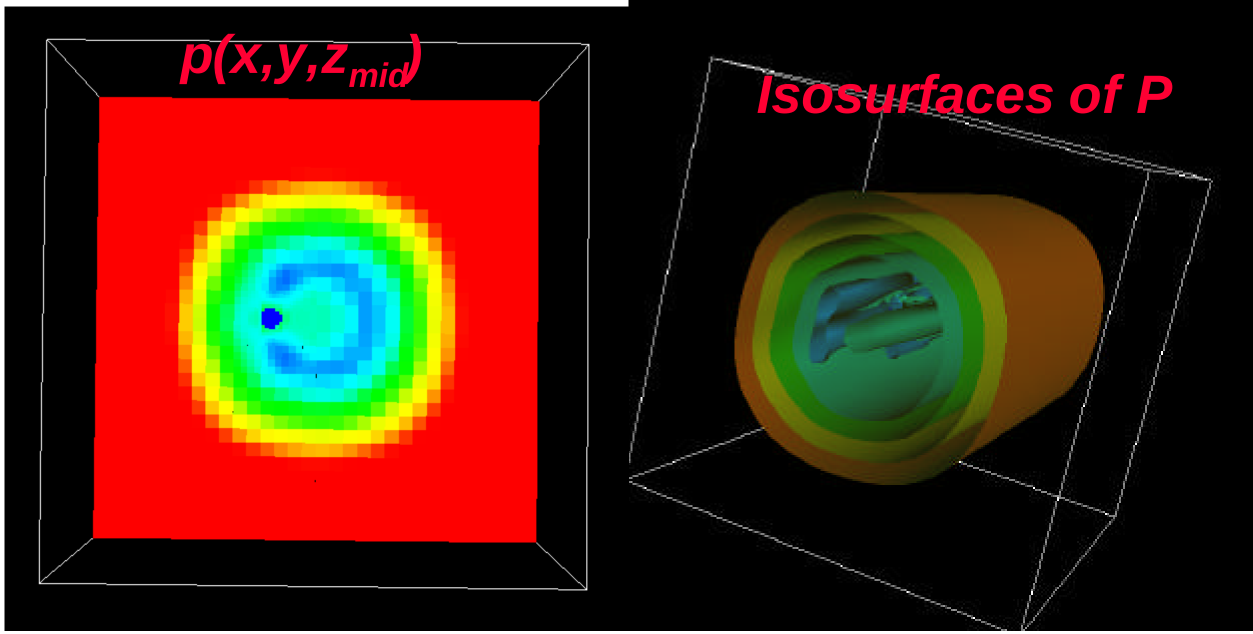
Results from AMR Simulations



Results from AMR Simulations

t~25





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Conclusion and Future Work

- First 3D AMR simulations of pellet injection in Cartesian geometry
  - *Includes model for pellet ablation and prescribed motion of pellet*
  - *Ablated mass is distributed along field lines*
  - *This preliminary study indicates that AMR is a viable approach to* efficiently *resolve the relatively small pellet*
- A conservative solenoidal B AMR MHD code was developed in 3D using the Chombo framework
  - *Unsplit upwinding method for hyperbolic fluxes*
  - **r¢** *B=0 achieved via projection*
- Future Work
  - *Toroidal forcing terms to mimic tokamak geometry*
  - *Investigate LFS and HFS pellet-launches*
  - *Better treatment of energy equation*
    - *Inclusion of anisotropic heat conduction*
  - *Realistic device parameters*