Two Fluid Numerical Experiments*

NIMROD Meeting

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Outline

- Physics of HMHD (Hall-MHD)
- Some formulation details
- 2^{nd} order s-i HMHD operator
- 0-D results
- The fly in the ointment
- Back to split Δt
- 1-D results (split Δt)
- NIMROD implementation status

Physics of HMHD

- We never learned this in graduate school (at least I didn't)
- Full (warm 2 fluid) dispersion relation has 3 waves (cubic)
- This was given by Stringer (1963) and is also discussed by Swanson

$$w = \omega^2$$

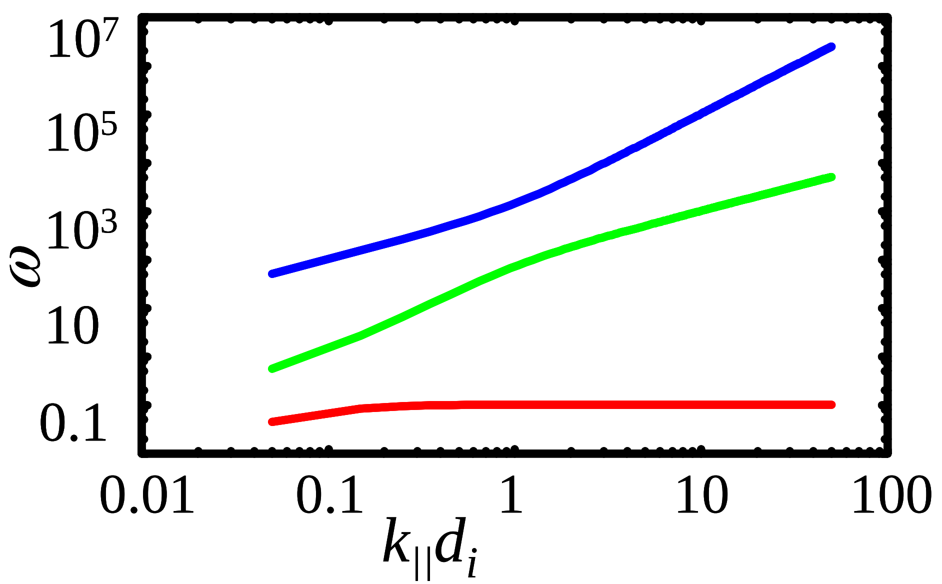
$$w^3 - A w^2 + B w - C = 0$$

$$A = k_\parallel^2 + \left(1 + \hat{\beta}\right) k^2 + H^2 k_\parallel^2 k^2$$

$$B = k_\parallel^2 k^2 \left(1 + 2\hat{\beta} + \hat{\beta} H^2 k^2\right)$$

$$C = \hat{\beta} k^2 k_\parallel^4$$

Physics of HMHD



Whistler, KAW, IIC $\quad k_\perp / k_{||} = 100, H = 0.1, \hat{\beta} = 0.01$

Some Formulation Details.1

- Convenient to define "field-line velocity" $\mathbf{u}_B$
- Almost electron velocity

$$\mathbf{u}_B = \mathbf{u} - \frac{1}{en}\left(\frac{\nabla\times\tilde{\mathbf{B}}}{\mu_0} - \frac{\lambda}{\mu_0}\tilde{\mathbf{B}} - \frac{\hat{\mathbf{b}}\times\nabla\,\tilde{P}_e}{B}\right);\ \lambda = \frac{\mathbf{B}\bullet\nabla\times\mathbf{B}}{B^2}$$

$$\Rightarrow \nabla\times\mathbf{u}_B\times\mathbf{B} = \nabla\times\left(\mathbf{u}\times\mathbf{B} - \frac{\nabla\times\tilde{\mathbf{B}}/\mu_0\times\mathbf{B} + \mathbf{J}_\parallel\times\tilde{\mathbf{B}} - \nabla_\perp\tilde{P}_e}{en}\right)$$

- Almost HMHD Ohm's law (and as close as we will come in this talk)

Some Formulation Details.2

- We will assume the induction equation becomes

$$\boxed{\frac{\partial\tilde{\mathbf{B}}}{\partial t} = \nabla\times\mathbf{u}_B\times\mathbf{B} + corrections}$$

and deal with corrections later

Some Formulation Details.3

- An interesting equation results for evolution of $\mathbf{u}_B$

$$\frac{\partial}{\partial t} Mn\,\mathbf{u}_B = \overleftrightarrow{\mathbf{F}} \bullet \boldsymbol{\xi} - \frac{\hat{\mathbf{b}}}{\Omega_i} \times \overleftrightarrow{\mathbf{F}} \bullet \mathbf{u}_B + corrections$$

$\overleftrightarrow{\mathbf{F}}$ is MHD force operator

- Which suggests several things
  - Use semi-impl differencing here and B equation as in NIMROD
  - Maybe it is important that Hall term is cross product with symmetric operator?

2nd-order SI HMHD operator

$$\tilde{\mathbf{B}}^{n+1/2} - \tilde{\mathbf{B}}^{n-1/2} = \Delta t \nabla \times \mathbf{u}_B \times \mathbf{B}$$

$$\left(Mn - \frac{\sigma \Delta t^2}{4} \overleftrightarrow{\mathbf{F}} + \frac{\sigma_H \Delta t}{2\Omega_i} \hat{\mathbf{b}} \times \overleftrightarrow{\mathbf{F}} \right) \bullet \left(\mathbf{u}_B^{n+1} - \mathbf{u}_B^{n} \right) =$$

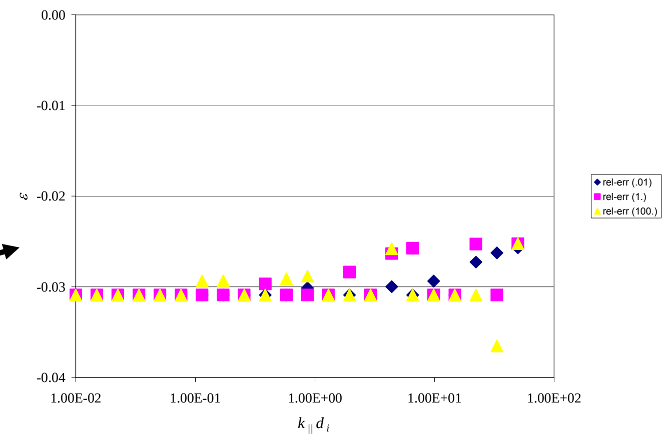
$$\Delta t \left(\mathbf{F}_{ex} - \frac{\hat{\mathbf{b}}}{\Omega_i} \times \overleftrightarrow{\mathbf{F}} \bullet \mathbf{u}_B^{n} \right)$$

Use asymmetric solver to invert operator

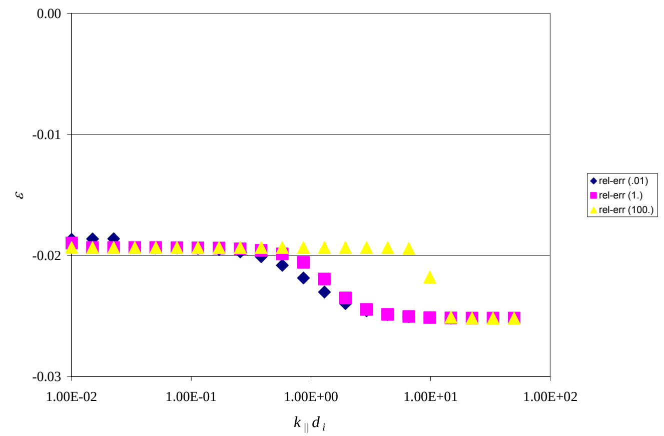
0-D Tests

- Replace all derivatives by k
- Time step exactly as outlined on previous slide

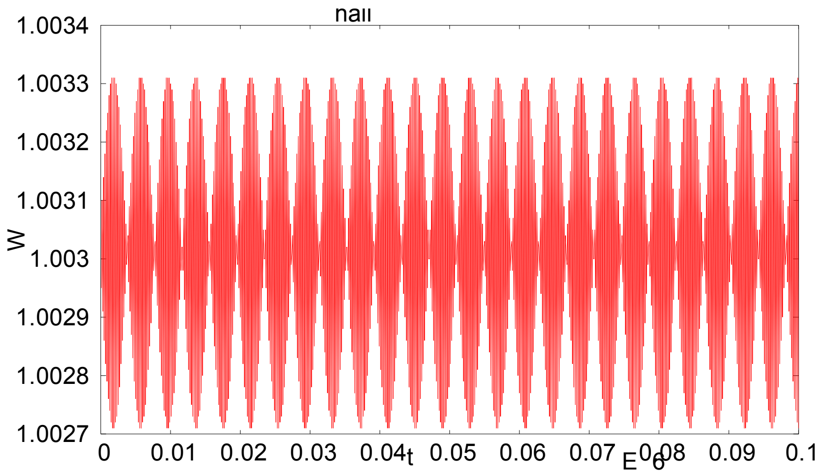
KAW mode



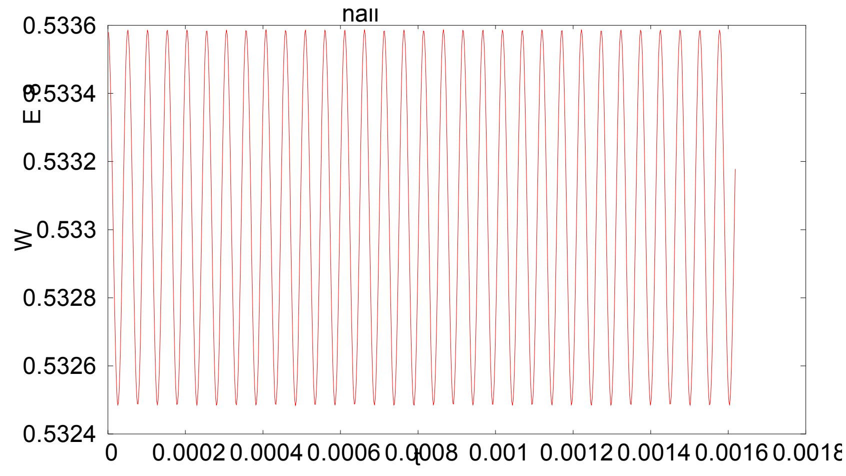
IIC mode



Energy Conservation



KAW



IIC

The Fly in the Ointment -1-D Tests

- 2nd-order does not work at all!
- High $k_\perp$ generated at boundary (inevitable, since spatial differencing is 1st-order there)
- Convectively unstable
- Cause is asymmetric operator
  - Any error in k between explicit and implicit terms causes instability (greater <u>or</u> lesser, in contrast to symmetric systems)

Back to the Future – Split Δt

- Use previous $\mathbf{u}_B$ advance, but split MHD and Hall terms
- Use 2nd-order si operator in Hall step
- PC on Hall step, if required

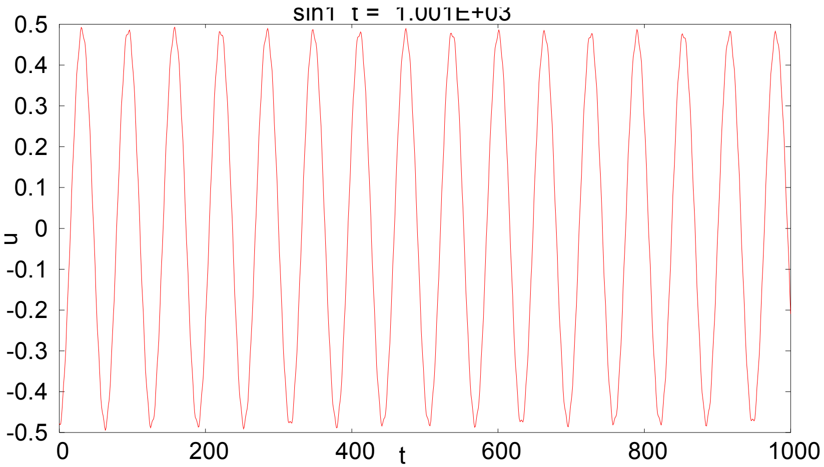
$$\left(Mn - \frac{\sigma \Delta t^2}{4} \overleftrightarrow{\mathbf{F}} \right) \bullet \left(\mathbf{u}_B^* - \mathbf{u}_B^n \right) = \Delta t \, \mathbf{F}_{ex}$$

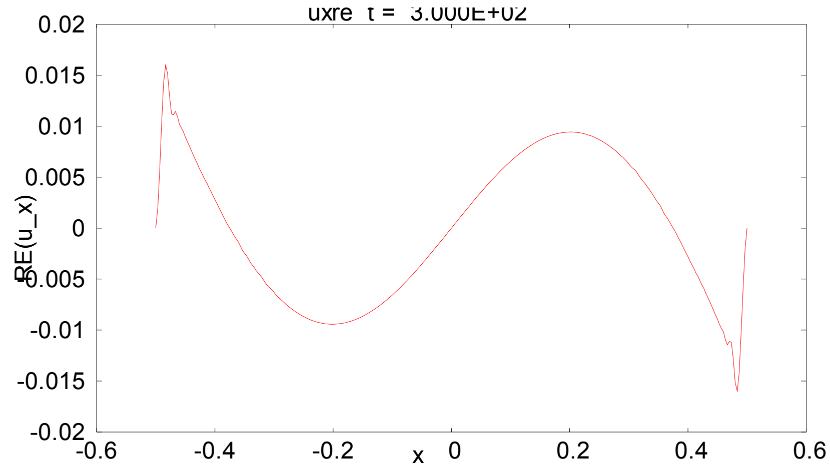
$$\left(Mn - \frac{\sigma \Delta t^2}{4} \overleftrightarrow{\mathbf{F}} \right) \bullet \left(\mathbf{u}_B^{n+1} - \mathbf{u}_B^* \right) = - \frac{\Delta t \hat{\mathbf{b}}}{\Omega_i} \times \overleftrightarrow{\mathbf{F}} \bullet \mathbf{u}_B^{pc}$$

where $pc = *$ or estimated 1/2 time level

That's right, it's the SI <u>MHD</u> operator here!!

1-D Results





$$\Delta t = 1\ \tau_A,\ N_x = 100,\ H = 0.1,\ k_{||} = 1,\ k_y = 0.1,$$

everything normalized to unity

NIMROD Implementation

- Yes, there is one
- It is 2^{nd}-order not split formulation
- Doesn't work (consistent with theory)
- Shows high k convective growth

Some Lessons Learned

- Don't assume asymmetric problems will be cooperative
- Energy conservation cures many ills (so does dissipation, but let's not go there)
- Most problems with HMHD step in explicit (rhs) evaluation
- *MUST MUST* use cross product with symmetric operator for explicit Hall piece
- Including asymmetric pieces in si operator breaks smoothing properties (not to mention hard to invert)
- Boundary condition for HMHD somewhat obscure
  - Can't have no penetration *and* conductor both when $H \neq 0$
  - Only one condition allowed (ideal case), *e.g.*
- It's easy to blame boundary for convective instabilities
- NIMROD may not be best for prototyping

Near-term Work

- Find dispersion for split scheme (0-D, 1-D)
- Implement successful 1-D algorithm in NIMROD
- Details about correction terms omitted so far
- Write-up
- Physics applications