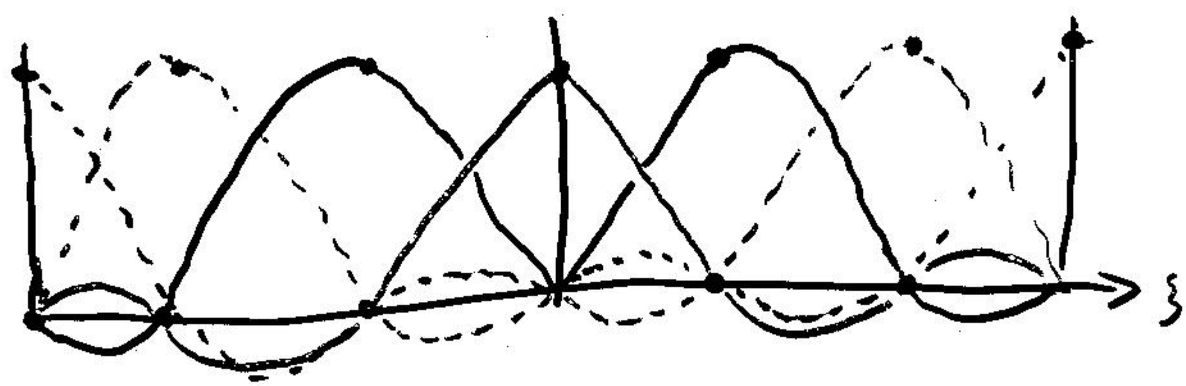
F. E. Basis Function Continuity Discussion

Carl Sovinec - CEMM mtg 4/5/04

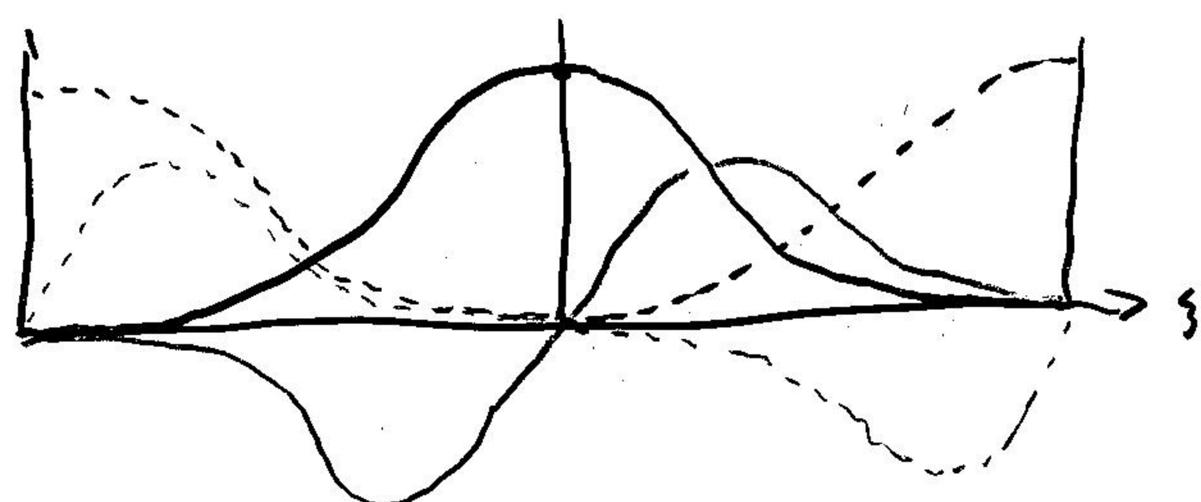
Example: 1D - cubics

C^0



(Lagrange or spectral)

C^1



(Hermite)

- C^0 has ~ 3 DOF / element
- C^1 has ~ 2 DOF / element

$\left.\right\}$ in 1D

Is one representation (C^0 or C^1) better than the other?

→ No obvious answer

→ Likely depends on the application

★ Advantages of C^1

1) Second derivatives have finite 'energy' (square integrable).

- reduced MHD + viscosity
- Hall s.i. operator without auxiliary field

$$\int d\vec{x}\ \vec{A}\cdot\Delta\vec{B} + \int d\vec{x}\ \nabla\times\left(\frac{\Delta t}{\mu_0 n e}(\nabla\times\Delta b)\times B_0\right)\cdot\nabla\times\left(\frac{\Delta t}{\mu_0 n e}(\nabla\times\vec{A})\times B_0\right)$$
$$= rhs$$

2) Continuity of derivatives seems desirable qualitatively.

★ Disadvantages of C^1

1) Possibly slower convergence if the solution isn't smooth.

2) Mappings are more restricted.

why?

1) → FE approach stems from variational / Galerkin formulation and the choice of solution space.

physical model → diff. eqns. → strong form

$$Lu = f$$

- mapping from space containing u to space containing f
- space containing u has higher continuity.

Strong form (continued)

- If $\int_A f^2 < \infty$ and

L is a second (fourth) - order diff op., then L is a mapping

from $\mathcal{H}^2$ $(\mathcal{H}^4)$ to $\mathcal{H}^0$

(here, superscript indicates degree of deriv. with finite energy) $[\mathcal{H}^s]$

- Can restrict to a space satisfying b.c.s to order s-1 only $\rightarrow$ $\mathcal{H}^s_{\mathcal{D}}$.

- Weak form

$$(Lu, v) = (f, v) \quad \text{for all } v \text{ in } \mathcal{H}^s_B ?$$

int by parts

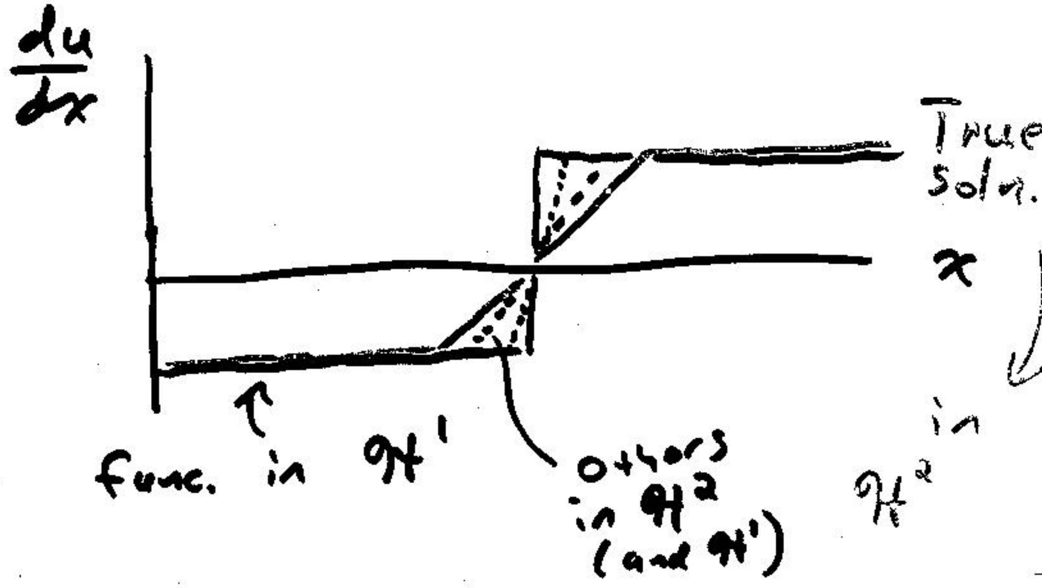
$$a(u, v) = (f, v)$$

→ energies of derivs of order m appear (only)

$s = 2m$ or $2m-1$

→ <u>Enlarge</u> Space to $\mathcal{H}^m_B$??

Yes if minimum / stationary 'point' (function) is the same.



- FE approximation uses a family of subspaces in $\mathcal{H}^M_\mathcal{B}$ → chooses 'best' approximation in each space (parameterized by h, p).

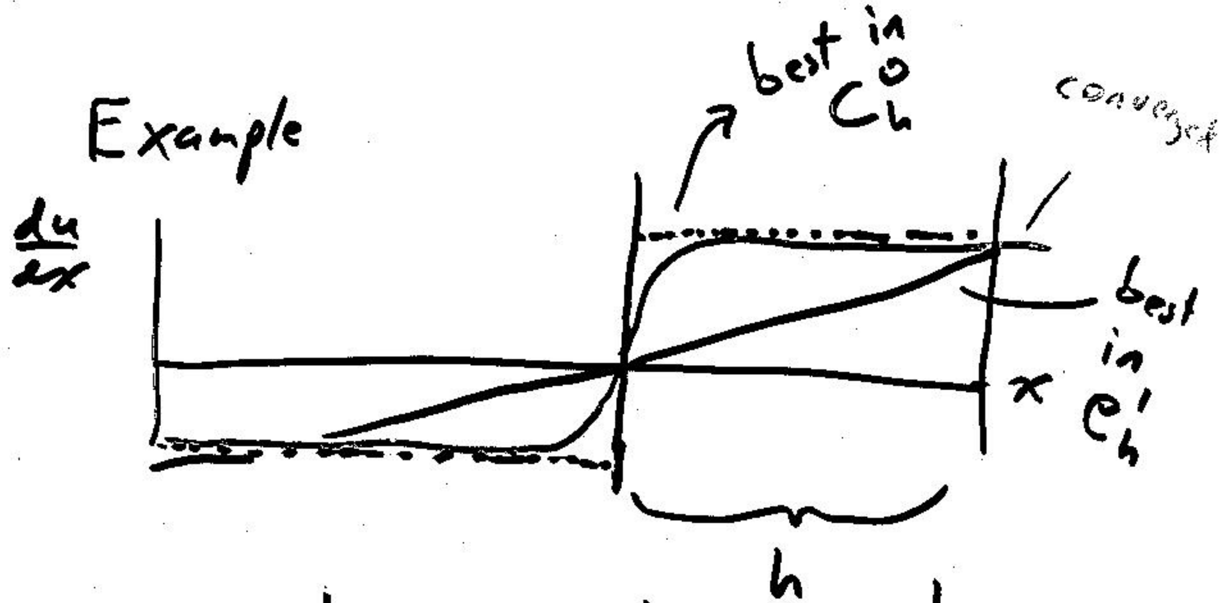
→ Resolve solution by progressing to subspaces that contain more of $\mathcal{H}^M_\mathcal{B}$

→ <u>C^1 vs. C^0</u> is a question of whether it helps to restrict in the process of converging.

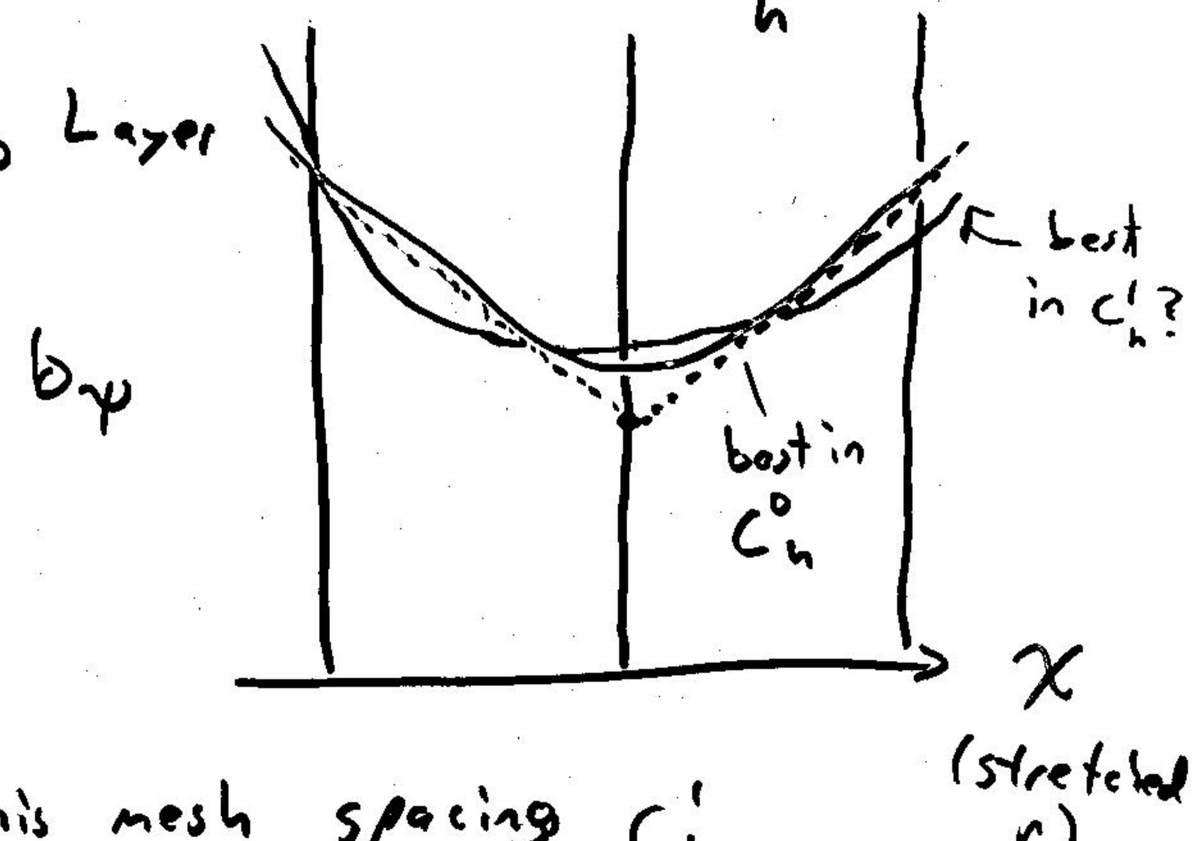
[Previous page argues converged solution is the same.

• If the converged solution has C^1 continuity, how can C^0 do better for the F.E.A.?

Generic Example



Tearing Layer



• For this mesh spacing C_h^1 solution would overshoot.

2) Mappings are more restricted with C^1.

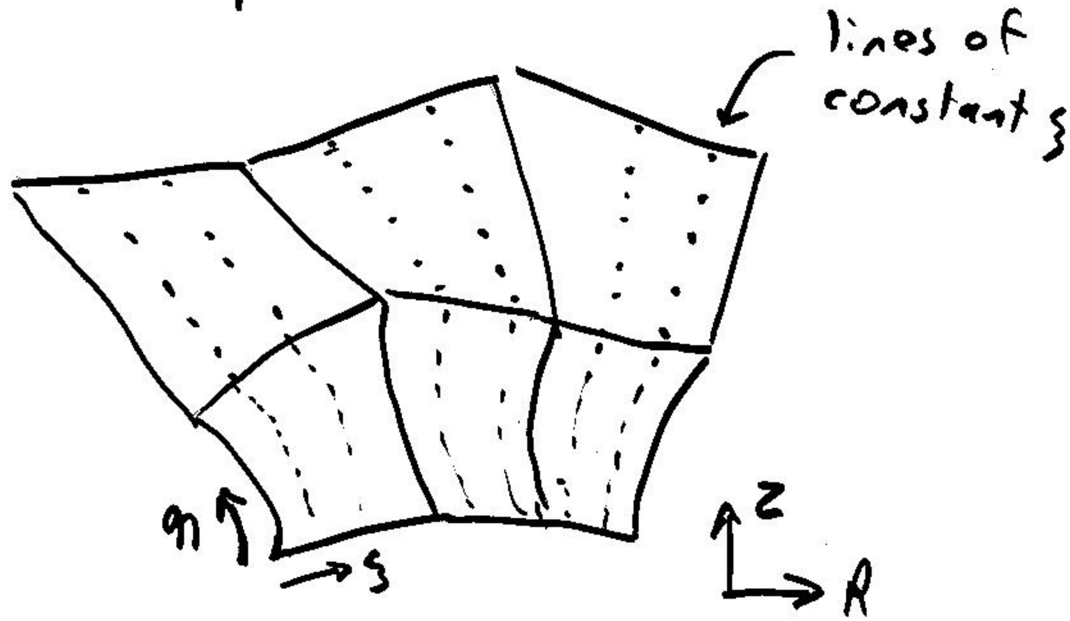
- In F.E., the physically motivated operator becomes a norm of the error, which is used to choose the best soln in S^h.
- In this norm, the F.E. soln is better than interpolate functions in the same space.

→ Makes the connection to Taylor approximation wrt ind. vars. of PDE (not logical or element coordinates).

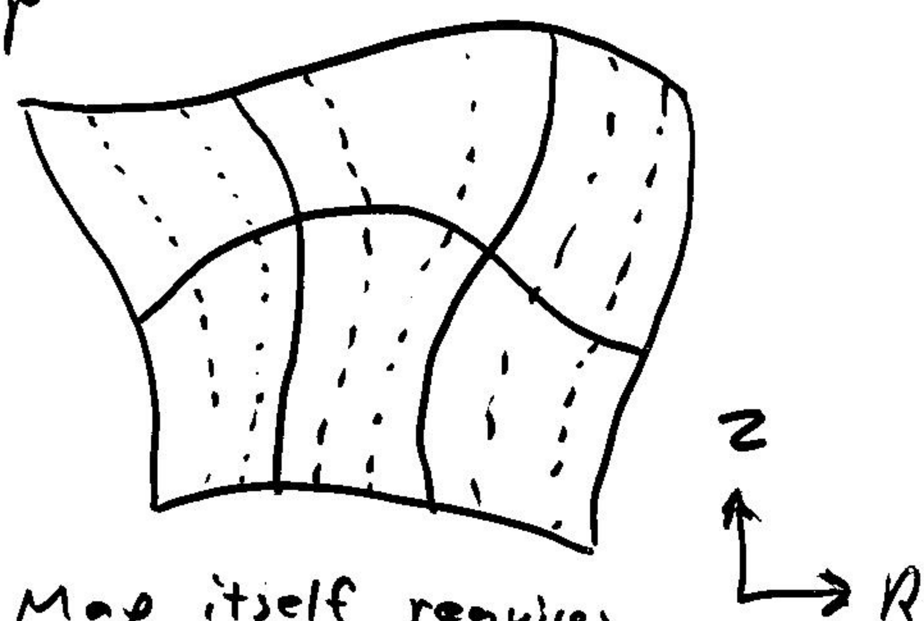
→ With the exception of special cases, (reduced quintic on triangles with linear maps), C^1 requires a C^1 map.

$$R(\xi,\eta) \quad , \quad Z(\xi,\eta)$$

C^0 map:



C^1 map



→ Map itself requires a global solve of $R(\xi,\eta)$, $Z(\xi,\eta)$.

→ Not used for structural shell elements.