M3D CDX-U Benchmark Status

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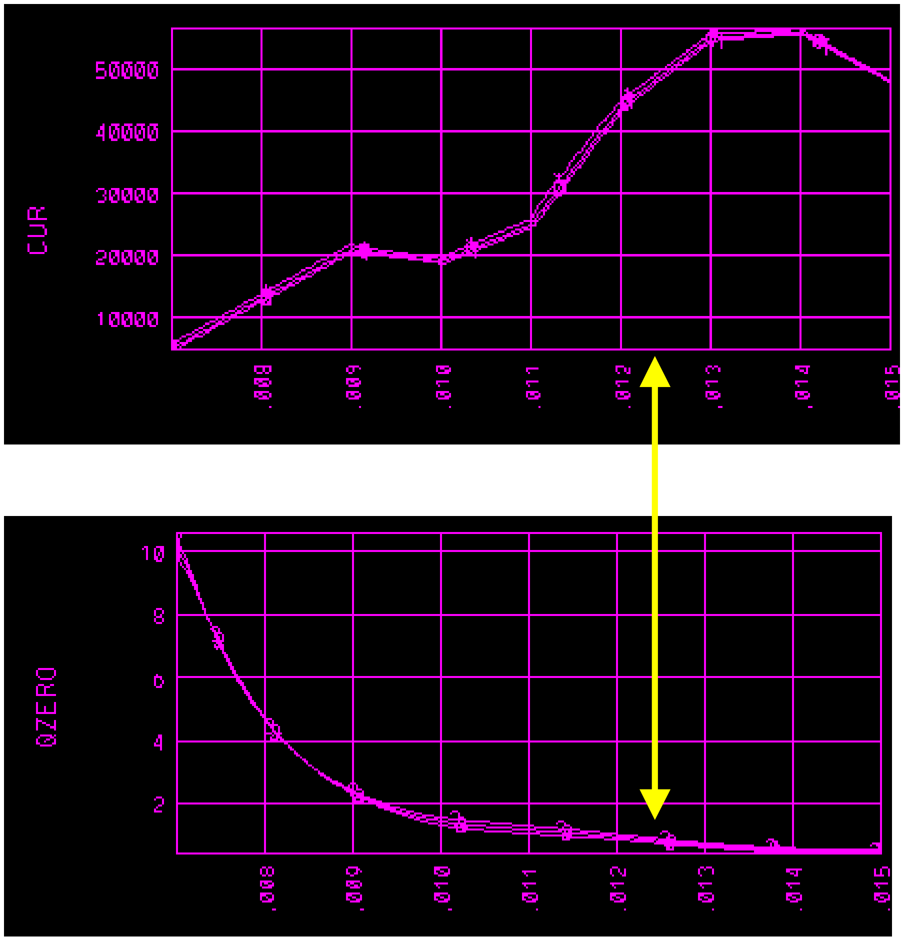
Characteristics of the Current Drive Experiment Upgrade (CDX-U)

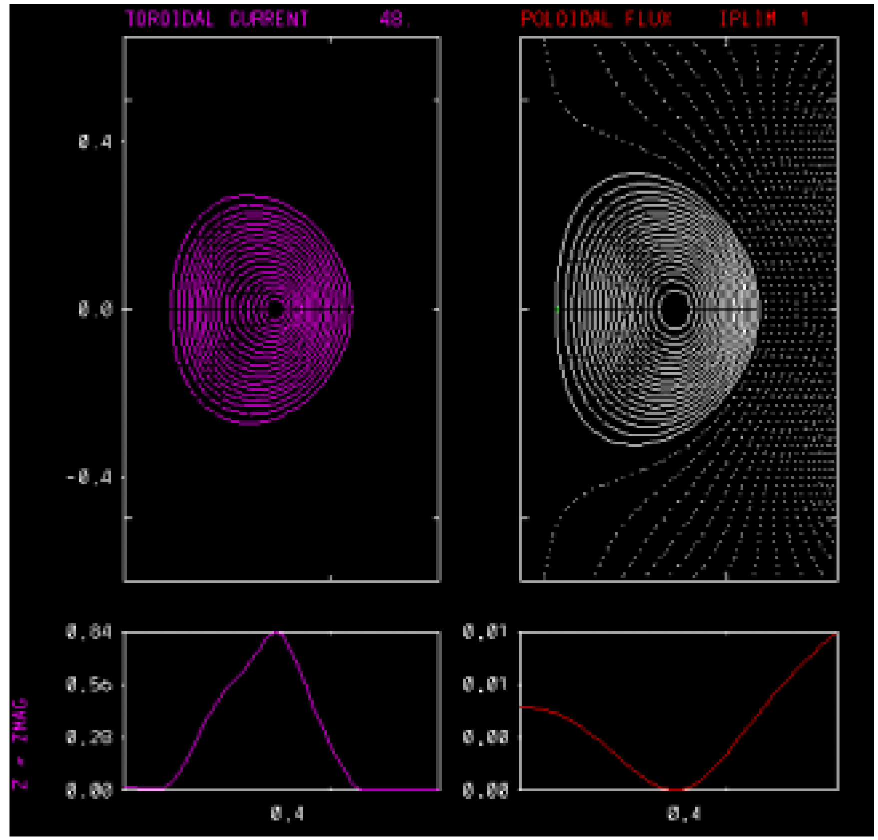
- Low aspect ratio tokamak (R_0/a = 1.4 – 1.5)
- Small (R_0 = 33.5 cm)
- Elongation $\kappa \sim 1.6$
- $B_T \sim 2300$ gauss
- $n_e \sim 4\times10^{13}$ cm^{-3}
- $T_e \sim 100$ eV
- $I_p \sim 70$ kA

- Soft X-ray signals from typical discharges indicate two predominant types of low-*n* MHD activity:
  - sawteeth
  - “snakes”

The TSC Sequence

TSC follows 2D (axisymmetric) evolution of typical CDX-U discharge

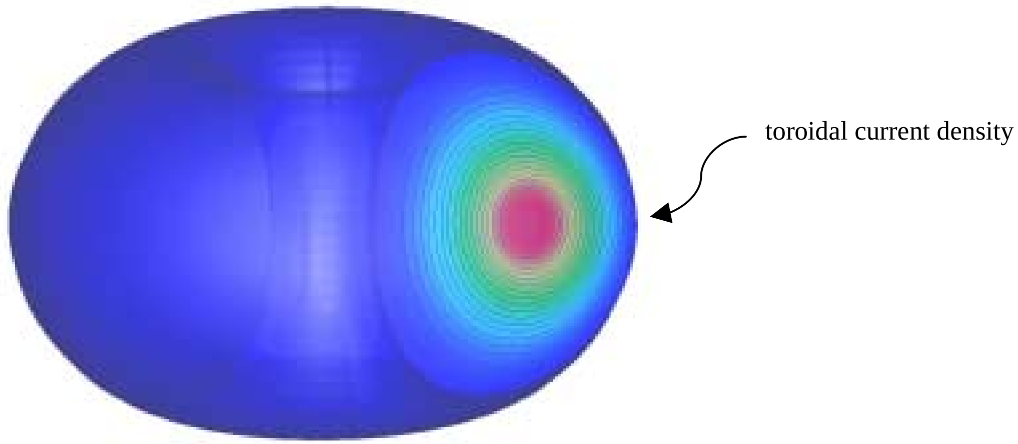


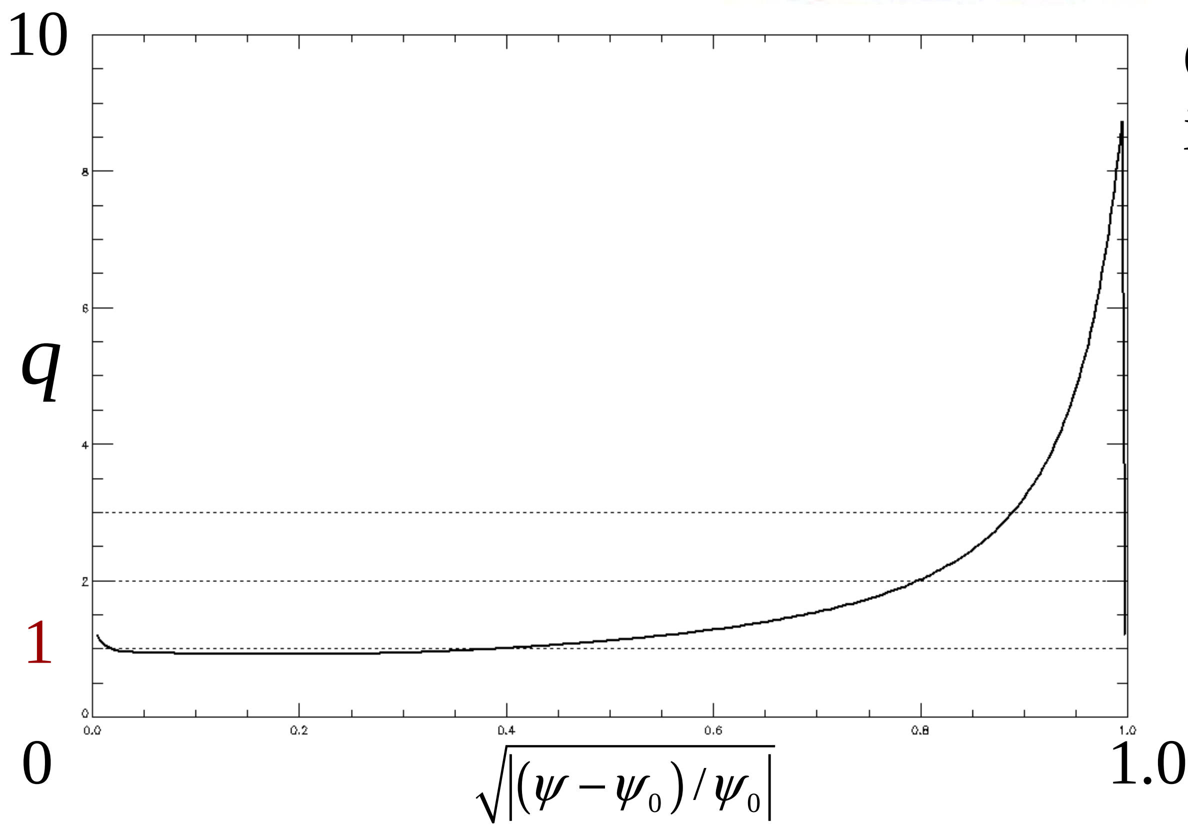


Equilibrium at t=12.40ms (as q_0 drops to 0.92) is used to initialize 3D runs

Device Aspect Ratio (Low-A) Case

- Equilibrium taken from a TSC sequence (Jsolver file).
- Shaped cross-section
- $R/a = 1.4$
- $q_{min} \approx 0.922$
- $q(a) \sim 9$



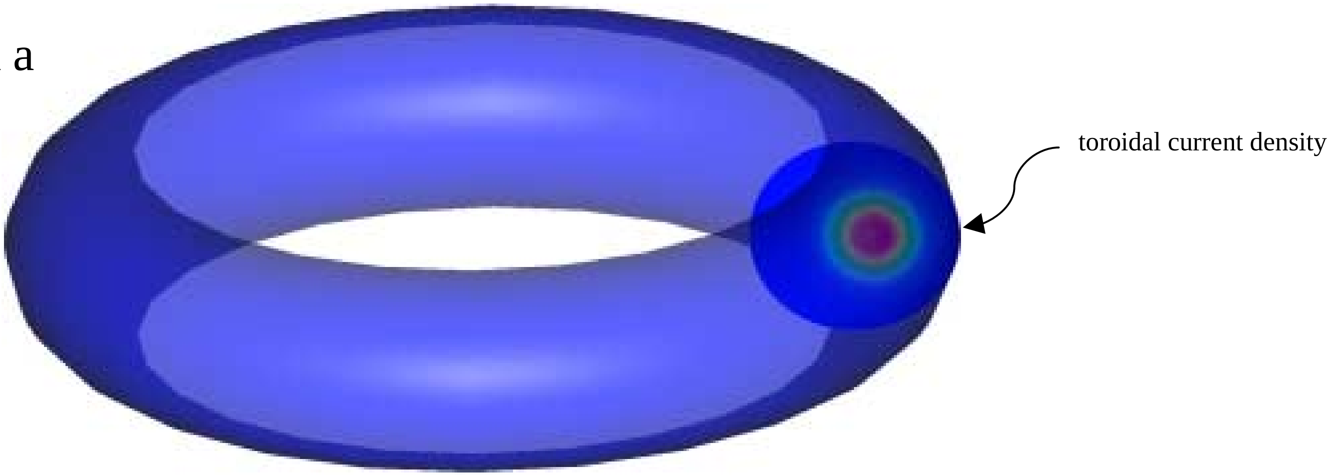


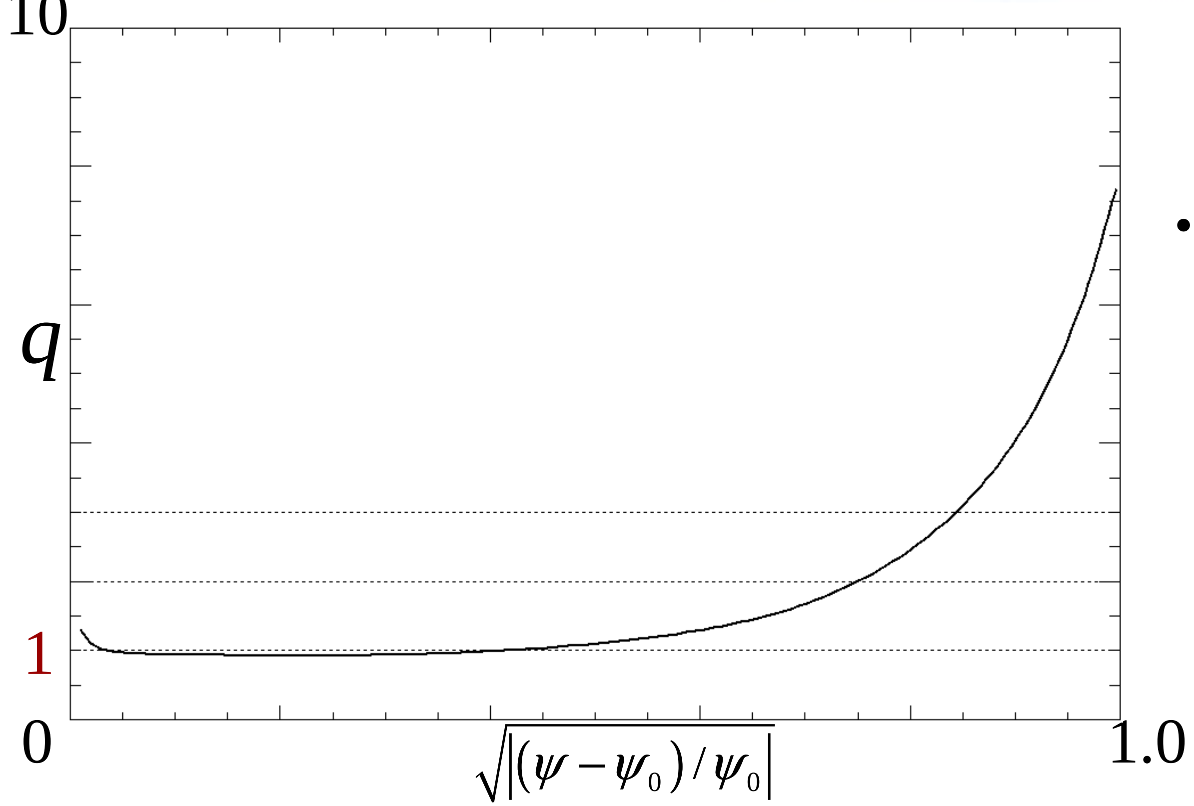
Questions to investigate:

- Linear growth rate and eigenfunctions
- Nonlinear evolution
  - disruption?
  - stagnation?
  - repeated reconnections?

Large Aspect Ratio Case

- Equilibrium taken from a TSC sequence (Jsolver file).
- Circular cross-section
- $R/a = 3.75$
- $q_{min} \approx 0.927$
- $q(a) \sim 8$





- Note: this does not correspond to any actual device. Chosen for ease of cross-code comparison.

Baseline Parameters for Cross-code Benchmark

Lundquist Number S	$\sim 2\times10^4$ on axis.
Resistivity η	Spitzer profile $\propto T_{eq}^{-3/2}$, cut off at $100\times\ \eta_0$
*Prandtl Number Pr	10 on axis.
*Viscosity μ	Constant in space and time.
*Perpendicular thermal conduction $\kappa_\perp$	0
*Parallel thermal conduction $\kappa_{\|\|}$	0
Peak Plasma β	$\sim 3\times10^{-2}$ (low-beta).
Density Evolution	Turned on for nonlinear phase.
Nonlinear initialization	Pure n=1 perturbation such that $\frac{\max(B^1_{\rm pol})}{\max(B^0_\phi)} = 10^{-4}$

*Non-physical values chosen to aid in cross-code benchmark.

Comparing the Codes

M3D and NIMROD are both parallel 3D nonlinear extended MHD codes in toroidal geometry maintained by multi-institutional collaborations, and comprise the two members of the Center for Extended MHD Modeling (CEMM) SciDAC.

M3D

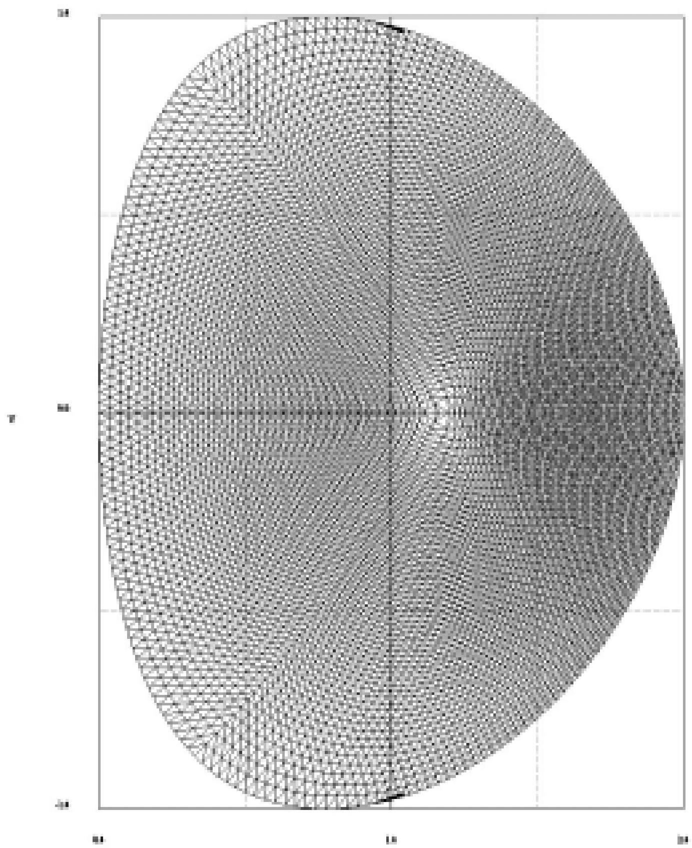
- Uses linear finite elements in-plane.
- Uses finite differences between planes or pseudo-spectral derivatives.
- Partially implicit treatment allows efficient time advance but requires small time steps.
- Linear operation: full nonlinear + filtering, active equilibrium maintenance.
- Nonlinear operation: all components of all quantities evolve nonlinearly.

NIMROD

- Uses high-order finite elements in-plane.
- Uses Fourier decomposition in toroidal direction.
- Fully implicit treatment requires costly matrix inversions but allows large time steps.
- Linear operation: evolve perturbations to particular modes only.
- Nonlinear operation: perturbations to fixed equilibrium are evolved, with nonlinear couplings between modes.

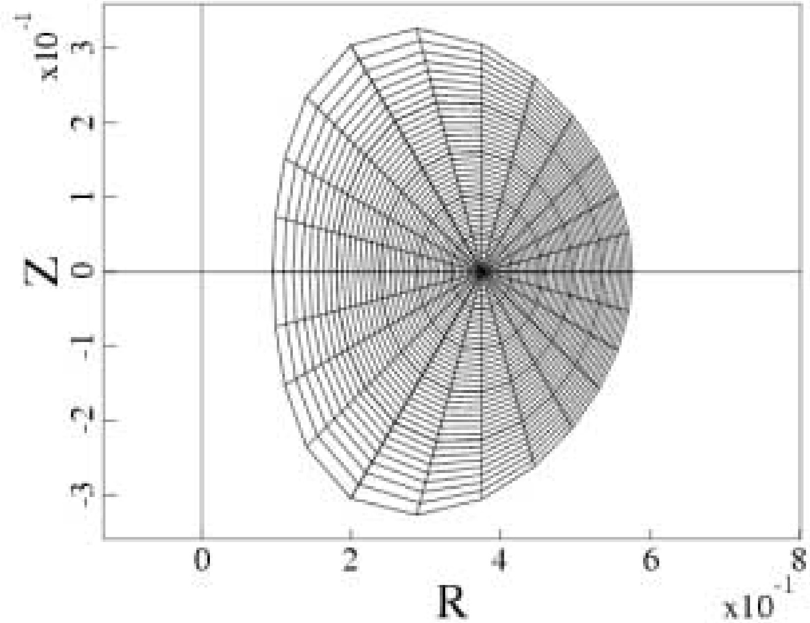
Poloidal Meshes for the low-*A* Case

M3D



- 89 radial zones, up to 267 in θ in unstructured mesh
- Linear basis functions on triangular elements
- Conducting wall
- Finite differences toroidally; 24 planes

NIMROD

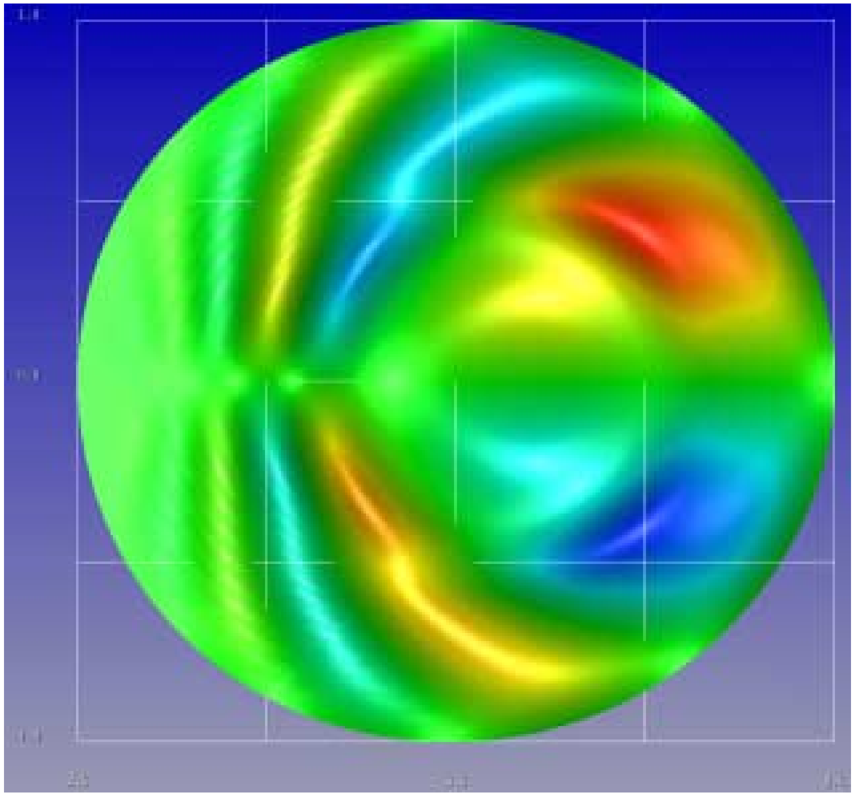


- 40×24 structured grid
- 4th order basis functions on quadrilateral elements
- Conducting wall
- Fourier decomposition toroidally; 10 or more modes retained

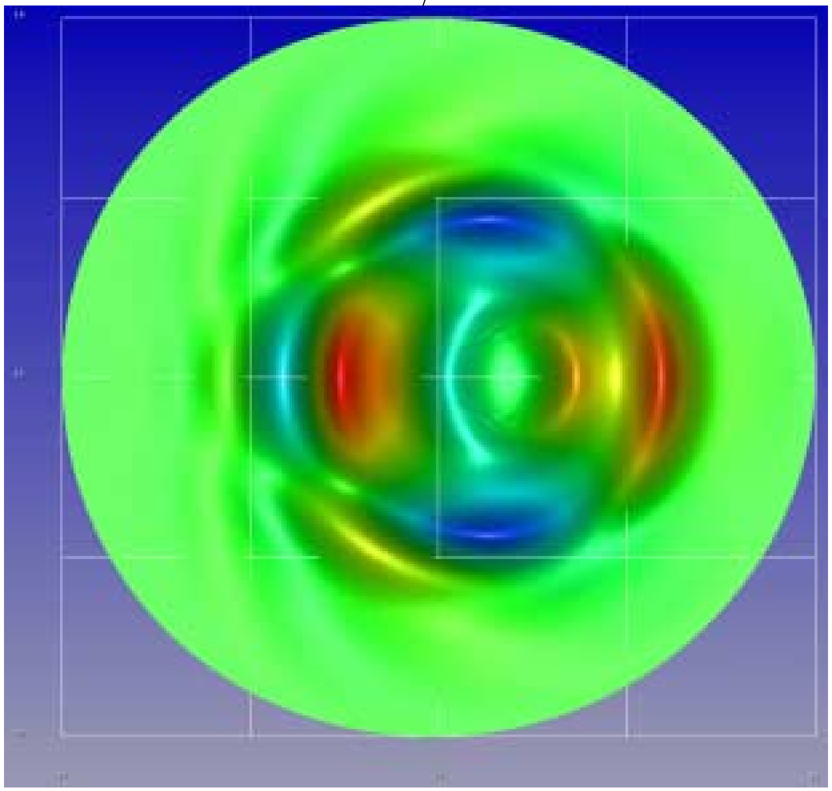
Large Aspect Ratio:

n=1 Eigenmode

Incompressible velocity stream function U



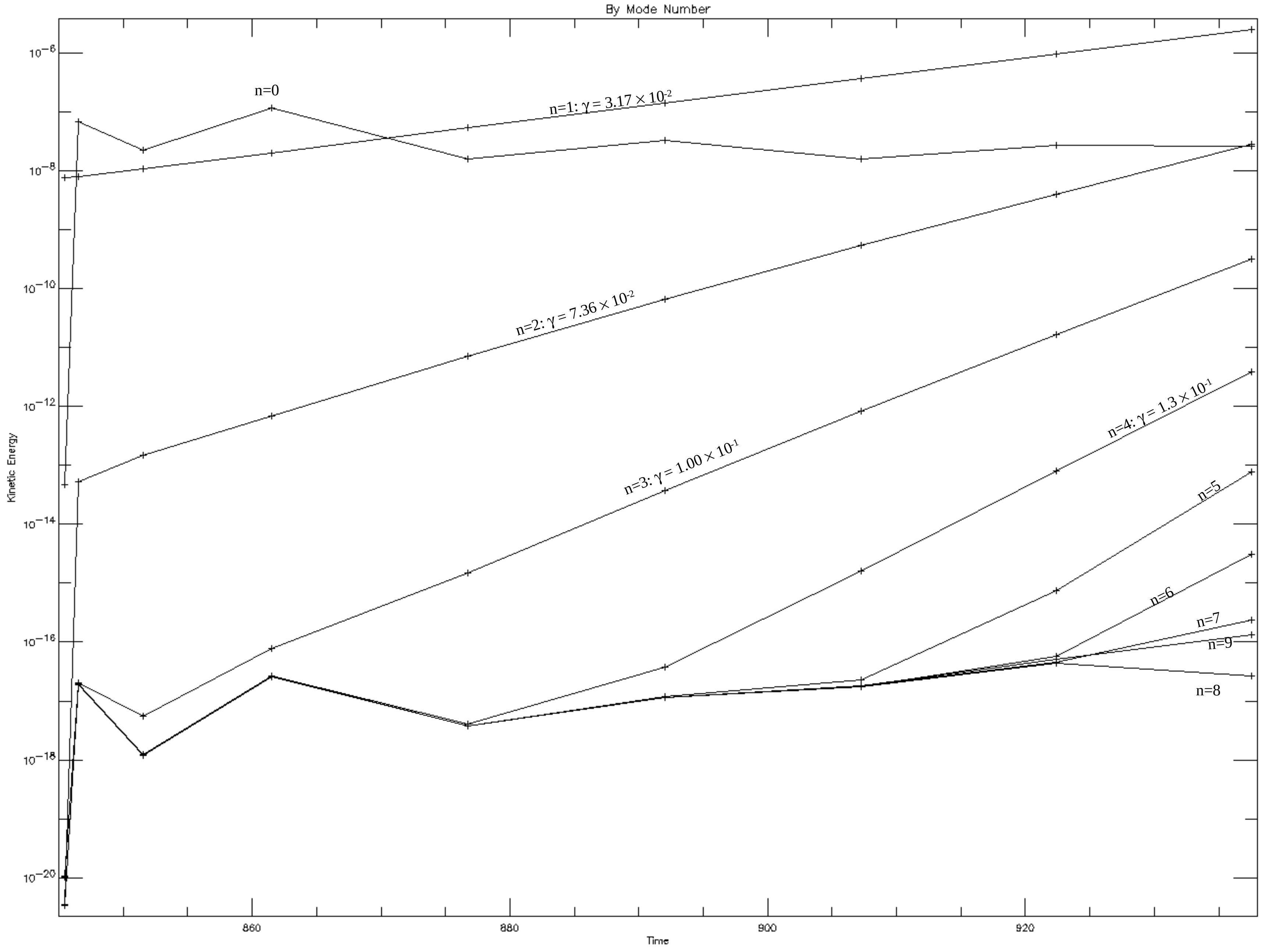
Toroidal current density J_ϕ



$\gamma\tau_A = 3.28 \times 10^{-2} \rightarrow$ growth time = 30.5 τ_A

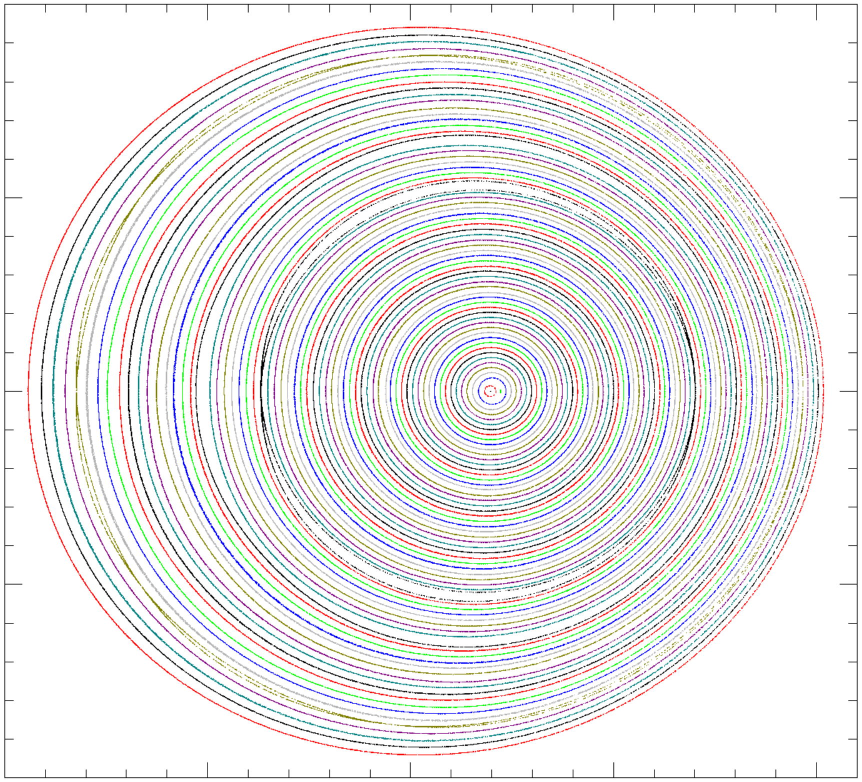
Agrees with NIMROD value to within 7%.

Large Aspect Ratio: Nonlinear Kinetic Energy History

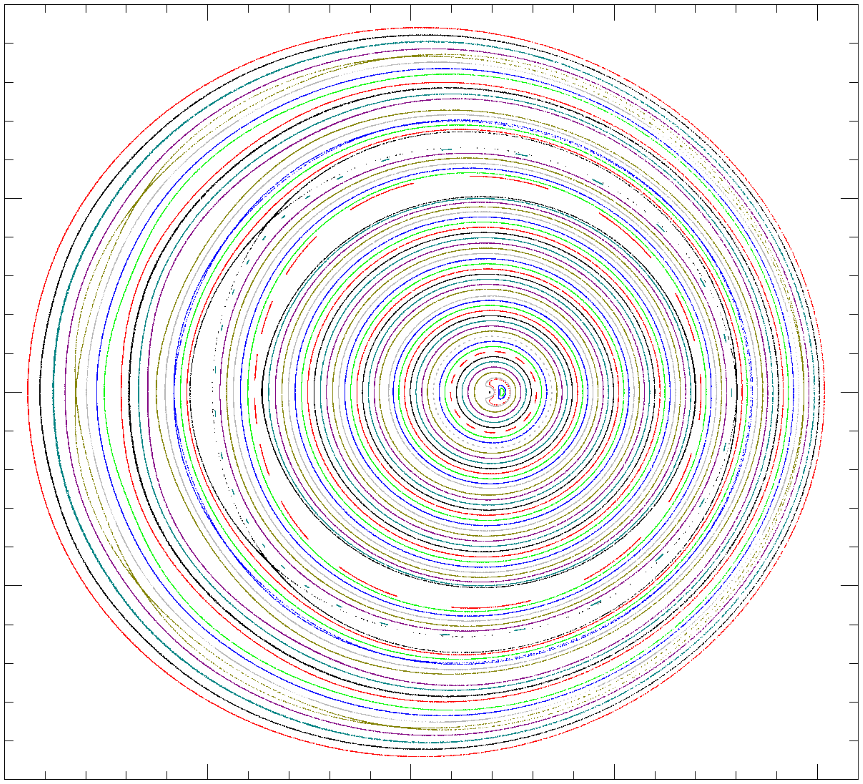


Large Aspect Ratio: Nonlinear Time Series

Poincaré Plots



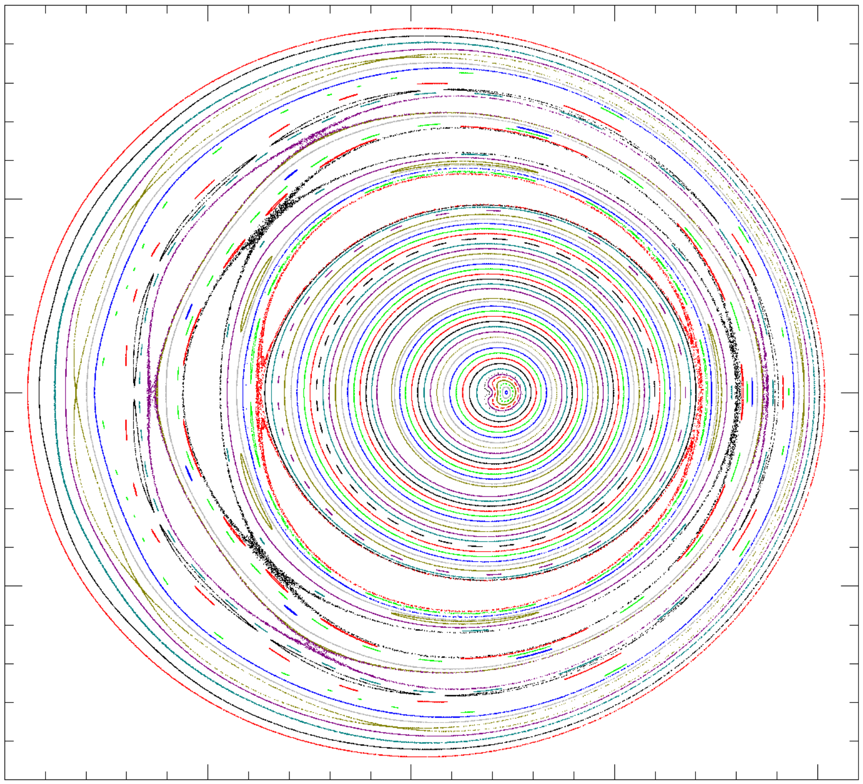
t = 845.45; chopped × 6.498543



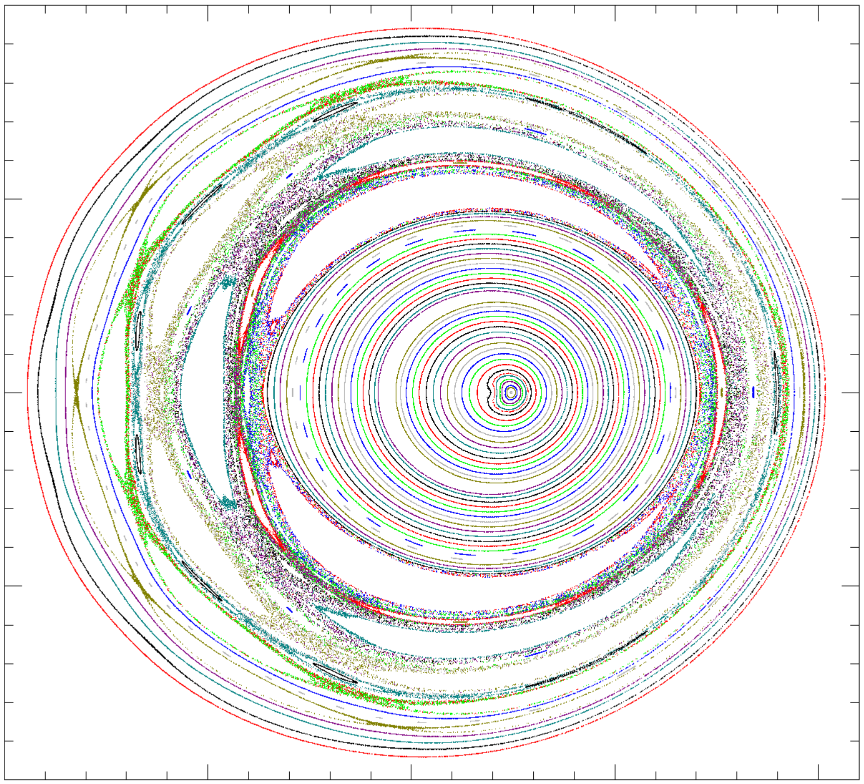
t = 892.00

Large Aspect Ratio: Nonlinear Time Series

Poincaré Plots, Continued



t = 922.37



t = 937.56

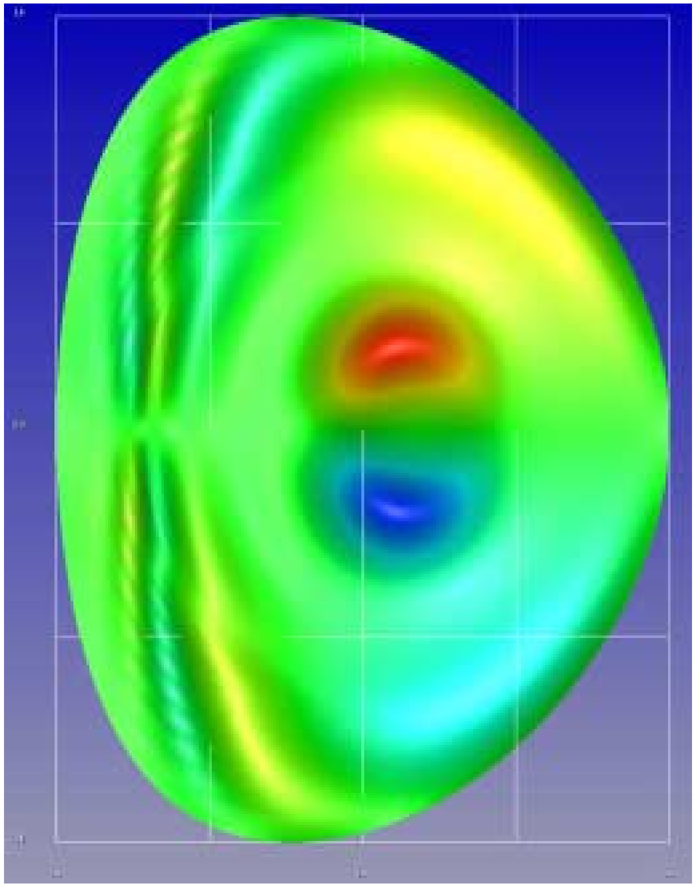
<u>All</u> islands visible in these plots have toroidal mode number *n*=1.

Note: Run halted because of onset of stochasticity.

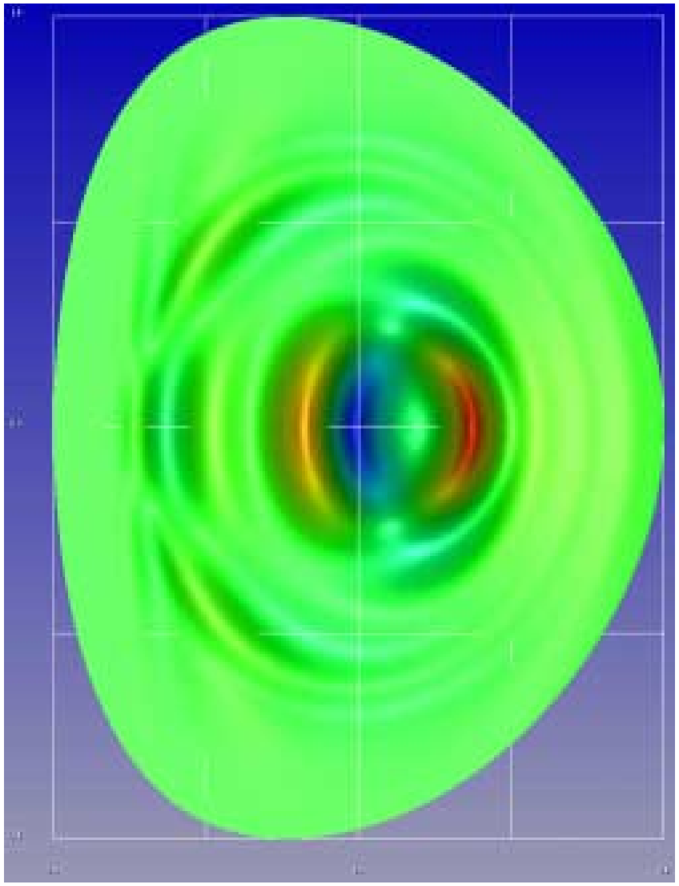
Low Aspect Ratio:

n=1 Eigenmode

Incompressible velocity stream function U

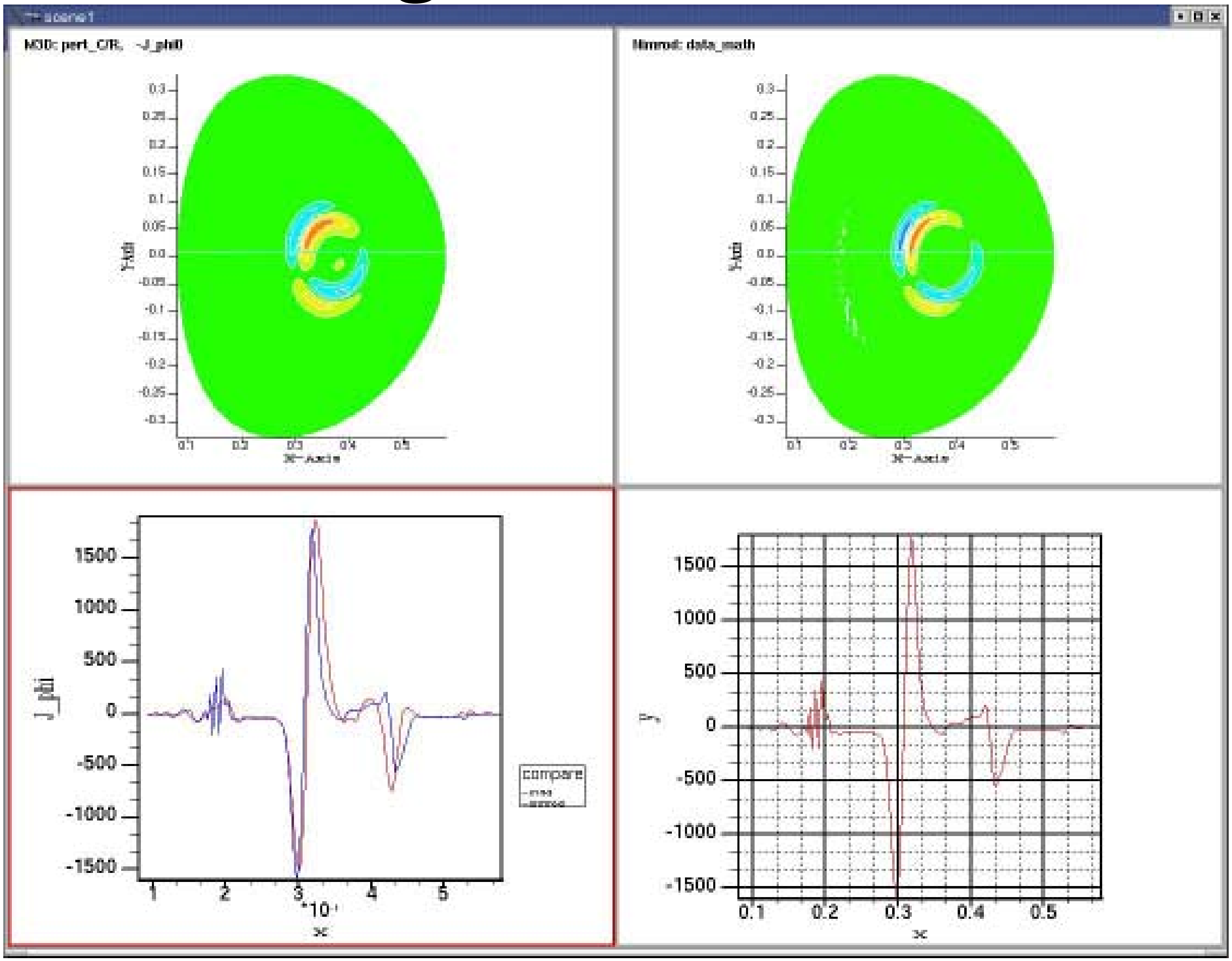


Toroidal current density J_ϕ



$\gamma\tau_A = 8.61\times 10^{-3} \rightarrow$ growth time = 116 τ_A

Predicted Eigenmode Shows Qualitative Agreement with NIMROD

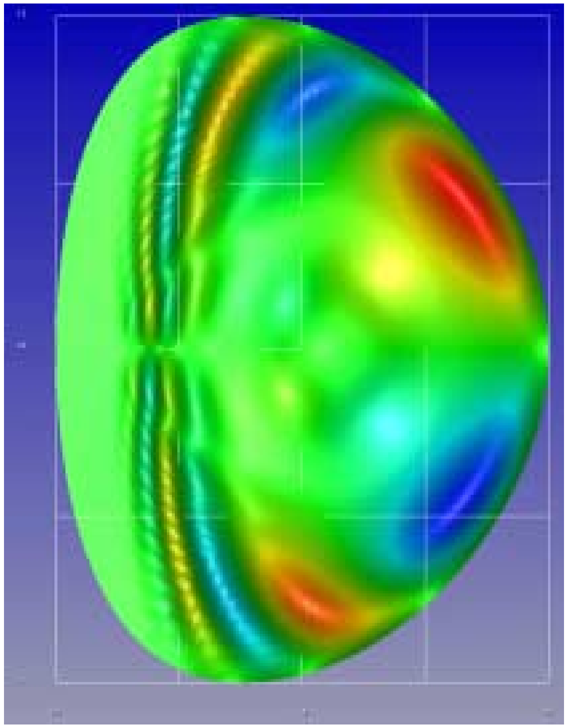


Low Aspect Ratio:

Higher n Eigenmodes

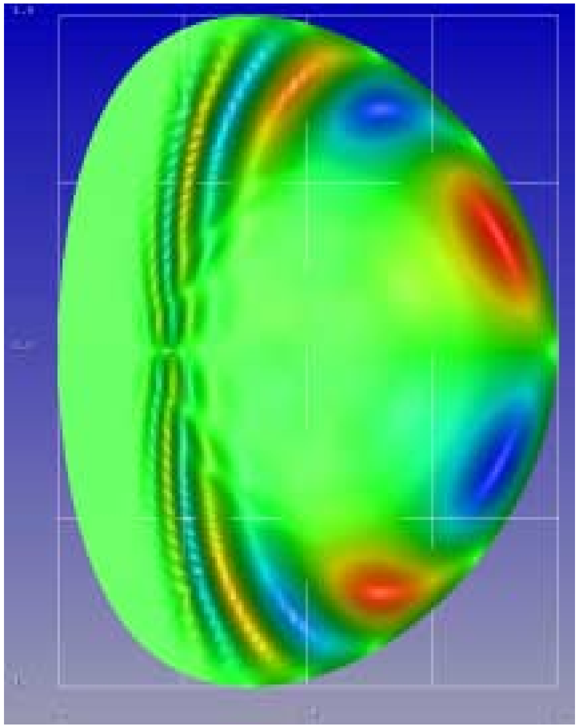
Incompressible velocity
stream function U

$n = 2$



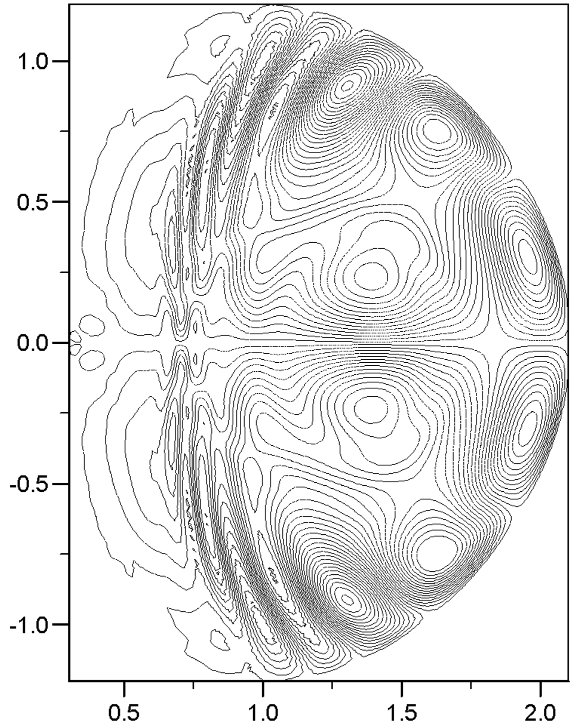
m ≥ 5
$\gamma\tau_A = 1.28 \times 10^{-2}$

$n = 3$



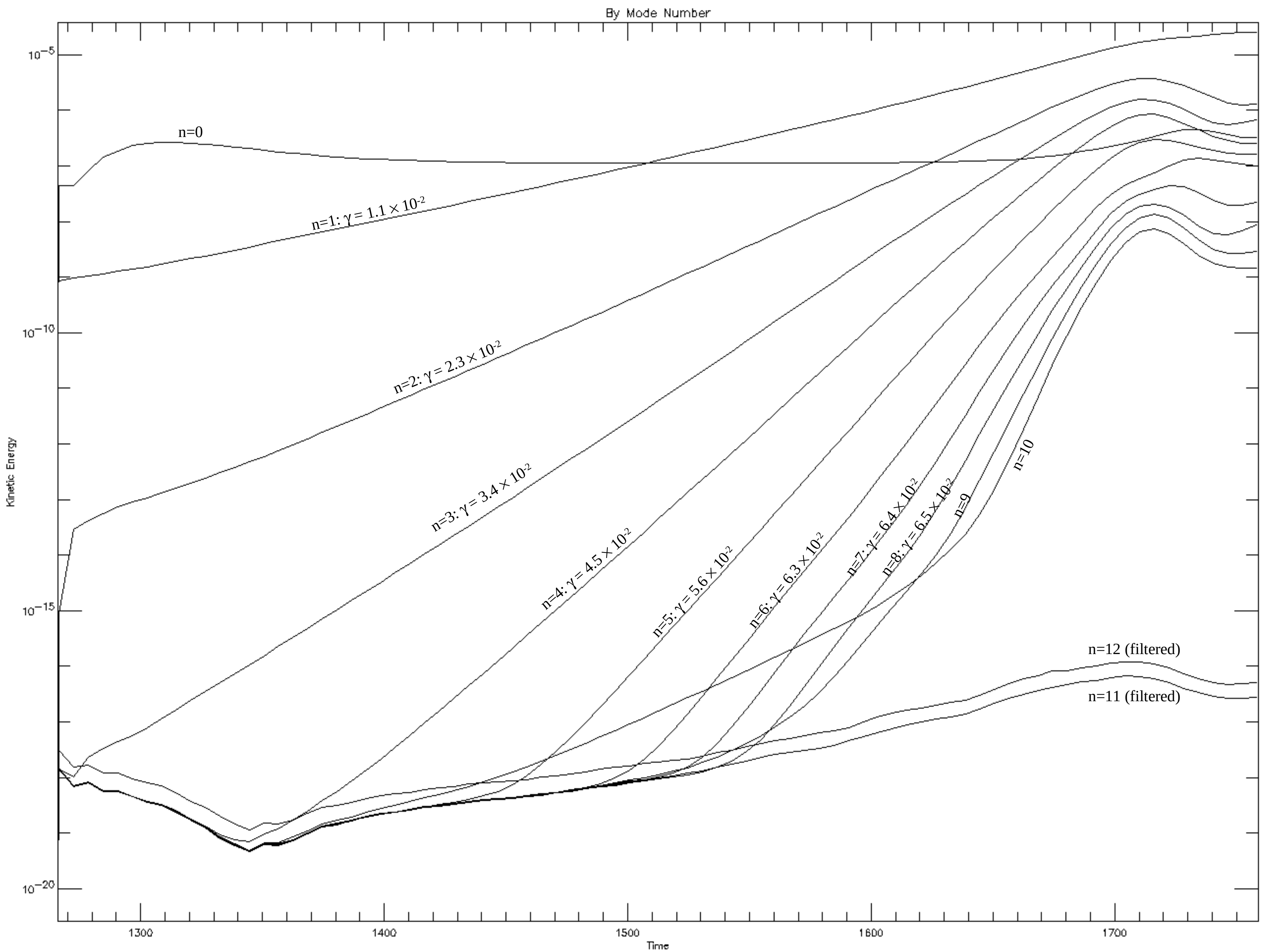
m ≥ 7
$\gamma\tau_A = 1.71 \times 10^{-2}$

$n = 4$ (projected)



m ≥ 8
not converged

Low Aspect Ratio: Nonlinear Kinetic Energy History

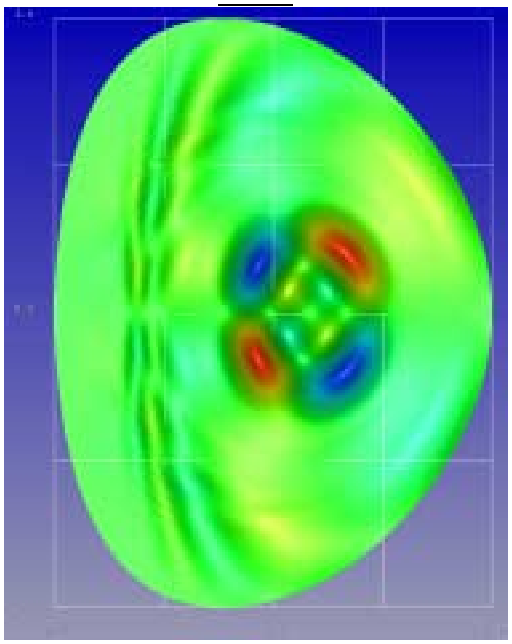


“Linear” high-*n* modes are driven, not eigenmodes

Incompressible velocity stream function *U*

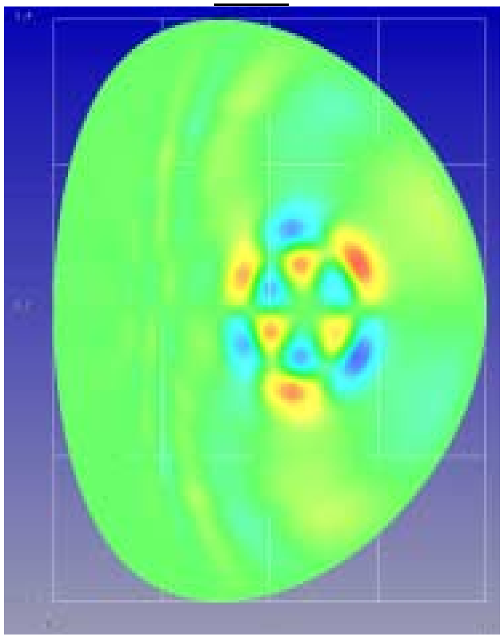
Component of “linear” mode
in nonlinear run

<u>*n*=2</u>



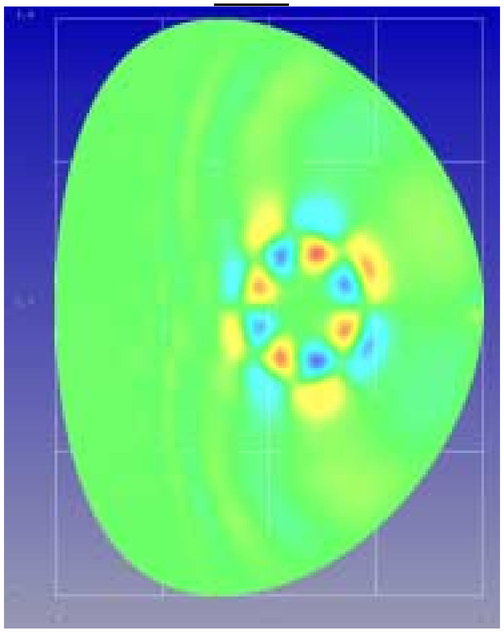
m = 2
$\gamma = 2.3 \times 10^{-2}$

<u>*n*=3</u>



m = 3
$\gamma = 3.4 \times 10^{-2}$

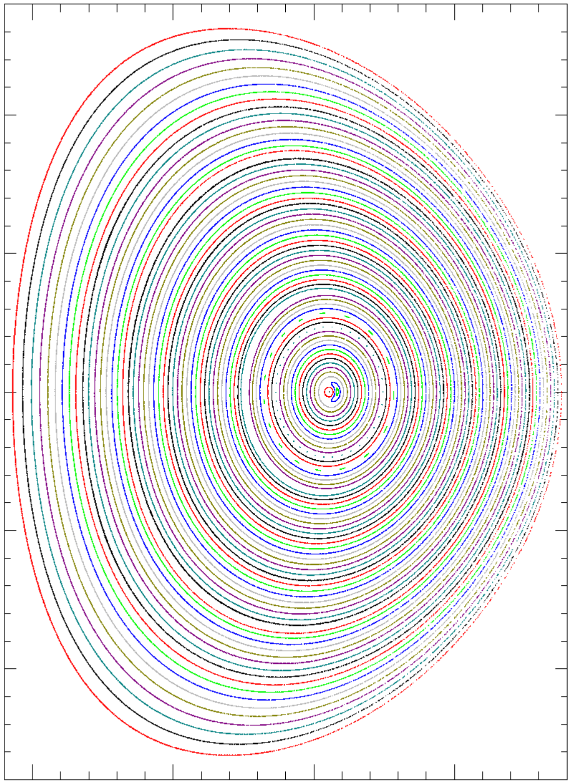
<u>*n*=4</u>



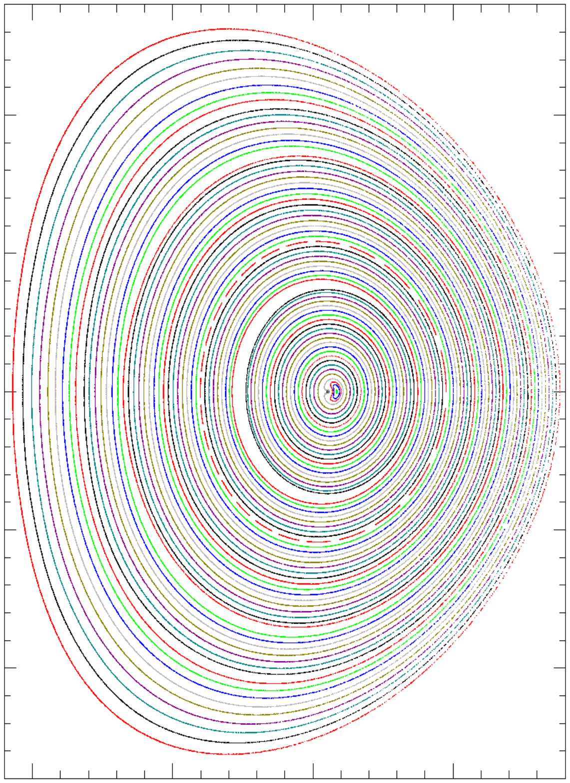
m = 4
$\gamma = 4.5 \times 10^{-2}$

Low Aspect Ratio: Nonlinear Time Series

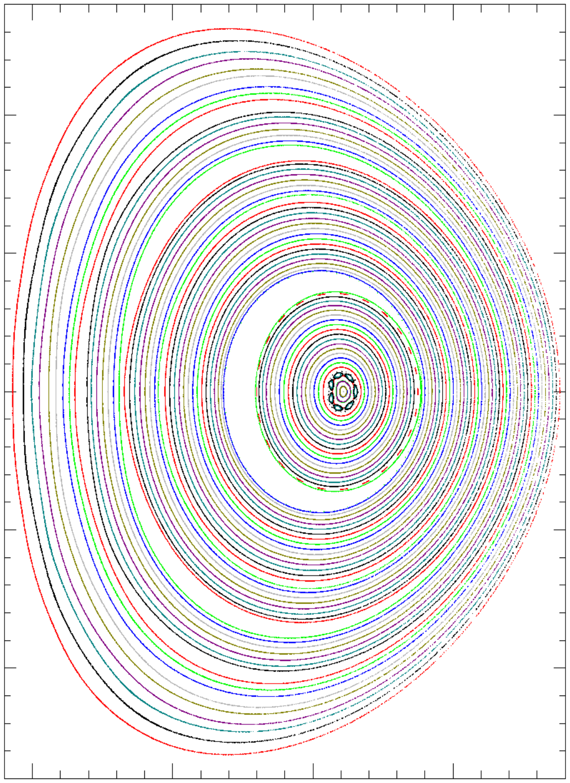
Poincaré Plots



t = 1266.17, chopped × 4.470358



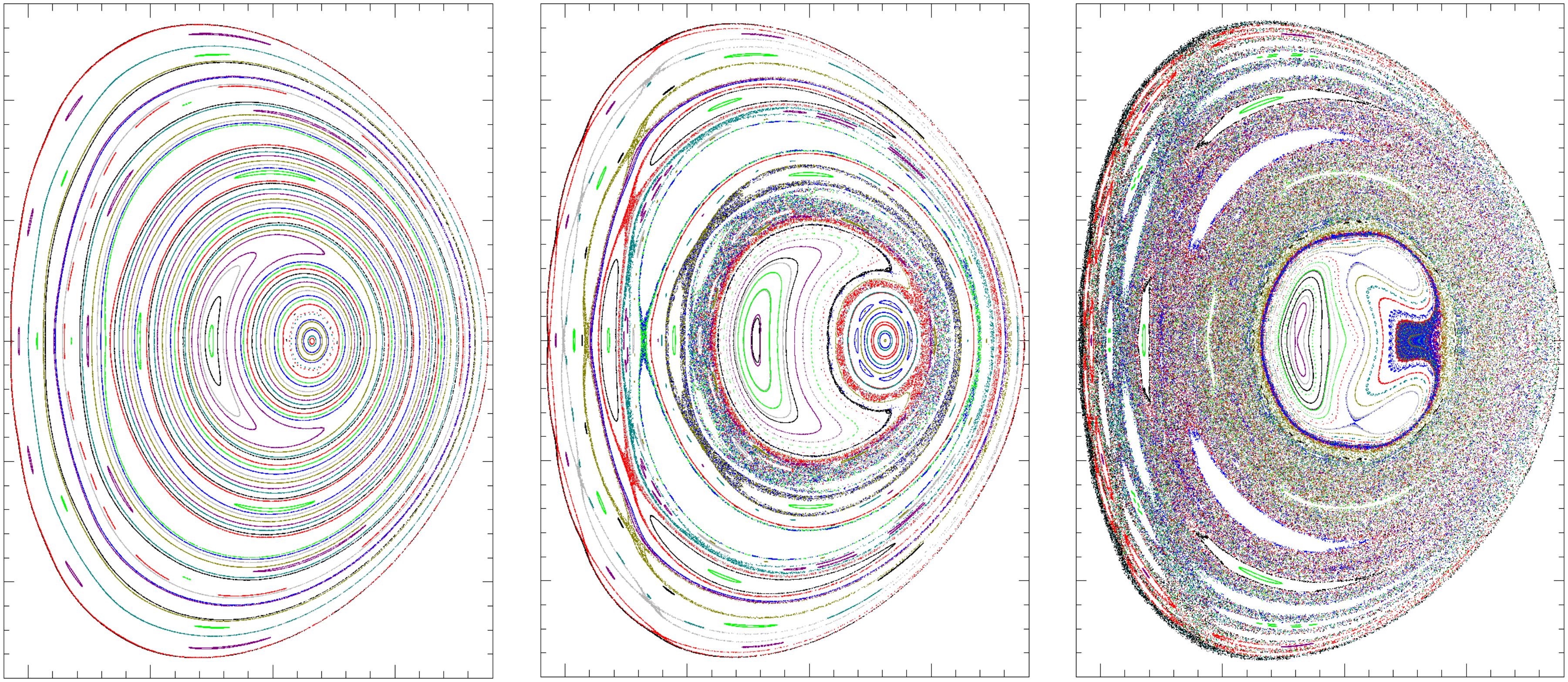
t = 1404.57



t = 1548.68

Low Aspect Ratio: Nonlinear Time Series

Poincaré Plots, Continued



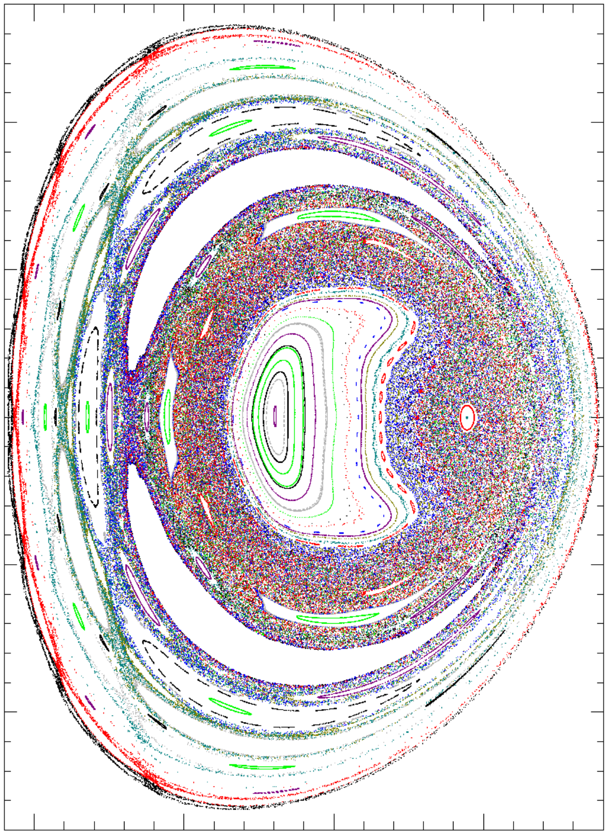
t = 1620.62 t = 1686.41 t = 1758.34

Disruption occurs before completion of sawtooth crash.

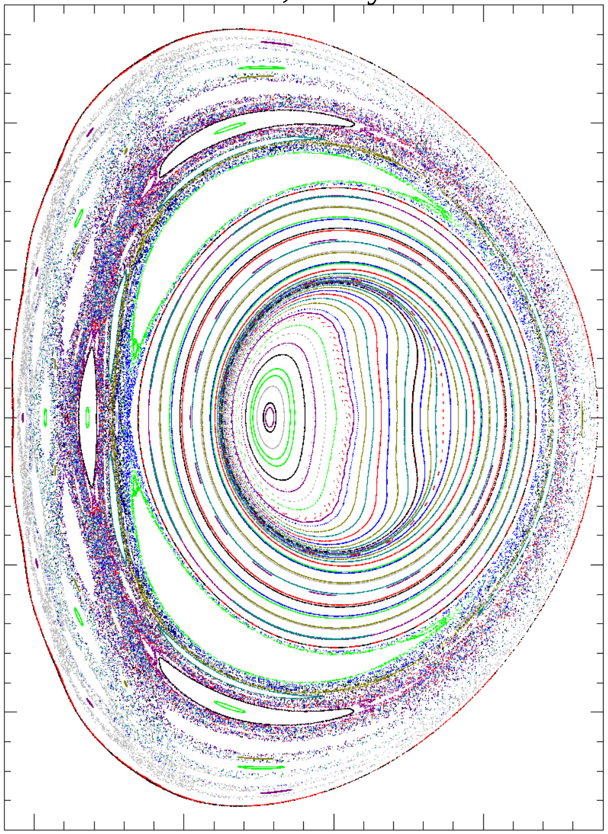
Focus on Saturation Time t=1710.38

(Time when n=2 kinetic energy peaks)

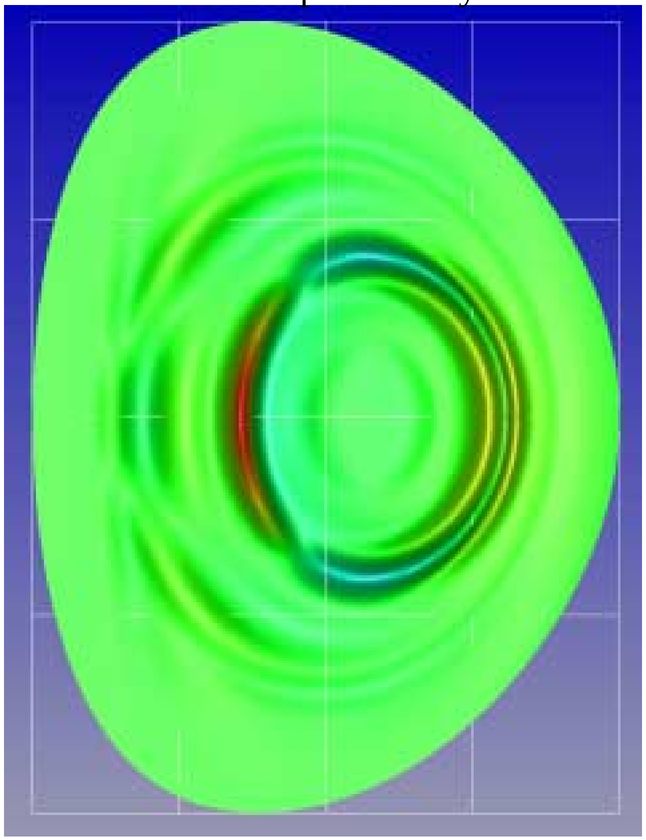
Poincaré plot,
all modes



Poincaré plot,
n=0,1 only



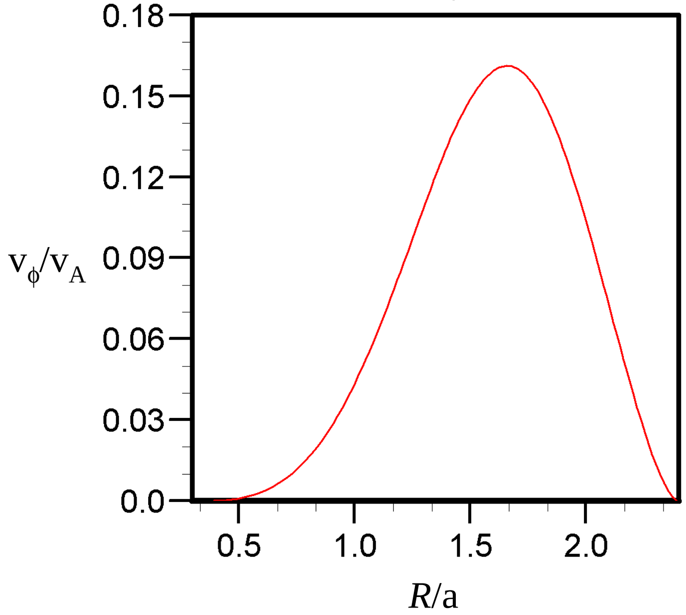
J_ϕ,
n=1 component only



Flow Stabilization of the n=1 Mode in the low-A MHD Case

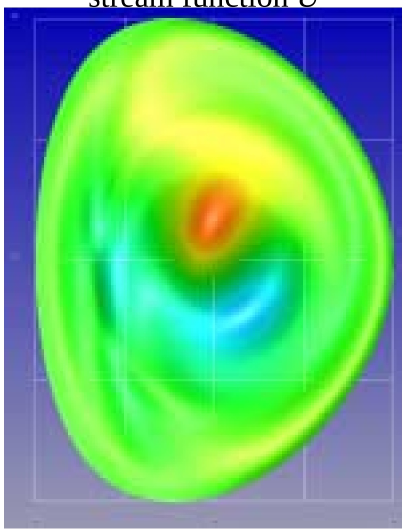
- Introduce steady-state 2D toroidal flow profile of the form

$$\frac{v_\phi(R,z)}{v_A} = 0.1\frac{R}{a}\left|\frac{\psi(R,z)}{\psi_0}\right|^{1.5} :$$

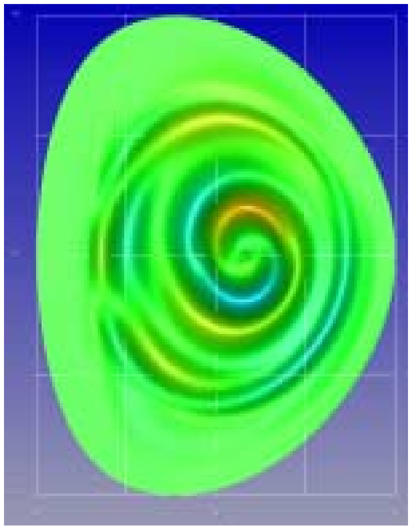


New n=1 eigenmode:

Incompressible velocity stream function U



Toroidal current density J_ϕ



Growth rate $\gamma\tau_A$ decreases from 8.61×10^{-3} to 2.4×10^{-3}.

Extended MHD Effects

(Initial density profile uniform)

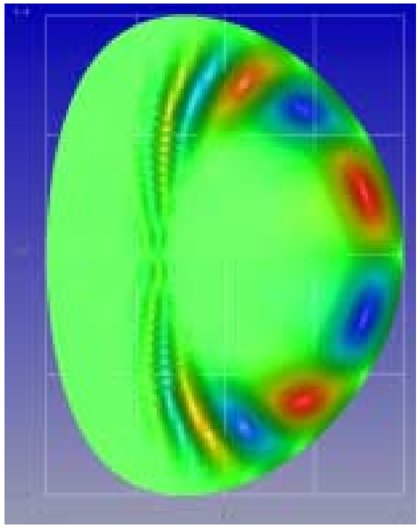
		MHD		MHD + ω_i^* term	
		$v_{\|\|}/v_A = 0$ ($\chi_{\|\|}$ off)	$v_{\|\|}/v_A = 2.5$ ($\chi_{\|\|}$ on)	$v_{\|\|}/v_A = 0$ ($\chi_{\|\|}$ off, H=0.10)	$v_{\|\|}/v_A = 2.5$ ($\chi_{\|\|}$ on, H=0.15)
Evolving Density	*n*=1	0.016	0.014	0.0165	0.0137
	n=4	0.0307	0.0075	0.0304	0.0095
		$v_{\|\|}/v_A = 0$ ($\chi_{\|\|}$ off)	$v_{\|\|}/v_A = 2.5$ ($\chi_{\|\|}$ on)	$v_{\|\|}/v_A = 0$ ($\chi_{\|\|}$ off, H=0.15)	$v_{\|\|}/v_A = 2.5$ ($\chi_{\|\|}$ on, H=0.15)
Constant Density	*n*=1	0.017	0.021	0.021	0.021
	n=4	0.0349	0.0132	0.0354	0.0123

- Large parallel thermal conductivity reduces growth rate of higher *n* modes.
- ω_i^* term alone does not have stabilizing effect on either mode.

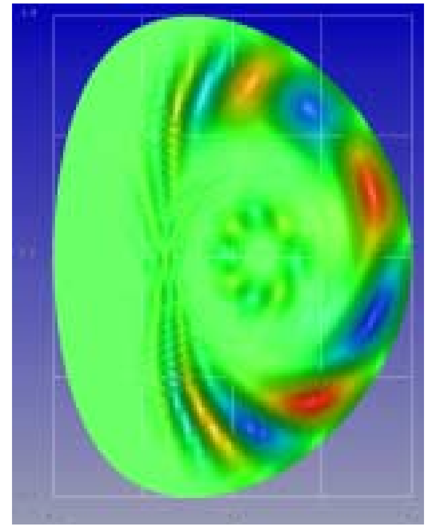
n=4 Eigenmodes, Evolving Density

MHD | MHD + ω*

$\chi_{\parallel} = 0$

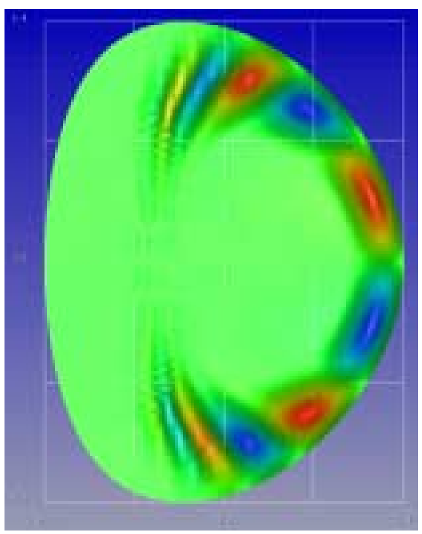


$\gamma\tau_A = 0.0307$

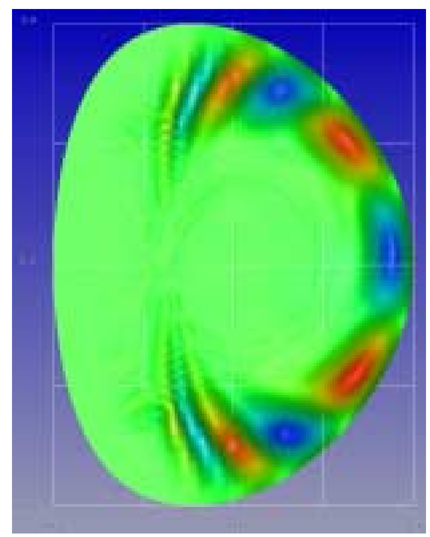


$\gamma\tau_A = 0.0304$

$\chi_{\parallel} \neq 0$



$\gamma\tau_A = 0.0075$



$\gamma\tau_A = 0.0095$

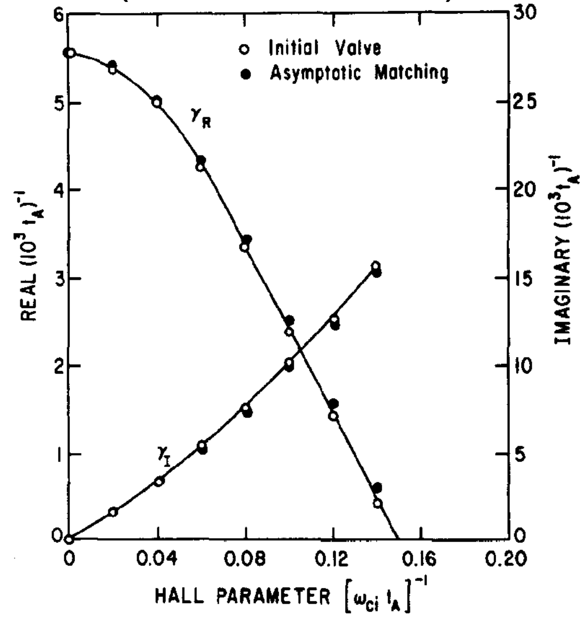
Summary

- All toroidal modes of the q_{min}= 0.92 CDX equilibrium are linearly MHD-unstable.
  - n =1 is an internal kink mode
  - n >1 are ballooning instabilities
  - Higher n modes have higher growth rates.
- Nonlinear MHD evolution beginning with just an n=1 perturbation disrupts within a sawtooth crash time.
  - High poloidal mode number m components of the n=1 mode interact to create islands, stochasticity in outer region.
  - n =1 mode couples to and drives higher n modes at q=1 rational surface to create stochasticity in inner region.
- Adding toroidal flow to the model reduces the growth rate of the n=1 mode.
- Adding large parallel thermal conductivity (via artificial sound wave) has a stabilizing effect on higher n modes, but not on n=1.
- Adding the ω* term to the MHD equations does not appreciably alter the growth rates of either the n =1 or the n >1 modes.

Future Work

Previous studies[1] indicate that resistive interchange modes can be stabilized with a sufficiently large Hall term:

GROWTH RATE OF
RESISTIVE INTERCHANGE MODE
(CYLINDRICAL SPHEROMAK)



- Run linear and nonlinear cases with Hall term on.
- Run linear and nonlinear cases with heat conduction on.
- Do convergence studies.

[1]J. DeLucia, S.C. Jardin, and A.H. Glasser, *Phys. Fluids* **27** (6), 1470 (1984).